# T-Run

[4, 3, 1, 2, 3, 4, 4, 3], [8, 7, 5, 6, 7, 8, 8,

---

$\tilde{\pi}$ = [1, 1, 2, 2, 1, 1, 2, 2]
$\boldsymbol{\delta}$ = [1, 1, 3, 3, 1, 1, 3, 3]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 21/256, .8203125000e-1

*NullSpace* of Δ

{2, 6}, {1, 5}, {4, 8}, {3, 7}

Nullspace of A

[{7},{3}] `,` [{8},{4}] `,` [{5},{1}] `,` [{6},{2}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[8] v[7] + v[4] v[6] + v[3] v[5]

Degree 3
v[1] v[3] v[8] + v[2] v[4] v[7]

Degree 4
v[3] v[4] v[5] v[6] + 2 v[4] v[6] v[8] v[7] + v[1] v[2] v[3] v[4] + 2 v[3] v[5] v[8] v[7]

Degree 5

2 v[1] v[3] v[4] v[6] v[8] + 2 v[2] v[3] v[4] v[5] v[7]

Degree 6
4 v[3] v[4] v[5] v[6] v[8] v[7] + 4 v[1] v[2] v[3] v[4] v[8] v[7]

=================================================

{}

R: [4, 3, 1, 2, 3, 4, 4, 3]
B: [8, 7, 5, 6, 7, 8, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512} (1 + s) (7 + 2s + s^2) (6 + 3s + s^2) (-1 + s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", 1 + v[8] v[7]

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 0, −1, 1, 0], [0, 1, 0, 0, 0, 0, 0, −1]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$
degree 2: $\frac{1}{6}\left( v[1]v[2] + 2\,v[3]v[4] + v[5]v[6] + 2\,v[8]v[7] \right)$

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT2" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [3, 1, 2, 3, 0, 0, 3, 2, 1, 0, 3, 3, 0, 3, 2, 1, 1, 2, 1, 2, 2, 1, 3, 3, 0, 0, 3, 3]

supp u2 = {1, 2, 3, 4, 7, 8, 9, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28}

Action of R on ranges, [[3], [3], [4], [3]]
Action of B on ranges, [[1], [1], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 1, 4, 2]

B−BLOCKS,

[4, 4, 1, 1]

with invariant measure, [2, 1, 1, 2]

N by blocks, N − check: true

$b_1$ = {1, 3, 6, 7}
$b_2$ = {1, 4, 6, 7}
$b_3$ = {2, 3, 5, 8}
$b_4$ = {2, 4, 5, 8}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $3 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4}}, true

$\Omega_B$ in Vec(K)? , {{7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

2, "partition", {{1, 3, 6, 7}, {2, 4, 5, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 8, 7], [7, 8, 7, 8, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 6, 5], [5, 6, 5, 6, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 4, 3], [3, 4, 3, 4, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 2, 1], [1, 2, 1, 2, 2, 1, 1, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$\mathrm{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 1 & 2 & 3 & 0 & 0 & 3 & 2 & 1 & 0 & 3 & 3 & 0 & 3 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 3 & 3 & 0 & 0 & 3 & 3 \end{pmatrix}$

{1, 2, 3, 4, 7, 8, 9, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28} picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 1 = \begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2}\end{pmatrix}$

picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3}\\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{9} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & \frac{1}{3} & 0 & \frac{1}{18} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{18} & \frac{1}{9} & 0 & \frac{1}{3} & \frac{1}{9} & \frac{1}{18} & \frac{1}{9} & \frac{2}{9} \\ 0 & \frac{1}{6} & \frac{1}{9} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{9} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{32}{9} & \frac{16}{9} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{16}{9} & \frac{32}{9} & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & \frac{16}{3} & 0 & \frac{8}{9} & \frac{16}{9} & \frac{32}{9} & \frac{16}{9} \\ \frac{8}{9} & \frac{16}{9} & 0 & \frac{16}{3} & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{32}{9} \\ 0 & \frac{8}{3} & \frac{16}{9} & \frac{32}{9} & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & \frac{32}{9} & \frac{16}{9} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ \frac{8}{3} & 0 & \frac{32}{9} & \frac{16}{9} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{16}{9} & \frac{32}{9} & \frac{8}{3} & 0 & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 2, 2, -1, -1, -2, -2]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & s & -s & -s & t & t & -t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & -2 & -1 & 2 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & -t & 0 & 0 \\ s & t & 0 & 0 \\ 0 & 0 & s & t \\ 0 & 0 & -s & -t \\ s & t & 0 & 0 \\ -s & -t & 0 & 0 \\ -s & -t & 0 & 0 \\ s & t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -s & 0 & s & s+t & 0 \\ s & 0 & t & 0 & 0 \\ 0 & t & s & s & -s \\ 0 & -t & t & t & s \\ s & 0 & t & 0 & 0 \\ -s & 0 & s & s+t & 0 \\ -s & 0 & s & s+t & 0 \\ s & 0 & t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & 1 & 0 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

$$T \begin{pmatrix} \frac{2}{9} & \frac{1}{18} & \frac{1}{3} & \frac{1}{9} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{32}{9}\ \ \frac{8}{9}\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{2}{9}\ \ \frac{34}{9}\ \ \frac{34}{9}\ \ \frac{2}{9}\ \ \frac{38}{27}\ \ \frac{70}{27}\ \ \frac{2}{9}\ \ \frac{34}{9}\right)$$

$\boldsymbol{\tau} = 32/1$ , rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$ , $r' = 1/2$ , $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$–check is positive? 208/9

max r = 36/5 , $r$–check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT2" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$
M_C\text{-scaled} = \begin{pmatrix}
1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\
1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\
\frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\
\frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\
\frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1
\end{pmatrix}
\quad N_C\text{-scaled} =
$$

$$
\begin{pmatrix}
1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\
\frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{19}{31} & \frac{7}{31} \\
\frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & \frac{7}{31} & \frac{19}{31} \\
\frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\
1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1
\end{pmatrix}
$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 2.888888889, 3.154700539, 0.845299461]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.354838710, 3.663523206, 0.981638084]

NullSpace $M_C$

{[0, 0, −1, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [1, 0, 1, 0, 1, 0, 1, 0], [0, 0, 0, 0, −1, 1, 0, 0], [1, 0, 1, 0, 1, 0, 0, 1] }

NullSpace $N_C$

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues $M_0$

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues $N_0$

[0., 0., 0., 0., 0., 4., 3.154700539, 0.845299461]

NullSpace $M_0$

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace $N_0$

{[1, 1, −1, −1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [0, 1, −1, −1, 0, 1, 0, 0], [0, 1, −1, −1, 0, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., 4., −0.845299461, −3.154700539]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

 

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 3 & 0 & 0 & 3 \\ 3 & 0 & 2 & 1 & 0 & 3 & 3 & 0 \\ 1 & 2 & 0 & 3 & 2 & 1 & 1 & 2 \\ 2 & 1 & 3 & 0 & 1 & 2 & 2 & 1 \\ 3 & 0 & 2 & 1 & 0 & 3 & 3 & 0 \\ 0 & 3 & 1 & 2 & 3 & 0 & 0 & 3 \\ 0 & 3 & 1 & 2 & 3 & 0 & 0 & 3 \\ 3 & 0 & 2 & 1 & 0 & 3 & 3 & 0 \end{pmatrix}$$

=================================================

{2, 5}

R: [4, 7, 1, 2, 7, 4, 4, 3]
B: [8, 3, 5, 6, 3, 8, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{1024}(-1+s)(-84-10s-s^2-2s^3+s^4)$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 3

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[2] v[4] v[7]
"B CYCLES", (1 + v[8] v[7]) (1 + v[3] v[5])

Eigenvalues

R: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[1, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 0, 0, 1, −1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 7 & 14 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 14 & 0 & 0 & 7 & 0 \\ 7 & 0 & 0 & 14 & 0 & 7 & 14 & 14 \\ 14 & 14 & 14 & 0 & 0 & 0 & 14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & 7 & 14 \\ 0 & 0 & 7 & 0 & 7 & 0 & 0 & 14 \\ 0 & 7 & 14 & 14 & 7 & 0 & 0 & 14 \\ 0 & 0 & 14 & 0 & 14 & 14 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{24}$ ( $v[1]v[2] + v[1]v[3] + 2v[1]v[4] + 2v[2]v[4] + v[2]v[7] + 2v[3]v[4] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + 2v[4]v[7] + v[5]v[6] + v[5]v[7] + 2v[5]v[8] + 2v[6]v[8] + 2v[8]v[7]$ )
degree 3 : $\frac{1}{8}$ ( $v[1]v[2]v[4] + v[1]v[3]v[4] + v[2]v[4]v[7] + v[3]v[4]v[7] + v[3]v[6]v[8] + v[3]v[8]v[7] + v[5]v[6]v[8] + v[5]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 4, 7}

"RG5" = {5, 6, 8}

"RG6" = {3, 6, 8}

"RG7" = {1, 3, 4}

"RG8" = {1, 2, 4}

$\pi3$ = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0]

supp $\pi3$ = {2, 7, 29, 39, 45, 46, 54, 55}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[3], [7], [8], [4], [3], [7], [8], [4]] Action of B on ranges, [[2], [1], [5], [6], [2], [1], [5], [6]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1 = \{4, 8\}$
$b_2 = \{1, 6, 7\}$
$b_3 = \{2, 3, 5\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 23, Shape: 3 ⊕ 20/18

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{2, 4, 7}}, true

$\Omega_B$ in Vec(K)? , {{7, 8}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{3}{16} & 0 & \frac{3}{16} & 0 & \frac{5}{16} & \frac{5}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{4, 8}, {1, 6, 7}, {2, 3, 5}}

1, "range", [5, 7, 8], [[8, 7, 7, 5, 7, 8, 8, 5], [8, 5, 5, 7, 5, 8, 8, 7], [7, 8, 8, 5, 8, 7, 7, 5], [7, 5, 5, 8, 5, 7, 7, 8], [5, 8, 8, 7, 8, 5, 5, 7], [5, 7, 7, 8, 7, 5, 5, 8]]

2, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

3, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

4, "range", [2, 4, 7], [[7, 4, 4, 2, 4, 7, 7, 2], [7, 2, 2, 4, 2, 7, 7, 4], [4, 7, 7, 2, 7, 4, 4, 2], [4, 2, 2, 7, 2, 4, 4, 7], [2, 7, 7, 4, 7, 2, 2, 4], [2, 4, 4, 7, 4, 2, 2, 7]]

5, "range", [5, 6, 8], [[8, 6, 6, 5, 6, 8, 8, 5], [8, 5, 5, 6, 5, 8, 8, 6], [6, 8, 8, 5, 8, 6, 6, 5], [6, 5, 5, 8, 5, 6, 6, 8], [5, 8, 8, 6, 8, 5, 5, 6], [5, 6, 6, 8, 6, 5, 5, 8]]

6, "range", [3, 6, 8], [[8, 6, 6, 3, 6, 8, 8, 3], [8, 3, 3, 6, 3, 8, 8, 6], [6, 8, 8, 3, 8, 6, 6, 3], [6, 3, 3, 8, 3, 6, 6, 8], [3, 8, 8, 6, 8, 3, 3, 6], [3, 6, 6, 8, 6, 3, 3, 8]]

7, "range", [1, 3, 4], [[4, 3, 3, 1, 3, 4, 4, 1], [4, 1, 1, 3, 1, 4, 4, 3], [3, 4, 4, 1, 4, 3, 3, 1], [3, 1, 1, 4, 1, 3, 3, 4], [1, 4, 4, 3, 4, 1, 1, 3], [1, 3, 3, 4, 3, 1, 1, 4]]

8, "range", [1, 2, 4], [[4, 2, 2, 1, 2, 4, 4, 1], [4, 1, 1, 2, 1, 4, 4, 2], [2, 4, 4, 1, 4, 2, 2, 1], [2, 1, 1, 4, 1, 2, 2, 4], [1, 4, 4, 2, 4, 1, 1, 2], [1, 2, 2, 4, 2, 1, 1, 4]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 = \quad [[1, 3, 2]]$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $\quad 1 + t + t^2 + t^3$

Molien Series to order 10: $\quad 1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12 t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{2, 7, 29, 39, 45, 46, 54, 55}

$u3 =$
$(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 =$
$\begin{pmatrix} 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 2 & 2 & 0 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 2 \end{pmatrix}$

$u2 =$
$\Big(\frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}$

picheck $\begin{pmatrix} 4 & 4 & 8 & 8 & 4 & 4 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 4 & 8 & 8 & 4 & 4 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 4 & 8 & 8 & 4 & 4 & 8 & 8 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, 1, 0, 2, −1, −1, 0, −2]

$$\ker N_C = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & 0 & t & 0 & -t & 0 & s & 0 \\ 0 & -s & s & 0 & 0 & -t & t & 0 \\ -s & 0 & t & 0 & -t & 0 & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -2 & -1 & 0 & -1 \end{pmatrix}$

M0 is invertible. det= 381271/104976

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{8} & \frac{7}{8} & \frac{7}{4} & 0 & 0 & 0 & 0 \\ \frac{7}{8} & \frac{11}{6} & 0 & \frac{7}{4} & 0 & 0 & \frac{7}{8} & 0 \\ \frac{7}{8} & 0 & \frac{11}{3} & \frac{7}{4} & 0 & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{11}{3} & 0 & 0 & \frac{7}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{11}{6} & \frac{7}{8} & \frac{7}{8} & \frac{7}{4} \\ 0 & 0 & \frac{7}{8} & 0 & \frac{7}{8} & \frac{11}{6} & 0 & \frac{7}{4} \\ 0 & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} & \frac{7}{8} & 0 & \frac{11}{3} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & 0 & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{3} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T\left(0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{3}{4} \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{49}{4}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{1841}{174}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2$ / $r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau}$ / $n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 6 vs no. of idems 8

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 4, 7}

"RG5" = {5, 6, 8}

"RG6" = {3, 6, 8}

"RG7" = {1, 3, 4}

"RG8" = {1, 2, 4}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{31}{72} & \frac{-1}{72} & \frac{31}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-8}{9} \\ \frac{-1}{72} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} & \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-1}{36} & \frac{17}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-1}{36} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{25}{18} & \frac{31}{72} & \frac{-1}{72} & \frac{31}{36} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-8}{9} & \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} \\ \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-1}{36} & \frac{-16}{9} & \frac{31}{36} & \frac{31}{36} & \frac{-1}{36} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{100} & \frac{-1}{100} & \frac{31}{50} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-16}{25} \\ \frac{31}{100} & 1 & \frac{-16}{25} & \frac{31}{50} & \frac{-8}{25} & \frac{-8}{25} & \frac{-1}{100} & \frac{-16}{25} \\ \frac{-1}{136} & \frac{-8}{17} & 1 & \frac{-1}{68} & \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & \frac{-1}{68} \\ \frac{31}{68} & \frac{31}{68} & \frac{-1}{68} & 1 & \frac{-8}{17} & \frac{-8}{17} & \frac{-1}{68} & \frac{-16}{17} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-16}{25} & 1 & \frac{31}{100} & \frac{-1}{100} & \frac{31}{50} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-1}{100} & \frac{-16}{25} & \frac{31}{100} & 1 & \frac{-16}{25} & \frac{31}{50} \\ \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & \frac{-1}{68} & \frac{-1}{136} & \frac{-8}{17} & 1 & \frac{-1}{68} \\ \frac{-8}{17} & \frac{-8}{17} & \frac{-1}{68} & \frac{-16}{17} & \frac{31}{68} & \frac{31}{68} & \frac{-1}{68} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 2.754892744, 0.147885034]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6900000000, 1.633400282, 0.0713056009, 3.473569042, 0.417607428, 1.631532414, 0.0825852329]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues $M_0$

[0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 9.145704965, 0.142342103, 2.503619601]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, 0, 1, 0, 0, 0, −1], [0, 1, 0, 0, −1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 1, 0, −1, 0, 0, 0]}

Eigenvalues M

[0., 0., −0.8750000000, −2.625000000, 2.950746158, −2.075746158, 5.795540962, −3.170540962]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{[−1, 0, 0, 0, 0, −1, 1, 0], [0, 1, −1, 0, 1, 0, 0, 0]}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==================================================

{3, 4}

R: [4, 3, 5, 6, 3, 4, 4, 3]
B: [8, 7, 1, 2, 7, 8, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512}(-3+s)(1+s)(-7+s^2)(2+s)(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5])
"B CYCLES", 1 + v[8] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$

degree 2: $\frac{1}{6}$ ( $v[1]v[2] + 2v[3]v[4] + v[5]v[6] + 2v[8]v[7]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]

supp u2 = {1, 2, 4, 7, 9, 11, 12, 14, 16, 17, 19, 22, 23, 24, 27, 28}

Action of R on ranges, [[3], [3], [2], [3]]
Action of B on ranges, [[1], [1], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N − check: true

$b_1 = \{1, 4, 6, 7\}$
$b_2 = \{2, 3, 5, 8\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 9, Shape: $0 \oplus 9/7$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$  $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"  Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$  [[1, 2]]

$g_2 =$  []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2$ =
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 14, 23, 28}

$u2$ =
$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

{1, 2, 4, 7, 9, 11, 12, 14, 16, 17, 19, 22, 23, 24, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

idem-checks

$$T = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ 0 & \frac{8}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ \frac{8}{3} & 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 2, 2, 1, 1, -2, -2]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t & 0 & -s & 0 & s & 0 & -t \\ t & 0 & -s & 0 & s & 0 & -t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & 2 & 2 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & t \\ 0 & -s & 0 & -t \\ -t & 0 & s & 0 \\ t & 0 & -s & 0 \\ 0 & -s & 0 & -t \\ 0 & s & 0 & t \\ 0 & s & 0 & t \\ 0 & -s & 0 & -t \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & t & 0 \\ s+t & 0 & -s & s & 0 \\ s+t & -t & 0 & s+t & -s \\ 0 & t & 0 & 0 & s \\ s+t & 0 & -s & s & 0 \\ 0 & 0 & s & t & 0 \\ 0 & 0 & s & t & 0 \\ s+t & 0 & -s & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \end{pmatrix}$$

$$Skew\ Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \left(\frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$

$T \left(0 \;\; \frac{1}{3} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; 0 \;\; \frac{1}{6}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(0 \;\; \frac{16}{3} \;\; \frac{8}{3} \;\; 0 \;\; \frac{16}{3} \;\; 0 \;\; 0 \;\; \frac{8}{3}\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left(0 \;\; 4 \;\; 4 \;\; 0 \;\; 4 \;\; 0 \;\; 0 \;\; 4\right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 0., 4., 2.888888889]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 0., 4.645161290, 3.354838710]

NullSpace $M_C$

{[−1, 1, 0, 0, 0, 0, 0, 0], [1, 0, 1, 0, 1, 0, 1, 0], [0, 0, −1, 1, 0, 0, 0, 0], [1, 0, 1, 0, 0, 1, 1, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues $N_0$

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace $N_0$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 0, −1, 1, 0], [0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 1, 0, 0, 0, 0, −1], [1, 0, 0, 0, 0, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==========================================

{3, 7}

R: [4, 3, 5, 2, 3, 4, 8, 3]
B: [8, 7, 1, 6, 7, 8, 4, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{512} ( -1 + s ) ( 6 + 3s + s^2 ) ( 7 + 2s + s^2 ) ( 1 + s )$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 2

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[3] v[5]
"B CYCLES", 1 + v[4] v[6] v[8] v[7]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 23 & 0 \\ 0 & 0 & 23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 23 & 0 & 0 & 23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46 \\ 0 & 0 & 23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 23 & 0 \\ 23 & 0 & 0 & 0 & 0 & 23 & 0 & 0 \\ 0 & 0 & 0 & 46 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & 1 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 \\ \frac{2}{3} & 1 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 \\ \frac{1}{3} & 0 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} \\ 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(\,v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]\,)$

degree 2: $\frac{1}{6}\,(\,v[1]v[7] + v[2]v[3] + v[3]v[5] + 2\,v[4]v[8] + v[6]v[7]\,)$

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 2, 5, 6, 8}, {3, 4, 7}}

"PT2" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {4, 8}

"RG2" = {6, 7}

"RG3" = {3, 5}

"RG4" = {2, 3}

"RG5" = {1, 7}

$$\pi 2 = [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0]$$

supp $\pi 2$ = {6, 8, 15, 22, 26}

$$u2 = [1, 2, 2, 1, 0, 3, 1, 3, 3, 0, 1, 2, 0, 0, 3, 2, 1, 3, 3, 2, 1, 3, 1, 2, 0, 3, 1, 2]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28}

Action of R on ranges, [[4], [1], [3], [3], [1]]
Action of B on ranges, [[2], [1], [5], [5], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1, 1, 2]

B−BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N − check: true

$b_1$ = {1, 2, 5, 6, 8}

$b_2$ = {3, 4, 7}
$b_3$ = {1, 3, 4, 6}
$b_4$ = {2, 5, 7, 8}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: 3 ⊕ 15/13

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{3, 5}}, true

$\Omega_B$ in Vec(K)? , {{4, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 2, 5, 6, 8}, {3, 4, 7}}

1, "range", [4, 8], [[8, 8, 4, 4, 8, 8, 4, 8], [4, 4, 8, 8, 4, 4, 8, 4]]

2, "range", [6, 7], [[7, 7, 6, 6, 7, 7, 6, 7], [6, 6, 7, 7, 6, 6, 7, 6]]

3, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 5], [3, 3, 5, 5, 3, 3, 5, 3]]

4, "range", [2, 3], [[3, 3, 2, 2, 3, 3, 2, 3], [2, 2, 3, 3, 2, 2, 3, 2]]

5, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 7], [1, 1, 7, 7, 1, 1, 7, 1]]

2, "partition", {{1, 3, 4, 6}, {2, 5, 7, 8}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [6, 7], [[7, 6, 7, 7, 6, 7, 6, 6], [6, 7, 6, 6, 7, 6, 7, 7]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 5, 3, 3], [3, 5, 3, 3, 5, 3, 5, 5]]

4, "range", [2, 3], [[3, 2, 3, 3, 2, 3, 2, 2], [2, 3, 2, 2, 3, 2, 3, 3]]

5, "range", [1, 7], [[7, 1, 7, 7, 1, 7, 1, 1], [1, 7, 1, 1, 7, 1, 7, 7]]

"group has", 2, "elements"    Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1$ =    []

$g_2$ =    [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

 

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$

{6, 8, 15, 22, 26}

$u2 =$
$\begin{pmatrix} 1 & 2 & 2 & 1 & 0 & 3 & 1 & 3 & 3 & 0 & 1 & 2 & 0 & 0 & 3 & 2 & 1 & 3 & 3 & 2 & 1 & 3 & 1 & 2 & 0 & 3 & 1 & 2 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks    N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$ idem-checks    N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{18} & \frac{2}{9} & 0 \\ \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{18} & \frac{2}{9} & 0 \\ \frac{1}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{2}{9} \\ 0 & \frac{1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{46}{27} & \frac{46}{27} & \frac{23}{9} & 0 & \frac{92}{27} \\ \frac{46}{27} & \frac{23}{9} & 0 & 0 & \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{46}{9} \\ \frac{23}{27} & 0 & \frac{46}{9} & \frac{46}{9} & 0 & \frac{23}{27} & \frac{92}{27} & 0 \\ \frac{23}{27} & 0 & \frac{46}{9} & \frac{46}{9} & 0 & \frac{23}{27} & \frac{92}{27} & 0 \\ \frac{46}{27} & \frac{23}{9} & 0 & 0 & \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{46}{9} \\ \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{46}{27} & \frac{46}{27} & \frac{23}{9} & 0 & \frac{92}{27} \\ 0 & \frac{23}{27} & \frac{92}{27} & \frac{92}{27} & \frac{23}{27} & 0 & \frac{46}{9} & \frac{46}{27} \\ \frac{46}{27} & \frac{23}{9} & 0 & 0 & \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{46}{9} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [-1, 1, 2, 0, 1, -1, -2, 0]

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s & s & -s-t & 0 & t & t & -s-t \\ t & 0 & s & -s-t & s & 0 & t & -s-t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 1 & 0 & 2 \end{pmatrix}$

M0 is invertible. det= 327680000/1594323

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&1&0&0&1&1&0&1\\1&1&0&0&1&1&0&1\\0&0&1&1&0&0&1&0\\0&0&1&1&0&0&1&0\\1&1&0&0&1&1&0&1\\1&1&0&0&1&1&0&1\\0&0&1&1&0&0&1&0\\1&1&0&0&1&1&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&\frac{1}{3}&\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}\\\frac{1}{3}&1&0&0&1&\frac{1}{3}&\frac{2}{3}&1\\\frac{2}{3}&0&1&1&0&\frac{2}{3}&\frac{1}{3}&0\\\frac{2}{3}&0&1&1&0&\frac{2}{3}&\frac{1}{3}&0\\\frac{1}{3}&1&0&0&1&\frac{1}{3}&\frac{2}{3}&1\\1&\frac{1}{3}&\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}\\0&\frac{2}{3}&\frac{1}{3}&\frac{1}{3}&\frac{2}{3}&0&1&\frac{2}{3}\\\frac{1}{3}&1&0&0&1&\frac{1}{3}&\frac{2}{3}&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{71}{27} & \frac{71}{27} & 0 & 0 & \frac{23}{9} & \frac{73}{27} \\ 0 & 0 & \frac{23}{9} & \frac{23}{9} & 0 & 0 & \frac{71}{27} & \frac{25}{9} \\ \frac{-71}{27} & \frac{-23}{9} & 0 & 0 & \frac{-23}{9} & \frac{-71}{27} & 0 & 0 \\ \frac{-71}{27} & \frac{-23}{9} & 0 & 0 & \frac{-23}{9} & \frac{-71}{27} & 0 & 0 \\ 0 & 0 & \frac{23}{9} & \frac{23}{9} & 0 & 0 & \frac{71}{27} & \frac{25}{9} \\ 0 & 0 & \frac{71}{27} & \frac{71}{27} & 0 & 0 & \frac{23}{9} & \frac{73}{27} \\ \frac{-23}{9} & \frac{-71}{27} & 0 & 0 & \frac{-71}{27} & \frac{-23}{9} & 0 & 0 \\ \frac{-73}{27} & \frac{-25}{9} & 0 & 0 & \frac{-25}{9} & \frac{-73}{27} & 0 & 0 \end{pmatrix} \text{ Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & 0 & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{18} & \frac{1}{6} \\ \frac{-1}{18} & 0 & 0 & 0 & 0 & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & 0 & 0 & 0 & 0 & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{18} & \frac{1}{6} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & 0 & \frac{1}{9} \\ 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & 0 & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & \frac{-1}{9} & 0 & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{25}{9} & 0 & 0 & 0 & 0 & 0 & \frac{23}{9} & 0 \\ 0 & \frac{25}{9} & \frac{23}{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{23}{9} & \frac{50}{9} & 0 & \frac{23}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{50}{9} & 0 & 0 & 0 & \frac{46}{9} \\ 0 & 0 & \frac{23}{9} & 0 & \frac{25}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{25}{9} & \frac{23}{9} & 0 \\ \frac{23}{9} & 0 & 0 & 0 & 0 & \frac{23}{9} & \frac{50}{9} & 0 \\ 0 & 0 & 0 & \frac{46}{9} & 0 & 0 & 0 & \frac{50}{9} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} \\ \frac{2}{3} & 1 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 \\ \frac{1}{3} & 0 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & 1 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 1 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{46}{3} T + 0\Omega$$

$$\Omega \left( \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \right)$$

$$T \left( 0 \ \frac{1}{18} \ \frac{1}{9} \ 0 \ 0 \ \frac{1}{6} \ \frac{1}{9} \ \frac{2}{9} \ 0 \ \frac{1}{6} \ \frac{1}{9} \ \frac{1}{9} \ \frac{1}{9} \ \frac{1}{9} \ \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

NM $\left(0 \quad \frac{23}{27} \quad \frac{46}{27} \quad 0 \quad 0 \quad \frac{23}{9} \quad \frac{46}{27} \quad \frac{92}{27} \quad 0 \quad \frac{23}{9} \quad \frac{46}{27} \quad \frac{46}{27} \quad \frac{46}{27} \quad \frac{46}{27} \quad \frac{23}{9}\right)$

"IS MN in Vec(K)?", false

MN
$\left(\frac{-506}{5427} \quad \frac{4393}{5427} \quad \frac{10304}{5427} \quad \frac{-23}{180} \quad \frac{-1012}{5427} \quad \frac{15272}{5427} \quad \frac{10304}{5427} \quad \frac{805}{324} \quad \frac{736}{5427} \quad \frac{14329}{5427} \quad \frac{10258}{5427} \quad \frac{437}{324}\right.$

$\boldsymbol{\tau} = 100/3$ , rank = 2 , ratio = 50/3 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 92/3$ , $r' = 1/2$ , $\boldsymbol{\tau} / n^2 = 25/48$

$p^2 = 5/36$ , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$−check is positive? 220/9

max r = 36/5 , $r$−check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{4}{3} T + \frac{92}{3} \Omega$$

There are, 2, partitions and, 5, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 15
out of total no. of elements equal to 20

dim span idems 10 vs no. of idems 10

"PT1" = {{1, 2, 5, 6, 8}, {3, 4, 7}}

"PT2" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {4, 8}

"RG2" = {6, 7}

"RG3" = {3, 5}

"RG4" = {2, 3}

"RG5" = {1, 7}

$$M_c = \begin{pmatrix} \frac{7}{3} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{5}{3} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{7}{3} & \frac{5}{3} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{5}{3} & \frac{34}{9} & \frac{-16}{9} & \frac{5}{3} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{34}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{10}{3} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{5}{3} & \frac{-8}{9} & \frac{7}{3} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{7}{3} & \frac{5}{3} & \frac{-8}{9} \\ \frac{5}{3} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{5}{3} & \frac{34}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{10}{3} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{34}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} \\ \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} & \frac{-4}{21} & \frac{-4}{21} & \frac{5}{7} & \frac{-8}{21} \\ \frac{-4}{21} & 1 & \frac{5}{7} & \frac{-8}{21} & \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} \\ \frac{-4}{17} & \frac{15}{34} & 1 & \frac{-8}{17} & \frac{15}{34} & \frac{-4}{17} & \frac{-8}{17} & \frac{-8}{17} \\ \frac{-4}{17} & \frac{-4}{17} & \frac{-8}{17} & 1 & \frac{-4}{17} & \frac{-4}{17} & \frac{-8}{17} & \frac{15}{17} \\ \frac{-4}{21} & \frac{-4}{21} & \frac{5}{7} & \frac{-8}{21} & 1 & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} \\ \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} & \frac{-4}{21} & 1 & \frac{5}{7} & \frac{-8}{21} \\ \frac{15}{34} & \frac{-4}{17} & \frac{-8}{17} & \frac{-8}{17} & \frac{-4}{17} & \frac{15}{34} & 1 & \frac{-8}{17} \\ \frac{-4}{17} & \frac{-4}{17} & \frac{-8}{17} & \frac{15}{17} & \frac{-4}{17} & \frac{-4}{17} & \frac{-8}{17} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{19}{31} & \frac{7}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} \\ \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 \\ \frac{7}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} \\ \frac{7}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} \\ \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 \\ 1 & \frac{19}{31} & \frac{7}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} \\ \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} \\ \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 \end{pmatrix}$$

 

$$N_C M_C = \begin{pmatrix} \frac{1}{9} & \frac{1}{27} & \frac{-2}{27} & \frac{-2}{27} & \frac{1}{27} & \frac{1}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{9} & \frac{1}{27} & \frac{-2}{27} & \frac{2}{9} \\ \frac{-1}{27} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{-1}{27} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{1}{27} & \frac{1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{9} & \frac{1}{27} & \frac{-2}{27} & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{27} & \frac{-2}{27} & \frac{-2}{27} & \frac{1}{27} & \frac{1}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{-1}{9} & \frac{-1}{27} & \frac{2}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{9} & \frac{1}{27} & \frac{-2}{27} & \frac{2}{9} \end{pmatrix} \quad M_C N_C =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{1}{27} & \frac{1}{9} & \frac{-1}{9} & \frac{1}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{27} & \frac{-1}{27} & \frac{1}{9} \\ \frac{-2}{27} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{-2}{27} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{1}{27} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{27} & \frac{-1}{27} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{1}{27} & \frac{1}{9} & \frac{-1}{9} & \frac{1}{27} \\ \frac{-2}{9} & \frac{-2}{27} & \frac{2}{27} & \frac{2}{27} & \frac{-2}{27} & \frac{-2}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{2}{27} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{27} & \frac{-2}{27} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{9} & \frac{-1}{27} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{27} & \frac{-1}{9} \\ \frac{-1}{27} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{27} & 0 & 0 \\ \frac{-1}{27} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{27} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{27} & \frac{-1}{9} \\ 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{9} & \frac{-1}{27} \\ \frac{-1}{9} & \frac{-1}{27} & 0 & 0 & \frac{-1}{27} & \frac{-1}{9} & 0 & 0 \\ \frac{1}{27} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{27} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.4444444444, 8.038454464, 0.294878870, 9.788223741, 0.322887371, 2.777777778, 2.777777778]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3.524988800, 1.153500726, 2.210399362]

Eigenvalues $M_C$–scaled

[0., 0.1176470588, 2.555851929, 0.105212497, 2.734275744, 0.106060390, 1.190476190, 1.190476190]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 4.093535381, 1.339549232, 2.566915388]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [1, −1, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 0.4444444444, 2.777777778, 8.038454464, 0.294878870, 2.777777778, 8.038454464, 0.294878870]

Eigenvalues $N_0$

[0., 0., 0., 0., 0., 4.401119421, 1.158798503, 2.440082075]

NullSpace $M_0$

{}

NullSpace $N_0$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 1, 0, −1, −1, 1], [0, 0, 1, 0, 0, −1, −1, 1]}

Eigenvalues M

[5.111111111, −5.111111111, 0., 0., 3.614101326, −3.614101326, 3.614101326, −3.614101326]

Eigenvalues N

[0., 0., 0., 0., 0., 3.897781351, −2.727013674, −1.170767678]

NullSpace M

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace N

{[0, −1, 0, 0, 0, 0, 0, 1], [1, −1, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 2 & 1 & 0 & 3 & 1 \\ 1 & 0 & 3 & 3 & 0 & 1 & 2 & 0 \\ 2 & 3 & 0 & 0 & 3 & 2 & 1 & 3 \\ 2 & 3 & 0 & 0 & 3 & 2 & 1 & 3 \\ 1 & 0 & 3 & 3 & 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 3 & 1 \\ 3 & 2 & 1 & 1 & 2 & 3 & 0 & 2 \\ 1 & 0 & 3 & 3 & 0 & 1 & 2 & 0 \end{pmatrix}$$

==========================================================

20, [1, 1, 1, –1, –1, 1, 1, 1]

==========================================================

{4, 8}

R: [4, 3, 1, 6, 3, 4, 4, 7]
B: [8, 7, 5, 2, 7, 8, 8, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{512}(1+s)(-1+s)(6+3s+s^2)(7+2s+s^2)$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 2

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[4] v[6]
"B CYCLES", 1 + v[3] v[5] v[8] v[7]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 23 \\ 0 & 0 & 0 & 0 & 0 & 0 & 46 & 0 \\ 23 & 0 & 0 & 0 & 0 & 23 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 23 \\ 0 & 0 & 0 & 23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 46 & 0 & 0 & 0 & 0 & 0 \\ 0 & 23 & 0 & 0 & 23 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{3} & 1 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 1 \\ 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 1 & 1 & \frac{1}{3} \\ 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & 1 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 1 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{6}$ ( $v[1]v[4] + v[2]v[8] + 2v[3]v[7] + v[4]v[6] + v[5]v[8]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{2, 3, 4, 5}, {1, 6, 7, 8}}

"PT2" = {{1, 2, 5, 6, 7}, {3, 4, 8}}

"RG1" = {5, 8}

"RG2" = {2, 8}

"RG3" = {3, 7}

"RG4" = {4, 6}

"RG5" = {1, 4}

$\pi 2$ = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0]

supp $\pi 2$ = {3, 13, 17, 20, 25}

$u2$ = [1, 3, 3, 1, 0, 0, 2, 2, 2, 0, 1, 1, 3, 0, 2, 3, 3, 1, 2, 3, 3, 1, 1, 1, 3, 0, 2, 2]

supp u2 = {1, 2, 3, 4, 7, 8, 9, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28}

Action of R on ranges, [[3], [3], [5], [4], [4]]
Action of B on ranges, [[3], [3], [1], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[4, 1, 4, 1]

B−BLOCKS,

[2, 4, 1, 3]

with invariant measure, [2, 1, 1, 2]

N by blocks, N − check: true

$b_1 = \{1, 2, 5, 6, 7\}$
$b_2 = \{2, 3, 4, 5\}$
$b_3 = \{1, 6, 7, 8\}$
$b_4 = \{3, 4, 8\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: 3 ⊕ 15/13

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}}, true

$\Omega_B$ in Vec(K)? , {{3, 5, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{2, 3, 4, 5}, {1, 6, 7, 8}}

1, "range", [5, 8], [[8, 5, 5, 5, 5, 8, 8, 8], [5, 8, 8, 8, 8, 5, 5, 5]]

2, "range", [2, 8], [[8, 2, 2, 2, 2, 8, 8, 8], [2, 8, 8, 8, 8, 2, 2, 2]]

3, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

4, "range", [4, 6], [[6, 4, 4, 4, 4, 6, 6, 6], [4, 6, 6, 6, 6, 4, 4, 4]]

5, "range", [1, 4], [[4, 1, 1, 1, 1, 4, 4, 4], [1, 4, 4, 4, 4, 1, 1, 1]]

2, "partition", {{1, 2, 5, 6, 7}, {3, 4, 8}}

1, "range", [5, 8], [[8, 8, 5, 5, 8, 8, 8, 5], [5, 5, 8, 8, 5, 5, 5, 8]]

2, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 8, 2], [2, 2, 8, 8, 2, 2, 2, 8]]

3, "range", [3, 7], [[7, 7, 3, 3, 7, 7, 7, 3], [3, 3, 7, 7, 3, 3, 3, 7]]

4, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 6, 4], [4, 4, 6, 6, 4, 4, 4, 6]]

5, "range", [1, 4], [[4, 4, 1, 1, 4, 4, 4, 1], [1, 1, 4, 4, 1, 1, 1, 4]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$   []

$g_2 =$   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2$ =

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

{3, 13, 17, 20, 25}

$u2 =$
$\begin{pmatrix} 1 & 3 & 3 & 1 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 3 & 0 & 2 & 3 & 3 & 1 & 2 & 3 & 3 & 1 & 1 & 1 & 3 & 0 & 2 & 2 \end{pmatrix}$

{1, 2, 3, 4, 7, 8, 9, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{6} & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & \frac{1}{9} & \frac{2}{9} & 0 \\ 0 & \frac{1}{18} & \frac{1}{3} & \frac{1}{3} & \frac{1}{18} & 0 & 0 & \frac{2}{9} \\ 0 & \frac{1}{18} & \frac{1}{3} & \frac{1}{3} & \frac{1}{18} & 0 & 0 & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & \frac{1}{9} & \frac{2}{9} & 0 \\ \frac{1}{6} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{6} & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{6} & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{18} & 0 & \frac{2}{9} & \frac{2}{9} & 0 & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{23}{9} & \frac{46}{27} & 0 & 0 & \frac{46}{27} & \frac{23}{9} & \frac{46}{9} & \frac{46}{27} \\ \frac{46}{27} & \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{23}{9} & \frac{46}{27} & \frac{92}{27} & 0 \\ 0 & \frac{23}{27} & \frac{46}{9} & \frac{46}{9} & \frac{23}{27} & 0 & 0 & \frac{92}{27} \\ 0 & \frac{23}{27} & \frac{46}{9} & \frac{46}{9} & \frac{23}{27} & 0 & 0 & \frac{92}{27} \\ \frac{46}{27} & \frac{23}{9} & \frac{46}{27} & \frac{46}{27} & \frac{23}{9} & \frac{46}{27} & \frac{92}{27} & 0 \\ \frac{23}{9} & \frac{46}{27} & 0 & 0 & \frac{46}{27} & \frac{23}{9} & \frac{46}{9} & \frac{46}{27} \\ \frac{23}{9} & \frac{46}{27} & 0 & 0 & \frac{46}{27} & \frac{23}{9} & \frac{46}{9} & \frac{46}{27} \\ \frac{23}{27} & 0 & \frac{92}{27} & \frac{92}{27} & 0 & \frac{23}{27} & \frac{46}{27} & \frac{46}{9} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 2, -1, 1, 0, -2]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s & 0 & s+t & -s & -t & 0 & s+t & -t \\ -s & t & 0 & 0 & -t & s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & -2 & 2 & -1 \end{pmatrix}$

M0 is invertible. det= 327680000/1594323

$$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 1 & 1 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 1 & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 1 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 1 & 1 & \frac{2}{3} \\ 1 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 1 & 1 & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{23}{9} & \frac{23}{9} & 0 & 0 & \frac{25}{9} & \frac{71}{27} \\ 0 & 0 & \frac{71}{27} & \frac{71}{27} & 0 & 0 & \frac{73}{27} & \frac{23}{9} \\ \frac{-23}{9} & \frac{-71}{27} & 0 & 0 & \frac{-71}{27} & \frac{-23}{9} & 0 & 0 \\ \frac{-23}{9} & \frac{-71}{27} & 0 & 0 & \frac{-71}{27} & \frac{-23}{9} & 0 & 0 \\ 0 & 0 & \frac{71}{27} & \frac{71}{27} & 0 & 0 & \frac{73}{27} & \frac{23}{9} \\ 0 & 0 & \frac{23}{9} & \frac{23}{9} & 0 & 0 & \frac{25}{9} & \frac{71}{27} \\ \frac{-25}{9} & \frac{-73}{27} & 0 & 0 & \frac{-73}{27} & \frac{-25}{9} & 0 & 0 \\ \frac{-71}{27} & \frac{-23}{9} & 0 & 0 & \frac{-23}{9} & \frac{-71}{27} & 0 & 0 \end{pmatrix} \quad \text{Skew } T =$$

 

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{9} & 0 \\ 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & 0 & 0 & 0 \\ 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{18} \\ \frac{-1}{6} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{18} & 0 & 0 & 0 & 0 & \frac{-1}{18} & 0 & 0 \end{pmatrix} \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

 

$$M_0 = \begin{pmatrix} \frac{25}{9} & 0 & 0 & \frac{23}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{25}{9} & 0 & 0 & 0 & 0 & 0 & \frac{23}{9} \\ 0 & 0 & \frac{50}{9} & 0 & 0 & 0 & \frac{46}{9} & 0 \\ \frac{23}{9} & 0 & 0 & \frac{50}{9} & 0 & \frac{23}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{25}{9} & 0 & 0 & \frac{23}{9} \\ 0 & 0 & 0 & \frac{23}{9} & 0 & \frac{25}{9} & 0 & 0 \\ 0 & 0 & \frac{46}{9} & 0 & 0 & 0 & \frac{50}{9} & 0 \\ 0 & \frac{23}{9} & 0 & 0 & \frac{23}{9} & 0 & 0 & \frac{50}{9} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 1 & 1 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 1 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 1 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & 0 \\ 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 1 & 1 & \frac{1}{3} \\ 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{46}{3} T + 0\Omega$$

