1. Let \( g(x) = e^{-x} - 1 + x \). Find \( g'(x) \), \( g''(x) \), \( g'(0) \), \( g''(0) \) and \( g''(\ln 2) \).

Use the second derivative test at the critical point of \( g \) to determine if \( g \) has a (local) maximum or minimum there.

2. Let \( f(x) = \frac{x^2}{2} - \ln x \). Find the domain and critical points. Use the second derivative test to decide if you have a (relative) minimum or maximum at the critical point. Find intervals where the function is concave up/down.

3. Let \( f(x) = 3\sin x + 2\cos x \), on the interval \( [-\frac{\pi}{2}, \frac{\pi}{2}] \). Find the critical point to two decimal places (hint: use \( \tan x \) after differentiating). Use the second derivative test to decide if you have a (relative) minimum or maximum at the critical point.

4. State the Mean Value Theorem:

If \( f \) is continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\), then
5. A bicycle moves on a straight path towards a wall 1000 ft away. It moves for 30 seconds such that its position $s(t)$, in feet, at time $t$ seconds is given by

$$s(t) = 100t - \frac{1}{3}t^3$$

a. Find the average velocity over the time interval $0 \leq t \leq 3$.

b. Find $v(t)$ the instantaneous velocity as a function of $t$.

c. Find the value of $t$, call it $c$, where the instantaneous velocity equals the average velocity (for $0 < t < 3$) found in part a.

d. What theorem guarantees the existence of such a value $c$ in part c.?
6. A point is moving along the graph of \( y = \tan x \) for \(-\frac{\pi}{4} \leq x < \frac{\pi}{2}\) such that \( \frac{dy}{dt} = 4 \) cm/sec.

Find \( \frac{dx}{dt} \) as:

a) the point crosses the origin,

b) when \( x = \pi/4 \). What happens to \( dx/dt \) in the limit \( x \to \pi/2 \)?

7. State the Fundamental Theorem of Calculus, Parts I and II.

I. Fundamental Theorem of Calculus, Part I:

If a function \( f \) is continuous on \([a, b]\), then the function \( g \) defined by \( g(x) = \int_{a}^{x} f(t) \, dt \),

\( a \leq x \leq b \), is continuous on ________, differentiable on _______ and _________.

II. Fundamental Theorem of Calculus, Part II:

If \( f \) is continuous on \([a, b]\), then

\[ \int_{a}^{b} f(x) \, dx = \]

where \( F \) is any ____________ of \( f \).
8. Quick Integrals. You need not write the “$+C$”.

\[
\int_1^x \frac{du}{u} = \\
\int_1^2 \frac{du}{u} = \\
\int \sin 3x \, dx = \\
\int \cos 2x \, dx = \\
\int e^{-4x} \, dx = \\
\int \left(\frac{1}{x} + 1 + x^2\right) \, dx = \\
\int_0^1 x \, dx = \\
\int \frac{dx}{2\sqrt{x}} = \\
\int \sqrt{x} \, dx = \\
\int 2x e^{x^2} \, dx =
\]
In these Problems, you must use calculus methods to solve.

9. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at the rate of 20 cu.ft./min. The diameter of the base of the cone is 4 times the altitude. At what rate is the height of the pile changing when it is 10 feet high?

10. Comet Iwanana moves along a hyperbola that near the earth looks like the straight line $x + y = 2$. Our space station is located at coordinates $(1, 0)$. Find the point on the line where the comet is closest to our space station.

11. George is planning a rectangular lot with the southwest (SW) corner at the origin, $(0,0)$, and bounded by a road along the line $y + 3x = 6$ at its northeast (NE) corner. Find the best location for the NE corner so that the lot has maximum area.

12. Consider a cylindrical can with a bottom, but no top, having a volume of 100 cm$^3$. Express the surface area $S$ in terms of the radius alone and find the shape (radius and height) of such a can with minimum surface area. What happens to the surface area as $r \to 0$? [Try similar problem for a box with a square base $x$ and height $h$.]
13. Find these integrals. Show details.

a. \[ \int \tan^5 x \sec^2 x \, dx \]

b. \[ \int_3^6 \frac{2x}{\sqrt{x^2 - 8}} \, dx \]

14. Find these integrals. Show details.

a. \[ \int \frac{x + 1 + x^{3/2}}{\sqrt{x}} \, dx \]

b. \[ \int_0^2 (e^x + 1)^2 \, dx \]
15. Let \( F(x) = \int_0^x \frac{1 - t}{(1 + t^2)^2} dt \).

a. \( F'(x) = \)

b. Find the critical numbers of the function \( F \).

16. Find these integrals:

a. \( \int_0^\pi \frac{4 \sin x}{1 + \cos^2 x} \, dx \)

b. \( \int \frac{t}{(1 + t^2)^3} \, dt \)

17. Evaluate the given integral: (Ans. 9/28)

\[ \int_0^1 x \sqrt{1 - x} \, dx \]

18. Find these indefinite integrals:

a. \( \int \frac{\ln t}{t} \, dt \)

b. \( \int (e^{2t} + e^{-2t})^2 \, dt \)
19. Let \( f(x) = 3e^{x\ln 2} \). Find

a. \( f'(x) = \)

b. \( f'(1) = \)

c. \( f'(0) = \)

d. \( \int f(x) \, dx = \)

20. A quantity \( y(t) \) varies so that \( \frac{dy}{dt} = -5y \), with \( y(0) = 9 \). (i) Write a formula for \( y(t) \). (ii) Discuss the behavior of \( y(t) \) as \( t \to +\infty \). (iii) Find the exact value of \( y(t) \) at \( t = \ln(10) \).

21. Let \( F(x) = \int_{0}^{x} \ln t \, dt \), for \( x > 0 \).

a. Find \( F'(x) \).

b. Find the critical numbers of the function \( F \) in the interval \((0, \infty)\).

c. Identify intervals on which \( F \) is increasing or decreasing.

22. Let \( g(x) = 4e^{x\ln 10} \). Find

(i) \( g'(x) = \)

(ii) \( g'\left(\log_{10} 2\right) = \)

(iii) \( \int g(x) \, dx = \)
23. Evaluate the given integrals:

\[ \int_{0}^{1} x(1 - x)^{5/2} \, dx \quad (\text{Ans. } \frac{4}{63}) \]

\[ \int_{0}^{1} x(1 - x^2)^{5/2} \, dx \quad (\text{Ans. } \frac{1}{7}) \]

24. Evaluate the given integrals:

\[ \int_{0}^{2} \frac{3e^t}{e^t + 5} \, dt \]

\[ \int_{e}^{e^2} \frac{dt}{t \ln t} \, dt \]
25. Find these indefinite integrals:

a. \[ \int \frac{2 + \sqrt{u}}{u^2} \, du \]

b. \[ \int (1 + \sin 2x)^5 \cos 2x \, dx \]

26. Quick Integrals. You must write the “+C” if it is necessary.

\[ \int_1^x \frac{du}{u} = \]

\[ \int_0^1 u \, du = \]

\[ \int \sin^2 x \cos x \, dx = \]

\[ \int e^{-4x} \, dx = \]

\[ \int \left( \frac{1}{x} + 1 + x^2 \right) \, dx = \]

\[ \int_0^1 dx = \]

\[ \int \frac{-1}{x^2} \, dx = \]

\[ \int \sqrt{t} \, dt = \]

\[ \int x e^{-x^2/2} \, dx = \]