Equation of a Tangent Line

First recall the slope-intercept form of a straight line. Given the slope $m$ at the point $(x_0, y_0)$, the equation of the line through that point having slope $m$ is $y - y_0 = m(x - x_0)$. To get this in the form $y = L(x)$ modify it to

$$y = y_0 + m(x - x_0)$$

Steps. To find an equation of the tangent line to the graph of $y = f(x)$ at the point $(x_0, y_0)$:

a. If only $x = x_0$ is given, find $y_0 = f(x_0)$.
b. Take the derivative $y' = f'(x)$.
c. Substitute $x = x_0$ to get the slope $m = f'(x_0)$.
d. Write the modified slope-intercept form to read $y = y_0 + m(x - x_0)$.

Example. Find an equation of the tangent line to the graph of $y = 2x^2 - \frac{3}{x}$ at $(1, -1)$.

Since $y_0 = -1$ is given, differentiate to get $y' = 4x + 3/x^2$. So $m = 4(1) + 3/1^2 = 7$ and the equation is $y = -1 + 7(x - 1)$.

Example. Find an equation of the tangent line to the graph of $y = 2e^{2x} - x$ at $x = \ln 2$.

First, find $y_0 = 2e^{2\ln 2} - \ln 2 = 2e^{\ln 4} - \ln 2 = 2 \times 4 - \ln 2 = 8 - \ln 2$. Next, $y' = 4e^{2x} - 1$ so $m = 4 \times 4 - 1 = 15$. Hence the equation of the tangent line is $y = 8 - \ln 2 + 15(x - \ln 2)$.