$\Omega \left( \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \right)$

$T \left( \frac{1}{18} \ 0 \ \frac{1}{6} \ \frac{1}{9} \ \frac{1}{9} \ \frac{1}{6} \ \frac{1}{9} \ \frac{1}{9} \ \frac{1}{3} \ \frac{1}{6} \ \frac{1}{9} \ 0 \ 0 \ \frac{1}{9} \ \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left( \frac{23}{27} \ 0 \ \frac{23}{9} \ \frac{46}{27} \ \frac{46}{27} \ \frac{23}{9} \ \frac{46}{27} \ \frac{46}{27} \ \frac{46}{9} \ \frac{23}{9} \ \frac{46}{27} \ 0 \ 0 \ \frac{46}{27} \ \frac{23}{9} \right)$

"IS MN in Vec(K)?", false

MN

$\left( \frac{4393}{5427} \ \frac{-506}{5427} \ \frac{14329}{5427} \ \frac{9016}{5427} \ \frac{437}{324} \ \frac{14329}{5427} \ \frac{10258}{5427} \ \frac{8786}{5427} \ \frac{713}{180} \ \frac{15272}{5427} \ \frac{10304}{5427} \ \frac{-1012}{5427} \right.$

$$\boldsymbol{\tau} = 100/3 \text{ , rank } = 2 \text{ , ratio } = 50/3 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 92/3 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 25/48$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 220/9$$

max r = 36/5 , $r$−check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{4}{3} T + \frac{92}{3} \Omega$$

There are, 2, partitions and, 5, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 15
out of total no. of elements equal to 20

dim span idems 10 vs no. of idems 10

"PT1" = {{2, 3, 4, 5}, {1, 6, 7, 8}}

"PT2" = {{1, 2, 5, 6, 7}, {3, 4, 8}}

"RG1" = {5, 8}

"RG2" = {2, 8}

"RG3" = {3, 7}

"RG4" = {4, 6}

"RG5" = {1, 4}

$$M_C = \begin{pmatrix} \frac{7}{3} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{3} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{7}{3} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{3} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{34}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{10}{3} & \frac{-16}{9} \\ \frac{5}{3} & \frac{-8}{9} & \frac{-16}{9} & \frac{34}{9} & \frac{-8}{9} & \frac{5}{3} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{7}{3} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{3} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{3} & \frac{-4}{9} & \frac{7}{3} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{10}{3} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{34}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{5}{3} & \frac{-16}{9} & \frac{-16}{9} & \frac{5}{3} & \frac{-8}{9} & \frac{-16}{9} & \frac{34}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{31}{36} & \frac{31}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{31}{36} & \frac{7}{36} & \frac{7}{36} & \frac{31}{36} & \frac{19}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{7}{36} & \frac{31}{36} & \frac{31}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{7}{36} & \frac{31}{36} & \frac{31}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{19}{36} & \frac{31}{36} & \frac{7}{36} & \frac{7}{36} & \frac{31}{36} & \frac{19}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{31}{36} & \frac{31}{36} & \frac{7}{36} \\ \frac{31}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{31}{36} & \frac{31}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{7}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-4}{21} & \frac{-8}{21} & \frac{5}{7} & \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} \\ \frac{-4}{21} & 1 & \frac{-8}{21} & \frac{-8}{21} & \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{5}{7} \\ \frac{-4}{17} & \frac{-4}{17} & 1 & \frac{-8}{17} & \frac{-4}{17} & \frac{-4}{17} & \frac{15}{17} & \frac{-8}{17} \\ \frac{15}{34} & \frac{-4}{17} & \frac{-8}{17} & 1 & \frac{-4}{17} & \frac{15}{34} & \frac{-8}{17} & \frac{-8}{17} \\ \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{-8}{21} & 1 & \frac{-4}{21} & \frac{-8}{21} & \frac{5}{7} \\ \frac{-4}{21} & \frac{-4}{21} & \frac{-8}{21} & \frac{5}{7} & \frac{-4}{21} & 1 & \frac{-8}{21} & \frac{-8}{21} \\ \frac{-4}{17} & \frac{-4}{17} & \frac{15}{17} & \frac{-8}{17} & \frac{-4}{17} & \frac{-4}{17} & 1 & \frac{-8}{17} \\ \frac{-4}{17} & \frac{15}{34} & \frac{-8}{17} & \frac{-8}{17} & \frac{15}{34} & \frac{-4}{17} & \frac{-8}{17} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

 

$$\begin{pmatrix} 1 & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & 1 & 1 & \frac{7}{31} \\ \frac{19}{31} & 1 & \frac{7}{31} & \frac{7}{31} & 1 & \frac{19}{31} & \frac{19}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{7}{31} & 1 & 1 & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} \\ \frac{-5}{31} & \frac{7}{31} & 1 & 1 & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} \\ \frac{19}{31} & 1 & \frac{7}{31} & \frac{7}{31} & 1 & \frac{19}{31} & \frac{19}{31} & \frac{-5}{31} \\ 1 & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & 1 & 1 & \frac{7}{31} \\ 1 & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & 1 & 1 & \frac{7}{31} \\ \frac{7}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{7}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{9} & \frac{1}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{27} & \frac{1}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-2}{27} & \frac{-2}{27} & \frac{1}{9} & \frac{1}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{27} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{27} & \frac{-1}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{-1}{9} & \frac{-1}{27} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{27} & \frac{-1}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-2}{27} & \frac{-2}{27} & \frac{1}{9} & \frac{1}{27} & \frac{2}{27} & \frac{-2}{9} \\ \frac{1}{9} & \frac{1}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{27} & \frac{1}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{1}{9} & \frac{1}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{1}{27} & \frac{1}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{-1}{27} & \frac{-1}{9} & \frac{2}{27} & \frac{2}{27} & \frac{-1}{9} & \frac{-1}{27} & \frac{-2}{27} & \frac{2}{9} \end{pmatrix} \quad M_C N_C =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{27} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{27} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-1}{27} & \frac{-1}{27} & \frac{1}{9} & \frac{1}{27} & \frac{1}{27} & \frac{-1}{9} \\ \frac{-2}{9} & \frac{-2}{27} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{-2}{9} & \frac{-2}{27} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{27} \\ \frac{1}{27} & \frac{1}{9} & \frac{-1}{27} & \frac{-1}{27} & \frac{1}{9} & \frac{1}{27} & \frac{1}{27} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{27} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{27} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{27} \\ \frac{2}{9} & \frac{2}{27} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{27} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{27} \\ \frac{-2}{27} & \frac{-2}{9} & \frac{2}{27} & \frac{2}{27} & \frac{-2}{9} & \frac{-2}{27} & \frac{-2}{27} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{-1}{9} & \frac{1}{27} \\ 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{-1}{27} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-1}{27} & 0 & 0 & \frac{-1}{27} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{27} & 0 & 0 & \frac{-1}{27} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{-1}{27} & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{-1}{9} & \frac{1}{27} \\ \frac{1}{9} & \frac{1}{27} & 0 & 0 & \frac{1}{27} & \frac{1}{9} & 0 & 0 \\ \frac{-1}{27} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{27} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.4444444444, 8.038454464, 0.294878870, 9.788223741, 0.322887371, 2.777777778, 2.777777778]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3.524988800, 1.153500726, 2.210399362]

Eigenvalues $M_C$–scaled

[0., 0.1176470588, 2.555851929, 0.105212497, 2.734275744, 0.106060390, 1.190476190, 1.190476190]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 4.093535381, 1.339549232, 2.566915388]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 1, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 0.4444444444, 2.777777778, 8.038454464, 0.294878870, 2.777777778, 8.038454464, 0.294878870]

Eigenvalues $N_0$

[0., 0., 0., 0., 0., 4.401119421, 1.158798503, 2.440082075]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, 0, 1, −1, 0, 1, −1], [0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 1, 0, −1, 0, 1, −1], [1, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues M

[5.111111111, −5.111111111, 0., 0., 3.614101326, −3.614101326, 3.614101326, −3.614101326]

Eigenvalues N

[0., 0., 0., 0., 0., 3.897781351, −2.727013674, −1.170767678]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N

{[0, 0, −1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 1, −1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 3 & 3 & 1 & 0 & 0 & 2 \\ 1 & 0 & 2 & 2 & 0 & 1 & 1 & 3 \\ 3 & 2 & 0 & 0 & 2 & 3 & 3 & 1 \\ 3 & 2 & 0 & 0 & 2 & 3 & 3 & 1 \\ 1 & 0 & 2 & 2 & 0 & 1 & 1 & 3 \\ 0 & 1 & 3 & 3 & 1 & 0 & 0 & 2 \\ 0 & 1 & 3 & 3 & 1 & 0 & 0 & 2 \\ 2 & 3 & 1 & 1 & 3 & 2 & 2 & 0 \end{pmatrix}$$

==================================================

{5, 6}

R: [4, 3, 1, 2, 7, 8, 4, 3]
B: [8, 7, 5, 6, 3, 4, 8, 7]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 6

Level 2 det = $\frac{-1}{4096}(-4+s)(2+s)(-14+4s+s^2)(-3+s)(-1+s)^2$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6]) (1 + v[3] v[5])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 1., −1., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 4 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 4 & 0 & 0 & 0 & 0 \\ 4 & 4 & 0 & 8 & 0 & 0 & 4 & 4 \\ 4 & 4 & 8 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 4 & 4 \\ 0 & 0 & 4 & 4 & 4 & 4 & 0 & 8 \\ 0 & 0 & 4 & 4 & 4 & 4 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 7

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{6}$ ( $2v[1]v[2] + v[1]v[3] + v[1]v[4] + v[2]v[3] + v[2]v[4] + 4v[3]v[4] + v[3]v[7] + v[3]v[8] + v[4]v[7] + v[4]v[8] + 2v[5]v[6] + v[5]v[7] + v[5]v[8] + v[6]v[7] + v[6]v[8] + 4v[8]v[7]$ )
degree 3 : $\frac{1}{12}$ ( $v[1]v[2]v[3] + v[1]v[2]v[4] + v[1]v[3]v[4] + v[2]v[3]v[4] + v[3]v[4]v[7] + v[3]v[4]v[8] + v[3]v[8]v[7] + v[4]v[8]v[7] + v[5]v[6]v[7] + v[5]v[6]v[8] + v[5]v[8]v[7] + v[6]v[8]v[7]$ )
degree 4 : $\frac{1}{3}$ ( $v[1]v[2]v[3]v[4] + v[3]v[4]v[8]v[7] + v[5]v[6]v[8]v[7]$ )

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

## KERNEL STRUCTURE

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"RG1" = {5, 6, 7, 8}

"RG2" = {3, 4, 7, 8}

"RG3" = {1, 2, 3, 4}

$\pi 4$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1] supp $\pi 4$ = {1, 61, 70}

$u4$ = [1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1]

supp u4 = {1, 3, 6, 10, 19, 24, 28, 33, 38, 43, 47, 52, 61, 65, 68, 70}

Action of R on ranges, [[2], [3], [3]]
Action of B on ranges, [[2], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 4, 2, 1]

B-BLOCKS,

[2, 1, 3, 4]

with invariant measure, [1, 1, 1, 1]

N by blocks, N – check: true

$b_1$ = {2, 8}
$b_2$ = {1, 7}
$b_3$ = {4, 6}
$b_4$ = {3, 5}

dim(span of partition vectors), rank($N_0$), rank($N$): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: $3 \oplus 11/9$

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {7, 8}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

1, "range", [5, 6, 7, 8], [[8, 7, 6, 5, 6, 5, 8, 7], [8, 7, 5, 6, 5, 6, 8, 7], [7, 8, 6, 5, 6, 5, 7, 8], [7, 8, 5, 6, 5, 6, 7, 8], [6, 5, 8, 7, 8, 7, 6, 5], [6, 5, 7, 8, 7, 8, 6, 5], [5, 6, 8, 7, 8, 7, 5, 6], [5, 6, 7, 8, 7, 8, 5, 6]]

2, "range", [3, 4, 7, 8], [[8, 7, 4, 3, 4, 3, 8, 7], [8, 7, 3, 4, 3, 4, 8, 7], [7, 8, 4, 3, 4, 3, 7, 8], [7, 8, 3, 4, 3, 4, 7, 8], [4, 3, 8, 7, 8, 7, 4, 3], [4, 3, 7, 8, 7, 8, 4, 3], [3, 4, 8, 7, 8, 7, 3, 4], [3, 4, 7, 8, 7, 8, 3, 4]]

3, "range", [1, 2, 3, 4], [[4, 3, 2, 1, 2, 1, 4, 3], [4, 3, 1, 2, 1, 2, 4, 3], [3, 4, 2, 1, 2, 1, 3, 4], [3, 4, 1, 2, 1, 2, 3, 4], [2, 1, 4, 3, 4, 3, 2, 1], [2, 1, 3, 4, 3, 4, 2, 1], [1, 2, 4, 3, 4, 3, 1, 2], [1, 2, 3, 4, 3, 4, 1, 2]]

"group has", 8, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2], [3, 4]]

$g_2 =$ [[3, 4]]

$g_3 =$ [[1, 2]]

$g_4 =$ []

$g_5 =$ [[1, 4], [2, 3]]

linear dimension, 6

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} -2h[1]+2h[2] & 2h[1] & 2h[1] & 0 & h[3] & h[3] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ 0 & 0 & 2. & -2. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 4 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 6 & 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 4 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1+t+3t^2+4t^3+8t^4+10t^5+16t^6+20t^7+29t^8+35t^9+47t^{10}$

n−choose−rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 4$ =
$(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

{1, 61, 70}

$u4$ =
$(1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0$

{1, 3, 6, 10, 19, 24, 28, 33, 38, 43, 47, 52, 61, 65, 68, 70}

picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 3$ =
$(1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

$u3$ =

$\left(\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad 0 \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad \frac{1}{4} \quad 0\right.$

picheck $(3 \quad 3 \quad 6 \quad 6 \quad 3 \quad 3 \quad 6 \quad 6)$

$\pi 2 =$
$(2 \quad 2 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0 \quad 0 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 4)$

$u2 =$
$\left(\frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad 0 \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad 0 \quad \frac{1}{8} \quad 0 \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad 0 \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8}\right.$

picheck $(6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12)$

$\pi 1 = (6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12)$

$u1 = \left(\frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32} \quad \frac{3}{32}\right)$

picheck $(6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3}&0&0&0&0&0&\frac{2}{3}&0\\ 0&\frac{1}{3}&0&0&0&0&0&\frac{2}{3}\\ 0&0&\frac{2}{3}&0&\frac{1}{3}&0&0&0\\ 0&0&0&\frac{2}{3}&0&\frac{1}{3}&0&0\\ 0&0&\frac{2}{3}&0&\frac{1}{3}&0&0&0\\ 0&0&0&\frac{2}{3}&0&\frac{1}{3}&0&0\\ \frac{1}{3}&0&0&0&0&0&\frac{2}{3}&0\\ 0&\frac{1}{3}&0&0&0&0&0&\frac{2}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 \\ \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 & \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 & \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} \\ 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 1, 1, -1, -1, -1, -1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & -t & 0 & t & 0 & -s & 0 \\ 0 & -s & 0 & t & 0 & -t & 0 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & 0 & 0 & t & 0 \\ -s & 0 & 0 & -s & -s+t \\ 0 & -t & s & -t & -t \\ 0 & t & -s & s & s \\ -t & 0 & 0 & -t & -t+s \\ t & 0 & 0 & s & 0 \\ s & 0 & 0 & t & 0 \\ -s & 0 & 0 & -s & -s+t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & -t & s & 0 & 0 & t \\ 0 & t & 0 & 0 & s & 0 \\ s & 0 & 0 & t & 0 & 0 \\ 0 & 0 & t & -t & t & s \\ 0 & s & 0 & 0 & t & 0 \\ s & -s & t & 0 & 0 & s \\ t & -t & s & 0 & 0 & t \\ 0 & t & 0 & 0 & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$$\Omega\left(\frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\right)$$

$$T\left(0\ \ \frac{1}{3}\ \ 0\ \ \frac{2}{3}\ \ 0\ \ 0\ \ 0\ \ \frac{2}{3}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{3}\right)$$

"IS NM in Vec(K)?", true

NM $\left(\frac{16}{3} \;\; 4 \;\; \frac{16}{3} \;\; 8 \;\; \frac{8}{3} \;\; \frac{8}{3} \;\; \frac{16}{3} \;\; 8 \;\; \frac{8}{3} \;\; \frac{8}{3} \;\; \frac{16}{3} \;\; \frac{16}{3} \;\; \frac{8}{3} \;\; 4\right)$

"IS MN in Vec(K)?", false

MN $\left(4 \;\; 6 \;\; 4 \;\; 6 \;\; 4 \;\; 4 \;\; 4 \;\; 6 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 6\right)$

$$\boldsymbol{\tau} = 16/1 \text{ , rank} = 4 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 8

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 24

dim span idems 3 vs no. of idems 3

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"RG1" = {5, 6, 7, 8}

"RG2" = {3, 4, 7, 8}

"RG3" = {1, 2, 3, 4}

$$M_c = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & -1 & -1 \\ 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & -1 & -1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & -1 & -1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & -1 & -1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & -1 & -1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & -1 & -1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & -1 & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 \\ -1 & -1 & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 0., 5.333333333, 1.777777778]

Eigenvalues $N_C$

[0.8888888889, 2., 2., 2., 0., 0., 0., 0.]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 0., 6., 2.]

Eigenvalues $N_C$–scaled

[1.032258065, 2.322580645, 2.322580645, 2.322580645, 0., 0., 0., 0.]

NullSpace $M_C$

{[0, 0, 0, 1, 1, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0, 0, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0]}

Eigenvalues $M_0$

[0., 0., 0., 0., 0., 5.333333333, 9.104569499, 1.562097167]

Eigenvalues $N_0$

[2., 2., 2., 2., 0., 0., 0., 0.]

NullSpace $M_0$

{[1, 0, −1, 0, −1, 0, 1, 0], [1, 0, −1, 0, −1, 0, 0, 1], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0]}

NullSpace $N_0$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0]}

Eigenvalues M

[0., 6.666666667, 3.415403751, −2.082070417, −1.333333333, −2.666666667, −1.333333333, −2.666666667]

Eigenvalues N

[6., −2., −2., −2., 0., 0., 0., 0.]

NullSpace M

{[−2, −2, 1, 1, −2, −2, 1, 1]}

NullSpace N

{[0, 0, 1, 0, −1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==========================================

{7, 8}

R: [4, 3, 1, 2, 3, 4, 8, 7]
B: [8, 7, 5, 6, 7, 8, 4, 3]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{3}{256}(-1+s)^2(7+3s+2s^2)(6+7s+3s^2)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[1] v[2] v[3] v[4]) (1 + v[8] v[7])
"B CYCLES", 1 + v[3] v[4] v[5] v[6] v[8] v[7]

Eigenvalues

R: [1. I, -1. I, 0., 0., 1., -1., 1., -1.]

B: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 6 & 5 & 5 & 0 & 0 & 5 & 5 \\ 6 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 12 & 5 & 5 & 10 & 10 \\ 5 & 5 & 12 & 0 & 5 & 5 & 10 & 10 \\ 0 & 0 & 5 & 5 & 0 & 6 & 5 & 5 \\ 0 & 0 & 5 & 5 & 6 & 0 & 5 & 5 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 12 \\ 5 & 5 & 10 & 10 & 5 & 5 & 12 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{6}(4v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + v[2]v[7] + v[2]v[8] + 8v[3]v[4] + v[3]v[5] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + v[4]v[5] + v[4]v[6] + 2v[4]v[7] + 2v[4]v[8] + 4v[5]v[6] + v[5]v[7] + v[5]v[8] + v[6]v[7] + v[6]v[8] + 8v[8]v[7])$

degree 3 : $\frac{1}{24}(3v[1]v[4]v[7] + 3v[3]v[5]v[7] + 3v[4]v[6]v[7] + 3v[3]v[6]v[7] + 3v[2]v[4]v[8] + 2v[5]v[8]v[7] + 3v[1]v[4]v[8] + 2v[6]v[8]v[7] + 4v[3]v[4]v[8] + 2v[1]v[2]v[4] + 3v[2]v[4]v[7] + 2v[1]v[3]v[4] + 3v[1]v[3]v[8] + 2v[1]v[8]v[7] + 2v[2]v[3]v[4] + 2v[3]v[4]v[5] + 2v[5]v[6]v[8] + 2v[1]v[2]v[8] + 4v[3]v[8]v[7] + 2v[3]v[4]v[6] + 3v[4]v[6]v[8] + 3v[3]v[5]v[8] + 3v[1]v[3]v[7] + 3v[2]v[3]v[7] + 2v[5]v[6]v[7] + 3v[4]v[$

5]$v$[8] + 2$v$[2]$v$[8]$v$[7] + 3$v$[2]$v$[3]$v$[8] + 4$v$[3]$v$[4]$v$[7] + 2$v$[3]$v$[5]$v$[6] + 3$v$[3]$v$[6]$v$[8] + 3$v$[4]$v$[5]$v$[7] + 4$v$[4]$v$[8]$v$[7] + 2$v$[1]$v$[2]$v$[7] + 2$v$[1]$v$[2]$v$[3] + 2$v$[4]$v$[5]$v$[6] )

degree 4 : $\frac{1}{6}$ ( $v$[3]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[6]$v$[7] + $v$[1]$v$[3]$v$[4]$v$[7] + 8$v$[3]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[3]$v$[8]$v$[7] + 4$v$[1]$v$[2]$v$[3]$v$[4] + $v$[4]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[8] + 4$v$[5]$v$[6]$v$[8]$v$[7] + 4$v$[3]$v$[4]$v$[5]$v$[6] + $v$[2]$v$[3]$v$[4]$v$[8] + $v$[1]$v$[2]$v$[4]$v$[7] + 4$v$[1]$v$[2]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[7] + $v$[2]$v$[3]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[8] + $v$[2]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[4]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[7] + $v$[2]$v$[3]$v$[4]$v$[7] + $v$[4]$v$[5]$v$[6]$v$[7] + $v$[4]$v$[5]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[4]$v$[8] + $v$[4]$v$[5]$v$[6]$v$[8] + $v$[1]$v$[3]$v$[4]$v$[8] + $v$[3]$v$[5]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[8] + $v$[3]$v$[4]$v$[6]$v$[8] )

degree 5 : $\frac{1}{12}$ ( $v$[1]$v$[2]$v$[3]$v$[4]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[4]$v$[8] + $v$[1]$v$[2]$v$[3]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[3]$v$[4]$v$[8]$v$[7] + $v$[2]$v$[3]$v$[4]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[6]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[6]$v$[8] + $v$[3]$v$[4]$v$[5]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[8]$v$[7] + $v$[4]$v$[5]$v$[6]$v$[8]$v$[7] )

degree 6 : $\frac{1}{2}$ ( $v$[4] ) ( $v$[7] ) ( $v$[1]$v$[2] + $v$[5]$v$[6] ) ( $v$[3] ) ( $v$[8] )

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

## KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3}, {4}, {8}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$\pi 6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

supp $\pi 6$ = {6, 28}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1]

supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 4, 2, 1, 6, 5]

B−BLOCKS,

[4, 3, 5, 6, 1, 2]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 6\}$
$b_2 = \{2, 5\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{8\}$
$b_6 = \{7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 19, Shape: $11 \oplus 8/6$

$$CLB = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{3, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} \frac{3}{16} & \frac{3}{16} & \frac{3}{16} & \frac{3}{16} & 0 & 0 & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3}, {4}, {8}, {7}}

1, "range", [3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 7, 8, 4, 3], [8, 7, 6, 5, 7, 8, 3, 4], [8, 7, 5, 6, 7, 8, 4, 3], [8, 7, 5, 6, 7, 8, 3, 4], [8, 7, 4, 3, 7, 8, 6, 5], [8, 7, 4, 3, 7, 8, 5, 6], [8, 7, 3, 4, 7, 8, 6, 5], [8, 7, 3, 4, 7, 8, 5, 6], [7, 8, 6, 5, 8, 7, 4, 3], [7, 8, 6, 5, 8, 7, 3, 4], [7, 8, 5, 6, 8, 7, 4, 3], [7, 8, 5, 6, 8, 7, 3, 4], [7, 8, 4, 3, 8, 7, 6, 5], [7, 8, 4, 3, 8, 7, 5, 6], [7, 8, 3, 4, 8, 7, 6, 5], [7, 8, 3, 4, 8, 7, 5, 6], [6, 5, 8, 7, 5, 6, 4, 3], [6, 5, 8, 7, 5, 6, 3, 4], [6, 5, 7, 8, 5, 6, 4, 3], [6, 5, 7, 8, 5, 6, 3, 4], [6, 5, 4, 3, 5, 6, 8, 7], [6, 5, 4, 3, 5, 6, 7, 8], [6, 5, 3, 4, 5, 6, 8, 7], [6, 5, 3, 4, 5, 6, 7, 8], [5, 6, 8, 7, 6, 5, 4, 3], [5, 6, 8, 7, 6, 5, 3, 4], [5, 6, 7, 8, 6, 5, 4, 3], [5, 6, 7, 8, 6, 5, 3, 4], [5, 6, 4, 3, 6, 5, 8, 7], [5, 6, 4, 3, 6, 5, 7, 8], [5, 6, 3, 4, 6, 5, 8, 7], [5, 6, 3, 4, 6, 5, 7, 8], [4, 3, 8, 7, 3, 4, 6, 5], [4, 3, 8, 7, 3, 4, 5, 6], [4, 3, 7, 8, 3, 4, 6, 5], [4, 3, 7, 8, 3, 4, 5, 6], [4, 3, 6, 5, 3, 4, 8, 7], [4, 3, 6, 5, 3, 4, 7, 8], [4, 3, 5, 6, 3, 4, 8, 7], [4, 3, 5, 6, 3, 4, 7, 8], [3, 4, 8, 7, 4, 3, 6, 5], [3, 4, 8, 7, 4, 3, 5, 6], [3, 4, 7, 8, 4, 3, 6, 5], [3, 4, 7, 8, 4, 3, 5, 6], [3, 4, 6, 5, 4, 3, 8, 7], [3, 4, 6, 5, 4, 3, 7, 8], [3, 4, 5, 6, 4, 3, 8, 7], [3, 4, 5, 6, 4, 3, 7, 8] ]

2, "range", [1, 2, 3, 4, 7, 8], [[8, 7, 4, 3, 7, 8, 2, 1], [8, 7, 4, 3, 7, 8, 1, 2], [8, 7, 3, 4, 7, 8, 2, 1], [8, 7, 3, 4, 7, 8, 1, 2], [8, 7, 2, 1, 7, 8, 4, 3], [8, 7, 2, 1, 7, 8, 3, 4], [8, 7, 1, 2, 7, 8, 4, 3], [8, 7, 1, 2, 7, 8, 3, 4], [7, 8, 4, 3, 8, 7, 2, 1], [7, 8, 4, 3, 8, 7, 1, 2], [7, 8, 3, 4, 8, 7, 2, 1], [7, 8, 3, 4, 8, 7, 1, 2], [7, 8, 2, 1, 8, 7, 4, 3], [7, 8, 2, 1, 8, 7, 3, 4], [7, 8, 1, 2, 8, 7, 4, 3], [7, 8, 1, 2, 8, 7, 3, 4], [4, 3, 8, 7, 3, 4, 2, 1], [4, 3, 8, 7, 3, 4, 1, 2], [4, 3, 7, 8, 3, 4, 2, 1], [4, 3, 7, 8, 3, 4, 1, 2], [4, 3, 2, 1, 3, 4, 8, 7], [4, 3, 2, 1, 3, 4, 7, 8], [4, 3, 1, 2, 3, 4, 8, 7], [4, 3, 1, 2, 3, 4, 7, 8], [3, 4, 8, 7, 4, 3, 2, 1], [3, 4, 8, 7, 4, 3, 1, 2], [3, 4, 7, 8, 4, 3, 2, 1], [3, 4, 7, 8, 4, 3, 1, 2], [3, 4, 2, 1, 4, 3, 8, 7], [3, 4, 2, 1, 4, 3, 7, 8], [3, 4, 1, 2, 4, 3, 8, 7], [3, 4, 1, 2, 4, 3, 7, 8], [2, 1, 8, 7, 1, 2, 4, 3], [2, 1, 8, 7, 1, 2, 3, 4], [2, 1, 7, 8, 1, 2, 4, 3], [2, 1, 7, 8, 1, 2, 3, 4], [2, 1, 4, 3, 1, 2, 8, 7], [2, 1, 4, 3, 1, 2, 7, 8], [2, 1, 3, 4, 1, 2, 8, 7], [2, 1, 3, 4, 1, 2, 7, 8], [1, 2, 8, 7, 2, 1, 4, 3], [1, 2, 8, 7, 2, 1, 3, 4], [1, 2, 7, 8, 2, 1, 4, 3], [1, 2, 7, 8, 2, 1, 3, 4], [1, 2, 4, 3, 2, 1, 8, 7], [1, 2, 4, 3, 2, 1, 7, 8], [1, 2, 3, 4, 2, 1, 8, 7], [1, 2, 3, 4, 2, 1, 7, 8] ]

"group has", 48, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$    [[1, 4, 6], [2, 3, 5]]

$g_2 =$    [[1, 4, 6, 2, 3, 5]]

$g_3 =$    [[1, 3, 5, 2, 4, 6]]

$g_4 =$    [[1, 3, 5], [2, 4, 6]]

$g_5 =$    [[1, 2], [3, 5, 4, 6]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true

$\big(-2h[3] \quad 2h[3] \quad 2h[3] \quad -8h[1]-2h[3] \quad 2h[3] \quad 8h[1] \quad 2h[3] \quad -8h[2]-2h[3] \quad 2h[3] \quad 8$

"Basis for Z(G)"

1, "coeff", 8

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 8

$$Z[2] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 2

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. & 1. & -1. \\ 4. & -2. & -2. & 0 & 0 & 0 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8$

$+84t^9 + 122t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 6 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

{6, 28}

$u6 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

{6, 18, 25, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 =$
$(0 \; 0 \; 1 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0 \; 0$

$u5 =$
$\Big(0 \; 0 \; \frac{1}{6} \; \frac{1}{6} \; 0 \; 0 \; 0 \; 0 \; 0 \; \frac{1}{6} \; 0 \; 0 \; 0 \; 0 \; 0 \; \frac{1}{6} \; 0 \; 0 \; 0 \; 0 \; 0 \; \frac{1}{6} \; \frac{1}{6} \; 0 \; 0 \; \frac{1}{6} \; 0 \; 0 \; \frac{1}{6}$

picheck $\begin{pmatrix} 5 & 5 & 10 & 10 & 5 & 5 & 10 & 10 \end{pmatrix}$

$\pi 4 =$
$(2 \; 0 \; 0 \; 2 \; 2 \; 0 \; 0 \; 2 \; 2 \; 0 \; 0 \; 0 \; 0 \; 0 \; 2 \; 0 \; 0 \; 2 \; 2 \; 0 \; 0 \; 0 \; 0 \; 0 \; 2 \; 0 \; 0 \; 0 \; 0$

$u4 =$

$\left(\frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\right.$

picheck $(20\ \ 20\ \ 40\ \ 40\ \ 20\ \ 20\ \ 40\ \ 40)$

$\pi 3 =$
$(6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6$

$u3 =$
$\left(\frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\right.$

picheck $(60\ \ 60\ \ 120\ \ 120\ \ 60\ \ 60\ \ 120\ \ 120)$

$\pi 2 =$
$(24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 48\ \ 24\ \ 24\ \ 48\ \ 48\ \ 24\ \ 24\ \ 4$

$u2 =$
$\left(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\right.$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$\pi 1 = (120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, 1, 0, 0, −1, −1, 0, 0]

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & 0 & -s & 0 \\ t & 0 & s & 0 \\ 0 & t & 0 & s \\ 0 & -t & 0 & -s \\ t & 0 & s & 0 \\ -t & 0 & -s & 0 \\ -s & 0 & -t & 0 \\ s & 0 & t & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & s+t & -s & s+t \\ 0 & t & 0 & s & 0 \\ t & 0 & 0 & 0 & s \\ -t & 0 & s+t & 0 & t \\ 0 & t & 0 & s & 0 \\ 0 & -t & s+t & -s & s+t \\ 0 & -s & s+t & -t & s+t \\ 0 & s & 0 & t & 0 \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 0 & 4 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{3} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

$$T \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$NM \left( \frac{26}{3} \;\; \frac{7}{2} \;\; 7 \;\; \frac{7}{2} \;\; \frac{7}{2} \;\; 7 \;\; \frac{46}{3} \;\; \frac{7}{2} \;\; \frac{20}{3} \;\; \frac{26}{3} \;\; \frac{7}{2} \;\; \frac{7}{2} \;\; 7 \;\; 7 \;\; \frac{13}{3} \;\; \frac{10}{3} \;\; 7 \;\; 7 \;\; \frac{10}{3} \;\; \frac{13}{3} \right)$

"IS MN in Vec(K)?", false

MN

$\left( \frac{551}{66} \;\; \frac{63}{22} \;\; \frac{147}{22} \;\; \frac{63}{22} \;\; \frac{63}{22} \;\; \frac{147}{22} \;\; \frac{485}{33} \;\; \frac{63}{22} \;\; \frac{419}{66} \;\; \frac{551}{66} \;\; \frac{63}{22} \;\; \frac{63}{22} \;\; \frac{63}{11} \;\; \frac{63}{11} \;\; \frac{307}{66} \;\; \frac{241}{66} \;\; \frac{63}{11} \;\; \frac{63}{11} \right.$

$\boldsymbol{\tau} = 12/1$ , rank = 6 , ratio = 2/1 , $n^2 / r = 32/3$

$\boldsymbol{\tau}' = 52/1$ , $r' = 5/6$ , $\boldsymbol{\tau} / n^2 = 3/16$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$–check is positive? 28/9

max r = 36/5 , $r$–check is positive? 1/6

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{14}{51} T + \frac{176}{17} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 96

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {3}, {4}, {8}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$$M_C = \begin{pmatrix} \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

 

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 2., 0.6666666667, 0.4444444444]

Eigenvalues $N_C$

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 3., 1.400000000, 3.600000000]

Eigenvalues $N_C$–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace $M_C$

{[0, 1, 1, 0, 0, 1, 0, 1], [0, 0, 0, 0, 0, 0, 1, −1], [0, 0, −1, 1, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, 1, 1, 0, 1, 0, 0, 1]}

NullSpace $N_C$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues $M_0$

[0., 0., 0., 0., 2., 0.6666666667, 8.935416159, 0.397917175]

Eigenvalues $N_0$

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace $M_0$

{[0, 0, 0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 1, −1, 0, 0, 0, 0]}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[1., −1.333333333, 7.142286814, −0.808953480, −1., −2., −1., −2.]

Eigenvalues N

[−2., 6.531128874, −1.531128874, 0., 0., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Commutator(s)

1, 2 : commutator = $\begin{pmatrix} 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

==================================================

{2, 3, 6}

R: [4, 7, 5, 2, 3, 8, 4, 3]
B: [8, 3, 1, 6, 7, 4, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{3}{4096}(-56 + 4s + s^2 + 2s^3 + s^4)(2 + s)(-1 + s)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 3, "vs", 5

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[3] v[5]) (1 + v[2] v[4] v[7])
"B CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6])

Eigenvalues

R: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 8 & 0 & 0 & 8 & 0 & 8 \\ 0 & 0 & 16 & 0 & 8 & 0 & 0 & 0 \\ 8 & 16 & 0 & 16 & 0 & 0 & 8 & 0 \\ 0 & 0 & 16 & 0 & 8 & 0 & 0 & 24 \\ 0 & 8 & 0 & 8 & 0 & 0 & 8 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 8 & 0 & 8 & 16 & 0 & 16 \\ 8 & 0 & 0 & 24 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3    "Trace mark", 3, "Rank mark", 3, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{18}$ ( $v[1]v[3] + v[1]v[6] + v[1]v[8] + 2v[2]v[3] + v[2]v[5] + 2v[3]v[4] + v[3]v[7] + v[4]v[5] + 3v[4]v[8] + v[5]v[7] + 2v[6]v[7] + 2v[8]v[7]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"RG1" = {7, 8}

"RG2" = {6, 7}

"RG3" = {5, 7}

"RG4" = {3, 7}

"RG5" = {4, 8}

"RG6" = {4, 5}

"RG7" = {3, 4}

"RG8" = {2, 5}

"RG9" = {2, 3}

"RG10" = {1, 8}

"RG11" = {1, 6}

"RG12" = {1, 3}

$\pi 2$ = [0, 1, 0, 0, 1, 0, 1, 2, 0, 1, 0, 0, 0, 2, 0, 0, 1, 0, 1, 0, 0, 3, 0, 1, 0, 2, 0, 2]

supp $\pi 2$ = {2, 5, 7, 8, 10, 14, 17, 19, 22, 24, 26, 28}

$u2$ = [0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1]

supp u2 = {2, 4, 5, 7, 8, 10, 11, 13, 14, 17, 19, 20, 22, 24, 26, 28}

Action of R on ranges, [[7], [5], [7], [6], [9], [9], [8], [4], [3], [7], [5], [6]]
Action of B on ranges, [[1], [5], [1], [10], [2], [2], [11], [4], [12], [1], [5], [10]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{18} & \frac{1}{18} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{18} & \frac{1}{9} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 2]

B–BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{1, 2, 4, 7\}$
$b_2 = \{3, 5, 6, 8\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 2, 2, 2

## LIE STRUCTURE

Dimension of Lie algebra: 32, Shape: 8 ⊕ 24/22

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{2, 4, 7}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {7, 8}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} & \frac{1}{4} & 0 & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{8} & 0 & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [7, 8], [[8, 8, 7, 8, 7, 7, 8, 7], [7, 7, 8, 7, 8, 8, 7, 8]]

2, "range", [6, 7], [[7, 7, 6, 7, 6, 6, 7, 6], [6, 6, 7, 6, 7, 7, 6, 7]]

3, "range", [5, 7], [[7, 7, 5, 7, 5, 5, 7, 5], [5, 5, 7, 5, 7, 7, 5, 7]]

4, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

5, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

6, "range", [4, 5], [[5, 5, 4, 5, 4, 4, 5, 4], [4, 4, 5, 4, 5, 5, 4, 5]]

7, "range", [3, 4], [[4, 4, 3, 4, 3, 3, 4, 3], [3, 3, 4, 3, 4, 4, 3, 4]]

8, "range", [2, 5], [[5, 5, 2, 5, 2, 2, 5, 2], [2, 2, 5, 2, 5, 5, 2, 5]]

9, "range", [2, 3], [[3, 3, 2, 3, 2, 2, 3, 2], [2, 2, 3, 2, 3, 3, 2, 3]]

10, "range", [1, 8], [[8, 8, 1, 8, 1, 1, 8, 1], [1, 1, 8, 1, 8, 8, 1, 8]]

11, "range", [1, 6], [[6, 6, 1, 6, 1, 1, 6, 1], [1, 1, 6, 1, 6, 6, 1, 6]]

12, "range", [1, 3], [[3, 3, 1, 3, 1, 1, 3, 1], [1, 1, 3, 1, 3, 3, 1, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 3 & 0 & 1 & 0 & 2 & 0 & 2 \end{pmatrix}$

{2, 5, 7, 8, 10, 14, 17, 19, 22, 24, 26, 28}

$u2 =$
$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{2, 4, 5, 7, 8, 10, 11, 13, 14, 17, 19, 20, 22, 24, 26, 28}

picheck $\begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_8 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} \\ 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & \frac{8}{3} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, 1, 1, 1, -1, -1, -1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -s & t & 0 & 0 & -t & s \\ 0 & -s & 0 & s & 0 & -t & 0 & t \\ t & 0 & -s & 0 & s & 0 & -t & 0 \\ 0 & -s & t & 0 & 0 & -t & s & 0 \\ 0 & -s & 0 & s & 0 & -t & 0 & t \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & -1 & 1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} -s+t \\ -s+t \\ -t+s \\ -s+t \\ -t+s \\ -t+s \\ -s+t \\ -t+s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s \\ t & s \\ s & t \\ t & s \\ s & t \\ s & t \\ t & s \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \end{pmatrix}$$

$$Skew\ Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

 

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & \frac{8}{9} & 0 & 0 & \frac{8}{9} & 0 & \frac{8}{9} \\ 0 & \frac{8}{3} & \frac{16}{9} & 0 & \frac{8}{9} & 0 & 0 & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & \frac{16}{9} & 0 & 0 & \frac{8}{9} & 0 \\ 0 & 0 & \frac{16}{9} & \frac{16}{3} & \frac{8}{9} & 0 & 0 & \frac{8}{3} \\ 0 & \frac{8}{9} & 0 & \frac{8}{9} & \frac{8}{3} & 0 & \frac{8}{9} & 0 \\ \frac{8}{9} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{9} & 0 \\ 0 & 0 & \frac{8}{9} & 0 & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & \frac{16}{9} \\ \frac{8}{9} & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{9} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \left( \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \right)$

$T \left( \frac{1}{6} \ \frac{1}{6} \ 0 \ \frac{1}{3} \ 0 \ 0 \ 0 \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{6} \ \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left( \frac{8}{3} \ \frac{8}{3} \ 0 \ \frac{16}{3} \ 0 \ 0 \ 0 \ \frac{16}{3} \ 0 \ 0 \ \frac{16}{3} \ 0 \ \frac{8}{3} \ \frac{8}{3} \right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left( 4 \ 4 \ 0 \ 4 \ 0 \ 0 \ 0 \ 4 \ 0 \ 0 \ 4 \ 0 \ 4 \ 4 \right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 12, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 24

dim span idems 7 vs no. of idems 12

"PT1" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"RG1" = {7, 8}

"RG2" = {6, 7}

"RG3" = {5, 7}

"RG4" = {3, 7}

"RG5" = {4, 8}

"RG6" = {4, 5}

"RG7" = {3, 4}

"RG8" = {2, 5}

"RG9" = {2, 3}

"RG10" = {1, 8}

"RG11" = {1, 6}

"RG12" = {1, 3}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & 0 & \frac{-8}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} & 0 \\ \frac{-4}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ 0 & \frac{8}{9} & \frac{32}{9} & 0 & \frac{-8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & 0 & \frac{32}{9} & 0 & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} & 0 & \frac{20}{9} & \frac{-4}{9} & 0 & \frac{-8}{9} \\ \frac{4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & 0 & \frac{8}{9} & \frac{32}{9} & 0 \\ 0 & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & 0 & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & 0 & \frac{-2}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{-2}{5} & 0 \\ \frac{-1}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 0 & \frac{1}{4} & 1 & 0 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 0 & 1 & 0 & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{-1}{5} & \frac{1}{5} & \frac{-2}{5} & 0 & 1 & \frac{-1}{5} & 0 & \frac{-2}{5} \\ \frac{1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 0 & \frac{1}{4} & 1 & 0 \\ 0 & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 \\ 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 \\ 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 2.666666667, 3.555555556, 6.222222222, 6.128983570, 1.517865258, 3.019817840]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 0., 4., 2.888888889]

Eigenvalues $M_C$–scaled

[0., 0., 1.875400564, 0.6378547175, 1.086744718, 1.990960343, 0.9222603030, 1.486779355]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 0., 4.645161290, 3.354838710]

NullSpace $M_C$

{[1, 1, 0, 1, 0, 0, 1, 0], [0, 0, 1, 0, 1, 1, 0, 1]}

NullSpace $N_C$

{[0, −1, 0, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0]}

Eigenvalues $M_0$

[0., 3.555555556, 6.222222222, 9.517531656, 1.511050604, 5.860306632, 2.666666667, 2.666666667]

Eigenvalues $N_0$

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{[−1, −1, 1, −1, 1, 1, −1, 1]}

NullSpace $N_0$

{[0, −1, 0, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0]}

Eigenvalues M

[0.5286569585, 1.752280775, −1.352734336, −4.483758953, 1.352734336, 4.483758953, −0.5286569585, −1.752280775]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[1, 0, 0, −1, 0, 0, 0, 0], [0, 1, 0, −1, 0, 0, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, −1, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

==================================================

{2, 4, 5}

R: [4, 7, 1, 6, 7, 4, 4, 3]
B: [8, 3, 5, 2, 3, 8, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{512}(2+s)(1+s)(-1+s)(-7+s^2)(-3+s)$

RANK of R is 5

 

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 2

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[4] v[6]
"B CYCLES", (1 + v[8] v[7]) (1 + v[3] v[5])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 8 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 8 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{6}$ ( $v[1]v[4] + v[2]v[8] + 2v[3]v[7] + v[4]v[6] + v[5]v[8]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"RG1" = {5, 8}

"RG2" = {2, 8}

"RG3" = {3, 7}

"RG4" = {4, 6}

"RG5" = {1, 4}

$\pi 2$ = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0]

supp $\pi 2$ = {3, 13, 17, 20, 25}

$u2$ = [1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0]

supp u2 = {1, 2, 3, 4, 11, 12, 13, 16, 17, 18, 20, 21, 22, 23, 24, 25}

Action of R on ranges, [[3], [3], [5], [4], [4]]
Action of B on ranges, [[3], [3], [1], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1]

B-BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N - check: true

$b_1 = \{1, 6, 7, 8\}$
$b_2 = \{2, 3, 4, 5\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 2, 2, 2

## LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $0 \oplus 13/11$

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}}, true

$\Omega_B$ in Vec(K)? , {{7, 8}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega−checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [5, 8], [[8, 5, 5, 5, 5, 8, 8, 8], [5, 8, 8, 8, 8, 5, 5, 5]]

2, "range", [2, 8], [[8, 2, 2, 2, 2, 8, 8, 8], [2, 8, 8, 8, 8, 2, 2, 2]]

3, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

4, "range", [4, 6], [[6, 4, 4, 4, 4, 6, 6, 6], [4, 6, 6, 6, 6, 4, 4, 4]]

5, "range", [1, 4], [[4, 1, 1, 1, 1, 4, 4, 4], [1, 4, 4, 4, 4, 1, 1, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

{3, 13, 17, 20, 25}

$u2 =$
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

{1, 2, 3, 4, 11, 12, 13, 16, 17, 18, 20, 21, 22, 23, 24, 25}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 0 & 0 & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 2, -1, 1, 0, -2]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t & -t & 0 & 0 & s & -s & 0 \\ 0 & 0 & s & -s & 0 & 0 & t & -t \\ s & 0 & -t & 0 & t & 0 & -s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 & 2 & -2 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & s & -t \\ -t+s & 0 & 0 \\ 0 & -s & t \\ 0 & -s & t \\ -t+s & 0 & 0 \\ 0 & s & -t \\ 0 & s & -t \\ -s+t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & -t & 0 & s+t \\ s & 0 & -s+t & s \\ s & t & 0 & 0 \\ s & t & 0 & 0 \\ s & 0 & -s+t & s \\ t & -t & 0 & s+t \\ t & -t & 0 & s+t \\ t & 0 & -t+s & t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \end{pmatrix}$$

$$Skew\ Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & \frac{8}{3} \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ \frac{8}{3} & 0 & 0 & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \left( \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \right)$

$T \left( \frac{1}{6} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{6} \ 0 \ 0 \ 0 \ 0 \ \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left( \frac{8}{3} \ 0 \ \frac{16}{3} \ \frac{16}{3} \ \frac{8}{3} \ 0 \ 0 \ 0 \ 0 \ \frac{8}{3} \right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left( \frac{8}{3} \ 0 \ \frac{16}{3} \ 4 \ \frac{8}{3} \ 0 \ 0 \ 0 \ 0 \ \frac{8}{3} \right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 5, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 10
out of total no. of elements equal to 10

dim span idems 5 vs no. of idems 5

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"RG1" = {5, 8}

"RG2" = {2, 8}

"RG3" = {3, 7}

"RG4" = {4, 6}

"RG5" = {1, 4}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{16}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{16}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{16}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{16}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{16}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{16}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

 

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{4}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 \\ \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[9.777777778, 8., 2.666666667, 2.666666667, 0., 0., 0., 0.]

 

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 0., 4., 2.888888889]

Eigenvalues $M_C$−scaled

[2.900000000, 2.700000000, 1.200000000, 1.200000000, 0., 0., 0., 0.]

Eigenvalues $N_C$−scaled

[0., 0., 0., 0., 0., 0., 4.645161290, 3.354838710]

NullSpace $M_C$

{[0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 1, 0, 0, 1, 1], [1, 0, 0, 0, 0, 1, 1, 1], [0, 1, 0, 0, 1, 0, 0, −1]}

NullSpace $N_C$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 8., 2.666666667, 8., 2.666666667, 0., 0., 0.]

Eigenvalues $N_0$

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{[1, 0, 0, −1, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, −1, 0, 0, 1]}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

Eigenvalues M

[5.333333333, −5.333333333, 0., 0., 3.771236166, −3.771236166, 3.771236166, −3.771236166]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace N

{[−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

==============================================

{2, 5, 6}

R: [4, 7, 1, 2, 7, 8, 4, 3]
B: [8, 3, 5, 6, 3, 4, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{-15}{4096}(-8+5s)(14-7s-s^2+2s^3)(-2+s)(-1+s)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 1, "vs", 3

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[2] v[4] v[7]
"B CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6]) (1 + v[3] v[5])

Eigenvalues

R: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 1., −1., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[1, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 14 & 14 & 0 & 0 & 0 & 7 \\ 7 & 0 & 0 & 21 & 0 & 0 & 14 & 0 \\ 14 & 0 & 0 & 14 & 0 & 14 & 14 & 28 \\ 14 & 21 & 14 & 0 & 7 & 0 & 28 & 0 \\ 0 & 0 & 0 & 7 & 0 & 7 & 14 & 14 \\ 0 & 0 & 14 & 0 & 7 & 0 & 0 & 21 \\ 0 & 14 & 14 & 28 & 14 & 0 & 0 & 14 \\ 7 & 0 & 28 & 0 & 14 & 21 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{36}$ ( $v[1]v[2] + 2v[1]v[3] + 2v[1]v[4] + v[1]v[8] + 3v[2]v[4] + 2v[2]v[7] + 2v[3]v[4] + 2v[3]v[6] + 2v[3]v[7] + 4v[3]v[8] + v[4]v[5] + 4v[4]v[7] + v[5]v[6] + 2v[5]v[7] + 2v[5]v[8] + 3v[6]v[8] + 2v[8]v[7]$ )
degree 3 : $\frac{1}{12}$ ( $v[1]v[2]v[4] + v[1]v[3]v[4] + v[1]v[3]v[8] + 2v[2]v[4]v[7] + v[3]v[4]v[7] + 2v[3]v[6]v[8] + v[3]v[8]v[7] + v[4]v[5]v[7] + v[5]v[6]v[8] + v[5]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 4, 7}

"RG6" = {5, 6, 8}

"RG7" = {3, 6, 8}

"RG8" = {1, 3, 8}

"RG9" = {1, 3, 4}

"RG10" = {1, 2, 4}

$\pi 3$ = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp $\pi 3$ = {2, 7, 11, 29, 39, 45, 46, 48, 54, 55}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[4], [5], [9], [10], [5], [3], [8], [9], [10], [5]]
Action of B on ranges, [[3], [7], [1], [6], [7], [4], [2], [1], [6], [7]]

$$\boldsymbol{\beta} = \left(\frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12}\right)$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {4, 8}
$b_2$ = {1, 6, 7}
$b_3$ = {2, 3, 5}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 30, Shape: 6 ⊕ 24/20

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{2, 4, 7}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {7, 8}, {3, 5}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{3}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{4, 8}, {1, 6, 7}, {2, 3, 5}}

1, "range", [5, 7, 8], [[8, 7, 7, 5, 7, 8, 8, 5], [8, 5, 5, 7, 5, 8, 8, 7], [7, 8, 8, 5, 8, 7, 7, 5], [7, 5, 5, 8, 5, 7, 7, 8], [5, 8, 8, 7, 8, 5, 5, 7], [5, 7, 7, 8, 7, 5, 5, 8]]

2, "range", [4, 5, 7], [[7, 5, 5, 4, 5, 7, 7, 4], [7, 4, 4, 5, 4, 7, 7, 5], [5, 7, 7, 4, 7, 5, 5, 4], [5, 4, 4, 7, 4, 5, 5, 7], [4, 7, 7, 5, 7, 4, 4, 5], [4, 5, 5, 7, 5, 4, 4, 7]]

3, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

4, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

5, "range", [2, 4, 7], [[7, 4, 4, 2, 4, 7, 7, 2], [7, 2, 2, 4, 2, 7, 7, 4], [4, 7, 7, 2, 7, 4, 4, 2], [4, 2, 2, 7, 2, 4, 4, 7], [2, 7, 7, 4, 7, 2, 2, 4], [2, 4, 4, 7, 4, 2, 2, 7]]

6, "range", [5, 6, 8], [[8, 6, 6, 5, 6, 8, 8, 5], [8, 5, 5, 6, 5, 8, 8, 6], [6, 8, 8, 5, 8, 6, 6, 5], [6, 5, 5, 8, 5, 6, 6, 8], [5, 8, 8, 6, 8, 5, 5, 6], [5, 6, 6, 8, 6, 5, 5, 8]]

7, "range", [3, 6, 8], [[8, 6, 6, 3, 6, 8, 8, 3], [8, 3, 3, 6, 3, 8, 8, 6], [6, 8, 8, 3, 8, 6, 6, 3], [6, 3, 3, 8, 3, 6, 6, 8], [3, 8, 8, 6, 8, 3, 3, 6], [3, 6, 6, 8, 6, 3, 3, 8]]

8, "range", [1, 3, 8], [[8, 3, 3, 1, 3, 8, 8, 1], [8, 1, 1, 3, 1, 8, 8, 3], [3, 8, 8, 1, 8, 3, 3, 1], [3, 1, 1, 8, 1, 3, 3, 8], [1, 8, 8, 3, 8, 1, 1, 3], [1, 3, 3, 8, 3, 1, 1, 8]]

9, "range", [1, 3, 4], [[4, 3, 3, 1, 3, 4, 4, 1], [4, 1, 1, 3, 1, 4, 4, 3], [3, 4, 4, 1, 4, 3, 3, 1], [3, 1, 1, 4, 1, 3, 3, 4], [1, 4, 4, 3, 4, 1, 1, 3], [1, 3, 3, 4, 3, 1, 1, 4]]

10, "range", [1, 2, 4], [[4, 2, 2, 1, 2, 4, 4, 1], [4, 1, 1, 2, 1, 4, 4, 2], [2, 4, 4, 1, 4, 2, 2, 1], [2, 1, 1, 4, 1, 2, 2, 4], [1, 4, 4, 2, 4, 1, 1, 2], [1, 2, 2, 4, 2, 1, 1, 4]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12$

 

$t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$\left(0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2\right.$

{2, 7, 11, 29, 39, 45, 46, 48, 54, 55}

$u3 =$
$\left(0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1\right.$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $\left(3 \ 3 \ 6 \ 6 \ 3 \ 3 \ 6 \ 6\right)$

$\pi = \left(\frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6}\right)$

$\pi 2 =$
$\left(1 \ 2 \ 2 \ 0 \ 0 \ 0 \ 1 \ 0 \ 3 \ 0 \ 0 \ 2 \ 0 \ 2 \ 0 \ 2 \ 2 \ 4 \ 1 \ 0 \ 4 \ 0 \ 1 \ 2 \ 2 \ 0 \ 3 \ 2\right)$

$u2 =$
$\left(\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3}\right.$

picheck $\left(6 \ 6 \ 12 \ 12 \ 6 \ 6 \ 12 \ 12\right)$

$\pi 1 = \left(6 \ 6 \ 12 \ 12 \ 6 \ 6 \ 12 \ 12\right)$

$$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$$

picheck $\begin{pmatrix} 6 & 6 & 12 & 12 & 6 & 6 & 12 & 12 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 0, 1, -1, -1, 0, -1]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & -t & 0 & t & 0 & -s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & s & 0 & 0 & -t & t & 0 \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & -1 & 0 & -1 \end{pmatrix}$

M0 is invertible. det= 31175/5832

 

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = (8)$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\1&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\1&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{12} & \frac{7}{6} & \frac{7}{6} & 0 & 0 & 0 & \frac{7}{12} \\ \frac{7}{12} & \frac{11}{6} & 0 & \frac{7}{4} & 0 & 0 & \frac{7}{6} & 0 \\ \frac{7}{6} & 0 & \frac{11}{3} & \frac{7}{6} & 0 & \frac{7}{6} & \frac{7}{6} & \frac{7}{3} \\ \frac{7}{6} & \frac{7}{4} & \frac{7}{6} & \frac{11}{3} & \frac{7}{12} & 0 & \frac{7}{3} & 0 \\ 0 & 0 & 0 & \frac{7}{12} & \frac{11}{6} & \frac{7}{12} & \frac{7}{6} & \frac{7}{6} \\ 0 & 0 & \frac{7}{6} & 0 & \frac{7}{12} & \frac{11}{6} & 0 & \frac{7}{4} \\ 0 & \frac{7}{6} & \frac{7}{6} & \frac{7}{3} & \frac{7}{6} & 0 & \frac{11}{3} & \frac{7}{6} \\ \frac{7}{12} & 0 & \frac{7}{3} & 0 & \frac{7}{6} & \frac{7}{4} & \frac{7}{6} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{5}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{2}\ 0\ 0\ 0\ \frac{3}{4}\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ 14\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{1120}{87}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\right.$$

$$\boldsymbol{\tau} = 22/1 \text{ , rank} = 3 \text{ , ratio} = 22/3 \text{ , } n^2 / r = 64/3$$

$$\boldsymbol{\tau}' = 42/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 11/32$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 118/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 7/12$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 10, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 60

dim span idems 6 vs no. of idems 10

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 4, 7}

"RG6" = {5, 6, 8}

"RG7" = {3, 6, 8}

"RG8" = {1, 3, 8}

"RG9" = {1, 3, 4}

"RG10" = {1, 2, 4}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{5}{36} & \frac{5}{18} & \frac{5}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-11}{36} \\ \frac{5}{36} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{5}{18} & \frac{-8}{9} \\ \frac{5}{18} & \frac{-8}{9} & \frac{17}{9} & \frac{-11}{18} & \frac{-8}{9} & \frac{5}{18} & \frac{-11}{18} & \frac{5}{9} \\ \frac{5}{18} & \frac{31}{36} & \frac{-11}{18} & \frac{17}{9} & \frac{-11}{36} & \frac{-8}{9} & \frac{5}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-11}{36} & \frac{25}{18} & \frac{5}{36} & \frac{5}{18} & \frac{5}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{5}{18} & \frac{-8}{9} & \frac{5}{36} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} \\ \frac{-8}{9} & \frac{5}{18} & \frac{-11}{18} & \frac{5}{9} & \frac{5}{18} & \frac{-8}{9} & \frac{17}{9} & \frac{-11}{18} \\ \frac{-11}{36} & \frac{-8}{9} & \frac{5}{9} & \frac{-16}{9} & \frac{5}{18} & \frac{31}{36} & \frac{-11}{18} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{5} & \frac{1}{5} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-11}{50} \\ \frac{1}{10} & 1 & \frac{-16}{25} & \frac{31}{50} & \frac{-8}{25} & \frac{-8}{25} & \frac{1}{5} & \frac{-16}{25} \\ \frac{5}{34} & \frac{-8}{17} & 1 & \frac{-11}{34} & \frac{-8}{17} & \frac{5}{34} & \frac{-11}{34} & \frac{5}{17} \\ \frac{5}{34} & \frac{31}{68} & \frac{-11}{34} & 1 & \frac{-11}{68} & \frac{-8}{17} & \frac{5}{17} & \frac{-16}{17} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-11}{50} & 1 & \frac{1}{10} & \frac{1}{5} & \frac{1}{5} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{1}{5} & \frac{-16}{25} & \frac{1}{10} & 1 & \frac{-16}{25} & \frac{31}{50} \\ \frac{-8}{17} & \frac{5}{34} & \frac{-11}{34} & \frac{5}{17} & \frac{5}{34} & \frac{-8}{17} & 1 & \frac{-11}{34} \\ \frac{-11}{68} & \frac{-8}{17} & \frac{5}{17} & \frac{-16}{17} & \frac{5}{34} & \frac{31}{68} & \frac{-11}{34} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_cM_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_cN_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 1.250000000, 1.880025657, 0.1477521207, 0.1105329052, 0.9893584859, 3.153895214, 5.579546726]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$−scaled

[0., 0.9000000000, 1.112732969, 0.0825611488, 0.07130601633, 0.6087502500, 1.993559700, 3.231089916]

Eigenvalues $N_C$−scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0]}

Eigenvalues $M_0$

[1.250000000, 9.046144669, 0.1421285585, 1.728393442, 0.1105329052, 0.9893584859, 3.153895214, 5.579546726]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1]}

Eigenvalues M

[−1.750000000, −0.5833333333, 2.529337770, −2.655317988, 0.7093135502, 5.740384102, −3.216306512, −0.774077592]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

40, [1, −1, 1, 1, −1, 1, −1, 1]

==========================================================

{2, 5, 7}

R: [4, 7, 1, 2, 7, 4, 8, 3]
B: [8, 3, 5, 6, 3, 8, 4, 7]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{3}{256} (21 + 32s + 27s^2 + 13s^3 + 3s^4)(-1 + s)^2$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[8] v[7]
"B CYCLES", (1 + v[3] v[5]) (1 + v[4] v[6] v[8] v[7])

Eigenvalues

R: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

B: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 21 & 21 & 21 & 0 & 0 & 20 & 21 \\ 21 & 0 & 20 & 21 & 0 & 0 & 21 & 21 \\ 21 & 20 & 0 & 42 & 20 & 21 & 42 & 42 \\ 21 & 21 & 42 & 0 & 21 & 21 & 42 & 40 \\ 0 & 0 & 20 & 21 & 0 & 21 & 21 & 21 \\ 0 & 0 & 21 & 21 & 21 & 0 & 20 & 21 \\ 20 & 21 & 42 & 42 & 21 & 20 & 0 & 42 \\ 21 & 21 & 42 & 40 & 21 & 21 & 42 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$

degree 2: $\frac{1}{24}$ ( $v[1]v[2] + v[1]v[3] + v[1]v[4] + 4v[1]v[7] + v[1]v[8] + 4v[2]v[3] + v[2]v[4] + v[2]v[7] + v[2]v[8] + 2v[3]v[4] + 4v[3]v[5] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + v[4]v[5] + v[4]v[6] + 2v[4]v[7] + 8v[4]v[8] + v[5]v[6] + v[5]v[7] + v[5]v[8] + 4v[6]v[7] + v[6]v[8] + 2v[8]v[7]$ )

degree 3 : $\frac{1}{24}$ ( $2v[1]v[4]v[7] + 2v[3]v[5]v[7] + 2v[4]v[6]v[7] + 2v[3]v[6]v[7] + 2v[2]v[4]v[8] + 3v[5]v[8]v[7] + 2v[1]v[4]v[8] + 2v[6]v[8]v[7] + 4v[3]v[4]v[8] + 3v[1]v[2]v[4] + 3v[2]v[4]v[7] + 3v[1]v[3]v[4] + 3v[1]v[3]v[8] + 2v[1]v[8]v[7] + 2v[2]v[3]v[4] + 2v[3]v[4]v[5] + 3v[5]v[6]v[8] + 3v[1]v[2]v[8] + 6v[3]v[8]v[7] + 3v[3]v[4]v[6] + 2v[4]v[6]v[8] + 2v[3]v[5]v[8] + 2v[1]v[3]v[7] + 2v[2]v[3]v[7] + 2v[5]v[6]v[7] + 2v[4]v[5]v[8] + 3v[2]v[8]v[7] + 2v[2]v[3]v[8] + 6v[3]v[4]v[7] + 2v[3]v[5]v[6] + 3v[3]v[6]v[8] + 3v[4]v[5]v[7] + 4v[4]v[8]v[7] + 2v[1]v[2]v[7] + 2v[1]v[2]v[3] + 3v[4]v[5]v[6]$ )

degree 4 : $\frac{1}{24}$ ( $v[3]v[6]v[8]v[7] + v[3]v[4]v[6]v[7] + v[1]v[3]v[4]v[7] + 2v[3]v[4]v[8]v[7] + v[1]v[3]v[8]v[7] + v[1]v[2]v[3]v[4] + 4v[4]v[6]v[8]v[7] + 4v[3]v[4]v[5]v[8] + v[5]v[6]v[8]v[7] + v[3]v[4]v[5]v[6] + 4v[2]v[3]v[4]v[8] + v[1]v[2]v[4]v[7] + v[1]v[2]v[8]v[7] + v[3]v[4]v[5]v[7] + v[2]v[3]v[8]v[7] + 4v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[8] + v[2]v[4]v[8]v[7] + 4v[1]v[4]v[8]v[7] + 4v[3]v[5]v[6]v[7] + v[2]v[3]v[4]v[7] + v[4]v[5]v[6]v[7] + v[4]v[5]v[8]v[7] + v[1]v[2]v[4]v[8] + v[4]v[5]v[6]v[8] + v[1]v[3]v[4]v[8] + v[3]v[5]v[8]v[7] + v[3]v[5]v[6]v[8] + v[3]v[4]v[6]v[8]$ )

degree 5 : $\frac{1}{12}$ ( $v[1]v[2]v[3]v[4]v[7] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[3]v[8]v[7] + v[1]v[2]v[4]v[8]v[7] + v[1]v[3]v[4]v[8]v[7] + v[2]v[3]v[4]v[8]v[7] + v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[8] + v[3]v[4]v[5]v[8]v[7] + v[3]v[4]v[6]v[8]v[7] + v[3]v[5]v[6]v[8]v[7] + v[4]v[5]v[6]v[8]v[7]$ )

degree 6 : $\frac{1}{2}$ ( $v[4]$ ) ( $v[7]$ ) ( $v[1]v[2] + v[5]v[6]$ ) ( $v[3]$ ) ( $v[8]$ )

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

## KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$\pi6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

supp $\pi6$ = {6, 28}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1] supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[4, 5, 6, 3, 1, 2]

B−BLOCKS,

[5, 4, 1, 2, 6, 3]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1$ = {1, 6}
$b_2$ = {2, 5}
$b_3$ = {8}
$b_4$ = {3}
$b_5$ = {4}
$b_6$ = {7}

dim(span of partition vectors), rank($N_0$), rank($N$): 6, 6, 6

## LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: 11 ⊕ 13/11

$$\text{CLB} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$\Omega_B$ in Vec(K)? , {{4, 6, 7, 8}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{vs} \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{5}{32} & \frac{5}{32} \end{pmatrix} \text{vs} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

1, "range", [3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 7, 8, 4, 3], [8, 7, 6, 3, 7, 8, 4, 5], [8, 6, 7, 5, 6, 8, 4, 3], [8, 6, 7, 3, 6, 8, 4, 5], [8, 5, 3, 7, 5, 8, 4, 6], [8, 5, 3, 6, 5, 8, 4, 7], [8, 3, 5, 7, 3, 8, 4, 6], [8, 3, 5, 6, 3, 8, 4, 7], [7, 8, 4, 5, 8, 7, 6, 3], [7, 8, 4, 3, 8, 7, 6, 5], [7, 5, 3, 8, 5, 7, 6, 4], [7, 5, 3, 4, 5, 7, 6, 8], [7, 4, 8, 5, 4, 7, 6, 3], [7, 4, 8, 3, 4, 7, 6, 5], [7, 3, 5, 8, 3, 7, 6, 4], [7, 3, 5, 4, 3, 7, 6, 8], [6, 8, 4, 5, 8, 6, 7, 3], [6, 8, 4, 3, 8, 6, 7, 5], [6, 5, 3, 8, 5, 6, 7, 4], [6, 5, 3, 4, 5, 6, 7, 8], [6, 4, 8, 5, 4, 6, 7, 3], [6, 4, 8, 3, 4, 6, 7, 5], [6, 3, 5, 8, 3, 6, 7, 4], [6, 3, 5, 4, 3, 6, 7, 8], [5, 8, 4, 7, 8, 5, 3, 6], [5, 8, 4, 6, 8, 5, 3, 7], [5, 7, 6, 8, 7, 5, 3, 4], [5, 7, 6, 4, 7, 5, 3, 8], [5, 6, 7, 8, 6, 5, 3, 4], [5, 6, 7, 4, 6, 5, 3, 8], [5, 4, 8, 7, 4, 5, 3, 6], [5, 4, 8, 6, 4, 5, 3, 7], [4, 7, 6, 5, 7, 4, 8, 3], [4, 7, 6, 3, 7, 4, 8, 5], [4, 6, 7, 5, 6, 4, 8, 3], [4, 6, 7, 3, 6, 4, 8, 5], [4, 5, 3, 7, 5, 4, 8, 6], [4, 5, 3, 6, 5, 4, 8, 7], [4, 3, 5, 7, 3, 4, 8, 6], [4, 3, 5, 6, 3, 4, 8, 7], [3, 8, 4, 7, 8, 3, 5, 6], [3, 8, 4, 6, 8, 3, 5, 7], [3, 7, 6, 8, 7, 3, 5, 4], [3, 7, 6, 4, 7, 3, 5, 8], [3, 6, 7, 8, 6, 3, 5, 4], [3, 6, 7, 4, 6, 3, 5, 8], [3, 4, 8, 7, 4, 3, 5, 6], [3, 4, 8, 6, 4, 3, 5, 7] ]

2, "range", [1, 2, 3, 4, 7, 8], [[8, 7, 1, 3, 7, 8, 4, 2], [8, 7, 1, 2, 7, 8, 4, 3], [8, 3, 2, 7, 3, 8, 4, 1], [8, 3, 2, 1, 3, 8, 4, 7], [8, 2, 3, 7, 2, 8, 4, 1], [8, 2, 3, 1, 2, 8, 4, 7], [8, 1, 7, 3, 1, 8, 4, 2], [8, 1, 7, 2, 1, 8, 4, 3], [7, 8, 4, 3, 8, 7, 1, 2], [7, 8, 4, 2, 8, 7, 1, 3], [7, 4, 8, 3, 4, 7, 1, 2], [7, 4, 8, 2, 4, 7, 1, 3], [7, 3, 2, 8, 3, 7, 1, 4], [7, 3, 2, 4, 3, 7, 1, 8], [7, 2, 3, 8, 2, 7, 1, 4], [7, 2, 3, 4, 2, 7, 1, 8], [4, 7, 1, 3, 7, 4, 8, 2], [4, 7, 1, 2, 7, 4, 8, 3], [4, 3, 2, 7, 3, 4, 8, 1], [4, 3, 2, 1, 3, 4, 8, 7], [4, 2, 3, 7, 2, 4, 8, 1], [4, 2, 3, 1, 2, 4, 8, 7], [4, 1, 7, 3, 1, 4, 8, 2], [4, 1, 7, 2, 1, 4, 8, 3], [3, 8, 4, 7, 8, 3, 2, 1], [3, 8, 4, 1, 8, 3, 2, 7], [3, 7, 1, 8, 7, 3, 2, 4], [3, 7, 1, 4, 7, 3, 2, 8], [3, 4, 8, 7, 4, 3, 2, 1], [3, 4, 8, 1, 4, 3, 2, 7], [3, 1, 7, 8, 1, 3, 2, 4], [3, 1, 7, 4, 1, 3, 2, 8], [2, 8, 4, 7, 8, 2, 3, 1], [2, 8, 4, 1, 8, 2, 3, 7], [2, 7, 1, 8, 7, 2, 3, 4], [2, 7, 1, 4, 7, 2, 3, 8], [2, 4, 8, 7, 4, 2, 3, 1], [2, 4, 8, 1, 4, 2, 3, 7], [2, 1, 7, 8, 1, 2, 3, 4], [2, 1, 7, 4, 1, 2, 3, 8], [1, 8, 4, 3, 8, 1, 7, 2], [1, 8, 4, 2, 8, 1, 7, 3], [1, 4, 8, 3, 4, 1, 7, 2], [1, 4, 8, 2, 4, 1, 7, 3], [1, 3, 2, 8, 3, 1, 7, 4], [1, 3, 2, 4, 3, 1, 7, 8], [1, 2, 3, 8, 2, 1, 7, 4], [1, 2, 3, 4, 2, 1, 7, 8] ]

"group has", 48, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 4, 6], [2, 3, 5]]

$g_2 =$ [[1, 4, 6, 3, 5, 2]]

$g_3 =$ [[1, 5, 2, 3, 4, 6]]

$g_4 =$ [[1, 5, 2], [3, 4, 6]]

$g_5 =$ [[2, 5], [4, 6]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true

$\big(-2h[2] \quad 2h[2] \quad 2h[2] \quad -8h[3]-2h[2] \quad 2h[2] \quad 8h[3] \quad -8h[1]-2h[2] \quad 8h[3] \quad 8h[1] \quad 2$

"Basis for Z(G)"

1, "coeff", 8

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

3, "coeff", 8

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 4. & -2. & -2. & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8$

$+84t^9 + 122t^{10}$

n−choose−rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 6 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

{6, 28}

$u6 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

{6, 18, 25, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 =$
$( 0 \ \ 0 \ \ 1 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 0 \ \ 0$

$u5 =$
$\Big( 0 \ \ 0 \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{1}{6} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{1}{6} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ 0 \ \ 0 \ \ \frac{1}{6} \ \ 0 \ \ 0 \ \ \frac{1}{6}$

picheck $\begin{pmatrix} 5 & 5 & 10 & 10 & 5 & 5 & 10 & 10 \end{pmatrix}$

$\pi 4 =$
$( 2 \ \ 0 \ \ 0 \ \ 2 \ \ 2 \ \ 0 \ \ 0 \ \ 2 \ \ 2 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 2 \ \ 0 \ \ 0 \ \ 2 \ \ 2 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 2 \ \ 0 \ \ 0 \ \ 0 \ \ 0$

$u4 =$

$\left(\frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\right.$

picheck $(20\ \ 20\ \ 40\ \ 40\ \ 20\ \ 20\ \ 40\ \ 40)$

$\pi 3 =$
$(6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6$

$u3 =$
$\left(\frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\right.$

picheck $(60\ \ 60\ \ 120\ \ 120\ \ 60\ \ 60\ \ 120\ \ 120)$

$\pi 2 =$
$(24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 48\ \ 24\ \ 24\ \ 48\ \ 48\ \ 24\ \ 24\ \ 4$

$u2 =$
$\left(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\right.$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$\pi 1 = (120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} & \frac{83}{24} & \frac{13}{3} & 7 & \frac{83}{12} \\ \frac{83}{24} & \frac{13}{3} & 7 & \frac{83}{12} & \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} \\ \frac{83}{24} & \frac{7}{2} & \frac{26}{3} & \frac{83}{12} & \frac{7}{2} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} \\ \frac{83}{24} & \frac{83}{24} & \frac{83}{12} & \frac{26}{3} & \frac{83}{24} & \frac{83}{24} & \frac{83}{12} & 7 \\ \frac{83}{24} & \frac{13}{3} & 7 & \frac{83}{12} & \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} \\ \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} & \frac{83}{24} & \frac{13}{3} & 7 & \frac{83}{12} \\ \frac{7}{2} & \frac{83}{24} & \frac{83}{12} & \frac{83}{12} & \frac{83}{24} & \frac{7}{2} & \frac{26}{3} & \frac{83}{12} \\ \frac{83}{24} & \frac{83}{24} & \frac{83}{12} & 7 & \frac{83}{24} & \frac{83}{24} & \frac{83}{12} & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, 1, 0, 0, −1, −1, 0, 0]

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 \end{pmatrix}$

M0 is invertible. det= 5/7776

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{5}{6} & \frac{7}{8} \\ 0 & 0 & \frac{5}{6} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ \frac{-7}{8} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-7}{8} & 0 & 0 \\ \frac{-7}{8} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-7}{8} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ 0 & 0 & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{5}{6} & \frac{7}{8} \\ \frac{-5}{6} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-5}{6} & 0 & 0 \\ \frac{-7}{8} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-7}{8} & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{5}{6} & \frac{7}{8} \\ \frac{7}{8} & 1 & \frac{5}{6} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{5}{6} & 2 & \frac{7}{4} & \frac{5}{6} & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} \\ \frac{7}{8} & \frac{7}{8} & \frac{7}{4} & 2 & \frac{7}{8} & \frac{7}{8} & \frac{7}{4} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{7}{8} & 1 & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} \\ 0 & 0 & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & 1 & \frac{5}{6} & \frac{7}{8} \\ \frac{5}{6} & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} & \frac{7}{8} & \frac{5}{6} & 2 & \frac{7}{4} \\ \frac{7}{8} & \frac{7}{8} & \frac{7}{4} & \frac{5}{3} & \frac{7}{8} & \frac{7}{8} & \frac{7}{4} & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$\Omega$

$$\left(0 \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{2} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6}\right.$$

$$T \left(\frac{-1}{2} \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{2}\right)$$

"IS NM in Vec(K)?", true

NM

$$\left(\frac{-11}{12}\ \frac{83}{24}\ \frac{7}{2}\ \frac{83}{24}\ \frac{26}{3}\ \frac{83}{12}\ \frac{83}{24}\ \frac{83}{24}\ \frac{7}{2}\ \frac{83}{24}\ \frac{271}{12}\ \frac{83}{24}\ \frac{13}{3}\ \frac{83}{12}\ 7\ \frac{13}{3}\ \frac{83}{24}\ \frac{83}{12}\ 7\ \frac{13}{3}\ \frac{8}{2}\right.$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{-54209}{45840}\ \frac{10793}{3820}\ \frac{32761}{11460}\ \frac{6971}{2292}\ \frac{6397}{764}\ \frac{6971}{1146}\ \frac{6971}{2292}\ \frac{6971}{2292}\ \frac{32761}{11460}\ \frac{10793}{3820}\ \frac{4042}{191}\ \frac{1}{3}\right.$$

$$\tau = 12/1 \text{ , rank} = 6 \text{ , ratio} = 2/1 \text{ , } n^2 / r = 32/3$$

$$\tau' = 52/1 \text{ , } r' = 5/6 \text{ , } \tau / n^2 = 3/16$$

$$p^2 = 5/36 \text{ , min } \tau = 80/9 \text{ , } \tau\text{-check is positive? } 28/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 1/6$$

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{109}{408} T + \frac{707}{68} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 25
out of total no. of elements equal to 96

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$$M_c = \begin{pmatrix} \frac{5}{9} & \frac{31}{72} & \frac{-1}{72} & \frac{-1}{72} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{72} \\ \frac{31}{72} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{72} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-1}{72} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{72} & \frac{5}{9} & \frac{31}{72} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{31}{72} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{72} \\ \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{40} & \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{40} \\ \frac{31}{40} & 1 & \frac{-1}{10} & \frac{-1}{40} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} \\ \frac{-1}{16} & \frac{-1}{4} & 1 & \frac{-1}{8} & \frac{-1}{4} & \frac{-1}{16} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{8} & 1 & \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{8} & \frac{-1}{2} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{40} & 1 & \frac{31}{40} & \frac{-1}{40} & \frac{-1}{40} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} & \frac{31}{40} & 1 & \frac{-1}{10} & \frac{-1}{40} \\ \frac{-1}{4} & \frac{-1}{16} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{16} & \frac{-1}{4} & 1 & \frac{-1}{8} \\ \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{8} & \frac{-1}{2} & \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{8} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{72} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-1}{36} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{72} \\ \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{72} & \frac{1}{72} & 0 & 0 & \frac{1}{18} & \frac{1}{72} \\ 0 & 0 & \frac{1}{18} & \frac{1}{72} & 0 & 0 & \frac{1}{72} & \frac{1}{72} \\ \frac{-1}{72} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{72} & 0 & 0 \\ \frac{-1}{72} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{72} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{72} & 0 & 0 & \frac{1}{72} & \frac{1}{72} \\ 0 & 0 & \frac{1}{72} & \frac{1}{72} & 0 & 0 & \frac{1}{18} & \frac{1}{72} \\ \frac{-1}{18} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{72} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{72} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.3333333333, 0.1250000000, 1.875000000, 0.2733980339, 0.1016019661, 0.2607222482, 0.1420555296]

Eigenvalues $N_C$

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues $M_C$–scaled

[0., 0.5000000000, 1.500000000, 3.375000000, 0.2250000000, 1.050000000, 1.155234318, 0.1947656822]

Eigenvalues $N_C$–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues $M_0$

[0.3333333333, 0.1250000000, 1.875000000, 0.2733980339, 0.1016019661, 8.908764470, 0.1331882736, 0.2497139224]

Eigenvalues $N_0$

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[-1, 0, 0, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[-1.666666667, -0.8750000000, 0.8750000000, -0.8710495813, -1.753950419, 7.122690612, -1.795935464, -1.035088480]

Eigenvalues N

[-2., 6.531128874, -1.531128874, 0., 0., -1., -1., -1.]

NullSpace M

{}

NullSpace N

{[-1, 0, 0, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

=============================================

{3, 4, 7}

R: [4, 3, 5, 6, 3, 4, 8, 3]
B: [8, 7, 1, 2, 7, 8, 4, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det $= \frac{-1}{1024} (84 + 12s - s^2 + s^4)(-1 + s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5])
"B CYCLES", 1 + v[2] v[4] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 0 & 0 & 0 & 0 & 14 & 7 \\ 7 & 0 & 0 & 7 & 0 & 0 & 14 & 0 \\ 0 & 0 & 0 & 14 & 14 & 14 & 0 & 14 \\ 0 & 7 & 14 & 0 & 7 & 0 & 14 & 14 \\ 0 & 0 & 14 & 7 & 0 & 7 & 0 & 0 \\ 0 & 0 & 14 & 0 & 7 & 0 & 0 & 7 \\ 14 & 14 & 0 & 14 & 0 & 0 & 0 & 14 \\ 7 & 0 & 14 & 14 & 0 & 7 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{24}$ ( $v[1]v[2] + 2v[1]v[7] + v[1]v[8] + v[2]v[4] + 2v[2]v[7] + 2v[3]v[4] + 2 v[3]v[5] + 2v[3]v[6] + 2v[3]v[8] + v[4]v[5] + 2v[4]v[7] + 2v[4]v[8] + v[5]v[6] + v[6] v[8] + 2v[8]v[7]$ )
degree 3 : $\frac{1}{8}$ ( $v[1]v[2]v[7] + v[1]v[8]v[7] + v[2]v[4]v[7] + v[3]v[4]v[5] + v[3]v[4]v[8] + v[3]v[5]v[6] + v[3]v[6]v[8] + v[4]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {3, 6, 8}

"RG2" = {3, 5, 6}

"RG3" = {4, 7, 8}

"RG4" = {3, 4, 8}

"RG5" = {3, 4, 5}

"RG6" = {2, 4, 7}

"RG7" = {1, 7, 8}

"RG8" = {1, 2, 7}

$\pi3$ = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0]

supp $\pi3$ = {5, 21, 29, 37, 40, 41, 45, 52}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[5], [5], [1], [2], [2], [1], [4], [4]] Action of B on ranges, [[7], [7], [6], [8], [8], [6], [3], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 3, 2]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 4, 6\}$
$b_2 = \{2, 5, 8\}$
$b_3 = \{3, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$):    3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 23, Shape: 3 ⊕ 20/18

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{2, 4, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{5}{16} & \frac{3}{16} & \frac{5}{16} & \frac{3}{16} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [3, 6, 8], [[8, 6, 3, 8, 6, 8, 3, 6], [8, 3, 6, 8, 3, 8, 6, 3], [6, 8, 3, 6, 8, 6, 3, 8], [6, 3, 8, 6, 3, 6, 8, 3], [3, 8, 6, 3, 8, 3, 6, 8], [3, 6, 8, 3, 6, 3, 8, 6]]

2, "range", [3, 5, 6], [[6, 5, 3, 6, 5, 6, 3, 5], [6, 3, 5, 6, 3, 6, 5, 3], [5, 6, 3, 5, 6, 5, 3, 6], [5, 3, 6, 5, 3, 5, 6, 3], [3, 6, 5, 3, 6, 3, 5, 6], [3, 5, 6, 3, 5, 3, 6, 5]]

3, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

4, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

5, "range", [3, 4, 5], [[5, 4, 3, 5, 4, 5, 3, 4], [5, 3, 4, 5, 3, 5, 4, 3], [4, 5, 3, 4, 5, 4, 3, 5], [4, 3, 5, 4, 3, 4, 5, 3], [3, 5, 4, 3, 5, 3, 4, 5], [3, 4, 5, 3, 4, 3, 5, 4]]

6, "range", [2, 4, 7], [[7, 4, 2, 7, 4, 7, 2, 4], [7, 2, 4, 7, 2, 7, 4, 2], [4, 7, 2, 4, 7, 4, 2, 7], [4, 2, 7, 4, 2, 4, 7, 2], [2, 7, 4, 2, 7, 2, 4, 7], [2, 4, 7, 2, 4, 2, 7, 4]]

7, "range", [1, 7, 8], [[8, 7, 1, 8, 7, 8, 1, 7], [8, 1, 7, 8, 1, 8, 7, 1], [7, 8, 1, 7, 8, 7, 1, 8], [7, 1, 8, 7, 1, 7, 8, 1], [1, 8, 7, 1, 8, 1, 7, 8], [1, 7, 8, 1, 7, 1, 8, 7]]

8, "range", [1, 2, 7], [[7, 2, 1, 7, 2, 7, 1, 2], [7, 1, 2, 7, 1, 7, 2, 1], [2, 7, 1, 2, 7, 2, 1, 7], [2, 1, 7, 2, 1, 2, 7, 1], [1, 7, 2, 1, 7, 1, 2, 7], [1, 2, 7, 1, 2, 1, 7, 2]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[2, 3]]

$g_2 =$ [[1, 2, 3]]

$g_3 =$ []

$g_4 =$ [[1, 3]]

$g_5 =$ [[1, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 2h[1]-h[2] & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & 1 & 1 & 0 \\ 1 & 0 & 3 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1+t+t^2+t^3$

Molien Series to order 10: $1+t+2t^2+3t^3+4t^4+5t^5+7t^6+8t^7+10t^8+12t^9+14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{5, 21, 29, 37, 40, 41, 45, 52}

$u3 =$
$(1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

picheck $\begin{pmatrix}2 & 2 & 4 & 4 & 2 & 2 & 4 & 4\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 2 =$
$\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2\end{pmatrix}$

$u2 =$
$\left(\frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\right.$

picheck $\begin{pmatrix}4 & 4 & 8 & 8 & 4 & 4 & 8 & 8\end{pmatrix}$

$\pi 1 = \begin{pmatrix}4 & 4 & 8 & 8 & 4 & 4 & 8 & 8\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9}\end{pmatrix}$

picheck $\begin{pmatrix}4 & 4 & 8 & 8 & 4 & 4 & 8 & 8\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0\end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix}0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&1&0&0&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 2, 0, 1, 1, -2, 0]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t & 0 & -s & 0 & s & 0 & -t \\ -t & 0 & 0 & t & -s & 0 & 0 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & -2 & 1 & 1 \end{pmatrix}$

M0 is invertible. det= 381271/104976

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 0&0&1&0&0&0&1&0\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&0&1&0&0&0&1&0\\ 0&1&0&0&1&0&0&1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&1&0&0&1\\ 1&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&1&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 0&0&1&0&0&0&1&0\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&0&1&0&0&0&1&0\\ 0&1&0&0&1&0&0&1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&1&0&0\\ 0&1&0&0&1&0&0&1\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&1&0&0&1\\ 1&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&1&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{8} & 0 & 0 & 0 & 0 & \frac{7}{4} & \frac{7}{8} \\ \frac{7}{8} & \frac{11}{6} & 0 & \frac{7}{8} & 0 & 0 & \frac{7}{4} & 0 \\ 0 & 0 & \frac{11}{3} & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & 0 & \frac{7}{4} \\ 0 & \frac{7}{8} & \frac{7}{4} & \frac{11}{3} & \frac{7}{8} & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{8} & \frac{11}{6} & \frac{7}{8} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & 0 & \frac{7}{8} & \frac{11}{6} & 0 & \frac{7}{8} \\ \frac{7}{4} & \frac{7}{4} & 0 & \frac{7}{4} & 0 & 0 & \frac{11}{3} & \frac{7}{4} \\ \frac{7}{8} & 0 & \frac{7}{4} & \frac{7}{4} & 0 & \frac{7}{8} & \frac{7}{4} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ 0\ \frac{1}{2}\ 0\ 0\ 0\ 0\ \frac{1}{4}\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ 0\ 0\ \frac{1}{4}\right)$$

 

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \frac{21}{2}\ \frac{21}{4}\ \frac{7}{4}\ \frac{7}{4}\ \frac{21}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ 7\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \frac{287}{29}\ \frac{140}{29}\ \frac{63}{29}\ \frac{63}{29}\ \frac{322}{29}\ \frac{133}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \ldots\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2 / r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau} / n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 6 vs no. of idems 8

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {3, 6, 8}

"RG2" = {3, 5, 6}

"RG3" = {4, 7, 8}

"RG4" = {3, 4, 8}

"RG5" = {3, 4, 5}

"RG6" = {2, 4, 7}

"RG7" = {1, 7, 8}

"RG8" = {1, 2, 7}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{31}{72} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-1}{72} \\ \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{-1}{72} & \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-16}{9} & \frac{-1}{36} \\ \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{17}{9} & \frac{-1}{72} & \frac{-8}{9} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-1}{72} & \frac{25}{18} & \frac{31}{72} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} & \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{-1}{72} \\ \frac{31}{36} & \frac{31}{36} & \frac{-16}{9} & \frac{-1}{36} & \frac{-8}{9} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-8}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{100} & \frac{-16}{25} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-1}{100} \\ \frac{31}{100} & 1 & \frac{-16}{25} & \frac{-1}{100} & \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} \\ \frac{-8}{17} & \frac{-8}{17} & 1 & \frac{-1}{68} & \frac{31}{68} & \frac{31}{68} & \frac{-16}{17} & \frac{-1}{68} \\ \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & 1 & \frac{-1}{136} & \frac{-8}{17} & \frac{-1}{68} & \frac{-1}{68} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-1}{100} & 1 & \frac{31}{100} & \frac{-16}{25} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} & \frac{31}{100} & 1 & \frac{-16}{25} & \frac{-1}{100} \\ \frac{31}{68} & \frac{31}{68} & \frac{-16}{17} & \frac{-1}{68} & \frac{-8}{17} & \frac{-8}{17} & 1 & \frac{-1}{68} \\ \frac{-1}{136} & \frac{-8}{17} & \frac{-1}{68} & \frac{-1}{68} & \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

 

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 2.754892744, 0.147885034]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6900000000, 1.633400282, 0.0713056009, 3.473569042, 0.417607428, 1.631532414, 0.0825852329]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

Eigenvalues $M_0$

[0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 9.145704965, 0.142342103, 2.503619601]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 1, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, 0, 0, −1]}

Eigenvalues M

[0., 0., −0.8750000000, −2.625000000, 2.950746158, −2.075746158, 5.795540962, −3.170540962]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{[0, −1, 0, 0, −1, 0, 0, 1], [1, 0, 0, −1, 0, 1, 0, 0]}

NullSpace N

{[0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

 

==========================================

{3, 4, 8}

R: [4, 3, 5, 6, 3, 4, 4, 7]
B: [8, 7, 1, 2, 7, 8, 8, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{1024}(84 + 12s - s^2 + s^4)(-1 + s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5])
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[1, 0, 0, 0, 0, 0, −1, 0], [0, 0, 0, 0, 0, 1, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 7 & 0 & 0 & 0 & 0 & 14 \\ 7 & 0 & 0 & 0 & 0 & 0 & 7 & 14 \\ 7 & 0 & 0 & 14 & 0 & 7 & 14 & 14 \\ 0 & 0 & 14 & 0 & 14 & 14 & 14 & 0 \\ 0 & 0 & 0 & 14 & 0 & 7 & 7 & 0 \\ 0 & 0 & 7 & 14 & 7 & 0 & 0 & 0 \\ 0 & 7 & 14 & 14 & 7 & 0 & 0 & 14 \\ 14 & 14 & 14 & 0 & 0 & 0 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )

degree 2: $\frac{1}{24}$ ( $v[1]v[2] + v[1]v[3] + 2v[1]v[8] + v[2]v[7] + 2v[2]v[8] + 2v[3]v[4] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + 2v[4]v[5] + 2v[4]v[6] + 2v[4]v[7] + v[5]v[6] + v[5]v$

[7] + 2 v[8]v[7] )

degree 3 : $\frac{1}{8}$ ( v[1]v[2]v[8] + v[1]v[3]v[8] + v[2]v[8]v[7] + v[3]v[4]v[6] + v[3]v[4]v[7] + v[3]v[8]v[7] + v[4]v[5]v[6] + v[4]v[5]v[7] )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {4, 5, 7}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 7, 8}

"RG5" = {4, 5, 6}

"RG6" = {3, 4, 6}

"RG7" = {1, 3, 8}

"RG8" = {1, 2, 8}

$\pi3$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi3$ = {6, 11, 36, 38, 39, 46, 47, 48}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[6], [1], [5], [3], [6], [5], [1], [3]]
Action of B on ranges, [[4], [7], [8], [2], [4], [8], [7], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1, 3]

B−BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1 = \{4, 8\}$
$b_2 = \{1, 6, 7\}$
$b_3 = \{2, 3, 5\}$

dim(span of partition vectors), rank($N_0$), rank($N$):    3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 23, Shape: 3 ⊕ 20/18

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{5}{16} & \frac{3}{16} & \frac{5}{16} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{4, 8}, {1, 6, 7}, {2, 3, 5}}

1, "range", [4, 5, 7], [[7, 5, 5, 4, 5, 7, 7, 4], [7, 4, 4, 5, 4, 7, 7, 5], [5, 7, 7, 4, 7, 5, 5, 4], [5, 4, 4, 7, 4, 5, 5, 7], [4, 7, 7, 5, 7, 4, 4, 5], [4, 5, 5, 7, 5, 4, 4, 7]]

2, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

3, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

4, "range", [2, 7, 8], [[8, 7, 7, 2, 7, 8, 8, 2], [8, 2, 2, 7, 2, 8, 8, 7], [7, 8, 8, 2, 8, 7, 7, 2], [7, 2, 2, 8, 2, 7, 7, 8], [2, 8, 8, 7, 8, 2, 2, 7], [2, 7, 7, 8, 7, 2, 2, 8]]

5, "range", [4, 5, 6], [[6, 5, 5, 4, 5, 6, 6, 4], [6, 4, 4, 5, 4, 6, 6, 5], [5, 6, 6, 4, 6, 5, 5, 4], [5, 4, 4, 6, 4, 5, 5, 6], [4, 6, 6, 5, 6, 4, 4, 5], [4, 5, 5, 6, 5, 4, 4, 6]]

6, "range", [3, 4, 6], [[6, 4, 4, 3, 4, 6, 6, 3], [6, 3, 3, 4, 3, 6, 6, 4], [4, 6, 6, 3, 6, 4, 4, 3], [4, 3, 3, 6, 3, 4, 4, 6], [3, 6, 6, 4, 6, 3, 3, 4], [3, 4, 4, 6, 4, 3, 3, 6]]

7, "range", [1, 3, 8], [[8, 3, 3, 1, 3, 8, 8, 1], [8, 1, 1, 3, 1, 8, 8, 3], [3, 8, 8, 1, 8, 3, 3, 1], [3, 1, 1, 8, 1, 3, 3, 8], [1, 8, 8, 3, 8, 1, 1, 3], [1, 3, 3, 8, 3, 1, 1, 8]]

8, "range", [1, 2, 8], [[8, 2, 2, 1, 2, 8, 8, 1], [8, 1, 1, 2, 1, 8, 8, 2], [2, 8, 8, 1, 8, 2, 2, 1], [2, 1, 1, 8, 1, 2, 2, 8], [1, 8, 8, 2, 8, 1, 1, 2], [1, 2, 2, 8, 2, 1, 1, 8]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

$g_3 =$ [[2, 3]]

$g_4 =$ [[1, 3, 2]]

$g_5 =$ [[1, 2, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} 2h[1] & 0 & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

{6, 11, 36, 38, 39, 46, 47, 48}

$u3 =$
$(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $(2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 2 =$
$(1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2\ \ 2\ \ 0\ \ 1\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 2)$

$u2 =$
$\left(\frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\right.$

picheck $(4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

$\pi 1 = (4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

$u1 = \left(\frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\right)$

picheck $(4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 2, 1, 1, 0, -2]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & 0 & -s & 0 & s & 0 & -t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -t & t & 0 & 0 & -s & s & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 1 & 0 & -2 \end{pmatrix}$

M0 is invertible. det= 381271/104976

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&1&1&0\\ 0&1&1&0&1&0&0&0\\ 0&1&1&0&1&0&0&0\\ 0&0&0&1&0&0&0&1\\ 0&1&1&0&1&0&0&0\\ 1&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&0&0&1&0&0&0&1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&1&1&0\\ 0&1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&1&0&0&0\\ 1&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&1&1&0\\ 0&1&1&0&1&0&0&0\\ 0&1&1&0&1&0&0&0\\ 0&0&0&1&0&0&0&1\\ 0&1&1&0&1&0&0&0\\ 1&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&0&0&1&0&0&0&1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&1&1&0\\ 0&1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&1&0&0&0\\ 1&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{8} & \frac{7}{8} & 0 & 0 & 0 & 0 & \frac{7}{4} \\ \frac{7}{8} & \frac{11}{6} & 0 & 0 & 0 & 0 & \frac{7}{8} & \frac{7}{4} \\ \frac{7}{8} & 0 & \frac{11}{3} & \frac{7}{4} & 0 & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{11}{3} & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & 0 \\ 0 & 0 & 0 & \frac{7}{4} & \frac{11}{6} & \frac{7}{8} & \frac{7}{8} & 0 \\ 0 & 0 & \frac{7}{8} & \frac{7}{4} & \frac{7}{8} & \frac{11}{6} & 0 & 0 \\ 0 & \frac{7}{8} & \frac{7}{4} & \frac{7}{4} & \frac{7}{8} & 0 & \frac{11}{3} & \frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & 0 & 0 & 0 & \frac{7}{4} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{12}\ \frac{5}{12}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{3}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{4}\ \frac{1}{2}\ 0\ 0\ \frac{3}{4}\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \frac{7}{2}\ \frac{49}{4}\ \frac{7}{4}\ \frac{7}{4}\ \frac{49}{4}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ 7\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\ 7\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \frac{217}{87}\ \frac{1183}{87}\ \frac{63}{29}\ \frac{63}{29}\ \frac{1841}{174}\ \frac{133}{58}\ \frac{805}{174}\ \frac{189}{58}\ \frac{805}{174}\ \frac{805}{174}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{805}{174}\ \frac{133}{58}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2 / r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau} / n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 6 vs no. of idems 8

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {4, 5, 7}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 7, 8}

"RG5" = {4, 5, 6}

"RG6" = {3, 4, 6}

"RG7" = {1, 3, 8}

"RG8" = {1, 2, 8}

$$M_C = \begin{pmatrix} \frac{25}{18} & \frac{31}{72} & \frac{-1}{72} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{31}{36} \\ \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{31}{36} \\ \frac{-1}{72} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} & \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-1}{36} & \frac{17}{9} & \frac{31}{36} & \frac{31}{36} & \frac{-1}{36} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{31}{36} & \frac{25}{18} & \frac{31}{72} & \frac{-1}{72} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{31}{36} & \frac{31}{72} & \frac{25}{18} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{72} & \frac{-8}{9} & \frac{17}{9} & \frac{-1}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-1}{36} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-1}{36} & \frac{17}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{100} & \frac{-1}{100} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{31}{50} \\ \frac{31}{100} & 1 & \frac{-16}{25} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-1}{100} & \frac{31}{50} \\ \frac{-1}{136} & \frac{-8}{17} & 1 & \frac{-1}{68} & \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & \frac{-1}{68} \\ \frac{-8}{17} & \frac{-8}{17} & \frac{-1}{68} & 1 & \frac{31}{68} & \frac{31}{68} & \frac{-1}{68} & \frac{-16}{17} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{31}{50} & 1 & \frac{31}{100} & \frac{-1}{100} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-1}{100} & \frac{31}{50} & \frac{31}{100} & 1 & \frac{-16}{25} & \frac{-16}{25} \\ \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & \frac{-1}{68} & \frac{-1}{136} & \frac{-8}{17} & 1 & \frac{-1}{68} \\ \frac{31}{68} & \frac{31}{68} & \frac{-1}{68} & \frac{-16}{17} & \frac{-8}{17} & \frac{-8}{17} & \frac{-1}{68} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 2.754892744, 0.147885034]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6900000000, 1.633400282, 0.0713056009, 3.473569042, 0.417607428, 1.631532414, 0.0825852329]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, 1, 0, −1, 0, 0, 0], [0, 0, 0, 0, 0, 1, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, −1], [1, 0, 0, 0, 0, 0, −1, 0]}

Eigenvalues $M_0$

[0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 9.145704965, 0.142342103, 2.503619601]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., −0.8750000000, −2.625000000, 2.950746158, −2.075746158, 5.795540962, −3.170540962]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{[−1, 0, 0, 0, 0, −1, 1, 0], [0, 1, −1, 0, 1, 0, 0, 0]}

NullSpace N

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==================================================

{3, 7, 8}

R: [4, 3, 5, 2, 3, 4, 8, 7]
B: [8, 7, 1, 6, 7, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{1}{256}\,(3+s^2)\,(-7+s)\,(-1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[3] v[5]) (1 + v[8] v[7])
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 0 & 5 & 5 & 10 & 10 \\ 5 & 5 & 0 & 0 & 5 & 5 & 10 & 10 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 0 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]$ )
degree 2: $\frac{1}{24}$ ( $v[1]v[3] + v[1]v[4] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + v[2]v[$
$7] + v[2]v[8] + v[3]v[5] + v[3]v[6] + 2\,v[3]v[7] + 2\,v[3]v[8] + v[4]v[5] + v[4]v[6] + 2\,v$

$[4]v[7] + 2v[4]v[8] + v[5]v[7] + v[5]v[8] + v[6]v[7] + v[6]v[8])$

degree 3 : $\frac{1}{16}(v[3] + v[4])(v[7] + v[8])(v[1] + v[2] + v[5] + v[6])$

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{3, 4}, {1, 2, 5, 6}, {7, 8}}

"RG1" = {4, 6, 8}

"RG2" = {4, 6, 7}

"RG3" = {3, 6, 8}

"RG4" = {3, 6, 7}

"RG5" = {4, 5, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 5, 8}

"RG8" = {3, 5, 7}

"RG9" = {2, 4, 8}

"RG10" = {2, 4, 7}

"RG11" = {2, 3, 8}

"RG12" = {2, 3, 7}

"RG13" = {1, 4, 8}

"RG14" = {1, 4, 7}

"RG15" = {1, 3, 8}

"RG16" = {1, 3, 7}

$\pi 3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0]

supp $\pi 3$ = {10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

$u3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0]

supp u3 = {10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

Action of R on ranges, [[10], [9], [6], [5], [12], [11], [8], [7], [12], [11], [8], [7], [10], [9], [6], [5]]
Action of B on ranges, [[3], [1], [15], [13], [4], [2], [16], [14], [4], [2], [16], [14], [3], [1], [15], [13]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1, 3]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1$ = {3, 4}
$b_2$ = {1, 2, 5, 6}
$b_3$ = {7, 8}

dim(span of partition vectors), rank($N_0$), rank($N$):    3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $3 \oplus 24/22$

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{7, 8}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} 0 & 0 & \frac{3}{8} & 0 & \frac{3}{8} & 0 & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \text{ vs } \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{3, 4}, {1, 2, 5, 6}, {7, 8}}

1, "range", [4, 6, 8], [[8, 8, 6, 6, 8, 8, 4, 4], [8, 8, 4, 4, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 4, 4], [6, 6, 4, 4, 6, 6, 8, 8], [4, 4, 8, 8, 4, 4, 6, 6], [4, 4, 6, 6, 4, 4, 8, 8]]

2, "range", [4, 6, 7], [[7, 7, 6, 6, 7, 7, 4, 4], [7, 7, 4, 4, 7, 7, 6, 6], [6, 6, 7, 7, 6, 6, 4, 4], [6, 6, 4, 4, 6, 6, 7, 7], [4, 4, 7, 7, 4, 4, 6, 6], [4, 4, 6, 6, 4, 4, 7, 7]]

3, "range", [3, 6, 8], [[8, 8, 6, 6, 8, 8, 3, 3], [8, 8, 3, 3, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 3, 3], [6, 6, 3, 3, 6, 6, 8, 8], [3, 3, 8, 8, 3, 3, 6, 6], [3, 3, 6, 6, 3, 3, 8, 8]]

4, "range", [3, 6, 7], [[7, 7, 6, 6, 7, 7, 3, 3], [7, 7, 3, 3, 7, 7, 6, 6], [6, 6, 7, 7, 6, 6, 3, 3], [6, 6, 3, 3, 6, 6, 7, 7], [3, 3, 7, 7, 3, 3, 6, 6], [3, 3, 6, 6, 3, 3, 7, 7]]

5, "range", [4, 5, 8], [[8, 8, 5, 5, 8, 8, 4, 4], [8, 8, 4, 4, 8, 8, 5, 5], [5, 5, 8, 8, 5, 5, 4, 4], [5, 5, 4, 4, 5, 5, 8, 8], [4, 4, 8, 8, 4, 4, 5, 5], [4, 4, 5, 5, 4, 4, 8, 8]]

6, "range", [4, 5, 7], [[7, 7, 5, 5, 7, 7, 4, 4], [7, 7, 4, 4, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 4, 4], [5, 5, 4, 4, 5, 5, 7, 7], [4, 4, 7, 7, 4, 4, 5, 5], [4, 4, 5, 5, 4, 4, 7, 7]]

7, "range", [3, 5, 8], [[8, 8, 5, 5, 8, 8, 3, 3], [8, 8, 3, 3, 8, 8, 5, 5], [5, 5, 8, 8, 5, 5, 3, 3], [5, 5, 3, 3, 5, 5, 8, 8], [3, 3, 8, 8, 3, 3, 5, 5], [3, 3, 5, 5, 3, 3, 8, 8]]

8, "range", [3, 5, 7], [[7, 7, 5, 5, 7, 7, 3, 3], [7, 7, 3, 3, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 3, 3], [5, 5, 3, 3, 5, 5, 7, 7], [3, 3, 7, 7, 3, 3, 5, 5], [3, 3, 5, 5, 3, 3, 7, 7]]

9, "range", [2, 4, 8], [[8, 8, 4, 4, 8, 8, 2, 2], [8, 8, 2, 2, 8, 8, 4, 4], [4, 4, 8, 8, 4, 4, 2, 2], [4, 4, 2, 2, 4, 4, 8, 8], [2, 2, 8, 8, 2, 2, 4, 4], [2, 2, 4, 4, 2, 2, 8, 8]]

10, "range", [2, 4, 7], [[7, 7, 4, 4, 7, 7, 2, 2], [7, 7, 2, 2, 7, 7, 4, 4], [4, 4, 7, 7, 4, 4, 2, 2], [4, 4, 2, 2, 4, 4, 7, 7], [2, 2, 7, 7, 2, 2, 4, 4], [2, 2, 4, 4, 2, 2, 7, 7]]

11, "range", [2, 3, 8], [[8, 8, 3, 3, 8, 8, 2, 2], [8, 8, 2, 2, 8, 8, 3, 3], [3, 3, 8, 8, 3, 3, 2, 2], [3, 3, 2, 2, 3, 3, 8, 8], [2, 2, 8, 8, 2, 2, 3, 3], [2, 2, 3, 3, 2, 2, 8, 8]]

12, "range", [2, 3, 7], [[7, 7, 3, 3, 7, 7, 2, 2], [7, 7, 2, 2, 7, 7, 3, 3], [3, 3, 7, 7, 3, 3, 2, 2], [3, 3, 2, 2, 3, 3, 7, 7], [2, 2, 7, 7, 2, 2, 3, 3], [2, 2, 3, 3, 2, 2, 7, 7]]

13, "range", [1, 4, 8], [[8, 8, 4, 4, 8, 8, 1, 1], [8, 8, 1, 1, 8, 8, 4, 4], [4, 4, 8, 8, 4, 4, 1, 1], [4, 4, 1, 1, 4, 4, 8, 8], [1, 1, 8, 8, 1, 1, 4, 4], [1, 1, 4, 4, 1, 1, 8, 8]]

14, "range", [1, 4, 7], [[7, 7, 4, 4, 7, 7, 1, 1], [7, 7, 1, 1, 7, 7, 4, 4], [4, 4, 7, 7, 4, 4, 1, 1], [4, 4, 1, 1, 4, 4, 7, 7], [1, 1, 7, 7, 1, 1, 4, 4], [1, 1, 4, 4, 1, 1, 7, 7]]

15, "range", [1, 3, 8], [[8, 8, 3, 3, 8, 8, 1, 1], [8, 8, 1, 1, 8, 8, 3, 3], [3, 3, 8, 8, 3, 3, 1, 1], [3, 3, 1, 1, 3, 3, 8, 8], [1, 1, 8, 8, 1, 1, 3, 3], [1, 1, 3, 3, 1, 1, 8, 8]]

16, "range", [1, 3, 7], [[7, 7, 3, 3, 7, 7, 1, 1], [7, 7, 1, 1, 7, 7, 3, 3], [3, 3, 7, 7, 3, 3, 1, 1], [3, 3, 1, 1, 3, 3, 7, 7], [1, 1, 7, 7, 1, 1, 3, 3], [1, 1, 3, 3, 1, 1, 7, 7]]

"group has", 6, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

 

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1$

{10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

$u3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1$

{10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

picheck $\begin{pmatrix} 4 & 4 & 8 & 8 & 4 & 4 & 8 & 8 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$\pi 2 =$
$\begin{pmatrix} 0 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 0 & 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 & 0 & 2 & 2 & 2 & 2 & 0 \end{pmatrix}$

$u2 =$
$\Big(0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0$

picheck $\begin{pmatrix} 8 & 8 & 16 & 16 & 8 & 8 & 16 & 16 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 16 & 16 & 8 & 8 & 16 & 16 \end{pmatrix}$

$$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$$

picheck $\begin{pmatrix} 8 & 8 & 16 & 16 & 8 & 8 & 16 & 16 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks   N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks   N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks   N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{15} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{16} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

idem−checks

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} & \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} & \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} & \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} & \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, 1, 0, 0, 1, −1, 0, 0]

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & -s & 0 & 0 & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t & t & 0 & 0 & -s & s \\ 0 & 0 & s & -s & 0 & 0 & t & -t \\ -t & s & 0 & 0 & -s & t & 0 & 0 \end{pmatrix} \text{ RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 0 & 1 & 0 \end{pmatrix}$

M0 is invertible. det= 8192/27

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^\dagger = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&1&1&0&0&0&0\\0&0&1&1&0&0&0&0\\1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&0&0&0&0&1&1\\0&0&0&0&0&0&1&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&1&1&0&0&0&0\\0&0&1&1&0&0&0&0\\1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&0&0&0&0&1&1\\0&0&0&0&0&0&1&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\,T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew}$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 2 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 4 & 0 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & 0 & 4 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 2 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 2 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 4 & 0 \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{20}{3} T + 20\Omega$$

$$\Omega \left( \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{3}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12} \right)$$

$$T \left( \frac{1}{2}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left( \frac{20}{3}\ \ \frac{5}{3}\ \ \frac{10}{3}\ \ \frac{5}{3}\ \ \frac{5}{3}\ \ \frac{10}{3}\ \ \frac{40}{3}\ \ \frac{5}{3}\ \ \frac{20}{3}\ \ \frac{20}{3}\ \ \frac{5}{3}\ \ \frac{5}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3}\ \ \frac{10}{3} \right)$$

"IS MN in Vec(K)?", false

MN

$$\left( \frac{215}{33}\ \ \frac{15}{11}\ \ \frac{35}{11}\ \ \frac{15}{11}\ \ \frac{15}{11}\ \ \frac{35}{11}\ \ \frac{430}{33}\ \ \frac{15}{11}\ \ \frac{215}{33}\ \ \frac{215}{33}\ \ \frac{15}{11}\ \ \frac{15}{11}\ \ \frac{30}{11}\ \ \frac{30}{11}\ \ \frac{115}{33}\ \ \frac{115}{33}\ \ \frac{30}{11}\ \ \frac{30}{11} \right.$$

$$\boldsymbol{\tau} = 24/1 \text{ , rank} = 3 \text{ , ratio} = 8/1 \text{ , } n^2 / r = 64/3$$

$$\boldsymbol{\tau}' = 40/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 3/8$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 136/9$$

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{4}{3} T + 20 \Omega$$

There are, 1, partitions and, 16, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 96

dim span idems 6 vs no. of idems 16

"PT1" = {{3, 4}, {1, 2, 5, 6}, {7, 8}}

"RG1" = {4, 6, 8}

"RG2" = {4, 6, 7}

"RG3" = {3, 6, 8}

"RG4" = {3, 6, 7}

"RG5" = {4, 5, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 5, 8}

"RG8" = {3, 5, 7}

"RG9" = {2, 4, 8}

"RG10" = {2, 4, 7}

"RG11" = {2, 3, 8}

"RG12" = {2, 3, 7}

"RG13" = {1, 4, 8}

"RG14" = {1, 4, 7}

"RG15" = {1, 3, 8}

"RG16" = {1, 3, 7}

$$M_C = \begin{pmatrix} \frac{14}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{14}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{20}{9} & \frac{-16}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-16}{9} & \frac{20}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{14}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{14}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{20}{9} & \frac{-16}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-16}{9} & \frac{20}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-2}{7} & 1 & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-1}{40} & \frac{-1}{40} & 1 & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & 1 & \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} \\ \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & 1 & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & 1 & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} & \frac{-1}{40} & \frac{-1}{40} & 1 & \frac{-4}{5} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} & \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix} \quad M_C N_C =$$

$$\begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.4444444444, 4., 4., 2., 2., 2.]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 2., 3.588403348, 1.300485540]

Eigenvalues $M_C$–scaled

[0., 0.3000000000, 0.2428571429, 1.800000000, 1.800000000, 1.285714286, 1.285714286, 1.285714286]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 2.322580645, 4.167178083, 1.510241271]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1], [1, 0, 0, 0, −1, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0]}

Eigenvalues $M_0$

[0.6666666667, 8.935416159, 0.397917175, 4., 4., 2., 2., 2.]

Eigenvalues $N_0$

[4., 2., 2., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

Eigenvalues M

[0., 0., 0., 0., 0., −3.333333333, 5.393446629, −2.060113295]

Eigenvalues N

[0., 0., 0., 0., 0., −2., 5.123105626, −3.123105626]

NullSpace M

{[0, 0, 0, 0, −1, 1, 0, 0], [1, 0, 0, 0, −1, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

NullSpace N

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==========================================

60, [1, 1, 1, −1, 1, 1, −1, −1]

==========================================================

{4, 7, 8}

R: [4, 3, 1, 6, 3, 4, 8, 7]
B: [8, 7, 5, 2, 7, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{1}{256}(-7+s)(3+s^2)(-1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6])
"B CYCLES", 1 + v[2] v[4] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 0 & 5 & 5 & 10 & 10 \\ 5 & 5 & 0 & 0 & 5 & 5 & 10 & 10 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 0 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )

degree 2: $\frac{1}{24}$ ( $v[1]v[3] + v[1]v[4] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + v[2]v[$
$7] + v[2]v[8] + v[3]v[5] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + v[4]v[5] + v[4]v[6] + 2v$

[4]$v$[7] + 2$v$[4]$v$[8] + $v$[5]$v$[7] + $v$[5]$v$[8] + $v$[6]$v$[7] + $v$[6]$v$[8] )
degree 3 : $\frac{1}{16}$ ( $v$[3] + $v$[4] ) ( $v$[7] + $v$[8] ) ( $v$[1] + $v$[2] + $v$[5] + $v$[6] )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{3, 4}, {1, 2, 5, 6}, {7, 8}}

"RG1" = {4, 6, 8}

"RG2" = {4, 6, 7}

"RG3" = {3, 6, 8}

"RG4" = {3, 6, 7}

"RG5" = {4, 5, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 5, 8}

"RG8" = {3, 5, 7}

"RG9" = {2, 4, 8}

"RG10" = {2, 4, 7}

"RG11" = {2, 3, 8}

"RG12" = {2, 3, 7}

"RG13" = {1, 4, 8}

"RG14" = {1, 4, 7}

"RG15" = {1, 3, 8}

"RG16" = {1, 3, 7}

$\pi$3 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0]

supp $\pi$3 = {10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

$u$3 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0]

supp u3 = {10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

Action of R on ranges, [[2], [1], [14], [13], [4], [3], [16], [15], [4], [3], [16], [15], [2], [1], [14], [13]]
Action of B on ranges, [[11], [9], [7], [5], [12], [10], [8], [6], [12], [10], [8], [6], [11], [9], [7], [5]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1, 3]

B−BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {3, 4}
$b_2$ = {1, 2, 5, 6}
$b_3$ = {7, 8}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 3 ⊕ 24/22

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{2, 4, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{8} & 0 & \frac{3}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{3, 4}, {1, 2, 5, 6}, {7, 8}}

1, "range", [4, 6, 8], [[8, 8, 6, 6, 8, 8, 4, 4], [8, 8, 4, 4, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 4, 4], [6, 6, 4, 4, 6, 6, 8, 8], [4, 4, 8, 8, 4, 4, 6, 6], [4, 4, 6, 6, 4, 4, 8, 8]]

2, "range", [4, 6, 7], [[7, 7, 6, 6, 7, 7, 4, 4], [7, 7, 4, 4, 7, 7, 6, 6], [6, 6, 7, 7, 6, 6, 4, 4], [6, 6, 4, 4, 6, 6, 7, 7], [4, 4, 7, 7, 4, 4, 6, 6], [4, 4, 6, 6, 4, 4, 7, 7]]

3, "range", [3, 6, 8], [[8, 8, 6, 6, 8, 8, 3, 3], [8, 8, 3, 3, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 3, 3], [6, 6, 3, 3, 6, 6, 8, 8], [3, 3, 8, 8, 3, 3, 6, 6], [3, 3, 6, 6, 3, 3, 8, 8]]

4, "range", [3, 6, 7], [[7, 7, 6, 6, 7, 7, 3, 3], [7, 7, 3, 3, 7, 7, 6, 6], [6, 6, 7, 7, 6, 6, 3, 3], [6, 6, 3, 3, 6, 6, 7, 7], [3, 3, 7, 7, 3, 3, 6, 6], [3, 3, 6, 6, 3, 3, 7, 7]]

5, "range", [4, 5, 8], [[8, 8, 5, 5, 8, 8, 4, 4], [8, 8, 4, 4, 8, 8, 5, 5], [5, 5, 8, 8, 5, 5, 4, 4], [5, 5, 4, 4, 5, 5, 8, 8], [4, 4, 8, 8, 4, 4, 5, 5], [4, 4, 5, 5, 4, 4, 8, 8]]

6, "range", [4, 5, 7], [[7, 7, 5, 5, 7, 7, 4, 4], [7, 7, 4, 4, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 4, 4], [5, 5, 4, 4, 5, 5, 7, 7], [4, 4, 7, 7, 4, 4, 5, 5], [4, 4, 5, 5, 4, 4, 7, 7]]

7, "range", [3, 5, 8], [[8, 8, 5, 5, 8, 8, 3, 3], [8, 8, 3, 3, 8, 8, 5, 5], [5, 5, 8, 8, 5, 5, 3, 3], [5, 5, 3, 3, 5, 5, 8, 8], [3, 3, 8, 8, 3, 3, 5, 5], [3, 3, 5, 5, 3, 3, 8, 8]]

8, "range", [3, 5, 7], [[7, 7, 5, 5, 7, 7, 3, 3], [7, 7, 3, 3, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 3, 3], [5, 5, 3, 3, 5, 5, 7, 7], [3, 3, 7, 7, 3, 3, 5, 5], [3, 3, 5, 5, 3, 3, 7, 7]]

9, "range", [2, 4, 8], [[8, 8, 4, 4, 8, 8, 2, 2], [8, 8, 2, 2, 8, 8, 4, 4], [4, 4, 8, 8, 4, 4, 2, 2], [4, 4, 2, 2, 4, 4, 8, 8], [2, 2, 8, 8, 2, 2, 4, 4], [2, 2, 4, 4, 2, 2, 8, 8]]

10, "range", [2, 4, 7], [[7, 7, 4, 4, 7, 7, 2, 2], [7, 7, 2, 2, 7, 7, 4, 4], [4, 4, 7, 7, 4, 4, 2, 2], [4, 4, 2, 2, 4, 4, 7, 7], [2, 2, 7, 7, 2, 2, 4, 4], [2, 2, 4, 4, 2, 2, 7, 7]]

11, "range", [2, 3, 8], [[8, 8, 3, 3, 8, 8, 2, 2], [8, 8, 2, 2, 8, 8, 3, 3], [3, 3, 8, 8, 3, 3, 2, 2], [3, 3, 2, 2, 3, 3, 8, 8], [2, 2, 8, 8, 2, 2, 3, 3], [2, 2, 3, 3, 2, 2, 8, 8]]

12, "range", [2, 3, 7], [[7, 7, 3, 3, 7, 7, 2, 2], [7, 7, 2, 2, 7, 7, 3, 3], [3, 3, 7, 7, 3, 3, 2, 2], [3, 3, 2, 2, 3, 3, 7, 7], [2, 2, 7, 7, 2, 2, 3, 3], [2, 2, 3, 3, 2, 2, 7, 7]]

13, "range", [1, 4, 8], [[8, 8, 4, 4, 8, 8, 1, 1], [8, 8, 1, 1, 8, 8, 4, 4], [4, 4, 8, 8, 4, 4, 1, 1], [4, 4, 1, 1, 4, 4, 8, 8], [1, 1, 8, 8, 1, 1, 4, 4], [1, 1, 4, 4, 1, 1, 8, 8]]

14, "range", [1, 4, 7], [[7, 7, 4, 4, 7, 7, 1, 1], [7, 7, 1, 1, 7, 7, 4, 4], [4, 4, 7, 7, 4, 4, 1, 1], [4, 4, 1, 1, 4, 4, 7, 7], [1, 1, 7, 7, 1, 1, 4, 4], [1, 1, 4, 4, 1, 1, 7, 7]]

15, "range", [1, 3, 8], [[8, 8, 3, 3, 8, 8, 1, 1], [8, 8, 1, 1, 8, 8, 3, 3], [3, 3, 8, 8, 3, 3, 1, 1], [3, 3, 1, 1, 3, 3, 8, 8], [1, 1, 8, 8, 1, 1, 3, 3], [1, 1, 3, 3, 1, 1, 8, 8]]

16, "range", [1, 3, 7], [[7, 7, 3, 3, 7, 7, 1, 1], [7, 7, 1, 1, 7, 7, 3, 3], [3, 3, 7, 7, 3, 3, 1, 1], [3, 3, 1, 1, 3, 3, 7, 7], [1, 1, 7, 7, 1, 1, 3, 3], [1, 1, 3, 3, 1, 1, 7, 7]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1$

{10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

$u3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1$

{10, 11, 14, 15, 25, 26, 29, 30, 42, 43, 44, 45, 48, 49, 50, 51}

picheck $(4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 2 =$
$(0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4\ \ 0\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0)$

$u2 =$
$\left(0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\right.$

picheck $(8\ \ 8\ \ 16\ \ 16\ \ 8\ \ 8\ \ 16\ \ 16)$

$\pi 1 = (8\ \ 8\ \ 16\ \ 16\ \ 8\ \ 8\ \ 16\ \ 16)$

$u1 = \left(\frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\right)$

picheck $(8\ \ 8\ \ 16\ \ 16\ \ 8\ \ 8\ \ 16\ \ 16)$

Column Projections

$$P_1 = \begin{pmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks    N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks    N0-checks

$$P_{15} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks    N0-checks

$$P_{16} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} & \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} & \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} & \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{10}{3} & \frac{10}{3} & \frac{5}{3} & \frac{5}{3} & \frac{20}{3} & \frac{20}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 0, -1, 1, 0, 0]$

$$\ker N_C = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & -s & 0 & 0 & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & -t & 0 & 0 & t & -s & 0 & 0 \\ 0 & 0 & s & -s & 0 & 0 & t & -t \\ 0 & 0 & t & -t & 0 & 0 & s & -s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 & -1 & 0 \end{pmatrix}$

M0 is invertible. det= 8192/27

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&1&1&0&0&0&0\\0&0&1&1&0&0&0&0\\1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&0&0&0&0&1&1\\0&0&0&0&0&0&1&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&1&1&0&0&0&0\\0&0&1&1&0&0&0&0\\1&1&0&0&1&1&0&0\\1&1&0&0&1&1&0&0\\0&0&0&0&0&0&1&1\\0&0&0&0&0&0&1&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{3} & \frac{5}{3} & 0 & 0 & \frac{5}{3} & \frac{5}{3} \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \\ \frac{-5}{3} & \frac{-5}{3} & 0 & 0 & \frac{-5}{3} & \frac{-5}{3} & 0 & 0 \end{pmatrix} \quad Skew\,T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 2 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 4 & 0 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & 0 & 4 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 2 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 2 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 4 & 0 \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{20}{3} T + 20 \Omega$$

$$\Omega \left( \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{3} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left( \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; \frac{1}{2} \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left( \frac{20}{3} \;\; \frac{5}{3} \;\; \frac{10}{3} \;\; \frac{5}{3} \;\; \frac{5}{3} \;\; \frac{10}{3} \;\; \frac{40}{3} \;\; \frac{5}{3} \;\; \frac{20}{3} \;\; \frac{20}{3} \;\; \frac{5}{3} \;\; \frac{5}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \;\; \frac{10}{3} \right)$$

"IS MN in Vec(K)?", false

MN

$$\left( \frac{215}{33} \;\; \frac{15}{11} \;\; \frac{35}{11} \;\; \frac{15}{11} \;\; \frac{15}{11} \;\; \frac{35}{11} \;\; \frac{430}{33} \;\; \frac{15}{11} \;\; \frac{215}{33} \;\; \frac{215}{33} \;\; \frac{15}{11} \;\; \frac{15}{11} \;\; \frac{30}{11} \;\; \frac{30}{11} \;\; \frac{115}{33} \;\; \frac{115}{33} \;\; \frac{30}{11} \;\; \frac{30}{11} \right.$$

$$\boldsymbol{\tau} = 24/1 \text{ , rank} = 3 \text{ , ratio} = 8/1 \text{ , } n^2 / r = 64/3$$

$$\boldsymbol{\tau}' = 40/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 3/8$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 136/9$$

max r = 36/5 , $r$−check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{4}{3} T + 20\Omega$$

There are, 1, partitions and, 16, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 96

dim span idems 6 vs no. of idems 16

"PT1" = {{3, 4}, {1, 2, 5, 6}, {7, 8}}

"RG1" = {4, 6, 8}

"RG2" = {4, 6, 7}

"RG3" = {3, 6, 8}

"RG4" = {3, 6, 7}

"RG5" = {4, 5, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 5, 8}

"RG8" = {3, 5, 7}

"RG9" = {2, 4, 8}

"RG10" = {2, 4, 7}

"RG11" = {2, 3, 8}

"RG12" = {2, 3, 7}

"RG13" = {1, 4, 8}

"RG14" = {1, 4, 7}

"RG15" = {1, 3, 8}

"RG16" = {1, 3, 7}

$$M_C = \begin{pmatrix} \frac{14}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{14}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{20}{9} & \frac{-16}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-16}{9} & \frac{20}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{14}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{14}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{20}{9} & \frac{-16}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-16}{9} & \frac{20}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-2}{7} & 1 & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-1}{40} & \frac{-1}{40} & 1 & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & 1 & \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} \\ \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & 1 & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-2}{7} & \frac{-2}{7} & \frac{-1}{28} & \frac{-1}{28} & \frac{-2}{7} & 1 & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} & \frac{-1}{40} & \frac{-1}{40} & 1 & \frac{-4}{5} \\ \frac{-1}{40} & \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{20} & \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.4444444444, 4., 4., 2., 2., 2.]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 2., 3.588403348, 1.300485540]

Eigenvalues $M_C$−scaled

[0., 0.3000000000, 0.2428571429, 1.800000000, 1.800000000, 1.285714286, 1.285714286, 1.285714286]

Eigenvalues $N_C$−scaled

[0., 0., 0., 0., 0., 2.322580645, 4.167178083, 1.510241271]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0]}

Eigenvalues $M_0$

[0.6666666667, 8.935416159, 0.397917175, 4., 4., 2., 2., 2.]

Eigenvalues $N_0$

[4., 2., 2., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., 0., −3.333333333, 5.393446629, −2.060113295]

Eigenvalues N

[0., 0., 0., 0., 0., −2., 5.123105626, −3.123105626]

NullSpace M

{[0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 0, 0, −1, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace N

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==========================================

{2, 3, 4, 7}

R: [4, 7, 5, 6, 3, 4, 8, 3]
B: [8, 3, 1, 2, 7, 8, 4, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{3}{4096}\left(-224 + 20s - 50s^2 + 19s^3 + 11s^4 + 6s^5 + 2s^6\right)\left(-1 + s\right)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 4, "vs", 6

"R CYCLES", (1 + v[3] v[5]) (1 + v[4] v[6])
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[8] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 35 & 7 & 0 & 0 & 0 & 56 & 28 \\ 35 & 0 & 14 & 28 & 0 & 0 & 49 & 0 \\ 7 & 14 & 0 & 70 & 56 & 49 & 0 & 56 \\ 0 & 28 & 70 & 0 & 28 & 0 & 56 & 70 \\ 0 & 0 & 56 & 28 & 0 & 35 & 7 & 0 \\ 0 & 0 & 49 & 0 & 35 & 0 & 14 & 28 \\ 56 & 49 & 0 & 56 & 7 & 14 & 0 & 70 \\ 28 & 0 & 56 & 70 & 0 & 28 & 70 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$

degree 2: $\frac{5}{108}( 5v[1]v[2] + v[1]v[3] + 8v[1]v[7] + 4v[1]v[8] + 2v[2]v[3] + 4v[2]v[4] + 7v[2]v[7] + 10v[3]v[4] + 8v[3]v[5] + 7v[3]v[6] + 8v[3]v[8] + 4v[4]v[5] + 8v[4]v[7] + 10v[4]v[8] + 5v[5]v[6] + v[5]v[7] + 2v[6]v[7] + 4v[6]v[8] + 10v[8]v[7] )$

degree 3 : $\frac{1}{54}$ ( $2v[1]v[2]v[3] + 13v[1]v[2]v[7] + v[1]v[3]v[8] + 11v[1]v[8]v[7] + 4v[2$

]$v[3]v[4] + 8v[2]v[4]v[7] + 11v[3]v[4]v[5] + 15v[3]v[4]v[8] + 13v[3]v[5]v[6] + 8v[3]$
$v[6]v[8] + v[4]v[5]v[7] + 15v[4]v[8]v[7] + 2v[5]v[6]v[7] + 4v[6]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {3, 5, 6}

"RG5" = {4, 7, 8}

"RG6" = {3, 4, 8}

"RG7" = {4, 5, 7}

"RG8" = {3, 4, 5}

"RG9" = {2, 4, 7}

"RG10" = {2, 3, 4}

"RG11" = {1, 7, 8}

"RG12" = {1, 3, 8}

"RG13" = {1, 2, 7}

"RG14" = {1, 2, 3}

$\pi3$ = [2, 0, 0, 0, 13, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11, 4, 0, 0, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 0, 11, 0, 0, 15, 13, 0, 0, 0, 8, 0, 0, 1, 0, 0, 0, 15, 2, 0, 0, 4]

supp $\pi3$ = {1, 5, 11, 21, 22, 29, 37, 40, 41, 45, 48, 52, 53, 56}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[6], [8], [6], [8], [2], [4], [2], [4], [1], [3], [6], [8], [5], [7]]
Action of B on ranges, [[5], [11], [5], [11], [9], [13], [9], [13], [10], [14], [5], [11], [6],

[12]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{27} & \frac{2}{27} & \frac{1}{54} & \frac{13}{108} & \frac{5}{36} & \frac{5}{36} & \frac{1}{108} & \frac{11}{108} & \frac{2}{27} & \frac{1}{27} & \frac{11}{108} & \frac{1}{108} & \frac{13}{108} & \frac{1}{54} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 3, 2]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 4, 6\}$
$b_2 = \{2, 5, 8\}$
$b_3 = \{3, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 34, Shape: 11 ⊕ 23/21

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega−checks true, true

"I−V is singular"

$$\pi_R = \begin{pmatrix} 0 & 0 & \frac{5}{16} & \frac{3}{16} & \frac{5}{16} & \frac{3}{16} & 0 & 0 \end{pmatrix} \text{vs} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{vs} \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [6, 7, 8], [[8, 7, 6, 8, 7, 8, 6, 7], [8, 6, 7, 8, 6, 8, 7, 6], [7, 8, 6, 7, 8, 7, 6, 8], [7, 6, 8, 7, 6, 7, 8, 6], [6, 8, 7, 6, 8, 6, 7, 8], [6, 7, 8, 6, 7, 6, 8, 7]]

2, "range", [3, 6, 8], [[8, 6, 3, 8, 6, 8, 3, 6], [8, 3, 6, 8, 3, 8, 6, 3], [6, 8, 3, 6, 8, 6, 3, 8], [6, 3, 8, 6, 3, 6, 8, 3], [3, 8, 6, 3, 8, 3, 6, 8], [3, 6, 8, 3, 6, 3, 8, 6]]

3, "range", [5, 6, 7], [[7, 6, 5, 7, 6, 7, 5, 6], [7, 5, 6, 7, 5, 7, 6, 5], [6, 7, 5, 6, 7, 6, 5, 7], [6, 5, 7, 6, 5, 6, 7, 5], [5, 7, 6, 5, 7, 5, 6, 7], [5, 6, 7, 5, 6, 5, 7, 6]]

4, "range", [3, 5, 6], [[6, 5, 3, 6, 5, 6, 3, 5], [6, 3, 5, 6, 3, 6, 5, 3], [5, 6, 3, 5, 6, 5, 3, 6], [5, 3, 6, 5, 3, 5, 6, 3], [3, 6, 5, 3, 6, 3, 5, 6], [3, 5, 6, 3, 5, 3, 6, 5]]

5, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

6, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

7, "range", [4, 5, 7], [[7, 5, 4, 7, 5, 7, 4, 5], [7, 4, 5, 7, 4, 7, 5, 4], [5, 7, 4, 5, 7, 5, 4, 7], [5, 4, 7, 5, 4, 5, 7, 4], [4, 7, 5, 4, 7, 4, 5, 7], [4, 5, 7, 4, 5, 4, 7, 5]]

8, "range", [3, 4, 5], [[5, 4, 3, 5, 4, 5, 3, 4], [5, 3, 4, 5, 3, 5, 4, 3], [4, 5, 3, 4, 5, 4, 3, 5], [4, 3, 5, 4, 3, 4, 5, 3], [3, 5, 4, 3, 5, 3, 4, 5], [3, 4, 5, 3, 4, 3, 5, 4]]

9, "range", [2, 4, 7], [[7, 4, 2, 7, 4, 7, 2, 4], [7, 2, 4, 7, 2, 7, 4, 2], [4, 7, 2, 4, 7, 4, 2, 7], [4, 2, 7, 4, 2, 4, 7, 2], [2, 7, 4, 2, 7, 2, 4, 7], [2, 4, 7, 2, 4, 2, 7, 4]]

10, "range", [2, 3, 4], [[4, 3, 2, 4, 3, 4, 2, 3], [4, 2, 3, 4, 2, 4, 3, 2], [3, 4, 2, 3, 4, 3, 2, 4], [3, 2, 4, 3, 2, 3, 4, 2], [2, 4, 3, 2, 4, 2, 3, 4], [2, 3, 4, 2, 3, 2, 4, 3]]

11, "range", [1, 7, 8], [[8, 7, 1, 8, 7, 8, 1, 7], [8, 1, 7, 8, 1, 8, 7, 1], [7, 8, 1, 7, 8, 7, 1, 8], [7, 1, 8, 7, 1, 7, 8, 1], [1, 8, 7, 1, 8, 1, 7, 8], [1, 7, 8, 1, 7, 1, 8, 7]]

12, "range", [1, 3, 8], [[8, 3, 1, 8, 3, 8, 1, 3], [8, 1, 3, 8, 1, 8, 3, 1], [3, 8, 1, 3, 8, 3, 1, 8], [3, 1, 8, 3, 1, 3, 8, 1], [1, 8, 3, 1, 8, 1, 3, 8], [1, 3, 8, 1, 3, 1, 8, 3]]

13, "range", [1, 2, 7], [[7, 2, 1, 7, 2, 7, 1, 2], [7, 1, 2, 7, 1, 7, 2, 1], [2, 7, 1, 2, 7, 2, 1, 7], [2, 1, 7, 2, 1, 2, 7, 1], [1, 7, 2, 1, 7, 1, 2, 7], [1, 2, 7, 1, 2, 1, 7, 2]]

14, "range", [1, 2, 3], [[3, 2, 1, 3, 2, 3, 1, 2], [3, 1, 2, 3, 1, 3, 2, 1], [2, 3, 1, 2, 3, 2, 1, 3], [2, 1, 3, 2, 1, 2, 3, 1], [1, 3, 2, 1, 3, 1, 2, 3], [1, 2, 3, 1, 2, 1, 3, 2]]

"group has", 6, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1$ =    [[1, 3, 2]]

$g_2$ =    [[1, 3]]

$g_3$ =    [[1, 2]]

$g_4$ =    [[1, 2, 3]]

$g_5$ =    []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & h[2] & 2\,h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3$ =
(2 0 0 0 13 0 0 0 0 0 1 0 0 0 0 0 0 0 0 11 4 0 0 0 0 0 0

{1, 5, 11, 21, 22, 29, 37, 40, 41, 45, 48, 52, 53, 56}

$u3$ =
(1 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 1 1 0 1 0 0 0 0 1

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56} picheck $(27 \;\; 27 \;\; 54 \;\; 54 \;\; 27 \;\; 27 \;\; 54 \;\; 54)$

$\pi = \left(\frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

$\pi 2 =$
$(15 \;\; 3 \;\; 0 \;\; 0 \;\; 0 \;\; 24 \;\; 12 \;\; 6 \;\; 12 \;\; 0 \;\; 0 \;\; 21 \;\; 0 \;\; 30 \;\; 24 \;\; 21 \;\; 0 \;\; 24 \;\; 12 \;\; 0 \;\; 24 \;\; 30 \;\; 15$

$u2 =$
$\left(\frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3}\right.$

picheck $(54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108)$

$\pi 1 = (54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108)$

$u1 = \left(\frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9}\right)$

picheck $(54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, −1, 1, 0, 1, 1, −1, 0]

$$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & 0 & -t & s & 0 & 0 & -s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t-s & 0 & 0 & 0 & s-t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t & 0 & -s & 0 & s & 0 & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & -1 & 1 & 0 \end{pmatrix}$

M0 is invertible. det= 79012645/12754584

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = (8)$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{35}{36} & \frac{7}{36} & 0 & 0 & 0 & \frac{14}{9} & \frac{7}{9} \\ \frac{35}{36} & \frac{11}{6} & \frac{7}{18} & \frac{7}{9} & 0 & 0 & \frac{49}{36} & 0 \\ \frac{7}{36} & \frac{7}{18} & \frac{11}{3} & \frac{35}{18} & \frac{14}{9} & \frac{49}{36} & 0 & \frac{14}{9} \\ 0 & \frac{7}{9} & \frac{35}{18} & \frac{11}{3} & \frac{7}{9} & 0 & \frac{14}{9} & \frac{35}{18} \\ 0 & 0 & \frac{14}{9} & \frac{7}{9} & \frac{11}{6} & \frac{35}{36} & \frac{7}{36} & 0 \\ 0 & 0 & \frac{49}{36} & 0 & \frac{35}{36} & \frac{11}{6} & \frac{7}{18} & \frac{7}{9} \\ \frac{14}{9} & \frac{49}{36} & 0 & \frac{14}{9} & \frac{7}{36} & \frac{7}{18} & \frac{11}{3} & \frac{35}{18} \\ \frac{7}{9} & 0 & \frac{14}{9} & \frac{35}{18} & 0 & \frac{7}{9} & \frac{35}{18} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega \left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T \left(0\ 0\ \frac{1}{2}\ 0\ 0\ 0\ 0\ \frac{1}{4}\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{21}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{322}{29}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{80}{17}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2$ / $r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau}$ / $n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 84

dim span idems 6 vs no. of idems 14

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {3, 5, 6}

"RG5" = {4, 7, 8}

"RG6" = {3, 4, 8}

"RG7" = {4, 5, 7}

"RG8" = {3, 4, 5}

"RG9" = {2, 4, 7}

"RG10" = {2, 3, 4}

"RG11" = {1, 7, 8}

"RG12" = {1, 3, 8}

"RG13" = {1, 2, 7}

"RG14" = {1, 2, 3}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{19}{36} & \frac{-25}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{2}{3} & \frac{-1}{9} \\ \frac{19}{36} & \frac{25}{18} & \frac{-1}{2} & \frac{-1}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{17}{36} & \frac{-8}{9} \\ \frac{-25}{36} & \frac{-1}{2} & \frac{17}{9} & \frac{1}{6} & \frac{2}{3} & \frac{17}{36} & \frac{-16}{9} & \frac{-2}{9} \\ \frac{-8}{9} & \frac{-1}{9} & \frac{1}{6} & \frac{17}{9} & \frac{-1}{9} & \frac{-8}{9} & \frac{-2}{9} & \frac{1}{6} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{2}{3} & \frac{-1}{9} & \frac{25}{18} & \frac{19}{36} & \frac{-25}{36} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{17}{36} & \frac{-8}{9} & \frac{19}{36} & \frac{25}{18} & \frac{-1}{2} & \frac{-1}{9} \\ \frac{2}{3} & \frac{17}{36} & \frac{-16}{9} & \frac{-2}{9} & \frac{-25}{36} & \frac{-1}{2} & \frac{17}{9} & \frac{1}{6} \\ \frac{-1}{9} & \frac{-8}{9} & \frac{-2}{9} & \frac{1}{6} & \frac{-8}{9} & \frac{-1}{9} & \frac{1}{6} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

 

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{19}{50} & \frac{-1}{2} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{12}{25} & \frac{-2}{25} \\ \frac{19}{50} & 1 & \frac{-9}{25} & \frac{-2}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{17}{50} & \frac{-16}{25} \\ \frac{-25}{68} & \frac{-9}{34} & 1 & \frac{3}{34} & \frac{6}{17} & \frac{1}{4} & \frac{-16}{17} & \frac{-2}{17} \\ \frac{-8}{17} & \frac{-1}{17} & \frac{3}{34} & 1 & \frac{-1}{17} & \frac{-8}{17} & \frac{-2}{17} & \frac{3}{34} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{12}{25} & \frac{-2}{25} & 1 & \frac{19}{50} & \frac{-1}{2} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{17}{50} & \frac{-16}{25} & \frac{19}{50} & 1 & \frac{-9}{25} & \frac{-2}{25} \\ \frac{6}{17} & \frac{1}{4} & \frac{-16}{17} & \frac{-2}{17} & \frac{-25}{68} & \frac{-9}{34} & 1 & \frac{3}{34} \\ \frac{-1}{17} & \frac{-8}{17} & \frac{-2}{17} & \frac{3}{34} & \frac{-8}{17} & \frac{-1}{17} & \frac{3}{34} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.8611111111, 3.046532935, 0.147911509, 0.1104320892, 1.445096208, 2.501945792, 4.998081466]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6200000000, 1.804468788, 0.0825900361, 0.07128478047, 0.8939777245, 1.505507151, 3.022171521]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues $M_0$

[0.8611111111, 9.184234303, 0.142384626, 2.756714404, 0.1104320892, 1.445096208, 2.501945792, 4.998081466]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0]}

Eigenvalues M

[−0.9722222222, 5.817495005, −3.159972983, 0.259144643, 2.237091539, −0.1670884876, −1.336848455, −2.677599040]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==================================================

{2, 3, 5, 8}

R: [4, 7, 5, 2, 7, 4, 4, 7]
B: [8, 3, 1, 6, 3, 8, 8, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{3}{256}(-1+s)(-7+s)(1+s)^3$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[4] v[7]
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 7 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 7 & 0 & 0 & 7 & 0 \\ 7 & 0 & 0 & 0 & 0 & 7 & 0 & 14 \\ 0 & 7 & 0 & 0 & 7 & 0 & 14 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 7 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 & 0 & 7 \\ 0 & 7 & 0 & 14 & 7 & 0 & 0 & 0 \\ 7 & 0 & 14 & 0 & 0 & 7 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{12}$ ( $v[1]v[3] + v[1]v[8] + v[2]v[4] + v[2]v[7] + v[3]v[6] + 2v[3]v[8] + v[4]$
$v[5] + 2v[4]v[7] + v[5]v[7] + v[6]v[8]$ )
degree 3 : $\frac{1}{4}$ ( $v[1]v[3]v[8] + v[2]v[4]v[7] + v[3]v[6]v[8] + v[4]v[5]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{3, 4}, {1, 6, 7}, {2, 5, 8}}

"RG1" = {4, 5, 7}

"RG2" = {2, 4, 7}

"RG3" = {3, 6, 8}

"RG4" = {1, 3, 8}

$\pi 3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 3$ = {11, 29, 45, 48}

$u3$ = [1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0]

supp u3 = {1, 2, 8, 11, 12, 15, 24, 25, 28, 29, 41, 42, 45, 46, 47, 48, 51, 52}

Action of R on ranges, [[2], [2], [1], [1]]
Action of B on ranges, [[3], [3], [4], [4]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 3, 1]

B−BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {3, 4}
$b_2$ = {1, 6, 7}
$b_3$ = {2, 5, 8}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: 0 ⊕ 11/9

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{2, 4, 7}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{3, 4}, {1, 6, 7}, {2, 5, 8}}

1, "range", [4, 5, 7], [[7, 5, 4, 4, 5, 7, 7, 5], [5, 4, 7, 7, 4, 5, 5, 4], [4, 7, 5, 5, 7, 4, 4, 7]]

2, "range", [2, 4, 7], [[7, 2, 4, 4, 2, 7, 7, 2], [4, 7, 2, 2, 7, 4, 4, 7], [2, 4, 7, 7, 4, 2, 2, 4]]

3, "range", [3, 6, 8], [[8, 3, 6, 6, 3, 8, 8, 3], [6, 8, 3, 3, 8, 6, 6, 8], [3, 6, 8, 8, 6, 3, 3, 6]]

4, "range", [1, 3, 8], [[8, 3, 1, 1, 3, 8, 8, 3], [3, 1, 8, 8, 1, 3, 3, 1], [1, 8, 3, 3, 8, 1, 1, 8]]

"group has", 3, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 3, 2]]

$g_3 =$ [[1, 2, 3]]

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[3] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{11, 29, 45, 48}

$u3 =$
$(1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1$

{1, 2, 8, 11, 12, 15, 24, 25, 28, 29, 41, 42, 45, 46, 47, 48, 51, 52}

picheck $(1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1\ \ 2\ \ 2)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 2 =$
$(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 2\ \ 1\ \ 0\ \ 2\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0)$

$u2 =$
$\left(\frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\right.$

picheck $(2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

$\pi 1 = (2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

$u1 = \left(\frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\right)$

picheck $(2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

Column Projections

 

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

 

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -2, 2, 1, -1, 2, -2]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t & -s & 0 & 0 & s & -t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & 1 & -2 & -1 & 2 \end{pmatrix}$

M0 is invertible. det= 484/6561

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & 0 & \frac{7}{4} & 0 & 0 & 0 & 0 & \frac{7}{4} \\ 0 & \frac{11}{6} & 0 & \frac{7}{4} & 0 & 0 & \frac{7}{4} & 0 \\ \frac{7}{4} & 0 & \frac{11}{3} & 0 & 0 & \frac{7}{4} & 0 & \frac{7}{2} \\ 0 & \frac{7}{4} & 0 & \frac{11}{3} & \frac{7}{4} & 0 & \frac{7}{2} & 0 \\ 0 & 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & \frac{7}{4} & 0 \\ 0 & 0 & \frac{7}{4} & 0 & 0 & \frac{11}{6} & 0 & \frac{7}{4} \\ 0 & \frac{7}{4} & 0 & \frac{7}{2} & \frac{7}{4} & 0 & \frac{11}{3} & 0 \\ \frac{7}{4} & 0 & \frac{7}{2} & 0 & 0 & \frac{7}{4} & 0 & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12}\right)$$

$$T\left(0 \ 0 \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{2} \ \frac{1}{4} \ 0 \ 0 \ 0 \ 0 \ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{3}{2}\ \ \frac{3}{2}\ \ \frac{42}{13}\ \ \frac{21}{13}\ \ 3\ \ \frac{84}{13}\ \ \frac{42}{13}\ \ \frac{21}{13}\ \ 3\ \ \frac{42}{13}\ \ \frac{21}{13}\ \ \frac{42}{13}\right)$$

$\boldsymbol{\tau} = 22/1$ , rank = 3 , ratio = 22/3 , $n^2 / r = 64/3$

$\boldsymbol{\tau}' = 42/1$ , $r' = 2/3$ , $\boldsymbol{\tau} / n^2 = 11/32$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 12

dim span idems 4 vs no. of idems 4

"PT1" = {{3, 4}, {1, 6, 7}, {2, 5, 8}}

"RG1" = {4, 5, 7}

"RG2" = {2, 4, 7}

"RG3" = {3, 6, 8}

"RG4" = {1, 3, 8}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{31}{36} \\ \frac{-4}{9} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} \\ \frac{31}{36} & \frac{-8}{9} & \frac{17}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{31}{36} & \frac{-16}{9} & \frac{31}{18} \\ \frac{-8}{9} & \frac{31}{36} & \frac{-16}{9} & \frac{17}{9} & \frac{31}{36} & \frac{-8}{9} & \frac{31}{18} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{31}{36} & \frac{25}{18} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{31}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{25}{18} & \frac{-8}{9} & \frac{31}{36} \\ \frac{-8}{9} & \frac{31}{36} & \frac{-16}{9} & \frac{31}{18} & \frac{31}{36} & \frac{-8}{9} & \frac{17}{9} & \frac{-16}{9} \\ \frac{31}{36} & \frac{-8}{9} & \frac{31}{18} & \frac{-16}{9} & \frac{-8}{9} & \frac{31}{36} & \frac{-16}{9} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{31}{50} \\ \frac{-8}{25} & 1 & \frac{-16}{25} & \frac{31}{50} & \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} \\ \frac{31}{68} & \frac{-8}{17} & 1 & \frac{-16}{17} & \frac{-8}{17} & \frac{31}{68} & \frac{-16}{17} & \frac{31}{34} \\ \frac{-8}{17} & \frac{31}{68} & \frac{-16}{17} & 1 & \frac{31}{68} & \frac{-8}{17} & \frac{31}{34} & \frac{-16}{17} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{31}{50} & 1 & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{31}{50} & \frac{-16}{25} & \frac{-8}{25} & 1 & \frac{-16}{25} & \frac{31}{50} \\ \frac{-8}{17} & \frac{31}{68} & \frac{-16}{17} & \frac{31}{34} & \frac{31}{68} & \frac{-8}{17} & 1 & \frac{-16}{17} \\ \frac{31}{68} & \frac{-8}{17} & \frac{31}{34} & \frac{-16}{17} & \frac{-8}{17} & \frac{31}{68} & \frac{-16}{17} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.1111111111, 8.900126261, 0.099873739, 1.833333333, 0.1666666667, 1.833333333, 0.1666666667]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.06941176471, 5.046983781, 0.067133867, 1.320000000, 0.08823529412, 1.320000000, 0.08823529412]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, −1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Eigenvalues $M_0$

[0.1666666667, 1.833333333, 8.900126261, 0.099873739, 0.1666666667, 1.833333333, 8.900126261, 0.099873739]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0, 0, 0]}

Eigenvalues M

[−3.500000000, 5.663118960, −2.163118960, −3.500000000, 5.663118960, −2.163118960, 0., 0.]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==========================================

{2, 3, 6, 8}

R: [4, 7, 5, 2, 3, 8, 4, 7]
B: [8, 3, 1, 6, 7, 4, 8, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{4096}(8+s^2)(-1+s)(3+s)(14-6s+s^2)(1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 2, "vs", 5

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 2, "vs", 5

"R CYCLES", (1 + v[3] v[5]) (1 + v[2] v[4] v[7])
"B CYCLES", (1 + v[4] v[6]) (1 + v[1] v[3] v[8])

Eigenvalues

R: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

B: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 2 & 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 4 & 2 & 0 & 4 & 0 \\ 4 & 2 & 0 & 4 & 0 & 4 & 4 & 6 \\ 2 & 4 & 4 & 0 & 4 & 0 & 6 & 4 \\ 0 & 2 & 0 & 4 & 0 & 0 & 4 & 2 \\ 2 & 0 & 4 & 0 & 0 & 0 & 2 & 4 \\ 0 & 4 & 4 & 6 & 4 & 2 & 0 & 4 \\ 4 & 0 & 6 & 4 & 2 & 4 & 4 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$

degree 2: $\frac{1}{18}$ ( $2\,v[1]v[3] + v[1]v[4] + v[1]v[6] + 2\,v[1]v[8] + v[2]v[3] + 2\,v[2]v[4] + v[$
$2]v[5] + 2\,v[2]v[7] + 2\,v[3]v[4] + 2\,v[3]v[6] + 2\,v[3]v[7] + 3\,v[3]v[8] + 2\,v[4]v[5] + 3\,v[$

4]$v$[7] + 2$v$[4]$v$[8] + 2$v$[5]$v$[7] + $v$[5]$v$[8] + $v$[6]$v$[7] + 2$v$[6]$v$[8] + 2$v$[8]$v$[7] )
degree 3 : $\frac{1}{24}$ ( $v$[1]$v$[3]$v$[4] + $v$[1]$v$[3]$v$[6] + 2$v$[1]$v$[3]$v$[8] + $v$[1]$v$[4]$v$[8] + $v$[1]$v$[6]$v$[
8] + $v$[2]$v$[3]$v$[4] + $v$[2]$v$[3]$v$[7] + $v$[2]$v$[4]$v$[5] + 2$v$[2]$v$[4]$v$[7] + $v$[2]$v$[5]$v$[7] + $v$[3]$v$
[4]$v$[7] + $v$[3]$v$[4]$v$[8] + $v$[3]$v$[6]$v$[7] + 2$v$[3]$v$[6]$v$[8] + $v$[3]$v$[8]$v$[7] + 2$v$[4]$v$[5]$v$[7] +
$v$[4]$v$[5]$v$[8] + $v$[4]$v$[8]$v$[7] + $v$[5]$v$[8]$v$[7] + $v$[6]$v$[8]$v$[7] )
degree 4 : $\frac{1}{6}$ ( $v$[1]$v$[3]$v$[4]$v$[8] + $v$[1]$v$[3]$v$[6]$v$[8] + $v$[2]$v$[3]$v$[4]$v$[7] + $v$[2]$v$[4]$v$[5]$v$[
7] + $v$[3]$v$[6]$v$[8]$v$[7] + $v$[4]$v$[5]$v$[8]$v$[7] )

Group spectrum $1 + t + t^2 + t^3 + t^4$

## KERNEL STRUCTURE

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"RG1" = {4, 5, 7, 8}

"RG2" = {3, 6, 7, 8}

"RG3" = {2, 4, 5, 7}

"RG4" = {2, 3, 4, 7}

"RG5" = {1, 3, 6, 8}

"RG6" = {1, 3, 4, 8}

$\pi$4 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0]

supp $\pi$4 = {19, 24, 38, 47, 65, 68}

$u$4 = [1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1]

supp u4 = {1, 3, 6, 10, 19, 24, 28, 33, 38, 43, 47, 52, 61, 65, 68, 70}

Action of R on ranges, [[4], [1], [4], [3], [1], [3]]
Action of B on ranges, [[2], [6], [2], [5], [6], [5]]

$\boldsymbol{\beta} = \left(\frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 1, 2, 4]

B−BLOCKS,

[2, 4, 3, 1]

with invariant measure, [1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{2, 8\}$
$b_2 = \{1, 7\}$
$b_3 = \{4, 6\}$
$b_4 = \{3, 5\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 4, 4, 4

## LIE STRUCTURE

Dimension of Lie algebra: 28, Shape: $8 \oplus 20/18$

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{2, 4, 7}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

1, "range", [4, 5, 7, 8], [[8, 7, 4, 5, 4, 5, 8, 7], [8, 5, 7, 4, 7, 4, 8, 5], [8, 4, 5, 7, 5, 7, 8, 4], [7, 8, 5, 4, 5, 4, 7, 8], [7, 5, 4, 8, 4, 8, 7, 5], [7, 4, 8, 5, 8, 5, 7, 4], [5, 8, 4, 7, 4, 7, 5, 8], [5, 7, 8, 4, 8, 4, 5, 7], [5, 4, 7, 8, 7, 8, 5, 4], [4, 8, 7, 5, 7, 5, 4, 8], [4, 7, 5, 8, 5, 8, 4, 7], [4, 5, 8, 7, 8, 7, 4, 5]]

2, "range", [3, 6, 7, 8], [[8, 7, 6, 3, 6, 3, 8, 7], [8, 6, 3, 7, 3, 7, 8, 6], [8, 3, 7, 6, 7, 6, 8, 3], [7, 8, 3, 6, 3, 6, 7, 8], [7, 6, 8, 3, 8, 3, 7, 6], [7, 3, 6, 8, 6, 8, 7, 3], [6, 8, 7, 3, 7, 3, 6, 8], [6, 7, 3, 8, 3, 8, 6, 7], [6, 3, 8, 7, 8, 7, 6, 3], [3, 8, 6, 7, 6, 7, 3, 8], [3, 7, 8, 6, 8, 6, 3, 7], [3, 6, 7, 8, 7, 8, 3, 6]]

3, "range", [2, 4, 5, 7], [[7, 5, 4, 2, 4, 2, 7, 5], [7, 4, 2, 5, 2, 5, 7, 4], [7, 2, 5, 4, 5, 4, 7, 2], [5, 7, 2, 4, 2, 4, 5, 7], [5, 4, 7, 2, 7, 2, 5, 4], [5, 2, 4, 7, 4, 7, 5, 2], [4, 7, 5, 2, 5, 2, 4, 7], [4, 5, 2, 7, 2, 7, 4, 5], [4, 2, 7, 5, 7, 5, 4, 2], [2, 7, 4, 5, 4, 5, 2, 7], [2, 5, 7, 4, 7, 4, 2, 5], [2, 4, 5, 7, 5, 7, 2, 4]]

4, "range", [2, 3, 4, 7], [[7, 4, 2, 3, 2, 3, 7, 4], [7, 3, 4, 2, 4, 2, 7, 3], [7, 2, 3, 4, 3, 4, 7, 2], [4, 7, 3, 2, 3, 2, 4, 7], [4, 3, 2, 7, 2, 7, 4, 3], [4, 2, 7, 3, 7, 3, 4, 2], [3, 7, 2, 4, 2, 4, 3, 7], [3, 4, 7, 2, 7, 2, 3, 4], [3, 2, 4, 7, 4, 7, 3, 2], [2, 7, 4, 3, 4, 3, 2, 7], [2, 4, 3, 7, 3, 7, 2, 4], [2, 3, 7, 4, 7, 4, 2, 3]]

5, "range", [1, 3, 6, 8], [[8, 6, 3, 1, 3, 1, 8, 6], [8, 3, 1, 6, 1, 6, 8, 3], [8, 1, 6, 3, 6, 3, 8, 1], [6, 8, 1, 3, 1, 3, 6, 8], [6, 3, 8, 1, 8, 1, 6, 3], [6, 1, 3, 8, 3, 8, 6, 1], [3, 8, 6, 1, 6, 1, 3, 8], [3, 6, 1, 8, 1, 8, 3, 6], [3, 1, 8, 6, 8, 6, 3, 1], [1, 8, 3, 6, 3, 6, 1, 8], [1, 6, 8, 3, 8, 3, 1, 6], [1, 3, 6, 8, 6, 8, 1, 3]]

6, "range", [1, 3, 4, 8], [[8, 4, 3, 1, 3, 1, 8, 4], [8, 3, 1, 4, 1, 4, 8, 3], [8, 1, 4, 3, 4, 3, 8, 1], [4, 8, 1, 3, 1, 3, 4, 8], [4, 3, 8, 1, 8, 1, 4, 3], [4, 1, 3, 8, 3, 8, 4, 1], [3, 8, 4, 1, 4, 1, 3, 8], [3, 4, 1, 8, 1, 8, 3, 4], [3, 1, 8, 4, 8, 4, 3, 1], [1, 8, 3, 4, 3, 4, 1, 8], [1, 4, 8, 3, 8, 3, 1, 4], [1, 3, 4, 8, 4, 8, 1, 3]]

"group has", 12, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2], [3, 4]]

$g_2 =$ [[2, 3, 4]]

$g_3 =$ [[1, 3, 4]]

$g_4 =$ []

$g_5 =$ [[1, 4, 2]]

linear dimension, 10

"Symmetric?", true

Is Z in Vec(K)? true

$$\begin{pmatrix} 0 & 0 & h[2] & 3h[1]-h[2] & h[2] & 0 & 0 & h[2] & 0 & h[2] \end{pmatrix}$$

"Basis for Z(G)"

1, "coeff", 3

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 3. & -1. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 4 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 4 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 6t^5 + 10t^6 + 12t^7 + 17t^8 + 21t^9 + 28t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 4$ =
$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0$

{19, 24, 38, 47, 65, 68}

$u4$ =
$(1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0$

{1, 3, 6, 10, 19, 24, 28, 33, 38, 43, 47, 52, 61, 65, 68, 70}

picheck $(2\ 2\ 4\ 4\ 2\ 2\ 4\ 4)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 3 =$
$(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 2$

$u3 =$
$\left(\frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ 0\right.$

picheck $(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

$\pi 2 =$
$(0\ \ 4\ \ 2\ \ 0\ \ 2\ \ 0\ \ 4\ \ 2\ \ 4\ \ 2\ \ 0\ \ 4\ \ 0\ \ 4\ \ 0\ \ 4\ \ 4\ \ 6\ \ 4\ \ 0\ \ 6\ \ 4\ \ 0\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

$u2 =$
$\left(\frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ 0\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ 0\ \ \frac{1}{8}\ \ 0\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ 0\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\right.$

picheck $(12\ \ 12\ \ 24\ \ 24\ \ 12\ \ 12\ \ 24\ \ 24)$

$\pi 1 = (12\ \ 12\ \ 24\ \ 24\ \ 12\ \ 12\ \ 24\ \ 24)$

$u1 = \left(\frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\ \ \frac{3}{32}\right)$

picheck $(12\ \ 12\ \ 24\ \ 24\ \ 12\ \ 12\ \ 24\ \ 24)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 \\ \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 & \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 & \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} \\ 4 & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, 1, 1, -1, 1, -1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & 0 & s & 0 & -s & 0 & t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & 0 & t & 0 & -t & 0 & s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t \\ -t & -t+s & -t \\ -s & t-s & -s \\ t & 0 & s \\ -s & t-s & -s \\ t & 0 & s \\ s & 0 & t \\ -t & -t+s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & s \\ s & 0 & t & 0 \\ t & 0 & s & 0 \\ 0 & s & 0 & t \\ t & 0 & s & 0 \\ 0 & s & 0 & t \\ 0 & t & 0 & s \\ s & 0 & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & 0 & \frac{4}{3} & \frac{2}{3} & 0 & \frac{2}{3} & 0 & \frac{4}{3} \\ 0 & \frac{4}{3} & \frac{2}{3} & \frac{4}{3} & \frac{2}{3} & 0 & \frac{4}{3} & 0 \\ \frac{4}{3} & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} & 0 & \frac{4}{3} & \frac{4}{3} & 2 \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{4}{3} & 0 & 2 & \frac{4}{3} \\ 0 & \frac{2}{3} & 0 & \frac{4}{3} & \frac{4}{3} & 0 & \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{2}{3} & \frac{4}{3} \\ 0 & \frac{4}{3} & \frac{4}{3} & 2 & \frac{4}{3} & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} \\ \frac{4}{3} & 0 & 2 & \frac{4}{3} & \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$\Omega$

$$\left(\frac{1}{12} \quad \frac{2}{3} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{5}{12} \quad -1 \quad 1 \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{7}{12} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12}\right.$$

$$T \left(\frac{2}{3} \quad 0 \quad 0 \quad 0 \quad \frac{1}{6} \quad \frac{-1}{3} \quad \frac{1}{3} \quad 0 \quad \frac{2}{3} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{3} \quad 0 \quad 0 \quad \frac{2}{3} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{3}\right)$$

"IS NM in Vec(K)?", true

NM

$\left(\frac{16}{3}\ \ \frac{64}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ 14\ \ \frac{-100}{3}\ \ \frac{100}{3}\ \ \frac{16}{3}\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{56}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ \frac{16}{3}\ \ 4\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ 8\ \ \frac{8}{3}\ \ \frac{8}{3}\right.$

"IS MN in Vec(K)?", false

MN

$(2\ \ 20\ \ 4\ \ 4\ \ 15\ \ -38\ \ 38\ \ 4\ \ 2\ \ 4\ \ 4\ \ 20\ \ 4\ \ 4\ \ 4\ \ 4\ \ 6\ \ 4\ \ 4\ \ 6\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4$

$$\boldsymbol{\tau} = 16/1 \text{ , rank} = 4 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 12

KERNEL HAS LINEAR DIMENSION 26
out of total no. of elements equal to 72

dim span idems 5 vs no. of idems 6

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"RG1" = {4, 5, 7, 8}

"RG2" = {3, 6, 7, 8}

"RG3" = {2, 4, 5, 7}

"RG4" = {2, 3, 4, 7}

"RG5" = {1, 3, 6, 8}

"RG6" = {1, 3, 4, 8}

$$M_c = \begin{pmatrix} \frac{8}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{2}{9} & \frac{-8}{9} & \frac{4}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{2}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} \\ \frac{4}{9} & \frac{-2}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{2}{9} \\ \frac{-2}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{4}{9} & \frac{-8}{9} & \frac{2}{9} & \frac{-4}{9} \\ \frac{-4}{9} & \frac{2}{9} & \frac{-8}{9} & \frac{4}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-2}{9} & \frac{4}{9} \\ \frac{-8}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{2}{9} & \frac{4}{9} & \frac{-2}{9} & \frac{8}{9} & \frac{-4}{9} \\ \frac{4}{9} & \frac{-8}{9} & \frac{2}{9} & \frac{-4}{9} & \frac{-2}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{8}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & -1 & \frac{1}{2} \\ \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{-1}{4} & 1 & \frac{-1}{2} & -1 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{2} & \frac{-1}{2} & 1 & \frac{1}{2} & -1 & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{4} & -1 & \frac{1}{2} & 1 & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{4} \\ \frac{1}{4} & \frac{-1}{2} & \frac{1}{2} & -1 & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{1}{2} \\ -1 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{1}{2} & -1 & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{1}{2} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[4., 0.4444444444, 1.333333333, 1.333333333, 0., 0., 0., 0.]

Eigenvalues $N_C$

[0.8888888889, 2., 2., 2., 0., 0., 0., 0.]

Eigenvalues $M_C$−scaled

[4.500000000, 0.5000000000, 1.500000000, 1.500000000, 0., 0., 0., 0.]

Eigenvalues $N_C$−scaled

[1.032258065, 2.322580645, 2.322580645, 2.322580645, 0., 0., 0., 0.]

NullSpace $M_C$

{[0, 1, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace $N_C$

{[0, 0, 0, 1, 0, −1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, −1, 0, 0, 0]}

Eigenvalues $M_0$

[4., 8.935416159, 0.397917175, 1.333333333, 1.333333333, 0., 0., 0.]

Eigenvalues $N_0$

[2., 2., 2., 2., 0., 0., 0., 0.]

NullSpace $M_0$

{[0, 1, 0, −1, 0, −1, 0, 1], [0, 0, 1, −1, 1, −1, 0, 0], [1, 0, 0, −1, 0, −1, 1, 0]}

NullSpace $N_0$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0]}

Eigenvalues M

[2.108185107, −2.108185107, 6.553967931, −1.220634597, −0.3905242916, −2.276142374, −0.3905242916, −2.276142374]

Eigenvalues N

[6., −2., −2., −2., 0., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 0, 1, 0, −1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==========================================

80, [1, −1, 1, −1, 1, 1, −1, −1]

==========================================

{2, 5, 6, 7}

R: [4, 7, 1, 2, 7, 8, 8, 3]
B: [8, 3, 5, 6, 3, 4, 4, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{3}{4096}(-224 + 20s - 50s^2 + 19s^3 + 11s^4 + 6s^5 + 2s^6)(-1 + s)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 4, "vs", 6

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[8] v[7]
"B CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5])

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [1., -1., 1., -1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, -1, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, -1, 0]}

NullSpace of $B^*$

{[0, -1, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, -1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 35 & 28 & 14 & 0 & 0 & 0 & 49 \\ 35 & 0 & 0 & 7 & 0 & 0 & 28 & 56 \\ 28 & 0 & 0 & 70 & 0 & 28 & 70 & 56 \\ 14 & 7 & 70 & 0 & 49 & 56 & 56 & 0 \\ 0 & 0 & 0 & 49 & 0 & 35 & 28 & 14 \\ 0 & 0 & 28 & 56 & 35 & 0 & 0 & 7 \\ 0 & 28 & 70 & 56 & 28 & 0 & 0 & 70 \\ 49 & 56 & 56 & 0 & 14 & 7 & 70 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3    "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{5}{108}$ ( $5v[1]v[2] + 4v[1]v[3] + 2v[1]v[4] + 7v[1]v[8] + v[2]v[4] + 4v[2]v[7] + 8v[2]v[8] + 10v[3]v[4] + 4v[3]v[6] + 10v[3]v[7] + 8v[3]v[8] + 7v[4]v[5] + 8v[4]v[6] + 8v[4]v[7] + 5v[5]v[6] + 4v[5]v[7] + 2v[5]v[8] + v[6]v[8] + 10v[8]v[7]$ )
degree 3 : $\frac{1}{54}$ ( $2v[1]v[2]v[4] + 13v[1]v[2]v[8] + 4v[1]v[3]v[4] + 8v[1]v[3]v[8] + v[2]v[4]v[7] + 11v[2]v[8]v[7] + 11v[3]v[4]v[6] + 15v[3]v[4]v[7] + v[3]v[6]v[8] + 15v[3]v[8]v[7] + 13v[4]v[5]v[6] + 8v[4]v[5]v[7] + 2v[5]v[6]v[8] + 4v[5]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 7, 8}

"RG6" = {2, 4, 7}

"RG7" = {5, 6, 8}

"RG8" = {4, 5, 6}

"RG9" = {3, 6, 8}

"RG10" = {3, 4, 6}

"RG11" = {1, 3, 8}

"RG12" = {1, 3, 4}

"RG13" = {1, 2, 8}

"RG14" = {1, 2, 4}

$\pi 3$ = [0, 2, 0, 0, 0, 13, 4, 0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 11, 0, 11, 15, 0, 0, 0, 0, 0, 1, 15, 13, 8, 0, 0, 0, 0, 0, 2, 4, 0]

supp $\pi 3$ = {2, 6, 7, 11, 29, 36, 38, 39, 45, 46, 47, 48, 54, 55}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[3], [5], [11], [13], [3], [5], [3], [5], [11], [13], [12], [14], [4], [6]]
Action of B on ranges, [[4], [10], [2], [8], [4], [10], [4], [10], [2], [8], [1], [7], [3], [9]]

$$\boldsymbol{\beta} = \left(\frac{1}{27} \quad \frac{2}{27} \quad \frac{5}{36} \quad \frac{5}{36} \quad \frac{11}{108} \quad \frac{1}{108} \quad \frac{1}{54} \quad \frac{13}{108} \quad \frac{1}{108} \quad \frac{11}{108} \quad \frac{2}{27} \quad \frac{1}{27} \quad \frac{13}{108} \quad \frac{1}{54}\right)$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 3, 1]

B−BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {4, 8}
$b_2$ = {1, 6, 7}
$b_3$ = {2, 3, 5}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 34, Shape: 11 ⊕ 23/21

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{5}{16} & \frac{3}{16} & \frac{5}{16} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{4, 8}, {1, 6, 7}, {2, 3, 5}}

1, "range", [5, 7, 8], [[8, 7, 7, 5, 7, 8, 8, 5], [8, 5, 5, 7, 5, 8, 8, 7], [7, 8, 8, 5, 8, 7, 7, 5], [7, 5, 5, 8, 5, 7, 7, 8], [5, 8, 8, 7, 8, 5, 5, 7], [5, 7, 7, 8, 7, 5, 5, 8]]

2, "range", [4, 5, 7], [[7, 5, 5, 4, 5, 7, 7, 4], [7, 4, 4, 5, 4, 7, 7, 5], [5, 7, 7, 4, 7, 5, 5, 4], [5, 4, 4, 7, 4, 5, 5, 7], [4, 7, 7, 5, 7, 4, 4, 5], [4, 5, 5, 7, 5, 4, 4, 7]]

3, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

4, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

5, "range", [2, 7, 8], [[8, 7, 7, 2, 7, 8, 8, 2], [8, 2, 2, 7, 2, 8, 8, 7], [7, 8, 8, 2, 8, 7, 7, 2], [7, 2, 2, 8, 2, 7, 7, 8], [2, 8, 8, 7, 8, 2, 2, 7], [2, 7, 7, 8, 7, 2, 2, 8]]

6, "range", [2, 4, 7], [[7, 4, 4, 2, 4, 7, 7, 2], [7, 2, 2, 4, 2, 7, 7, 4], [4, 7, 7, 2, 7, 4, 4, 2], [4, 2, 2, 7, 2, 4, 4, 7], [2, 7, 7, 4, 7, 2, 2, 4], [2, 4, 4, 7, 4, 2, 2, 7]]

7, "range", [5, 6, 8], [[8, 6, 6, 5, 6, 8, 8, 5], [8, 5, 5, 6, 5, 8, 8, 6], [6, 8, 8, 5, 8, 6, 6, 5], [6, 5, 5, 8, 5, 6, 6, 8], [5, 8, 8, 6, 8, 5, 5, 6], [5, 6, 6, 8, 6, 5, 5, 8]]

8, "range", [4, 5, 6], [[6, 5, 5, 4, 5, 6, 6, 4], [6, 4, 4, 5, 4, 6, 6, 5], [5, 6, 6, 4, 6, 5, 5, 4], [5, 4, 4, 6, 4, 5, 5, 6], [4, 6, 6, 5, 6, 4, 4, 5], [4, 5, 5, 6, 5, 4, 4, 6]]

9, "range", [3, 6, 8], [[8, 6, 6, 3, 6, 8, 8, 3], [8, 3, 3, 6, 3, 8, 8, 6], [6, 8, 8, 3, 8, 6, 6, 3], [6, 3, 3, 8, 3, 6, 6, 8], [3, 8, 8, 6, 8, 3, 3, 6], [3, 6, 6, 8, 6, 3, 3, 8]]

10, "range", [3, 4, 6], [[6, 4, 4, 3, 4, 6, 6, 3], [6, 3, 3, 4, 3, 6, 6, 4], [4, 6, 6, 3, 6, 4, 4, 3], [4, 3, 3, 6, 3, 4, 4, 6], [3, 6, 6, 4, 6, 3, 3, 4], [3, 4, 4, 6, 4, 3, 3, 6]]

11, "range", [1, 3, 8], [[8, 3, 3, 1, 3, 8, 8, 1], [8, 1, 1, 3, 1, 8, 8, 3], [3, 8, 8, 1, 8, 3, 3, 1], [3, 1, 1, 8, 1, 3, 3, 8], [1, 8, 8, 3, 8, 1, 1, 3], [1, 3, 3, 8, 3, 1, 1, 8]]

12, "range", [1, 3, 4], [[4, 3, 3, 1, 3, 4, 4, 1], [4, 1, 1, 3, 1, 4, 4, 3], [3, 4, 4, 1, 4, 3, 3, 1], [3, 1, 1, 4, 1, 3, 3, 4], [1, 4, 4, 3, 4, 1, 1, 3], [1, 3, 3, 4, 3, 1, 1, 4]]

13, "range", [1, 2, 8], [[8, 2, 2, 1, 2, 8, 8, 1], [8, 1, 1, 2, 1, 8, 8, 2], [2, 8, 8, 1, 8, 2, 2, 1], [2, 1, 1, 8, 1, 2, 2, 8], [1, 8, 8, 2, 8, 1, 1, 2], [1, 2, 2, 8, 2, 1, 1, 8]]

14, "range", [1, 2, 4], [[4, 2, 2, 1, 2, 4, 4, 1], [4, 1, 1, 2, 1, 4, 4, 2], [2, 4, 4, 1, 4, 2, 2, 1], [2, 1, 1, 4, 1, 2, 2, 4], [1, 4, 4, 2, 4, 1, 1, 2], [1, 2, 2, 4, 2, 1, 1, 4]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$\left(0 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 13 \;\; 4 \;\; 0 \;\; 0 \;\; 0 \;\; 8 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1\right.$

{2, 6, 7, 11, 29, 36, 38, 39, 45, 46, 47, 48, 54, 55}

$u3 =$
$\left(0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1\right.$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $\left(27 \;\; 27 \;\; 54 \;\; 54 \;\; 27 \;\; 27 \;\; 54 \;\; 54\right)$

$\pi = \left(\frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

$\pi 2 =$
$\left(15 \;\; 12 \;\; 6 \;\; 0 \;\; 0 \;\; 0 \;\; 21 \;\; 0 \;\; 3 \;\; 0 \;\; 0 \;\; 12 \;\; 24 \;\; 30 \;\; 0 \;\; 12 \;\; 30 \;\; 24 \;\; 21 \;\; 24 \;\; 24 \;\; 0 \;\; 15\right.$

$u2 =$
$\left(\frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3}\right.$

picheck $\left(54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108\right)$

$\pi 1 = \left(54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108\right)$

$u1 = \left(\frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9}\right)$

picheck $\left(54 \;\; 54 \;\; 108 \;\; 108 \;\; 54 \;\; 54 \;\; 108 \;\; 108\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks NO-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks NO-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks NO-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 0, -1, -1, -1, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & -t & 0 & t & 0 & -s & 0 \\ 0 & s & -s & 0 & 0 & t & -t & 0 \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & -1 & -1 & 0 \end{pmatrix}$

M0 is invertible. det= 79012645/12754584

$$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = (8)$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{35}{36} & \frac{7}{9} & \frac{7}{18} & 0 & 0 & 0 & \frac{49}{36} \\ \frac{35}{36} & \frac{11}{6} & 0 & \frac{7}{36} & 0 & 0 & \frac{7}{9} & \frac{14}{9} \\ \frac{7}{9} & 0 & \frac{11}{3} & \frac{35}{18} & 0 & \frac{7}{9} & \frac{35}{18} & \frac{14}{9} \\ \frac{7}{18} & \frac{7}{36} & \frac{35}{18} & \frac{11}{3} & \frac{49}{36} & \frac{14}{9} & \frac{14}{9} & 0 \\ 0 & 0 & 0 & \frac{49}{36} & \frac{11}{6} & \frac{35}{36} & \frac{7}{9} & \frac{7}{18} \\ 0 & 0 & \frac{7}{9} & \frac{14}{9} & \frac{35}{36} & \frac{11}{6} & 0 & \frac{7}{36} \\ 0 & \frac{7}{9} & \frac{35}{18} & \frac{14}{9} & \frac{7}{9} & 0 & \frac{11}{3} & \frac{35}{18} \\ \frac{49}{36} & \frac{14}{9} & \frac{14}{9} & 0 & \frac{7}{18} & \frac{7}{36} & \frac{35}{18} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{2}\ 0\ 0\ 0\ 1\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{35}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{35}{2}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{18}{58}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2$ / $r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau}$ / $n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$–check is positive? 118/9

max r = 36/5 , $r$–check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 84

dim span idems 6 vs no. of idems 14

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 7, 8}

"RG6" = {2, 4, 7}

"RG7" = {5, 6, 8}

"RG8" = {4, 5, 6}

"RG9" = {3, 6, 8}

"RG10" = {3, 4, 6}

"RG11" = {1, 3, 8}

"RG12" = {1, 3, 4}

"RG13" = {1, 2, 8}

"RG14" = {1, 2, 4}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{19}{36} & \frac{-1}{9} & \frac{-1}{2} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{17}{36} \\ \frac{19}{36} & \frac{25}{18} & \frac{-8}{9} & \frac{-25}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{9} & \frac{2}{3} \\ \frac{-1}{9} & \frac{-8}{9} & \frac{17}{9} & \frac{1}{6} & \frac{-8}{9} & \frac{-1}{9} & \frac{1}{6} & \frac{-2}{9} \\ \frac{-1}{2} & \frac{-25}{36} & \frac{1}{6} & \frac{17}{9} & \frac{17}{36} & \frac{2}{3} & \frac{-2}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{17}{36} & \frac{25}{18} & \frac{19}{36} & \frac{-1}{9} & \frac{-1}{2} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{9} & \frac{2}{3} & \frac{19}{36} & \frac{25}{18} & \frac{-8}{9} & \frac{-25}{36} \\ \frac{-8}{9} & \frac{-1}{9} & \frac{1}{6} & \frac{-2}{9} & \frac{-1}{9} & \frac{-8}{9} & \frac{17}{9} & \frac{1}{6} \\ \frac{17}{36} & \frac{2}{3} & \frac{-2}{9} & \frac{-16}{9} & \frac{-1}{2} & \frac{-25}{36} & \frac{1}{6} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{19}{50} & \frac{-2}{25} & \frac{-9}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{17}{50} \\ \frac{19}{50} & 1 & \frac{-16}{25} & \frac{-1}{2} & \frac{-8}{25} & \frac{-8}{25} & \frac{-2}{25} & \frac{12}{25} \\ \frac{-1}{17} & \frac{-8}{17} & 1 & \frac{3}{34} & \frac{-8}{17} & \frac{-1}{17} & \frac{3}{34} & \frac{-2}{17} \\ \frac{-9}{34} & \frac{-25}{68} & \frac{3}{34} & 1 & \frac{1}{4} & \frac{6}{17} & \frac{-2}{17} & \frac{-16}{17} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{17}{50} & 1 & \frac{19}{50} & \frac{-2}{25} & \frac{-9}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-2}{25} & \frac{12}{25} & \frac{19}{50} & 1 & \frac{-16}{25} & \frac{-1}{2} \\ \frac{-8}{17} & \frac{-1}{17} & \frac{3}{34} & \frac{-2}{17} & \frac{-1}{17} & \frac{-8}{17} & 1 & \frac{3}{34} \\ \frac{1}{4} & \frac{6}{17} & \frac{-2}{17} & \frac{-16}{17} & \frac{-9}{34} & \frac{-25}{68} & \frac{3}{34} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

 

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.8611111111, 3.046532935, 0.147911509, 0.1104320892, 1.445096208, 2.501945792, 4.998081466]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6200000000, 1.804468788, 0.0825900361, 0.07128478047, 0.8939777245, 1.505507151, 3.022171521]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues $M_0$

[0.8611111111, 9.184234303, 0.142384626, 2.756714404, 0.1104320892, 1.445096208, 2.501945792, 4.998081466]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Eigenvalues M

[−0.9722222222, 5.817495005, −3.159972983, 0.259144643, 2.237091539, −0.1670884876, −1.336848455, −2.677599040]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==================================================

{3, 4, 5, 6}

R: [4, 3, 5, 6, 7, 8, 4, 3]
B: [8, 7, 1, 2, 3, 4, 8, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{3}{4096}(14 + 2s + 3s^2)(2 + s + s^2)(-12 - 3s + s^2)(-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[3] v[4] v[5] v[6] v[8] v[7]
"B CYCLES", 1 + v[8] v[7]

Eigenvalues

R: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, −1, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]$ )
degree 2: $\frac{1}{6}$ ( $v[1]v[2] + 2\,v[3]v[4] + v[5]v[6] + 2\,v[8]v[7]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT2" = {{2, 3, 6, 8}, {1, 4, 5, 7}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [4, 3, 1, 1, 3, 0, 4, 1, 3, 3, 1, 4, 0, 4, 2, 2, 3, 1, 2, 2, 1, 3, 4, 1, 3, 3, 1, 4]

supp u2 = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [2], [3]]
Action of B on ranges, [[1], [3], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 3, 1]

BPARTS [2, 2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[5, 4, 2, 6, 3, 1]

B−BLOCKS,

[2, 1, 1, 6, 3, 2]

with invariant measure, [2, 2, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{2, 3, 6, 8\}$
$b_2 = \{1, 4, 5, 7\}$
$b_3 = \{1, 4, 6, 7\}$
$b_4 = \{1, 3, 5, 7\}$
$b_5 = \{2, 4, 6, 8\}$
$b_6 = \{2, 3, 5, 8\}$

dim(span of partition vectors), rank($N_0$), rank($N$):    4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 21, Shape: $11 \oplus 10/8$

 

$$CLB = \begin{pmatrix} 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{3, 4, 5, 6, 7, 8}}, true

$\Omega_B$ in Vec(K)? , {{7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

2, "partition", {{2, 3, 6, 8}, {1, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 8, 7, 8, 7], [7, 8, 8, 7, 7, 8, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 6, 5, 6, 5], [5, 6, 6, 5, 5, 6, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3, 4, 3], [3, 4, 4, 3, 3, 4, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 2, 1, 2, 1], [1, 2, 2, 1, 1, 2, 1, 2]]

3, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 8, 7, 8, 7], [7, 8, 7, 8, 7, 8, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 6, 5, 6, 5], [5, 6, 5, 6, 5, 6, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 4, 3, 4, 3], [3, 4, 3, 4, 3, 4, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 2, 1, 2, 1], [1, 2, 1, 2, 1, 2, 1, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$\begin{pmatrix} 4 & 3 & 1 & 1 & 3 & 0 & 4 & 1 & 3 & 3 & 1 & 4 & 0 & 4 & 2 & 2 & 3 & 1 & 2 & 2 & 1 & 3 & 4 & 1 & 3 & 3 & 1 & 4 \end{pmatrix}$

{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks NO-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks NO-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{12} & \frac{1}{4} & \frac{1}{8} & \frac{1}{24} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{12} & \frac{1}{24} & \frac{1}{8} & 0 & \frac{1}{3} \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{3} & 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{24} & 0 & \frac{1}{3} & \frac{1}{12} & \frac{1}{12} & \frac{1}{4} & \frac{1}{12} \\ \frac{1}{8} & \frac{1}{24} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{12} \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{12} & \frac{1}{4} \\ \frac{1}{6} & 0 & \frac{1}{12} & \frac{1}{4} & \frac{1}{8} & \frac{1}{24} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{12} & \frac{1}{24} & \frac{1}{8} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{4}{3} & 4 & 2 & \frac{2}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 4 & \frac{4}{3} & \frac{2}{3} & 2 & 0 & \frac{16}{3} \\ \frac{2}{3} & 2 & \frac{16}{3} & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 4 \\ 2 & \frac{2}{3} & 0 & \frac{16}{3} & \frac{4}{3} & \frac{4}{3} & 4 & \frac{4}{3} \\ 2 & \frac{2}{3} & \frac{8}{3} & \frac{8}{3} & \frac{8}{3} & 0 & 4 & \frac{4}{3} \\ \frac{2}{3} & 2 & \frac{8}{3} & \frac{8}{3} & 0 & \frac{8}{3} & \frac{4}{3} & 4 \\ \frac{8}{3} & 0 & \frac{4}{3} & 4 & 2 & \frac{2}{3} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 4 & \frac{4}{3} & \frac{2}{3} & 2 & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 1, 1, 1, -1, -1]$

$$\ker N_C = \begin{pmatrix} -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & t & -s & -s & s & s & -t & -t \\ 0 & 0 & -s+t & -s+t & 0 & 0 & -t+s & -t+s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & t \\ -s & 0 & 0 & -t \\ 0 & -s & -t & 0 \\ 0 & s & t & 0 \\ -t & 0 & 0 & -s \\ t & 0 & 0 & s \\ s & 0 & 0 & t \\ -s & 0 & 0 & -t \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & t & 0 & 0 & 0 \\ t & -t & 0 & s+t & 0 \\ t & 0 & -t & t & s \\ s & 0 & t & s & -s \\ s & -s & 0 & s+t & 0 \\ t & s & 0 & 0 & 0 \\ s & t & 0 & 0 & 0 \\ t & -t & 0 & s+t & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 1 & 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ 0 & 1 & 1 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ 1 & 0 & 0 & 1 & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 1 & 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ 1 & 0 & 0 & 1 & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{24} & \frac{1}{8} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{8} & \frac{1}{24} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{-1}{24} & \frac{-1}{8} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{8} & \frac{-1}{24} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{8} & \frac{1}{24} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{24} & \frac{1}{8} \\ \frac{-1}{6} & 0 & 0 & 0 & \frac{-1}{8} & \frac{-1}{24} & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{24} & \frac{-1}{8} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \\ \frac{1}{4} & \frac{3}{4} & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega \left( \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \right)$$

$$T \left( \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{8} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{3} \ \frac{1}{24} \ 0 \ \frac{1}{3} \ \frac{1}{24} \ \frac{1}{8} \ \frac{1}{4} \ \frac{1}{12} \ 0 \ \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(4\ \ \tfrac{8}{3}\ \ \tfrac{8}{3}\ \ 2\ \ \tfrac{4}{3}\ \ \tfrac{4}{3}\ \ \tfrac{16}{3}\ \ \tfrac{2}{3}\ \ 0\ \ \tfrac{16}{3}\ \ \tfrac{2}{3}\ \ 2\ \ 4\ \ \tfrac{4}{3}\ \ 0\ \ \tfrac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\tfrac{8}{3}\ \ \tfrac{10}{3}\ \ 2\ \ \tfrac{8}{3}\ \ \tfrac{2}{3}\ \ 2\ \ \tfrac{14}{3}\ \ \tfrac{2}{3}\ \ 0\ \ 4\ \ 1\ \ 3\ \ 3\ \ 1\ \ 0\ \ 4\right)$$

$$\boldsymbol{\tau} = 32/1\ ,\ \text{rank} = 2\ ,\ \text{ratio} = 16/1\ ,\ n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\ ,\ r' = 1/2\ ,\ \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36\ ,\ \text{min}\ \boldsymbol{\tau} = 80/9\ ,\ \boldsymbol{\tau}\text{–check is positive? } 208/9$$

$$\text{max r} = 36/5\ ,\ r\text{–check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 3, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 24

dim span idems 12 vs no. of idems 12

"PT1" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT2" = {{2, 3, 6, 8}, {1, 4, 5, 7}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{1}{9} & \frac{11}{18} & \frac{11}{18} & \frac{1}{9} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{11}{18} & \frac{1}{9} & \frac{1}{9} & \frac{11}{18} & \frac{-5}{36} & \frac{31}{36} \\ \frac{1}{9} & \frac{11}{18} & \frac{31}{36} & \frac{-5}{36} & \frac{13}{36} & \frac{13}{36} & \frac{1}{9} & \frac{11}{18} \\ \frac{11}{18} & \frac{1}{9} & \frac{-5}{36} & \frac{31}{36} & \frac{13}{36} & \frac{13}{36} & \frac{11}{18} & \frac{1}{9} \\ \frac{11}{18} & \frac{1}{9} & \frac{13}{36} & \frac{13}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{11}{18} & \frac{1}{9} \\ \frac{1}{9} & \frac{11}{18} & \frac{13}{36} & \frac{13}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{1}{9} & \frac{11}{18} \\ \frac{31}{36} & \frac{-5}{36} & \frac{1}{9} & \frac{11}{18} & \frac{11}{18} & \frac{1}{9} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{11}{18} & \frac{1}{9} & \frac{1}{9} & \frac{11}{18} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{4}{31} & \frac{22}{31} & \frac{22}{31} & \frac{4}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{22}{31} & \frac{4}{31} & \frac{4}{31} & \frac{22}{31} & \frac{-5}{31} & 1 \\ \frac{4}{31} & \frac{22}{31} & 1 & \frac{-5}{31} & \frac{13}{31} & \frac{13}{31} & \frac{4}{31} & \frac{22}{31} \\ \frac{22}{31} & \frac{4}{31} & \frac{-5}{31} & 1 & \frac{13}{31} & \frac{13}{31} & \frac{22}{31} & \frac{4}{31} \\ \frac{22}{31} & \frac{4}{31} & \frac{13}{31} & \frac{13}{31} & 1 & \frac{-5}{31} & \frac{22}{31} & \frac{4}{31} \\ \frac{4}{31} & \frac{22}{31} & \frac{13}{31} & \frac{13}{31} & \frac{-5}{31} & 1 & \frac{4}{31} & \frac{22}{31} \\ 1 & \frac{-5}{31} & \frac{4}{31} & \frac{22}{31} & \frac{22}{31} & \frac{4}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{22}{31} & \frac{4}{31} & \frac{4}{31} & \frac{22}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues $N_C$

[0., 0., 0., 0., 1., 2.888888889, 2.618033988, 0.381966012]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 1.161290323, 3.354838710, 3.040297535, 0.443573433]

NullSpace $M_C$

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [1, 0, 1, 0, 1, 0, 1, 0], [1, 0, 1, 0, 1, 0, 0, 1] }

NullSpace $N_C$

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues $M_0$

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues $N_0$

[0., 0., 0., 0., 1., 4., 2.618033988, 0.381966012]

NullSpace $M_0$

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace $N_0$

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., −1., 4., −0.381966012, −2.618033988]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 0, 0, 1, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 4 & 3 & 1 & 1 & 3 & 0 & 4 \\ 4 & 0 & 1 & 3 & 3 & 1 & 4 & 0 \\ 3 & 1 & 0 & 4 & 2 & 2 & 3 & 1 \\ 1 & 3 & 4 & 0 & 2 & 2 & 1 & 3 \\ 1 & 3 & 2 & 2 & 0 & 4 & 1 & 3 \\ 3 & 1 & 2 & 2 & 4 & 0 & 3 & 1 \\ 0 & 4 & 3 & 1 & 1 & 3 & 0 & 4 \\ 4 & 0 & 1 & 3 & 3 & 1 & 4 & 0 \end{pmatrix}$$

========================================================

{3, 4, 5, 7}

R: [4, 3, 5, 6, 7, 4, 8, 3]
B: [8, 7, 1, 2, 3, 8, 4, 7]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{-7}{2048}(672 + 284s - 10s^2 + 63s^3 + 43s^4 + 28s^5)(-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5] v[8] v[7])
"B CYCLES", 1 + v[2] v[4] v[7]

Eigenvalues

R: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[0, 1, 0, 0, 0, 0, 0, −1], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 49 & 35 & 0 & 0 & 0 & 70 & 56 \\ 49 & 0 & 28 & 56 & 0 & 0 & 77 & 0 \\ 35 & 28 & 0 & 98 & 70 & 77 & 0 & 112 \\ 0 & 56 & 98 & 0 & 56 & 0 & 112 & 98 \\ 0 & 0 & 70 & 56 & 0 & 49 & 35 & 0 \\ 0 & 0 & 77 & 0 & 49 & 0 & 28 & 56 \\ 70 & 77 & 0 & 112 & 35 & 28 & 0 & 98 \\ 56 & 0 & 112 & 98 & 0 & 56 & 98 & 0 \end{pmatrix} \quad N =$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{7}{180}$ ( $7v[1]v[2] + 5v[1]v[3] + 10v[1]v[7] + 8v[1]v[8] + 4v[2]v[3] + 8v[2]v[4] + 11v[2]v[7] + 14v[3]v[4] + 10v[3]v[5] + 11v[3]v[6] + 16v[3]v[8] + 8v[4]v[5] + 16v[4]v[7] + 14v[4]v[8] + 7v[5]v[6] + 5v[5]v[7] + 4v[6]v[7] + 8v[6]v[8] + 14v[8]v[7]$ )
degree 3 : $\frac{1}{20}$ ( $3v[1]v[2]v[3] + 4v[1]v[2]v[7] + 2v[1]v[3]v[8] + 6v[1]v[8]v[7] + v[2]v[3]v[4] + 7v[2]v[4]v[7] + 6v[3]v[4]v[5] + 7v[3]v[4]v[8] + 4v[3]v[5]v[6] + 7v[3]v[6]v[8] + 2v[4]v[5]v[7] + 7v[4]v[8]v[7] + 3v[5]v[6]v[7] + v[6]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {3, 5, 6}

"RG5" = {4, 7, 8}

"RG6" = {3, 4, 8}

"RG7" = {4, 5, 7}

"RG8" = {3, 4, 5}

"RG9" = {2, 4, 7}

"RG10" = {2, 3, 4}

"RG11" = {1, 7, 8}

"RG12" = {1, 3, 8}

"RG13" = {1, 2, 7}

"RG14" = {1, 2, 3}

$\pi 3$ = [3, 0, 0, 0, 4, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 1, 0, 0, 0, 0, 0, 0, 7, 0, 0, 0, 0, 0, 0, 0, 6, 0, 0, 7, 4, 0, 0, 0, 7, 0, 0, 2, 0, 0, 0, 7, 3, 0, 0, 1]

supp $\pi 3$ = {1, 5, 11, 21, 22, 29, 37, 40, 41, 45, 48, 52, 53, 56}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[6], [8], [5], [7], [2], [4], [1], [3], [2], [4], [6], [8], [6], [8]]
Action of B on ranges, [[5], [11], [6], [12], [9], [13], [10], [14], [9], [13], [5], [11], [5], [11]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{60} & \frac{7}{60} & \frac{1}{20} & \frac{1}{15} & \frac{7}{60} & \frac{7}{60} & \frac{1}{30} & \frac{1}{10} & \frac{7}{60} & \frac{1}{60} & \frac{1}{10} & \frac{1}{30} & \frac{1}{15} & \frac{1}{20} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[1, 3, 2]

B−BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {1, 4, 6}
$b_2$ = {2, 5, 8}
$b_3$ = {3, 7}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 30, Shape: 6 ⊕ 24/20

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {3, 5, 7, 8}}, true

$\Omega_B$ in Vec(K)? , {{2, 4, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{5}{32} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [6, 7, 8], [[8, 7, 6, 8, 7, 8, 6, 7], [8, 6, 7, 8, 6, 8, 7, 6], [7, 8, 6, 7, 8, 7, 6, 8], [7, 6, 8, 7, 6, 7, 8, 6], [6, 8, 7, 6, 8, 6, 7, 8], [6, 7, 8, 6, 7, 6, 8, 7]]

2, "range", [3, 6, 8], [[8, 6, 3, 8, 6, 8, 3, 6], [8, 3, 6, 8, 3, 8, 6, 3], [6, 8, 3, 6, 8, 6, 3, 8], [6, 3, 8, 6, 3, 6, 8, 3], [3, 8, 6, 3, 8, 3, 6, 8], [3, 6, 8, 3, 6, 3, 8, 6]]

3, "range", [5, 6, 7], [[7, 6, 5, 7, 6, 7, 5, 6], [7, 5, 6, 7, 5, 7, 6, 5], [6, 7, 5, 6, 7, 6, 5, 7], [6, 5, 7, 6, 5, 6, 7, 5], [5, 7, 6, 5, 7, 5, 6, 7], [5, 6, 7, 5, 6, 5, 7, 6]]

4, "range", [3, 5, 6], [[6, 5, 3, 6, 5, 6, 3, 5], [6, 3, 5, 6, 3, 6, 5, 3], [5, 6, 3, 5, 6, 5, 3, 6], [5, 3, 6, 5, 3, 5, 6, 3], [3, 6, 5, 3, 6, 3, 5, 6], [3, 5, 6, 3, 5, 3, 6, 5]]

5, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

6, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

7, "range", [4, 5, 7], [[7, 5, 4, 7, 5, 7, 4, 5], [7, 4, 5, 7, 4, 7, 5, 4], [5, 7, 4, 5, 7, 5, 4, 7], [5, 4, 7, 5, 4, 5, 7, 4], [4, 7, 5, 4, 7, 4, 5, 7], [4, 5, 7, 4, 5, 4, 7, 5]]

8, "range", [3, 4, 5], [[5, 4, 3, 5, 4, 5, 3, 4], [5, 3, 4, 5, 3, 5, 4, 3], [4, 5, 3, 4, 5, 4, 3, 5], [4, 3, 5, 4, 3, 4, 5, 3], [3, 5, 4, 3, 5, 3, 4, 5], [3, 4, 5, 3, 4, 3, 5, 4]]

9, "range", [2, 4, 7], [[7, 4, 2, 7, 4, 7, 2, 4], [7, 2, 4, 7, 2, 7, 4, 2], [4, 7, 2, 4, 7, 4, 2, 7], [4, 2, 7, 4, 2, 4, 7, 2], [2, 7, 4, 2, 7, 2, 4, 7], [2, 4, 7, 2, 4, 2, 7, 4]]

10, "range", [2, 3, 4], [[4, 3, 2, 4, 3, 4, 2, 3], [4, 2, 3, 4, 2, 4, 3, 2], [3, 4, 2, 3, 4, 3, 2, 4], [3, 2, 4, 3, 2, 3, 4, 2], [2, 4, 3, 2, 4, 2, 3, 4], [2, 3, 4, 2, 3, 2, 4, 3]]

11, "range", [1, 7, 8], [[8, 7, 1, 8, 7, 8, 1, 7], [8, 1, 7, 8, 1, 8, 7, 1], [7, 8, 1, 7, 8, 7, 1, 8], [7, 1, 8, 7, 1, 7, 8, 1], [1, 8, 7, 1, 8, 1, 7, 8], [1, 7, 8, 1, 7, 1, 8, 7]]

12, "range", [1, 3, 8], [[8, 3, 1, 8, 3, 8, 1, 3], [8, 1, 3, 8, 1, 8, 3, 1], [3, 8, 1, 3, 8, 3, 1, 8], [3, 1, 8, 3, 1, 3, 8, 1], [1, 8, 3, 1, 8, 1, 3, 8], [1, 3, 8, 1, 3, 1, 8, 3]]

13, "range", [1, 2, 7], [[7, 2, 1, 7, 2, 7, 1, 2], [7, 1, 2, 7, 1, 7, 2, 1], [2, 7, 1, 2, 7, 2, 1, 7], [2, 1, 7, 2, 1, 2, 7, 1], [1, 7, 2, 1, 7, 1, 2, 7], [1, 2, 7, 1, 2, 1, 7, 2]]

14, "range", [1, 2, 3], [[3, 2, 1, 3, 2, 3, 1, 2], [3, 1, 2, 3, 1, 3, 2, 1], [2, 3, 1, 2, 3, 2, 1, 3], [2, 1, 3, 2, 1, 2, 3, 1], [1, 3, 2, 1, 3, 1, 2, 3], [1, 2, 3, 1, 2, 1, 3, 2]]

"group has", 6, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 3]]

$g_3 =$ [[1, 2]]

$g_4 =$ [[1, 2, 3]]

$g_5 =$ []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\left(h[2] \quad 0 \quad 0 \quad h[2] \quad 2\,h[1]\right)$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2\,t^2 + 3\,t^3 + 4\,t^4 + 5\,t^5 + 7t^6 + 8t^7 + 10t^8 + 12\,t^9 + 14\,t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi3 =$
$(3\ \ 0\ \ 0\ \ 0\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 6\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 7$

{1, 5, 11, 21, 22, 29, 37, 40, 41, 45, 48, 52, 53, 56}

$u3 =$
$(1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

picheck $(15\ \ 15\ \ 30\ \ 30\ \ 15\ \ 15\ \ 30\ \ 30)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi2 =$
$(7\ \ 5\ \ 0\ \ 0\ \ 0\ \ 10\ \ 8\ \ 4\ \ 8\ \ 0\ \ 0\ \ 11\ \ 0\ \ 14\ \ 10\ \ 11\ \ 0\ \ 16\ \ 8\ \ 0\ \ 16\ \ 14\ \ 7\ \ 5\ \ 0$

$u2 =$
$\left(\frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{3}\right.$

picheck $(30\ \ 30\ \ 60\ \ 60\ \ 30\ \ 30\ \ 60\ \ 60)$

$\pi1 = (30\ \ 30\ \ 60\ \ 60\ \ 30\ \ 30\ \ 60\ \ 60)$

$u1 = \left(\frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\right)$

picheck $(30\ \ 30\ \ 60\ \ 60\ \ 30\ \ 30\ \ 60\ \ 60)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{10} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, −1, 1, 0, 1, 1, −1, 0]

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s-t & 0 & 0 & 0 & t-s & 0 \\ t & 0 & 0 & -t & s & 0 & 0 & -s \\ 0 & t & 0 & -s & 0 & s & 0 & -t \\ 0 & 0 & s-t & 0 & 0 & 0 & t-s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & 1 & 0 & 0 \end{pmatrix}$

M0 is invertible. det= 95520449/10935000

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{49}{60} & \frac{7}{12} & 0 & 0 & 0 & \frac{7}{6} & \frac{14}{15} \\ \frac{49}{60} & \frac{11}{6} & \frac{7}{15} & \frac{14}{15} & 0 & 0 & \frac{77}{60} & 0 \\ \frac{7}{12} & \frac{7}{15} & \frac{11}{3} & \frac{49}{30} & \frac{7}{6} & \frac{77}{60} & 0 & \frac{28}{15} \\ 0 & \frac{14}{15} & \frac{49}{30} & \frac{11}{3} & \frac{14}{15} & 0 & \frac{28}{15} & \frac{49}{30} \\ 0 & 0 & \frac{7}{6} & \frac{14}{15} & \frac{11}{6} & \frac{49}{60} & \frac{7}{12} & 0 \\ 0 & 0 & \frac{77}{60} & 0 & \frac{49}{60} & \frac{11}{6} & \frac{7}{15} & \frac{14}{15} \\ \frac{7}{6} & \frac{77}{60} & 0 & \frac{28}{15} & \frac{7}{12} & \frac{7}{15} & \frac{11}{3} & \frac{49}{30} \\ \frac{14}{15} & 0 & \frac{28}{15} & \frac{49}{30} & 0 & \frac{14}{15} & \frac{49}{30} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega \left( \frac{1}{12} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{12} \; \frac{1}{12} \; \frac{1}{2} \; \frac{1}{12} \; \frac{1}{12} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{12} \; \frac{1}{12} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{12} \; \frac{1}{12} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{12} \; \frac{1}{12} \right)$$

$$T \left( 0 \; 0 \; \frac{1}{2} \; 0 \; 0 \; 0 \; 0 \; \frac{1}{4} \; 0 \; 0 \; \frac{1}{4} \; 0 \; 0 \; 0 \; \frac{1}{4} \; 0 \; \frac{1}{2} \; 0 \; 0 \; \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \frac{7}{2}\ 7\ \frac{7}{4}\ \frac{7}{4}\ \frac{21}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ 7\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \frac{63}{29}\ \frac{413}{58}\ \frac{63}{29}\ \frac{63}{29}\ \frac{322}{29}\ \frac{133}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{80}{17}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2$ / $r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau}$ / $n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 84

dim span idems 6 vs no. of idems 14

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {3, 5, 6}

"RG5" = {4, 7, 8}

"RG6" = {3, 4, 8}

"RG7" = {4, 5, 7}

"RG8" = {3, 4, 5}

"RG9" = {2, 4, 7}

"RG10" = {2, 3, 4}

"RG11" = {1, 7, 8}

"RG12" = {1, 3, 8}

"RG13" = {1, 2, 7}

"RG14" = {1, 2, 3}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{67}{180} & \frac{-11}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{5}{18} & \frac{2}{45} \\ \frac{67}{180} & \frac{25}{18} & \frac{-19}{45} & \frac{2}{45} & \frac{-4}{9} & \frac{-4}{9} & \frac{71}{180} & \frac{-8}{9} \\ \frac{-11}{36} & \frac{-19}{45} & \frac{17}{9} & \frac{-13}{90} & \frac{5}{18} & \frac{71}{180} & \frac{-16}{9} & \frac{4}{45} \\ \frac{-8}{9} & \frac{2}{45} & \frac{-13}{90} & \frac{17}{9} & \frac{2}{45} & \frac{-8}{9} & \frac{4}{45} & \frac{-13}{90} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{5}{18} & \frac{2}{45} & \frac{25}{18} & \frac{67}{180} & \frac{-11}{36} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{71}{180} & \frac{-8}{9} & \frac{67}{180} & \frac{25}{18} & \frac{-19}{45} & \frac{2}{45} \\ \frac{5}{18} & \frac{71}{180} & \frac{-16}{9} & \frac{4}{45} & \frac{-11}{36} & \frac{-19}{45} & \frac{17}{9} & \frac{-13}{90} \\ \frac{2}{45} & \frac{-8}{9} & \frac{4}{45} & \frac{-13}{90} & \frac{-8}{9} & \frac{2}{45} & \frac{-13}{90} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{67}{250} & \frac{-11}{50} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{1}{5} & \frac{4}{125} \\ \frac{67}{250} & 1 & \frac{-38}{125} & \frac{4}{125} & \frac{-8}{25} & \frac{-8}{25} & \frac{71}{250} & \frac{-16}{25} \\ \frac{-11}{68} & \frac{-19}{85} & 1 & \frac{-13}{170} & \frac{5}{34} & \frac{71}{340} & \frac{-16}{17} & \frac{4}{85} \\ \frac{-8}{17} & \frac{2}{85} & \frac{-13}{170} & 1 & \frac{2}{85} & \frac{-8}{17} & \frac{4}{85} & \frac{-13}{170} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{1}{5} & \frac{4}{125} & 1 & \frac{67}{250} & \frac{-11}{50} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{71}{250} & \frac{-16}{25} & \frac{67}{250} & 1 & \frac{-38}{125} & \frac{4}{125} \\ \frac{5}{34} & \frac{71}{340} & \frac{-16}{17} & \frac{4}{85} & \frac{-11}{68} & \frac{-19}{85} & 1 & \frac{-13}{170} \\ \frac{2}{85} & \frac{-8}{17} & \frac{4}{85} & \frac{-13}{170} & \frac{-8}{17} & \frac{2}{85} & \frac{-13}{170} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 1.016666667, 2.579911677, 0.147866101, 0.1105604697, 2.020648420, 2.920731937, 4.314725840]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.7320000000, 1.527771142, 0.0825817995, 0.07131218568, 1.284943178, 1.749037130, 2.552354566]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues $M_0$

[1.016666667, 9.123969865, 0.142311690, 2.350385110, 0.1105604697, 2.020648420, 2.920731937, 4.314725840]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[−0.8166666667, 5.783310716, −3.177781374, −0.155529340, 0.1980110731, 1.497108168, −0.7182862981, −2.610166276]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

===============================================

{3, 4, 6, 8}

R: [4, 3, 5, 6, 3, 8, 4, 7]
B: [8, 7, 1, 2, 7, 4, 8, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{-7}{2048}(672 + 284s - 10s^2 + 63s^3 + 43s^4 + 28s^5)(-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[3] v[5]) (1 + v[4] v[6] v[8] v[7])
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R:   [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

B:   [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $B^*$

{[0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, 0, −1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 49 & 56 & 28 & 0 & 0 & 0 & 77 \\ 49 & 0 & 0 & 35 & 0 & 0 & 56 & 70 \\ 56 & 0 & 0 & 98 & 0 & 56 & 98 & 112 \\ 28 & 35 & 98 & 0 & 77 & 70 & 112 & 0 \\ 0 & 0 & 0 & 77 & 0 & 49 & 56 & 28 \\ 0 & 0 & 56 & 70 & 49 & 0 & 0 & 35 \\ 0 & 56 & 98 & 112 & 56 & 0 & 0 & 98 \\ 77 & 70 & 112 & 0 & 28 & 35 & 98 & 0 \end{pmatrix} \quad N =$$

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{7}{180}$ ( $7v[1]v[2] + 8v[1]v[3] + 4v[1]v[4] + 11v[1]v[8] + 5v[2]v[4] + 8v[2]v[7] + 10v[2]v[8] + 14v[3]v[4] + 8v[3]v[6] + 14v[3]v[7] + 16v[3]v[8] + 11v[4]v[5] + 10v[4]v[6] + 16v[4]v[7] + 7v[5]v[6] + 8v[5]v[7] + 4v[5]v[8] + 5v[6]v[8] + 14v[8]v[7]$ )
degree 3 : $\frac{1}{20}$ ( $3v[1]v[2]v[4] + 4v[1]v[2]v[8] + v[1]v[3]v[4] + 7v[1]v[3]v[8] + 2v[2]v[4]v[7] + 6v[2]v[8]v[7] + 6v[3]v[4]v[6] + 7v[3]v[4]v[7] + 2v[3]v[6]v[8] + 7v[3]v[8]v[7] + 4v[4]v[5]v[6] + 7v[4]v[5]v[7] + 3v[5]v[6]v[8] + v[5]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 6, 7}, {4, 8}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 7, 8}

"RG6" = {2, 4, 7}

"RG7" = {5, 6, 8}

"RG8" = {4, 5, 6}

"RG9" = {3, 6, 8}

"RG10" = {3, 4, 6}

"RG11" = {1, 3, 8}

"RG12" = {1, 3, 4}

"RG13" = {1, 2, 8}

"RG14" = {1, 2, 4}

$\pi 3$ = [0, 3, 0, 0, 0, 4, 1, 0, 0, 0, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 6, 0, 6, 7, 0, 0, 0, 0, 0, 2, 7, 4, 7, 0, 0, 0, 0, 0, 3, 1, 0]

supp $\pi 3$ = {2, 6, 7, 11, 29, 36, 38, 39, 45, 46, 47, 48, 54, 55}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[4], [10], [2], [8], [4], [10], [3], [9], [1], [7], [2], [8], [4], [10]]
Action of B on ranges, [[3], [5], [11], [13], [3], [5], [4], [6], [12], [14], [11], [13], [3], [5]]

$$\boldsymbol{\beta} = \left(\frac{1}{60}\ \frac{7}{60}\ \frac{7}{60}\ \frac{7}{60}\ \frac{1}{10}\ \frac{1}{30}\ \frac{1}{20}\ \frac{1}{15}\ \frac{1}{30}\ \frac{1}{10}\ \frac{7}{60}\ \frac{1}{60}\ \frac{1}{15}\ \frac{1}{20}\right)$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1, 3]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1$ = {1, 6, 7}
$b_2$ = {4, 8}
$b_3$ = {2, 3, 5}

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 30, Shape: $6 \oplus 24/20$

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6, 7, 8}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{5}{32} & \frac{5}{32} \end{pmatrix} \text{vs} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \text{vs} \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{1, 6, 7}, {4, 8}, {2, 3, 5}}

1, "range", [5, 7, 8], [[8, 7, 7, 5, 7, 8, 8, 5], [8, 5, 5, 7, 5, 8, 8, 7], [7, 8, 8, 5, 8, 7, 7, 5], [7, 5, 5, 8, 5, 7, 7, 8], [5, 8, 8, 7, 8, 5, 5, 7], [5, 7, 7, 8, 7, 5, 5, 8]]

2, "range", [4, 5, 7], [[7, 5, 5, 4, 5, 7, 7, 4], [7, 4, 4, 5, 4, 7, 7, 5], [5, 7, 7, 4, 7, 5, 5, 4], [5, 4, 4, 7, 4, 5, 5, 7], [4, 7, 7, 5, 7, 4, 4, 5], [4, 5, 5, 7, 5, 4, 4, 7]]

3, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

4, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

5, "range", [2, 7, 8], [[8, 7, 7, 2, 7, 8, 8, 2], [8, 2, 2, 7, 2, 8, 8, 7], [7, 8, 8, 2, 8, 7, 7, 2], [7, 2, 2, 8, 2, 7, 7, 8], [2, 8, 8, 7, 8, 2, 2, 7], [2, 7, 7, 8, 7, 2, 2, 8]]

6, "range", [2, 4, 7], [[7, 4, 4, 2, 4, 7, 7, 2], [7, 2, 2, 4, 2, 7, 7, 4], [4, 7, 7, 2, 7, 4, 4, 2], [4, 2, 2, 7, 2, 4, 4, 7], [2, 7, 7, 4, 7, 2, 2, 4], [2, 4, 4, 7, 4, 2, 2, 7]]

7, "range", [5, 6, 8], [[8, 6, 6, 5, 6, 8, 8, 5], [8, 5, 5, 6, 5, 8, 8, 6], [6, 8, 8, 5, 8, 6, 6, 5], [6, 5, 5, 8, 5, 6, 6, 8], [5, 8, 8, 6, 8, 5, 5, 6], [5, 6, 6, 8, 6, 5, 5, 8]]

8, "range", [4, 5, 6], [[6, 5, 5, 4, 5, 6, 6, 4], [6, 4, 4, 5, 4, 6, 6, 5], [5, 6, 6, 4, 6, 5, 5, 4], [5, 4, 4, 6, 4, 5, 5, 6], [4, 6, 6, 5, 6, 4, 4, 5], [4, 5, 5, 6, 5, 4, 4, 6]]

9, "range", [3, 6, 8], [[8, 6, 6, 3, 6, 8, 8, 3], [8, 3, 3, 6, 3, 8, 8, 6], [6, 8, 8, 3, 8, 6, 6, 3], [6, 3, 3, 8, 3, 6, 6, 8], [3, 8, 8, 6, 8, 3, 3, 6], [3, 6, 6, 8, 6, 3, 3, 8]]

10, "range", [3, 4, 6], [[6, 4, 4, 3, 4, 6, 6, 3], [6, 3, 3, 4, 3, 6, 6, 4], [4, 6, 6, 3, 6, 4, 4, 3], [4, 3, 3, 6, 3, 4, 4, 6], [3, 6, 6, 4, 6, 3, 3, 4], [3, 4, 4, 6, 4, 3, 3, 6]]

11, "range", [1, 3, 8], [[8, 3, 3, 1, 3, 8, 8, 1], [8, 1, 1, 3, 1, 8, 8, 3], [3, 8, 8, 1, 8, 3, 3, 1], [3, 1, 1, 8, 1, 3, 3, 8], [1, 8, 8, 3, 8, 1, 1, 3], [1, 3, 3, 8, 3, 1, 1, 8]]

12, "range", [1, 3, 4], [[4, 3, 3, 1, 3, 4, 4, 1], [4, 1, 1, 3, 1, 4, 4, 3], [3, 4, 4, 1, 4, 3, 3, 1], [3, 1, 1, 4, 1, 3, 3, 4], [1, 4, 4, 3, 4, 1, 1, 3], [1, 3, 3, 4, 3, 1, 1, 4]]

13, "range", [1, 2, 8], [[8, 2, 2, 1, 2, 8, 8, 1], [8, 1, 1, 2, 1, 8, 8, 2], [2, 8, 8, 1, 8, 2, 2, 1], [2, 1, 1, 8, 1, 2, 2, 8], [1, 8, 8, 2, 8, 1, 1, 2], [1, 2, 2, 8, 2, 1, 1, 8]]

14, "range", [1, 2, 4], [[4, 2, 2, 1, 2, 4, 4, 1], [4, 1, 1, 2, 1, 4, 4, 2], [2, 4, 4, 1, 4, 2, 2, 1], [2, 1, 1, 4, 1, 2, 2, 4], [1, 4, 4, 2, 4, 1, 1, 2], [1, 2, 2, 4, 2, 1, 1, 4]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}}

## KERNEL HIERARCHY

$\pi 3$ =
$\left(0 \;\; 3 \;\; 0 \;\; 0 \;\; 0 \;\; 4 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 7 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 2\right.$

{2, 6, 7, 11, 29, 36, 38, 39, 45, 46, 47, 48, 54, 55}

$u3$ =
$\left(0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1\right.$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $\left(15 \;\; 15 \;\; 30 \;\; 30 \;\; 15 \;\; 15 \;\; 30 \;\; 30\right)$

$\pi = \left(\frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

$\pi 2$ =
$\left(7 \;\; 8 \;\; 4 \;\; 0 \;\; 0 \;\; 0 \;\; 11 \;\; 0 \;\; 5 \;\; 0 \;\; 0 \;\; 8 \;\; 10 \;\; 14 \;\; 0 \;\; 8 \;\; 14 \;\; 16 \;\; 11 \;\; 10 \;\; 16 \;\; 0 \;\; 7 \;\; 8 \;\; 4\right.$

$u2$ =
$\left(\frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{3}\right.$

picheck $\left(30 \;\; 30 \;\; 60 \;\; 60 \;\; 30 \;\; 30 \;\; 60 \;\; 60\right)$

$\pi 1 = \left(30 \;\; 30 \;\; 60 \;\; 60 \;\; 30 \;\; 30 \;\; 60 \;\; 60\right)$

$u1 = \left(\frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{2}{9}\right)$

picheck $\left(30 \;\; 30 \;\; 60 \;\; 60 \;\; 30 \;\; 30 \;\; 60 \;\; 60\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks    N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

idem-checks    N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks    N0-checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 1, 1, 1, 0, -1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & -t & 0 & 0 & s & -s & 0 \\ t & 0 & -s & 0 & s & 0 & -t & 0 \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

M0 is invertible. det= 95520449/10935000

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\1&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&0&1&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\1&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{49}{60} & \frac{14}{15} & \frac{7}{15} & 0 & 0 & 0 & \frac{77}{60} \\ \frac{49}{60} & \frac{11}{6} & 0 & \frac{7}{12} & 0 & 0 & \frac{14}{15} & \frac{7}{6} \\ \frac{14}{15} & 0 & \frac{11}{3} & \frac{49}{30} & 0 & \frac{14}{15} & \frac{49}{30} & \frac{28}{15} \\ \frac{7}{15} & \frac{7}{12} & \frac{49}{30} & \frac{11}{3} & \frac{77}{60} & \frac{7}{6} & \frac{28}{15} & 0 \\ 0 & 0 & 0 & \frac{77}{60} & \frac{11}{6} & \frac{49}{60} & \frac{14}{15} & \frac{7}{15} \\ 0 & 0 & \frac{14}{15} & \frac{7}{6} & \frac{49}{60} & \frac{11}{6} & 0 & \frac{7}{12} \\ 0 & \frac{14}{15} & \frac{49}{30} & \frac{28}{15} & \frac{14}{15} & 0 & \frac{11}{3} & \frac{49}{30} \\ \frac{77}{60} & \frac{7}{6} & \frac{28}{15} & 0 & \frac{7}{15} & \frac{7}{12} & \frac{49}{30} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{2}\ 0\ 0\ 0\ 1\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{35}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{35}{2}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{18}{58}\right.$$

$\boldsymbol{\tau} = 22/1$ , rank = 3 , ratio = 22/3 , $n^2 / r = 64/3$

$\boldsymbol{\tau}' = 42/1$ , $r' = 2/3$ , $\boldsymbol{\tau} / n^2 = 11/32$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 84

dim span idems 6 vs no. of idems 14

"PT1" = {{1, 6, 7}, {4, 8}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {4, 5, 7}

"RG3" = {3, 7, 8}

"RG4" = {3, 4, 7}

"RG5" = {2, 7, 8}

"RG6" = {2, 4, 7}

"RG7" = {5, 6, 8}

"RG8" = {4, 5, 6}

"RG9" = {3, 6, 8}

"RG10" = {3, 4, 6}

"RG11" = {1, 3, 8}

"RG12" = {1, 3, 4}

"RG13" = {1, 2, 8}

"RG14" = {1, 2, 4}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{67}{180} & \frac{2}{45} & \frac{-19}{45} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{71}{180} \\ \frac{67}{180} & \frac{25}{18} & \frac{-8}{9} & \frac{-11}{36} & \frac{-4}{9} & \frac{-4}{9} & \frac{2}{45} & \frac{5}{18} \\ \frac{2}{45} & \frac{-8}{9} & \frac{17}{9} & \frac{-13}{90} & \frac{-8}{9} & \frac{2}{45} & \frac{-13}{90} & \frac{4}{45} \\ \frac{-19}{45} & \frac{-11}{36} & \frac{-13}{90} & \frac{17}{9} & \frac{71}{180} & \frac{5}{18} & \frac{4}{45} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{71}{180} & \frac{25}{18} & \frac{67}{180} & \frac{2}{45} & \frac{-19}{45} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{2}{45} & \frac{5}{18} & \frac{67}{180} & \frac{25}{18} & \frac{-8}{9} & \frac{-11}{36} \\ \frac{-8}{9} & \frac{2}{45} & \frac{-13}{90} & \frac{4}{45} & \frac{2}{45} & \frac{-8}{9} & \frac{17}{9} & \frac{-13}{90} \\ \frac{71}{180} & \frac{5}{18} & \frac{4}{45} & \frac{-16}{9} & \frac{-19}{45} & \frac{-11}{36} & \frac{-13}{90} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$
M_C\text{-scaled} = \begin{pmatrix}
1 & \frac{67}{250} & \frac{4}{125} & \frac{-38}{125} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{71}{250} \\
\frac{67}{250} & 1 & \frac{-16}{25} & \frac{-11}{50} & \frac{-8}{25} & \frac{-8}{25} & \frac{4}{125} & \frac{1}{5} \\
\frac{2}{85} & \frac{-8}{17} & 1 & \frac{-13}{170} & \frac{-8}{17} & \frac{2}{85} & \frac{-13}{170} & \frac{4}{85} \\
\frac{-19}{85} & \frac{-11}{68} & \frac{-13}{170} & 1 & \frac{71}{340} & \frac{5}{34} & \frac{4}{85} & \frac{-16}{17} \\
\frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{71}{250} & 1 & \frac{67}{250} & \frac{4}{125} & \frac{-38}{125} \\
\frac{-8}{25} & \frac{-8}{25} & \frac{4}{125} & \frac{1}{5} & \frac{67}{250} & 1 & \frac{-16}{25} & \frac{-11}{50} \\
\frac{-8}{17} & \frac{2}{85} & \frac{-13}{170} & \frac{4}{85} & \frac{2}{85} & \frac{-8}{17} & 1 & \frac{-13}{170} \\
\frac{71}{340} & \frac{5}{34} & \frac{4}{85} & \frac{-16}{17} & \frac{-19}{85} & \frac{-11}{68} & \frac{-13}{170} & 1
\end{pmatrix} \quad N_C\text{-scaled} =
$$

$$
\begin{pmatrix}
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1
\end{pmatrix}
$$

 

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 1.016666667, 2.579911677, 0.147866101, 0.1105604697, 2.020648420, 2.920731937, 4.314725840]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.7320000000, 1.527771142, 0.0825817995, 0.07131218568, 1.284943178, 1.749037130, 2.552354566]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 0, 0, 0, 1, −1, 0], [1, 0, 0, 0, 0, 0, −1, 0], [0, 0, 1, 0, −1, 0, 0, 0]}

Eigenvalues $M_0$

[1.016666667, 9.123969865, 0.142311690, 2.350385110, 0.1105604697, 2.020648420, 2.920731937, 4.314725840]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Eigenvalues M

[−0.8166666667, 5.783310716, −3.177781374, −0.155529340, 0.1980110731, 1.497108168, −0.7182862981, −2.610166276]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

{3, 4, 7, 8}

R: [4, 3, 5, 6, 3, 4, 8, 7]
B: [8, 7, 1, 2, 7, 8, 4, 3]

TRACE TWO = 4

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{3}{256}(7+2s)(-1+s)^4(2+s)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6]) (1 + v[3] v[5])
"B CYCLES", (1 + v[2] v[4] v[7]) (1 + v[1] v[3] v[8])

Eigenvalues

R: [0., 0., 1., −1., 1., −1., 1., −1.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 4

$$\text{PROTO-M} = \begin{pmatrix} 0 & 6 & 5 & 5 & 0 & 0 & 5 & 5 \\ 6 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 12 & 5 & 5 & 10 & 10 \\ 5 & 5 & 12 & 0 & 5 & 5 & 10 & 10 \\ 0 & 0 & 5 & 5 & 0 & 6 & 5 & 5 \\ 0 & 0 & 5 & 5 & 6 & 0 & 5 & 5 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 12 \\ 5 & 5 & 10 & 10 & 5 & 5 & 12 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]\right)$

degree 2: $\frac{1}{6}\left(2v[1]v[2] + v[1]v[3] + 2v[1]v[4] + 2v[1]v[7] + v[1]v[8] + 2v[2]v[3] + v[2]v[4] + v[2]v[7] + 2v[2]v[8] + 4v[3]v[4] + 2v[3]v[5] + v[3]v[6] + 4v[3]v[7] + 2v[3]\right.$

$v[8] + v[4]v[5] + 2v[4]v[6] + 2v[4]v[7] + 4v[4]v[8] + 2v[5]v[6] + v[5]v[7] + 2v[5]v[8] + 2v[6]v[7] + v[6]v[8] + 4v[8]v[7])$

degree 3 : $\frac{1}{12}(v[1]v[4]v[7] + v[3]v[5]v[7] + v[4]v[6]v[7] + v[3]v[6]v[7] + v[2]v[4]v[8] + v[5]v[8]v[7] + v[1]v[4]v[8] + v[6]v[8]v[7] + 2v[3]v[4]v[8] + v[1]v[2]v[4] + 3v[2]v[4]v[7] + v[1]v[3]v[4] + 3v[1]v[3]v[8] + v[1]v[8]v[7] + v[2]v[3]v[4] + v[3]v[4]v[5] + v[5]v[6]v[8] + v[1]v[2]v[8] + 2v[3]v[8]v[7] + v[3]v[4]v[6] + v[4]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[7] + v[2]v[3]v[7] + v[5]v[6]v[7] + v[4]v[5]v[8] + v[2]v[8]v[7] + v[2]v[3]v[8] + 2v[3]v[4]v[7] + v[3]v[5]v[6] + 3v[3]v[6]v[8] + 3v[4]v[5]v[7] + 2v[4]v[8]v[7] + v[1]v[2]v[7] + v[1]v[2]v[3] + v[4]v[5]v[6])$

degree 4 : $\frac{1}{6}(v[3]v[6]v[8]v[7] + 2v[3]v[4]v[6]v[7] + 2v[1]v[3]v[4]v[7] + 4v[3]v[4]v[8]v[7] + v[1]v[3]v[8]v[7] + 2v[1]v[2]v[3]v[4] + 2v[4]v[6]v[8]v[7] + 2v[3]v[4]v[5]v[8] + 2v[5]v[6]v[8]v[7] + 2v[3]v[4]v[5]v[6] + 2v[2]v[3]v[4]v[8] + v[1]v[2]v[4]v[7] + 2v[1]v[2]v[8]v[7] + v[3]v[4]v[5]v[7] + 2v[2]v[3]v[8]v[7] + 2v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[8] + v[2]v[4]v[8]v[7] + 2v[1]v[4]v[8]v[7] + 2v[3]v[5]v[6]v[7] + v[2]v[3]v[4]v[7] + v[4]v[5]v[6]v[7] + v[4]v[5]v[8]v[7] + 2v[1]v[2]v[4]v[8] + 2v[4]v[5]v[6]v[8] + v[1]v[3]v[4]v[8] + 2v[3]v[5]v[8]v[7] + v[3]v[5]v[6]v[8] + v[3]v[4]v[6]v[8])$

degree 5 : $\frac{1}{12}(v[1]v[2]v[3]v[4]v[7] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[3]v[8]v[7] + v[1]v[2]v[4]v[8]v[7] + v[1]v[3]v[4]v[8]v[7] + v[2]v[3]v[4]v[8]v[7] + v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[8] + v[3]v[4]v[5]v[8]v[7] + v[3]v[4]v[6]v[8]v[7] + v[3]v[5]v[6]v[8]v[7] + v[4]v[5]v[6]v[8]v[7])$

degree 6 : $\frac{1}{2}(v[4])(v[7])(v[1]v[2] + v[5]v[6])(v[3])(v[8])$

Group spectrum $1 + t + 4t^2 + 4t^3 + 4t^4 + t^5 + t^6$

## KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$\pi 6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

supp $\pi 6$ = {6, 28}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1]

supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[1], [1]]

Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[5, 4, 6, 2, 1, 3]

B−BLOCKS,

[4, 5, 1, 3, 6, 2]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 6\}$
$b_2 = \{2, 5\}$
$b_3 = \{8\}$
$b_4 = \{3\}$
$b_5 = \{4\}$
$b_6 = \{7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: $3 \oplus 8/6$

$$CLB = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}, {2, 4, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{3}{16} & \frac{3}{16} & \frac{3}{16} & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

1, "range", [3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 7, 8, 4, 3], [7, 8, 4, 3, 8, 7, 6, 5], [6, 5, 3, 4, 5, 6, 7, 8], [5, 6, 7, 8, 6, 5, 3, 4], [4, 3, 5, 6, 3, 4, 8, 7], [3, 4, 8, 7, 4, 3, 5, 6] ]

2, "range", [1, 2, 3, 4, 7, 8], [[8, 7, 1, 2, 7, 8, 4, 3], [7, 8, 4, 3, 8, 7, 1, 2], [4, 3, 2, 1, 3, 4, 8, 7], [3, 4, 8, 7, 4, 3, 2, 1], [2, 1, 7, 8, 1, 2, 3, 4], [1, 2, 3, 4, 2, 1, 7, 8] ] "group has", 6, "elements"  Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1$ = [[1, 4, 6], [2, 3, 5]]

$g_2$ = [[1, 2], [3, 6], [4, 5]]

$g_3$ = []

$g_4$ = [[1, 5], [2, 6], [3, 4]]

$g_5$ = [[1, 3], [2, 4], [5, 6]]

linear dimension, 6

"Symmetric?", false

Is Z in Vec(K)? false

$\left( \frac{||(h[4]+h[6])||}{||2||} \quad \frac{||(h[2]+h[3]+h[5])||}{||3||} \quad h[1] \quad \frac{||(h[2]+h[3]+h[5])||}{||3||} \quad \frac{||(h[2]+h[3]+h[5])||}{||3||} \right.$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5, "coeff", 1

$$Z[\ 5\ ] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

6, "coeff", 1

 

$$Z[6] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
1, 5, true
1, 6, true
2, 3, false
2, 4, false
2, 5, false
2, 6, false
3, 4, false
3, 5, false
3, 6, false
4, 5, false
4, 6, true
5, 6, false

EIGS =

$$\begin{pmatrix} 1. & 1. & 1. & 1. & \\ 1. & -1. & 1. & -1. & \\ 1. & -1. & 1. & -1. & \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I & 1. & -0 \\ 1. & -1. & 1. & -1. & \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I & 1. & -0 \end{pmatrix}$$

PermChars :=

 

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 3 & 15 & 3 & 3 & 0 \\ 2 & 0 & 20 & 0 & 0 & 2 \\ 0 & 3 & 15 & 3 & 3 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 4t^2 + 4t^3 + 4t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 5t^2 + 10t^3 + 24t^4 + 42t^5 + 83t^6 + 132t^7 + 222t^8 + 335t^9 + 511t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 6 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

{6, 28}

$u6 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

{6, 18, 25, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5$ =
$(0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0$

$u5$ =
$\Big(0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}$

picheck $(5\ \ 5\ \ 10\ \ 10\ \ 5\ \ 5\ \ 10\ \ 10)$

$\pi 4$ =
$(2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0$

$u4$ =
$\Big(\frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0$

picheck $(20\ \ 20\ \ 40\ \ 40\ \ 20\ \ 20\ \ 40\ \ 40)$

$\pi 3$ =
$(6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6\ \ 6\ \ 0\ \ 0\ \ 6$

$u3$ =
$\Big(\frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}$

picheck $(60\ \ 60\ \ 120\ \ 120\ \ 60\ \ 60\ \ 120\ \ 120)$

$\pi 2$ =
$(24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 24\ \ 24\ \ 0\ \ 0\ \ 24\ \ 24\ \ 48\ \ 24\ \ 24\ \ 48\ \ 48\ \ 24\ \ 24\ \ 4$

$u2$ =
$\Big(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$\pi 1 = (120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck $(120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 0, 1, 1, 0, 0]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s & t & 0 & 0 \\ -s & -t & 0 & 0 \\ 0 & 0 & -t & -s \\ 0 & 0 & t & s \\ -s & -t & 0 & 0 \\ s & t & 0 & 0 \\ t & s & 0 & 0 \\ -t & -s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & s & 0 & 0 \\ 0 & -t & t & 0 & s+t \\ -s & 0 & s+t & -t & s+t \\ s & 0 & 0 & t & 0 \\ 0 & -t & t & 0 & s+t \\ 0 & t & s & 0 & 0 \\ 0 & s & t & 0 & 0 \\ 0 & -s & s & 0 & s+t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

 

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \end{pmatrix} \quad Skew\,T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$$\Omega \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

$$T \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", false

$$\mathrm{NM} \begin{pmatrix} \frac{7}{2} & \frac{145}{42} & \frac{145}{42} & \frac{7}{2} & 7 & \frac{145}{21} & \frac{13}{3} & \frac{31}{9} & \frac{145}{21} & 7 & \frac{31}{9} & \frac{13}{3} \end{pmatrix}$$

"IS MN in Vec(K)?", false

$$\mathrm{MN} \begin{pmatrix} 3 & \frac{62}{21} & \frac{62}{21} & 3 & 6 & \frac{124}{21} & \frac{13}{3} & \frac{31}{9} & \frac{124}{21} & 6 & \frac{31}{9} & \frac{13}{3} \end{pmatrix}$$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 6 \text{ , ratio} = 2/1 \text{ , } n^2 / r = 32/3$$

$$\boldsymbol{\tau}' = 52/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 3/16$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 28/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 1/6$$

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{14}{51} T + \frac{176}{17} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 12

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$$M_c = \begin{pmatrix} \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad N_c =$$

 

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$$

commutator =

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 2., 0.6666666667, 0.4444444444]

Eigenvalues $N_C$

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 3., 1.400000000, 3.600000000]

Eigenvalues $N_C$–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace $M_C$

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [1, 0, 1, 0, 1, 0, 1, 0], [0, 1, 1, 0, 1, 0, 1, 0], [0, 0, −1, 1, 0, 0, 0, 0] }

NullSpace $N_C$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues $M_0$

[0., 0., 0., 0., 2., 0.6666666667, 8.935416159, 0.397917175]

Eigenvalues $N_0$

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace $M_0$

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace $N_0$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues M

[1., −1.333333333, 7.142286814, −0.808953480, −1., −2., −1., −2.]

Eigenvalues N

[−2., 6.531128874, −1.531128874, 0., 0., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix}0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&1&0&0&0\end{pmatrix} \begin{pmatrix}0&0&0&0&0&0&1&0\\0&0&1&0&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&0&0&0&1&0\\1&0&0&0&0&1&0&0\\0&0&0&1&0&0&0&0\end{pmatrix} \begin{pmatrix}0&0&1&0&0&0&0&1\\0&0&0&1&0&0&1&0\\1&0&0&0&0&1&0&1\\0&1&0&0&1&0&1&0\\0&0&0&1&0&0&1&0\\0&0&1&0&0&0&0&1\\0&1&0&1&1&0&0&0\\1&0&1&0&0&1&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix}0&0&-1&0&0&0&0&1\\0&0&0&1&0&0&-1&0\\1&0&0&0&0&1&0&-2\\0&-1&0&0&-1&0&2&0\\0&0&0&1&0&0&-1&0\\0&0&-1&0&0&0&0&1\\0&1&0&-2&1&0&0&0\\-1&0&2&0&0&-1&0&0\end{pmatrix}$$

$$1, 3: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$1, 4: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 2 & 0 & 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$2, 3: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$2, 4 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

$$3, 4 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

==========================================

{5, 6, 7, 8}

R: [4, 3, 1, 2, 7, 8, 8, 7]
B: [8, 7, 5, 6, 3, 4, 4, 3]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 6

Level 2 det = $\frac{-3}{4096}(12+5s+s^2)(-14-4s+3s^2)(-1+s)^2(2+s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[8] v[7]) (1 + v[1] v[2] v[3] v[4])
"B CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5])

Eigenvalues

R: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of $R^*$

{[0, 0, 0, 0, 0, −1, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of $B^*$

{[0, 0, 0, 0, 0, −1, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \\ 4 & 0 & 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 8 & 4 & 4 & 4 & 4 \\ 0 & 0 & 8 & 0 & 4 & 4 & 4 & 4 \\ 0 & 0 & 4 & 4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 4 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 & 0 & 0 & 8 \\ 4 & 4 & 4 & 4 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 & 1 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & 1 & 1 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 7

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{6}$ ( $2v[1]v[2] + v[1]v[7] + v[1]v[8] + v[2]v[7] + v[2]v[8] + 4v[3]v[4] + v[3]v[5] + v[3]v[6] + v[3]v[7] + v[3]v[8] + v[4]v[5] + v[4]v[6] + v[4]v[7] + v[4]v[8] + 2v[5]v[6] + 4v[8]v[7]$ )
degree 3 : $\frac{1}{12}$ ( $v[1]v[2]v[7] + v[1]v[2]v[8] + v[1]v[8]v[7] + v[2]v[8]v[7] + v[3]v[4]v[5] + v[3]v[4]v[6] + v[3]v[4]v[7] + v[3]v[4]v[8] + v[3]v[5]v[6] + v[3]v[8]v[7] + v[4]v[5]v[6] + v[4]v[8]v[7]$ )
degree 4 : $\frac{1}{3}$ ( $v[1]v[2]v[8]v[7] + v[3]v[4]v[5]v[6] + v[3]v[4]v[8]v[7]$ )

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

## KERNEL STRUCTURE

"PT1" = {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

"PT2" = {{2, 4}, {5, 8}, {6, 7}, {1, 3}}

"RG1" = {3, 4, 7, 8}

"RG2" = {3, 4, 5, 6}

"RG3" = {1, 2, 7, 8}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 4$ = {15, 56, 61}

$u4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 0, 0, 3, 3, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 2, 2, 0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 2, 2, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 3, 3, 0, 0, 3, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp u4 = {10, 11, 14, 15, 20, 21, 24, 25, 26, 27, 30, 31, 40, 41, 44, 45, 46, 47, 50, 51, 56, 57, 60, 61}

Action of R on ranges, [[3], [3], [1]]
Action of B on ranges, [[2], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[6, 4, 1, 2, 3, 5]

B-BLOCKS,

[4, 5, 2, 1, 2, 4]

with invariant measure, [2, 3, 1, 3, 2, 1]

N by blocks, N – check: true

$b_1 = \{2, 4\}$
$b_2 = \{5, 8\}$
$b_3 = \{2, 3\}$
$b_4 = \{6, 7\}$
$b_5 = \{1, 3\}$
$b_6 = \{1, 4\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: $8 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{1, 2, 3, 4}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{4, 6}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

1, "range", [3, 4, 7, 8], [[8, 7, 7, 8, 4, 3, 3, 4], [8, 7, 7, 8, 3, 4, 4, 3], [7, 8, 8, 7, 4, 3, 3, 4], [7, 8, 8, 7, 3, 4, 4, 3], [4, 3, 3, 4, 8, 7, 7, 8], [4, 3, 3, 4, 7, 8, 8, 7], [3, 4, 4, 3, 8, 7, 7, 8], [3, 4, 4, 3, 7, 8, 8, 7]]

2, "range", [3, 4, 5, 6], [[6, 5, 5, 6, 4, 3, 3, 4], [6, 5, 5, 6, 3, 4, 4, 3], [5, 6, 6, 5, 4, 3, 3, 4], [5, 6, 6, 5, 3, 4, 4, 3], [4, 3, 3, 4, 6, 5, 5, 6], [4, 3, 3, 4, 5, 6, 6, 5], [3, 4, 4, 3, 6, 5, 5, 6], [3, 4, 4, 3, 5, 6, 6, 5]]

3, "range", [1, 2, 7, 8], [[8, 7, 7, 8, 2, 1, 1, 2], [8, 7, 7, 8, 1, 2, 2, 1], [7, 8, 8, 7, 2, 1, 1, 2], [7, 8, 8, 7, 1, 2, 2, 1], [2, 1, 1, 2, 8, 7, 7, 8], [2, 1, 1, 2, 7, 8, 8, 7], [1, 2, 2, 1, 8, 7, 7, 8], [1, 2, 2, 1, 7, 8, 8, 7]]

2, "partition", {{2, 4}, {5, 8}, {6, 7}, {1, 3}}

1, "range", [3, 4, 7, 8], [[8, 7, 8, 7, 4, 3, 3, 4], [8, 7, 8, 7, 3, 4, 4, 3], [7, 8, 7, 8, 4, 3, 3, 4], [7, 8, 7, 8, 3, 4, 4, 3], [4, 3, 4, 3, 8, 7, 7, 8], [4, 3, 4, 3, 7, 8, 8, 7], [3, 4, 3, 4, 8, 7, 7, 8], [3, 4, 3, 4, 7, 8, 8, 7]]

2, "range", [3, 4, 5, 6], [[6, 5, 6, 5, 4, 3, 3, 4], [6, 5, 6, 5, 3, 4, 4, 3], [5, 6, 5, 6, 4, 3, 3, 4], [5, 6, 5, 6, 3, 4, 4, 3], [4, 3, 4, 3, 6, 5, 5, 6], [4, 3, 4, 3, 5, 6, 6, 5], [3, 4, 3, 4, 6, 5, 5, 6], [3, 4, 3, 4, 5, 6, 6, 5]]

3, "range", [1, 2, 7, 8], [[8, 7, 8, 7, 2, 1, 1, 2], [8, 7, 8, 7, 1, 2, 2, 1], [7, 8, 7, 8, 2, 1, 1, 2], [7, 8, 7, 8, 1, 2, 2, 1], [2, 1, 2, 1, 8, 7, 7, 8], [2, 1, 2, 1, 7, 8, 8, 7], [1, 2, 1, 2, 8, 7, 7, 8], [1, 2, 1, 2, 7, 8, 8, 7]]

"group has", 8, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3], [2, 4]]

$g_2 =$ [[1, 3, 2, 4]]

$g_3 =$ [[1, 4, 2, 3]]

$g_4 =$ [[1, 4], [2, 3]]

$g_5 =$ []

linear dimension, 6

"Symmetric?", true

Is Z in Vec(K)? true

$$\begin{pmatrix} 0 & h[3] & h[3] & 2h[1]-2h[2] & 2h[2] & 2h[2] \end{pmatrix}$$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ 0 & 0 & 2. & -2. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 & 6 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 4 & 2 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 3t^2 + 4t^3 + 8t^4 + 10t^5 + 16t^6 + 20t^7 + 29t^8 + 35t^9 + 47t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}}

## KERNEL HIERARCHY

$\pi 4 =$

(0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

{15, 56, 61}

$u4$ =
$\left(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 3\ \ 3\ \ 0\ \ 0\ \ 3\ \ 3\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 2\ \ 2\ \ 0\ \ 0\right.$

{10, 11, 14, 15, 20, 21, 24, 25, 26, 27, 30, 31, 40, 41, 44, 45, 46, 47, 50, 51, 56, 57, 60, 61}

picheck $\left(1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 1\ \ 2\ \ 2\right)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 3$ =
$\left(0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right.$

$u3$ =
$\left(0\ \ 0\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ 0\ \ 0\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ 0\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{4}\right.$

picheck $\left(3\ \ 3\ \ 6\ \ 6\ \ 3\ \ 3\ \ 6\ \ 6\right)$

$\pi 2$ =
$\left(2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 2\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4\right)$

$u2$ =
$\left(\frac{3}{8}\ \ \frac{1}{8}\ \ \frac{1}{4}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{1}{4}\ \ \frac{1}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ 0\ \ 0\ \ \frac{3}{8}\ \ \frac{3}{8}\right)$

picheck $\left(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12\right)$

$\pi 1 = \left(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12\right)$

$u1 = \left(\frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\right)$

picheck $\left(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{4}{9} & \frac{2}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{9} & \frac{4}{9} & 0 & 0 & 0 & 0 \\ \frac{2}{9} & \frac{1}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{9} & \frac{2}{9} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{64}{9} & \frac{56}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{56}{9} & \frac{64}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{32}{9} & \frac{28}{9} & 8 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{28}{9} & \frac{32}{9} & \frac{16}{3} & 8 & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & 8 \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 4 & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 4 & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, -1, -1, -1, 1, 1]$

$$\ker N_C = \begin{pmatrix} -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & s & -s & -s & t & t & -t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 & -1 \\ -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & -t & s & 0 & t \\ -s & t & -s & 0 & -s \\ 0 & 0 & 0 & s & t \\ -s+t & 0 & 0 & -s & -s \\ -t & s & -t & 0 & -t \\ s & -s & t & 0 & s \\ s & -s & t & 0 & s \\ -t & s & -t & 0 & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & 0 & 0 & s \\ s & 0 & t & s & 0 & -s \\ s & 0 & 0 & 0 & t & 0 \\ 0 & t & t & s & -t & 0 \\ t & 0 & s & t & 0 & -t \\ 0 & s & 0 & 0 & 0 & t \\ 0 & s & 0 & 0 & 0 & t \\ t & 0 & s & t & 0 & -t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 2 & 2 & 2 & 2 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{2}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{2}{9} & 0 & 0 & 0 & 0 \\ \frac{-2}{9} & \frac{-1}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & \frac{-2}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$$\Omega\left(\frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\right)$$

$$T\left(0\ \ 0\ \ \frac{1}{3}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{2}{3}\ \ \frac{2}{9}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{2}{9}\ \ \frac{4}{9}\ \ 0\ \ \frac{1}{3}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{16}{3}\ \ \frac{8}{3}\ \ 4\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ 8\ \ \frac{32}{9}\ \ \frac{16}{3}\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{56}{9}\ \ \frac{64}{9}\ \ \frac{8}{3}\ \ 4\right)$$

"IS MN in Vec(K)?", false

$$MN\left(4\ \ 4\ \ 6\ \ 4\ \ 4\ \ 4\ \ 4\ \ 4\ \ \frac{19}{3}\ \ \frac{49}{9}\ \ 4\ \ 4\ \ 4\ \ 4\ \ \frac{43}{9}\ \ \frac{47}{9}\ \ \frac{13}{3}\ \ \frac{17}{3}\right)$$

$\boldsymbol{\tau} = 16/1$ , rank = 4 , ratio = 4/1 , $n^2 / r = 16/1$

$\boldsymbol{\tau}' = 48/1$ , $r' = 3/4$ , $\boldsymbol{\tau} / n^2 = 1/4$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$–check is positive? 64/9

max r = 36/5 , $r$–check is positive? 4/9

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 8

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 5 vs no. of idems 6

"PT1" = {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

"PT2" = {{2, 4}, {5, 8}, {6, 7}, {1, 3}}

"RG1" = {3, 4, 7, 8}

"RG2" = {3, 4, 5, 6}

"RG3" = {1, 2, 7, 8}

$$M_c = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & -1 & -1 & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ -1 & -1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ -1 & -1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & -1 & -1 \\ \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & -1 & -1 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & -1 & -1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 0., 5.333333333, 1.777777778]

Eigenvalues $N_C$

[0.6666666667, 1.333333333, 0.8888888889, 2., 2., 0., 0., 0.]

Eigenvalues $M_C$–scaled

[0., 0., 0., 0., 0., 0., 6., 2.]

Eigenvalues $N_C$–scaled

[0.7741935484, 1.032258065, 1.548387097, 2.322580645, 2.322580645, 0., 0., 0.]

NullSpace $M_C$

{[0, 0, 0, 0, 0, 1, 1, 0], [0, 0, 0, 0, 0, 1, 0, 1], [0, 1, 0, 1, 0, 0, 0, 0], [0, 1, 1, 0, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, −1, 0, 0]}

NullSpace $N_C$

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0]}

Eigenvalues $M_0$

[0., 0., 0., 0., 0., 5.333333333, 9.104569499, 1.562097167]

Eigenvalues $N_0$

[1.333333333, 0.6666666667, 2., 2., 2., 0., 0., 0.]

NullSpace $M_0$

{[0, 0, 0, 0, −1, 1, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 1, 0, 0, 1], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0]}

NullSpace $N_0$

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

Eigenvalues M

[0., 6.666666667, 3.415403751, −2.082070417, −1.333333333, −2.666666667, −1.333333333, −2.666666667]

Eigenvalues N

[−0.6666666667, −1.333333333, 6., −2., −2., 0., 0., 0.]

NullSpace M

{[−2, −2, 1, 1, −2, −2, 1, 1]}

NullSpace N

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 0, 1, 0, 0, −1], [1, 1, −1, −1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 3 & 0 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

===========================================

100, [1, −1, −1, −1, −1, −1, 1, 1]

===========================================

{2, 3, 4, 5, 7}

R: [4, 7, 5, 6, 7, 4, 8, 3]
B: [8, 3, 1, 2, 3, 8, 4, 7]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{3}{256}\left(21 + 32s + 27s^2 + 13s^3 + 3s^4\right)\left(-1 + s\right)^2$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[4] v[6]) (1 + v[3] v[5] v[8] v[7])
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[8] v[7]

Eigenvalues

R: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

B: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 21 & 21 & 20 & 0 & 0 & 21 & 21 \\ 21 & 0 & 21 & 21 & 0 & 0 & 21 & 20 \\ 21 & 21 & 0 & 42 & 21 & 21 & 40 & 42 \\ 20 & 21 & 42 & 0 & 21 & 20 & 42 & 42 \\ 0 & 0 & 21 & 21 & 0 & 21 & 21 & 20 \\ 0 & 0 & 21 & 20 & 21 & 0 & 21 & 21 \\ 21 & 21 & 40 & 42 & 21 & 21 & 0 & 42 \\ 21 & 20 & 42 & 42 & 20 & 21 & 42 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )

degree 2: $\frac{1}{24}$ ( $v[1]v[2] + v[1]v[3] + 4v[1]v[4] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + v[2]v[7] + 4v[2]v[8] + 2v[3]v[4] + v[3]v[5] + v[3]v[6] + 8v[3]v[7] + 2v[3]v[8] + v[4]v[5] + 4v[4]v[6] + 2v[4]v[7] + 2v[4]v[8] + v[5]v[6] + v[5]v[7] + 4v[5]v[8] + v[6]v[7] + v[6]v[8] + 2v[8]v[7]$ )

degree 3 : $\frac{1}{16}$ ( $2v[1]v[4]v[7] + 2v[3]v[5]v[7] + 2v[4]v[6]v[7] + 2v[3]v[6]v[7] + 2v[2]v[4]v[8] + 2v[5]v[8]v[7] + 2v[1]v[4]v[8] + 3v[6]v[8]v[7] + 6v[3]v[4]v[8] + 2v[1]v[2]v[4] + 3v[2]v[4]v[7] + 2v[1]v[3]v[4] + 3v[1]v[3]v[8] + 3v[1]v[8]v[7] + 3v[2]v[3]v[4] + 3v[3]v[4]v[5] + 2v[5]v[6]v[8] + 2v[1]v[2]v[8] + 4v[3]v[8]v[7] + 2v[3]v[4]v[6] + 2v[4]v[6]v[8] + 2v[3]v[5]v[8] + 2v[1]v[3]v[7] + 2v[2]v[3]v[7] + 3v[5]v[6]v[7] + 2v[4]v[$

5]$v$[8] + 2$v$[2]$v$[8]$v$[7] + 2$v$[2]$v$[3]$v$[8] + 4$v$[3]$v$[4]$v$[7] + 3$v$[3]$v$[5]$v$[6] + 3$v$[3]$v$[6]$v$[8] + 3$v$[4]$v$[5]$v$[7] + 6$v$[4]$v$[8]$v$[7] + 3$v$[1]$v$[2]$v$[7] + 3$v$[1]$v$[2]$v$[3] + 2$v$[4]$v$[5]$v$[6] )

degree 4 : $\frac{1}{24}$ ( $v$[3]$v$[6]$v$[8]$v$[7] + 4$v$[3]$v$[4]$v$[6]$v$[7] + 4$v$[1]$v$[3]$v$[4]$v$[7] + 2$v$[3]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[3]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[4] + $v$[4]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[8] + $v$[5]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[6] + $v$[2]$v$[3]$v$[4]$v$[8] + $v$[1]$v$[2]$v$[4]$v$[7] + $v$[1]$v$[2]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[7] + 4$v$[2]$v$[3]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[8] + $v$[2]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[4]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[7] + $v$[2]$v$[3]$v$[4]$v$[7] + $v$[4]$v$[5]$v$[6]$v$[7] + $v$[4]$v$[5]$v$[8]$v$[7] + 4$v$[1]$v$[2]$v$[4]$v$[8] + 4$v$[4]$v$[5]$v$[6]$v$[8] + $v$[1]$v$[3]$v$[4]$v$[8] + 4$v$[3]$v$[5]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[8] + $v$[3]$v$[4]$v$[6]$v$[8] )

degree 5 : $\frac{1}{12}$ ( $v$[1]$v$[2]$v$[3]$v$[4]$v$[7] + $v$[1]$v$[2]$v$[3]$v$[4]$v$[8] + $v$[1]$v$[2]$v$[3]$v$[8]$v$[7] + $v$[1]$v$[2]$v$[4]$v$[8]$v$[7] + $v$[1]$v$[3]$v$[4]$v$[8]$v$[7] + $v$[2]$v$[3]$v$[4]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[6]$v$[7] + $v$[3]$v$[4]$v$[5]$v$[6]$v$[8] + $v$[3]$v$[4]$v$[5]$v$[8]$v$[7] + $v$[3]$v$[4]$v$[6]$v$[8]$v$[7] + $v$[3]$v$[5]$v$[6]$v$[8]$v$[7] + $v$[4]$v$[5]$v$[6]$v$[8]$v$[7] )

degree 6 : $\frac{1}{2}$ ( $v$[4] ) ( $v$[7] ) ( $v$[1]$v$[2] + $v$[5]$v$[6] ) ( $v$[3] ) ( $v$[8] )

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

## KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$\pi 6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

supp $\pi 6$ = {6, 28}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1]

supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[5, 4, 6, 3, 1, 2]

B−BLOCKS,

[4, 5, 1, 2, 6, 3]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 6\}$
$b_2 = \{2, 5\}$
$b_3 = \{8\}$
$b_4 = \{3\}$
$b_5 = \{4\}$
$b_6 = \{7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 6, 6, 6

## LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: 11 ⊕ 13/11

$$CLB = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{3, 5, 7, 8}, {4, 6}}, true

$\Omega_B$ in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{3}{16} & \frac{5}{32} & \frac{5}{32} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$ vs $\left(\frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

1, "range", [3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 7, 8, 4, 3], [8, 7, 4, 5, 7, 8, 6, 3], [8, 6, 7, 5, 6, 8, 3, 4], [8, 6, 3, 5, 6, 8, 7, 4], [8, 4, 7, 5, 4, 8, 3, 6], [8, 4, 3, 5, 4, 8, 7, 6], [8, 3, 6, 5, 3, 8, 4, 7], [8, 3, 4, 5, 3, 8, 6, 7], [7, 8, 6, 3, 8, 7, 4, 5], [7, 8, 4, 3, 8, 7, 6, 5], [7, 6, 8, 3, 6, 7, 5, 4], [7, 6, 5, 3, 6, 7, 8, 4], [7, 5, 6, 3, 5, 7, 4, 8], [7, 5, 4, 3, 5, 7, 6, 8], [7, 4, 8, 3, 4, 7, 5, 6], [7, 4, 5, 3, 4, 7, 8, 6], [6, 8, 7, 4, 8, 6, 3, 5], [6, 8, 3, 4, 8, 6, 7, 5], [6, 7, 8, 4, 7, 6, 5, 3], [6, 7, 5, 4, 7, 6, 8, 3], [6, 5, 7, 4, 5, 6, 3, 8], [6, 5, 3, 4, 5, 6, 7, 8], [6, 3, 8, 4, 3, 6, 5, 7], [6, 3, 5, 4, 3, 6, 8, 7], [5, 7, 6, 8, 7, 5, 4, 3], [5, 7, 4, 8, 7, 5, 6, 3], [5, 6, 7, 8, 6, 5, 3, 4], [5, 6, 3, 8, 6, 5, 7, 4], [5, 4, 7, 8, 4, 5, 3, 6], [5, 4, 3, 8, 4, 5, 7, 6], [5, 3, 6, 8, 3, 5, 4, 7], [5, 3, 4, 8, 3, 5, 6, 7], [4, 8, 7, 6, 8, 4, 3, 5], [4, 8, 3, 6, 8, 4, 7, 5], [4, 7, 8, 6, 7, 4, 5, 3], [4, 7, 5, 6, 7, 4, 8, 3], [4, 5, 7, 6, 5, 4, 3, 8], [4, 5, 3, 6, 5, 4, 7, 8], [4, 3, 8, 6, 3, 4, 5, 7], [4, 3, 5, 6, 3, 4, 8, 7], [3, 8, 6, 7, 8, 3, 4, 5], [3, 8, 4, 7, 8, 3, 6, 5], [3, 6, 8, 7, 6, 3, 5, 4], [3, 6, 5, 7, 6, 3, 8, 4], [3, 5, 6, 7, 5, 3, 4, 8], [3, 5, 4, 7, 5, 3, 6, 8], [3, 4, 8, 7, 4, 3, 5, 6], [3, 4, 5, 7, 4, 3, 8, 6] ]

2, "range", [1, 2, 3, 4, 7, 8], [[8, 7, 4, 2, 7, 8, 1, 3], [8, 7, 1, 2, 7, 8, 4, 3], [8, 4, 7, 2, 4, 8, 3, 1], [8, 4, 3, 2, 4, 8, 7, 1], [8, 3, 4, 2, 3, 8, 1, 7], [8, 3, 1, 2, 3, 8, 4, 7], [8, 1, 7, 2, 1, 8, 3, 4], [8, 1, 3, 2, 1, 8, 7, 4], [7, 8, 4, 3, 8, 7, 1, 2], [7, 8, 1, 3, 8, 7, 4, 2], [7, 4, 8, 3, 4, 7, 2, 1], [7, 4, 2, 3, 4, 7, 8, 1], [7, 2, 4, 3, 2, 7, 1, 8], [7, 2, 1, 3, 2, 7, 4, 8], [7, 1, 8, 3, 1, 7, 2, 4], [7, 1, 2, 3, 1, 7, 8, 4], [4, 8, 7, 1, 8, 4, 3, 2], [4, 8, 3, 1, 8, 4, 7, 2], [4, 7, 8, 1, 7, 4, 2, 3], [4, 7, 2, 1, 7, 4, 8, 3], [4, 3, 8, 1, 3, 4, 2, 7], [4, 3, 2, 1, 3, 4, 8, 7], [4, 2, 7, 1, 2, 4, 3, 8], [4, 2, 3, 1, 2, 4, 7, 8], [3, 8, 4, 7, 8, 3, 1, 2], [3, 8, 1, 7, 8, 3, 4, 2], [3, 4, 8, 7, 4, 3, 2, 1], [3, 4, 2, 7, 4, 3, 8, 1], [3, 2, 4, 7, 2, 3, 1, 8], [3, 2, 1, 7, 2, 3, 4, 8], [3, 1, 8, 7, 1, 3, 2, 4], [3, 1, 2, 7, 1, 3, 8, 4], [2, 7, 4, 8, 7, 2, 1, 3], [2, 7, 1, 8, 7, 2, 4, 3], [2, 4, 7, 8, 4, 2, 3, 1], [2, 4, 3, 8, 4, 2, 7, 1], [2, 3, 4, 8, 3, 2, 1, 7], [2, 3, 1, 8, 3, 2, 4, 7], [2, 1, 7, 8, 1, 2, 3, 4], [2, 1, 3, 8, 1, 2, 7, 4], [1, 8, 7, 4, 8, 1, 3, 2], [1, 8, 3, 4, 8, 1, 7, 2], [1, 7, 8, 4, 7, 1, 2, 3], [1, 7, 2, 4, 7, 1, 8, 3], [1, 3, 8, 4, 3, 1, 2, 7], [1, 3, 2, 4, 3, 1, 8, 7], [1, 2, 7, 4, 2, 1, 3, 8], [1, 2, 3, 4, 2, 1, 7, 8] ]

"group has", 48, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 4, 6], [2, 3, 5]]

$g_2 =$ [[1, 2, 3, 5, 4, 6]]

$g_3 =$ [[1, 5], [2, 3, 4, 6]]

$g_4 =$ [[2, 3, 4, 6]]

$g_5 =$ [[1, 5], [2, 3], [4, 6]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true

$\left(-2h[2] \quad 2h[2] \quad -8h[1]-2h[2] \quad 8h[1] \quad 2h[2] \quad 2h[2] \quad -8h[1]-2h[2] \quad 2h[2] \quad 2h[2] \quad 8\right.$

"Basis for Z(G)"

1, "coeff", 8

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

3, "coeff", 8

$$Z[3] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 4. & -2. & -2. & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8 + 84t^9 + 122t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

## KERNEL HIERARCHY

$\pi 6 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

{6, 28}

$u6 =$

$(0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1)$

$\{6, 18, 25, 28\}$

picheck $(1\ 1\ 2\ 2\ 1\ 1\ 2\ 2)$

$\pi = \left(\frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\right)$

$\pi 5 =$
$(0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0$

$u5 =$
$\left(0\ 0\ \frac{1}{6}\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ \frac{1}{6}\ 0\ 0\ \frac{1}{6}\ 0\ 0\ \frac{1}{6}\right.$

picheck $(5\ 5\ 10\ 10\ 5\ 5\ 10\ 10)$

$\pi 4 =$
$(2\ 0\ 0\ 2\ 2\ 0\ 0\ 2\ 2\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 2\ 2\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 0$

$u4 =$
$\left(\frac{1}{18}\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\right.$

picheck $(20\ 20\ 40\ 40\ 20\ 20\ 40\ 40)$

$\pi 3 =$
$(6\ 6\ 0\ 0\ 6\ 6\ 6\ 0\ 0\ 6\ 6\ 0\ 0\ 6\ 6\ 0\ 0\ 0\ 0\ 0\ 6\ 6\ 0\ 0\ 6\ 6\ 0\ 0\ 6$

$u3 =$
$\left(\frac{1}{36}\ \frac{1}{36}\ 0\ 0\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ 0\ 0\ \frac{1}{36}\ \frac{1}{36}\right.$

picheck $(60\ 60\ 120\ 120\ 60\ 60\ 120\ 120)$

$\pi 2 =$
$(24\ 24\ 24\ 0\ 0\ 24\ 24\ 24\ 24\ 0\ 0\ 24\ 24\ 48\ 24\ 24\ 48\ 48\ 24\ 24\ 4$

$u2 =$
$\left(\frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ 0\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ 0\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\right.$

picheck $(120\ 120\ 240\ 240\ 120\ 120\ 240\ 240)$

$\pi 1 = \begin{pmatrix} 120 & 120 & 240 & 240 & 120 & 120 & 240 & 240 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} \end{pmatrix}$

picheck $\begin{pmatrix} 120 & 120 & 240 & 240 & 120 & 120 & 240 & 240 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

 

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & 7 & \frac{83}{24} & \frac{13}{3} & \frac{83}{12} & \frac{83}{12} \\ \frac{83}{24} & \frac{13}{3} & \frac{83}{12} & \frac{83}{12} & \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & 7 \\ \frac{83}{24} & \frac{83}{24} & \frac{26}{3} & \frac{83}{12} & \frac{83}{24} & \frac{83}{24} & 7 & \frac{83}{12} \\ \frac{7}{2} & \frac{83}{24} & \frac{83}{12} & \frac{26}{3} & \frac{83}{24} & \frac{7}{2} & \frac{83}{12} & \frac{83}{12} \\ \frac{83}{24} & \frac{13}{3} & \frac{83}{12} & \frac{83}{12} & \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & 7 \\ \frac{13}{3} & \frac{83}{24} & \frac{83}{12} & 7 & \frac{83}{24} & \frac{13}{3} & \frac{83}{12} & \frac{83}{12} \\ \frac{83}{24} & \frac{83}{24} & 7 & \frac{83}{12} & \frac{83}{24} & \frac{83}{24} & \frac{26}{3} & \frac{83}{12} \\ \frac{83}{24} & \frac{7}{2} & \frac{83}{12} & \frac{83}{12} & \frac{7}{2} & \frac{83}{24} & \frac{83}{12} & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [-1, -1, 0, 0, 1, 1, 0, 0]

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 \end{pmatrix}$

M0 is invertible. det= 5/7776

 

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{8} & \frac{5}{6} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ 0 & 0 & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{5}{6} \\ \frac{-7}{8} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-7}{8} & 0 & 0 \\ \frac{-5}{6} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{5}{6} \\ 0 & 0 & \frac{7}{8} & \frac{5}{6} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ \frac{-7}{8} & \frac{-7}{8} & 0 & 0 & \frac{-7}{8} & \frac{-7}{8} & 0 & 0 \\ \frac{-7}{8} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-7}{8} & 0 & 0 \end{pmatrix} \quad Skew\,T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{7}{8} & \frac{7}{8} & \frac{5}{6} & 0 & 0 & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & 1 & \frac{7}{8} & \frac{7}{8} & 0 & 0 & \frac{7}{8} & \frac{5}{6} \\ \frac{7}{8} & \frac{7}{8} & 2 & \frac{7}{4} & \frac{7}{8} & \frac{7}{8} & \frac{5}{3} & \frac{7}{4} \\ \frac{5}{6} & \frac{7}{8} & \frac{7}{4} & 2 & \frac{7}{8} & \frac{5}{6} & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{8} & \frac{7}{8} & 1 & \frac{7}{8} & \frac{7}{8} & \frac{5}{6} \\ 0 & 0 & \frac{7}{8} & \frac{5}{6} & \frac{7}{8} & 1 & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{5}{3} & \frac{7}{4} & \frac{7}{8} & \frac{7}{8} & 2 & \frac{7}{4} \\ \frac{7}{8} & \frac{5}{6} & \frac{7}{4} & \frac{7}{4} & \frac{5}{6} & \frac{7}{8} & \frac{7}{4} & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$\Omega$

$$\left(\frac{1}{4} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{2} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6}\right.$$

$$T \left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2}\right)$$

"IS NM in Vec(K)?", true

NM

$$\left(\frac{83}{8} \;\; \frac{83}{24} \;\; \frac{83}{24} \;\; \frac{83}{24} \;\; \frac{7}{2} \;\; \frac{83}{24} \;\; \frac{83}{12} \;\; \frac{26}{3} \;\; \frac{83}{24} \;\; \frac{83}{24} \;\; \frac{83}{4} \;\; \frac{83}{24} \;\; \frac{13}{3} \;\; \frac{83}{12} \;\; \frac{83}{12} \;\; \frac{13}{3} \;\; \frac{83}{24} \;\; \frac{83}{12} \;\; \frac{83}{12} \;\; \frac{13}{3} \;\; \frac{8}{2}\right.$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{88147}{9168} \;\; \frac{6971}{2292} \;\; \frac{37331}{11460} \;\; \frac{10793}{3820} \;\; \frac{32761}{11460} \;\; \frac{6971}{2292} \;\; \frac{6971}{1146} \;\; \frac{6397}{764} \;\; \frac{6971}{2292} \;\; \frac{6971}{2292} \;\; \frac{88147}{4584} \;\; \frac{107}{30}\right.$$

$\boldsymbol{\tau}$ = 12/1 , rank = 6 , ratio = 2/1 , $n^2 / r$ = 32/3

$\boldsymbol{\tau}'$ = 52/1 , $r'$ = 5/6 , $\boldsymbol{\tau} / n^2$ = 3/16

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 28/9

max r = 36/5 , $r$-check is positive? 1/6

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{109}{408} T + \frac{707}{68} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 25
out of total no. of elements equal to 96

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {8}, {3}, {4}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$$M_c = \begin{pmatrix} \frac{5}{9} & \frac{31}{72} & \frac{-1}{72} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{31}{72} & \frac{5}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{18} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{9} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{72} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{5}{9} & \frac{31}{72} & \frac{-1}{72} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{72} & \frac{-1}{18} & \frac{31}{72} & \frac{5}{9} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{72} & \frac{2}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{40} & \frac{-1}{40} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} \\ \frac{31}{40} & 1 & \frac{-1}{40} & \frac{-1}{40} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{10} \\ \frac{-1}{16} & \frac{-1}{16} & 1 & \frac{-1}{8} & \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{2} & \frac{-1}{8} \\ \frac{-1}{4} & \frac{-1}{16} & \frac{-1}{8} & 1 & \frac{-1}{16} & \frac{-1}{4} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{40} & 1 & \frac{31}{40} & \frac{-1}{40} & \frac{-1}{10} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{40} & \frac{-1}{10} & \frac{31}{40} & 1 & \frac{-1}{40} & \frac{-1}{40} \\ \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{2} & \frac{-1}{8} & \frac{-1}{16} & \frac{-1}{16} & 1 & \frac{-1}{8} \\ \frac{-1}{16} & \frac{-1}{4} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{4} & \frac{-1}{16} & \frac{-1}{8} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix}
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1
\end{pmatrix}$$

$$N_c M_c = \begin{pmatrix}
\frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} \\
\frac{-1}{72} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} \\
\frac{-1}{72} & \frac{-1}{72} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{9} & \frac{-1}{36} \\
\frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{72} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} \\
\frac{-1}{72} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} \\
\frac{1}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} \\
\frac{-1}{72} & \frac{-1}{72} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{72} & \frac{-1}{72} & \frac{2}{9} & \frac{-1}{36} \\
\frac{-1}{72} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{72} & \frac{-1}{36} & \frac{2}{9}
\end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{18} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} \\ \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} & \frac{-1}{18} & \frac{-1}{72} & \frac{1}{9} & \frac{-1}{72} & \frac{-1}{72} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{2}{9} \end{pmatrix} \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{72} & \frac{1}{18} & 0 & 0 & \frac{1}{72} & \frac{1}{72} \\ 0 & 0 & \frac{1}{72} & \frac{1}{72} & 0 & 0 & \frac{1}{72} & \frac{1}{18} \\ \frac{-1}{72} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{72} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{72} & \frac{1}{72} & 0 & 0 & \frac{1}{72} & \frac{1}{18} \\ 0 & 0 & \frac{1}{72} & \frac{1}{18} & 0 & 0 & \frac{1}{72} & \frac{1}{72} \\ \frac{-1}{72} & \frac{-1}{72} & 0 & 0 & \frac{-1}{72} & \frac{-1}{72} & 0 & 0 \\ \frac{-1}{72} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{72} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.3333333333, 0.1250000000, 1.875000000, 0.2733980339, 0.1016019661, 0.2607222482, 0.1420555296]

Eigenvalues $N_C$

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues $M_C$–scaled

[0., 0.5000000000, 1.500000000, 3.375000000, 0.2250000000, 1.050000000, 1.155234318, 0.1947656822]

Eigenvalues $N_c$–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace $M_c$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_c$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues $M_0$

[0.3333333333, 0.1250000000, 1.875000000, 0.2733980339, 0.1016019661, 8.908764470, 0.1331882736, 0.2497139224]

Eigenvalues $N_0$

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[−1.666666667, −0.8750000000, 0.8750000000, −0.8710495813, −1.753950419, 7.122690612, −1.795935464, −1.035088480]

Eigenvalues N

[−2., 6.531128874, −1.531128874, 0., 0., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&1&0&0&1&0&0&0 \end{pmatrix} \quad \begin{pmatrix} 0&1&1&0&1&0&1&1 \\ 1&0&1&1&0&1&1&0 \\ 1&1&0&1&1&1&0&1 \\ 0&1&1&0&1&0&1&1 \\ 1&0&1&1&0&1&1&0 \\ 0&1&1&0&1&0&1&1 \\ 1&1&0&1&1&1&0&1 \\ 1&0&1&1&0&1&1&0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&-1 \\ 0&0&0&-1&0&0&0&0 \\ 0&0&0&-1&0&0&0&-1 \\ 0&1&1&0&1&0&1&0 \\ 0&0&0&-1&0&0&0&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&-1&0&0&0&-1 \\ 1&0&1&0&0&1&1&0 \end{pmatrix}$$

==========================================

{2, 3, 4, 7, 8}

R: [4, 7, 5, 6, 3, 4, 8, 7]
B: [8, 3, 1, 2, 7, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{-15}{4096}(-2+s)(-8+5s)(-1+s)(14-7s-s^2+2s^3)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6]) (1 + v[3] v[5])
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R: [0., 0., 1., −1., 1., −1., 1., −1.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of $B^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 21 & 0 & 0 & 0 & 0 & 14 \\ 7 & 0 & 14 & 14 & 0 & 0 & 7 & 0 \\ 21 & 14 & 0 & 14 & 0 & 7 & 0 & 28 \\ 0 & 14 & 14 & 0 & 14 & 0 & 28 & 14 \\ 0 & 0 & 0 & 14 & 0 & 7 & 21 & 0 \\ 0 & 0 & 7 & 0 & 7 & 0 & 14 & 14 \\ 0 & 7 & 0 & 28 & 21 & 14 & 0 & 14 \\ 14 & 0 & 28 & 14 & 0 & 14 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3    "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{36}$ ( $v[1]v[2] + 3v[1]v[3] + 2v[1]v[8] + 2v[2]v[3] + 2v[2]v[4] + v[2]v[7] + 2v[3]v[4] + v[3]v[6] + 4v[3]v[8] + 2v[4]v[5] + 4v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 3v[5]v[7] + 2v[6]v[7] + 2v[6]v[8] + 2v[8]v[7]$ )
degree 3 : $\frac{1}{12}$ ( $v[1]v[2]v[3] + 2v[1]v[3]v[8] + v[2]v[3]v[4] + v[2]v[4]v[7] + v[3]v[4]v[8] + v[3]v[6]v[8] + 2v[4]v[5]v[7] + v[4]v[8]v[7] + v[5]v[6]v[7] + v[6]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

 

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {4, 7, 8}

"RG5" = {3, 4, 8}

"RG6" = {4, 5, 7}

"RG7" = {2, 4, 7}

"RG8" = {2, 3, 4}

"RG9" = {1, 3, 8}

"RG10" = {1, 2, 3}

$\pi 3$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 1, 1, 0, 0, 1]

supp $\pi 3$ = {1, 11, 22, 29, 40, 45, 48, 52, 53, 56}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[4], [6], [5], [1], [3], [2], [1], [3], [6], [6]]
Action of B on ranges, [[5], [9], [4], [8], [10], [7], [8], [10], [9], [9]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 3, 2]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1$ = {1, 4, 6}
$b_2$ = {2, 5, 8}
$b_3$ = {3, 7}

dim(span of partition vectors), rank($N_0$), rank($N$):    3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 30, Shape: 6 ⊕ 24/20

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

 

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}, {3, 5}}, false

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{3}{16} & \frac{3}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$  $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [6, 7, 8], [[8, 7, 6, 8, 7, 8, 6, 7], [8, 6, 7, 8, 6, 8, 7, 6], [7, 8, 6, 7, 8, 7, 6, 8], [7, 6, 8, 7, 6, 7, 8, 6], [6, 8, 7, 6, 8, 6, 7, 8], [6, 7, 8, 6, 7, 6, 8, 7]]

2, "range", [3, 6, 8], [[8, 6, 3, 8, 6, 8, 3, 6], [8, 3, 6, 8, 3, 8, 6, 3], [6, 8, 3, 6, 8, 6, 3, 8], [6, 3, 8, 6, 3, 6, 8, 3], [3, 8, 6, 3, 8, 3, 6, 8], [3, 6, 8, 3, 6, 3, 8, 6]]

3, "range", [5, 6, 7], [[7, 6, 5, 7, 6, 7, 5, 6], [7, 5, 6, 7, 5, 7, 6, 5], [6, 7, 5, 6, 7, 6, 5, 7], [6, 5, 7, 6, 5, 6, 7, 5], [5, 7, 6, 5, 7, 5, 6, 7], [5, 6, 7, 5, 6, 5, 7, 6]]

4, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

5, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

6, "range", [4, 5, 7], [[7, 5, 4, 7, 5, 7, 4, 5], [7, 4, 5, 7, 4, 7, 5, 4], [5, 7, 4, 5, 7, 5, 4, 7], [5, 4, 7, 5, 4, 5, 7, 4], [4, 7, 5, 4, 7, 4, 5, 7], [4, 5, 7, 4, 5, 4, 7, 5]]

7, "range", [2, 4, 7], [[7, 4, 2, 7, 4, 7, 2, 4], [7, 2, 4, 7, 2, 7, 4, 2], [4, 7, 2, 4, 7, 4, 2, 7], [4, 2, 7, 4, 2, 4, 7, 2], [2, 7, 4, 2, 7, 2, 4, 7], [2, 4, 7, 2, 4, 2, 7, 4]]

8, "range", [2, 3, 4], [[4, 3, 2, 4, 3, 4, 2, 3], [4, 2, 3, 4, 2, 4, 3, 2], [3, 4, 2, 3, 4, 3, 2, 4], [3, 2, 4, 3, 2, 3, 4, 2], [2, 4, 3, 2, 4, 2, 3, 4], [2, 3, 4, 2, 3, 2, 4, 3]]

9, "range", [1, 3, 8], [[8, 3, 1, 8, 3, 8, 1, 3], [8, 1, 3, 8, 1, 8, 3, 1], [3, 8, 1, 3, 8, 3, 1, 8], [3, 1, 8, 3, 1, 3, 8, 1], [1, 8, 3, 1, 8, 1, 3, 8], [1, 3, 8, 1, 3, 1, 8, 3]]

10, "range", [1, 2, 3], [[3, 2, 1, 3, 2, 3, 1, 2], [3, 1, 2, 3, 1, 3, 2, 1], [2, 3, 1, 2, 3, 2, 1, 3], [2, 1, 3, 2, 1, 2, 3, 1], [1, 3, 2, 1, 3, 1, 2, 3], [1, 2, 3, 1, 2, 1, 3, 2]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 3]]

$g_3 =$ [[1, 2]]

$g_4 =$ [[1, 2, 3]]

$g_5 =$ []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & 0 & 0 & h[2] & 2h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12$

 

$t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1$

{1, 11, 22, 29, 40, 45, 48, 52, 53, 56}

$u3 =$
$(1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1$

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

picheck $(3\ \ 3\ \ 6\ \ 6\ \ 3\ \ 3\ \ 6\ \ 6)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 2 =$
$(1\ 3\ 0\ 0\ 0\ 0\ 2\ 2\ 2\ 0\ 0\ 1\ 0\ 2\ 0\ 1\ 0\ 4\ 2\ 0\ 4\ 2\ 1\ 3\ 0\ 2\ 2\ 2)$

$u2 =$
$\left(\frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\right.$

picheck $(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

$\pi 1 = (6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 6 & 6 & 12 & 12 & 6 & 6 & 12 & 12 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -1, 0, 1, 1, 1, 0]$

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t-s & 0 & 0 & 0 & s-t & 0 \\ t & 0 & 0 & -t & s & 0 & 0 & -s \\ 0 & t & 0 & -s & 0 & s & 0 & -t \\ 0 & 0 & t-s & 0 & 0 & 0 & s-t & 0 \end{pmatrix} \quad RB$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & -1 & 0 & 0 \end{pmatrix}$

M0 is invertible. det= 31175/5832

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&1\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{12} & \frac{7}{4} & 0 & 0 & 0 & 0 & \frac{7}{6} \\ \frac{7}{12} & \frac{11}{6} & \frac{7}{6} & \frac{7}{6} & 0 & 0 & \frac{7}{12} & 0 \\ \frac{7}{4} & \frac{7}{6} & \frac{11}{3} & \frac{7}{6} & 0 & \frac{7}{12} & 0 & \frac{7}{3} \\ 0 & \frac{7}{6} & \frac{7}{6} & \frac{11}{3} & \frac{7}{6} & 0 & \frac{7}{3} & \frac{7}{6} \\ 0 & 0 & 0 & \frac{7}{6} & \frac{11}{6} & \frac{7}{12} & \frac{7}{4} & 0 \\ 0 & 0 & \frac{7}{12} & 0 & \frac{7}{12} & \frac{11}{6} & \frac{7}{6} & \frac{7}{6} \\ 0 & \frac{7}{12} & 0 & \frac{7}{3} & \frac{7}{4} & \frac{7}{6} & \frac{11}{3} & \frac{7}{6} \\ \frac{7}{6} & 0 & \frac{7}{3} & \frac{7}{6} & 0 & \frac{7}{6} & \frac{7}{6} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega \left( \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{2} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left( 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{21}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{322}{29}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{80}{17}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2$ / $r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau}$ / $n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 10, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 60

dim span idems 6 vs no. of idems 10

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {3, 6, 8}

"RG3" = {5, 6, 7}

"RG4" = {4, 7, 8}

"RG5" = {3, 4, 8}

"RG6" = {4, 5, 7}

"RG7" = {2, 4, 7}

"RG8" = {2, 3, 4}

"RG9" = {1, 3, 8}

"RG10" = {1, 2, 3}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{5}{36} & \frac{31}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{18} \\ \frac{5}{36} & \frac{25}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-11}{36} & \frac{-8}{9} \\ \frac{31}{36} & \frac{5}{18} & \frac{17}{9} & \frac{-11}{18} & \frac{-8}{9} & \frac{-11}{36} & \frac{-16}{9} & \frac{5}{9} \\ \frac{-8}{9} & \frac{5}{18} & \frac{-11}{18} & \frac{17}{9} & \frac{5}{18} & \frac{-8}{9} & \frac{5}{9} & \frac{-11}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{5}{18} & \frac{25}{18} & \frac{5}{36} & \frac{31}{36} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-11}{36} & \frac{-8}{9} & \frac{5}{36} & \frac{25}{18} & \frac{5}{18} & \frac{5}{18} \\ \frac{-8}{9} & \frac{-11}{36} & \frac{-16}{9} & \frac{5}{9} & \frac{31}{36} & \frac{5}{18} & \frac{17}{9} & \frac{-11}{18} \\ \frac{5}{18} & \frac{-8}{9} & \frac{5}{9} & \frac{-11}{18} & \frac{-8}{9} & \frac{5}{18} & \frac{-11}{18} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{1}{10} & \frac{31}{50} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{1}{5} \\ \frac{1}{10} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{-8}{25} & \frac{-8}{25} & \frac{-11}{50} & \frac{-16}{25} \\ \frac{31}{68} & \frac{5}{34} & 1 & \frac{-11}{34} & \frac{-8}{17} & \frac{-11}{68} & \frac{-16}{17} & \frac{5}{17} \\ \frac{-8}{17} & \frac{5}{34} & \frac{-11}{34} & 1 & \frac{5}{34} & \frac{-8}{17} & \frac{5}{17} & \frac{-11}{34} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{1}{5} & 1 & \frac{1}{10} & \frac{31}{50} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-11}{50} & \frac{-16}{25} & \frac{1}{10} & 1 & \frac{1}{5} & \frac{1}{5} \\ \frac{-8}{17} & \frac{-11}{68} & \frac{-16}{17} & \frac{5}{17} & \frac{31}{68} & \frac{5}{34} & 1 & \frac{-11}{34} \\ \frac{5}{34} & \frac{-8}{17} & \frac{5}{17} & \frac{-11}{34} & \frac{-8}{17} & \frac{5}{34} & \frac{-11}{34} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 1.250000000, 1.880025657, 0.1477521207, 0.1105329052, 0.9893584859, 3.153895214, 5.579546726]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.9000000000, 1.112732969, 0.0825611488, 0.07130601633, 0.6087502500, 1.993559700, 3.231089916]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, 0, 0, −1, 0, 0, 1], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 0, −1, 0, 0, 0, 0]}

Eigenvalues $M_0$

[1.250000000, 9.046144669, 0.1421285585, 1.728393442, 0.1105329052, 0.9893584859, 3.153895214, 5.579546726]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0]}

Eigenvalues M

[−1.750000000, −0.5833333333, 2.529337770, −2.655317988, 0.7093135502, 5.740384102, −3.216306512, −0.774077592]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1], [0, 1, 0, 0, 0, 0, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

{2, 3, 6, 7, 8}

R: [4, 7, 5, 2, 3, 8, 8, 7]
B: [8, 3, 1, 6, 7, 4, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{3}{4096}(-1+s)(-56+4s+s^2+2s^3+s^4)(2+s)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 2, "vs", 4

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 3, "vs", 5

"R CYCLES", (1 + v[8] v[7]) (1 + v[3] v[5])
"B CYCLES", (1 + v[4] v[6]) (1 + v[1] v[3] v[8])

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, 0, 0, 0, 0, −1, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of $B^*$

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 1, 0, 0, 0, 0, 0, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 16 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 16 & 0 & 8 & 24 & 0 \\ 16 & 8 & 16 & 0 & 0 & 0 & 0 & 8 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 8 & 24 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 8 & 16 & 8 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3    "Trace mark", 3, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{12}\left( v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$

degree 2: $\frac{1}{9}\left( 2\,v[1]v[4] + v[1]v[6] + v[2]v[4] + v[2]v[5] + v[2]v[7] + 2\,v[3]v[4] + v[3]v[6] + 3\,v[3]v[7] + v[4]v[8] + 2\,v[5]v[8] + v[6]v[8] + 2\,v[8]v[7] \right)$

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"RG1" = {7, 8}

"RG2" = {6, 8}

"RG3" = {5, 8}

"RG4" = {4, 8}

"RG5" = {3, 7}

"RG6" = {3, 6}

"RG7" = {3, 4}

"RG8" = {2, 7}

"RG9" = {2, 5}

"RG10" = {2, 4}

"RG11" = {1, 6}

"RG12" = {1, 4}

$\pi 2$ = [0, 0, 2, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 2, 0, 1, 3, 0, 0, 0, 0, 1, 0, 0, 2, 0, 1, 2]

supp $\pi 2$ = {3, 5, 9, 10, 12, 14, 16, 17, 22, 25, 27, 28}

$u2$ = [0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1]

supp u2 = {3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 17, 22, 25, 27, 28}

Action of R on ranges, [[1], [1], [5], [8], [3], [3], [9], [1], [5], [8], [4], [10]]
Action of B on ranges, [[7], [7], [5], [6], [12], [12], [11], [7], [5], [6], [4], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{9} & \frac{1}{18} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{9} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1]

B−BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N − check: true

$b_1 = \{1, 2, 3, 8\}$
$b_2 = \{4, 5, 6, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 2, 2, 2

## LIE STRUCTURE

Dimension of Lie algebra: 32, Shape: $8 \oplus 24/22$

$$CLB = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{7, 8}, {3, 5}}, false

$\Omega_B$ in Vec(K)? , {{4, 6}, {1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{3}{8} & \frac{3}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 2, 3, 8}, {4, 5, 6, 7}}

1, "range", [7, 8], [[8, 8, 8, 7, 7, 7, 7, 8], [7, 7, 7, 8, 8, 8, 8, 7]]

2, "range", [6, 8], [[8, 8, 8, 6, 6, 6, 6, 8], [6, 6, 6, 8, 8, 8, 8, 6]]

3, "range", [5, 8], [[8, 8, 8, 5, 5, 5, 5, 8], [5, 5, 5, 8, 8, 8, 8, 5]]

4, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

5, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

6, "range", [3, 6], [[6, 6, 6, 3, 3, 3, 3, 6], [3, 3, 3, 6, 6, 6, 6, 3]]

7, "range", [3, 4], [[4, 4, 4, 3, 3, 3, 3, 4], [3, 3, 3, 4, 4, 4, 4, 3]]

8, "range", [2, 7], [[7, 7, 7, 2, 2, 2, 2, 7], [2, 2, 2, 7, 7, 7, 7, 2]]

9, "range", [2, 5], [[5, 5, 5, 2, 2, 2, 2, 5], [2, 2, 2, 5, 5, 5, 5, 2]]

10, "range", [2, 4], [[4, 4, 4, 2, 2, 2, 2, 4], [2, 2, 2, 4, 4, 4, 4, 2]]

11, "range", [1, 6], [[6, 6, 6, 1, 1, 1, 1, 6], [1, 1, 1, 6, 6, 6, 6, 1]]

12, "range", [1, 4], [[4, 4, 4, 1, 1, 1, 1, 4], [1, 1, 1, 4, 4, 4, 4, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2$ =

$\begin{pmatrix} 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 2 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 2 \end{pmatrix}$

{3, 5, 9, 10, 12, 14, 16, 17, 22, 25, 27, 28}

$u2 =$
$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$

{3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 17, 22, 25, 27, 28}

picheck $\begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 3 & 3 & 6 & 6 & 3 & 3 & 6 & 6 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, -1, 1, -1, 1, 1]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & -s & 0 & 0 & s & -t \\ 0 & 0 & t & -s & 0 & 0 & s & -t \\ t & 0 & 0 & -s & s & 0 & 0 & -t \\ 0 & -s & s & 0 & 0 & -t & t & 0 \\ 0 & -s & 0 & t & 0 & -t & 0 & s \\ 0 & -s & 0 & t & 0 & -t & 0 & s \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} s-t \\ s-t \\ s-t \\ -s+t \\ -s+t \\ -s+t \\ -s+t \\ s-t \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s \\ t & s \\ t & s \\ s & t \\ s & t \\ s & t \\ s & t \\ t & s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Skew\ Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & \frac{16}{9} & 0 & \frac{8}{9} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & \frac{8}{9} & \frac{8}{9} & 0 & \frac{8}{9} & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{9} & 0 & \frac{8}{9} & \frac{8}{3} & 0 \\ \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{9} \\ 0 & \frac{8}{9} & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{16}{9} \\ \frac{8}{9} & 0 & \frac{8}{9} & 0 & 0 & \frac{8}{3} & 0 & \frac{8}{9} \\ 0 & \frac{8}{9} & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{9} \\ 0 & 0 & 0 & \frac{8}{9} & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left( \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \right)$$

$$T \left( \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{3} \quad 0 \quad 0 \quad 0 \quad \frac{1}{3} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left( \frac{8}{3} \quad \frac{8}{3} \quad \frac{16}{3} \quad 0 \quad 0 \quad 0 \quad \frac{16}{3} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{16}{3} \quad \frac{8}{3} \quad \frac{8}{3} \right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN} \left( 4 \quad 4 \quad 4 \quad 0 \quad 0 \quad 0 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4 \quad 4 \quad 4 \right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 12, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 24

dim span idems 7 vs no. of idems 12

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"RG1" = {7, 8}

"RG2" = {6, 8}

"RG3" = {5, 8}

"RG4" = {4, 8}

"RG5" = {3, 7}

"RG6" = {3, 6}

"RG7" = {3, 4}

"RG8" = {2, 7}

"RG9" = {2, 5}

"RG10" = {2, 4}

"RG11" = {1, 6}

"RG12" = {1, 4}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & 0 & \frac{4}{9} & \frac{-4}{9} & 0 & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & 0 & \frac{-8}{9} & 0 & \frac{8}{9} & \frac{-16}{9} \\ \frac{8}{9} & 0 & 0 & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{8}{9} \\ \frac{4}{9} & \frac{-4}{9} & 0 & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & 0 \\ \frac{-8}{9} & 0 & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & 0 \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{8}{9} & 0 & 0 & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & 0 & \frac{1}{5} & \frac{-1}{5} & 0 & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 0 & \frac{-1}{4} & 0 & \frac{1}{4} & \frac{-1}{2} \\ \frac{1}{4} & 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{4} \\ \frac{-1}{5} & \frac{1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{-1}{5} & 0 & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & 0 \\ \frac{-1}{4} & 0 & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 0 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{1}{4} & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & 1 & 1 & \frac{-5}{31} \\ 1 & 1 & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 2.666666667, 3.555555556, 6.222222222, 6.128983570, 1.517865258, 3.019817840]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 0., 4., 2.888888889]

Eigenvalues $M_C$−scaled

[0., 0., 1.875400564, 0.6378547175, 1.086744718, 1.990960343, 0.9222603030, 1.486779355]

Eigenvalues $N_C$−scaled

[0., 0., 0., 0., 0., 0., 4.645161290, 3.354838710]

NullSpace $M_C$

{[1, 1, 1, 0, 0, 0, 0, 1], [0, 0, 0, 1, 1, 1, 1, 0]}

NullSpace $N_C$

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, −1, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 1, 0, 0, 0, 0, 0, 0]}

Eigenvalues $M_0$

[0., 3.555555556, 6.222222222, 9.517531656, 1.511050604, 5.860306632, 2.666666667, 2.666666667]

Eigenvalues $N_0$

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{[1, 1, 1, −1, −1, −1, −1, 1]}

NullSpace $N_0$

{[0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0]}

Eigenvalues M

[0.5286569585, 1.752280775, −1.352734336, −4.483758953, 1.352734336, 4.483758953, −0.5286569585, −1.752280775]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

{2, 4, 5, 7, 8}

R: [4, 7, 1, 6, 7, 4, 8, 7]
B: [8, 3, 5, 2, 3, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{512}(-7+s^2)(-3+s)(1+s)(2+s)(-1+s)$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 2, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6])
"B CYCLES", 1 + v[3] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of $R^*$

{[0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 8 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3    "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )
degree 2: $\frac{1}{6}$ ( $v[1]v[7] + v[2]v[3] + v[3]v[5] + 2v[4]v[8] + v[6]v[7]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {6, 7}

"RG2" = {4, 8}

"RG3" = {3, 5}

"RG4" = {2, 3}

"RG5" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0]

supp $\pi 2$ = {6, 8, 15, 22, 26}

$u2$ = [1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0]

supp u2 = {1, 4, 6, 7, 8, 9, 11, 15, 17, 18, 19, 21, 22, 23, 26, 27}

Action of R on ranges, [[2], [1], [5], [5], [2]]
Action of B on ranges, [[2], [4], [3], [3], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N - check: true

$b_1 = \{1, 3, 4, 6\}$
$b_2 = \{2, 5, 7, 8\}$

dim(span of partition vectors), rank($N_0$), rank($N$):    2, 2, 2

## LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 0 ⊕ 13/11

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-7}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 3, 4, 6}, {2, 5, 7, 8}}

1, "range", [6, 7], [[7, 6, 7, 7, 6, 7, 6, 6], [6, 7, 6, 6, 7, 6, 7, 7]]

2, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 5, 3, 3], [3, 5, 3, 3, 5, 3, 5, 5]]

4, "range", [2, 3], [[3, 2, 3, 3, 2, 3, 2, 2], [2, 3, 2, 2, 3, 2, 3, 3]]

5, "range", [1, 7], [[7, 1, 7, 7, 1, 7, 1, 1], [1, 7, 1, 1, 7, 1, 7, 7]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

{6, 8, 15, 22, 26}

$$u2 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

{1, 4, 6, 7, 8, 9, 11, 15, 17, 18, 19, 21, 22, 23, 26, 27}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 \\ \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, -2, 0, -1, 1, 2, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -t & 0 & 0 & t & -s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t & t & 0 & 0 & -s & s \\ 0 & -t & 0 & s & 0 & -s & 0 & t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -t & 0 & s & 0 & -s & 0 & t \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & 0 & 2 & 1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s-t & 0 & 0 \\ 0 & -t & -s \\ 0 & t & s \\ 0 & t & s \\ 0 & -t & -s \\ s-t & 0 & 0 \\ t-s & 0 & 0 \\ 0 & -t & -s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t-s & s & 0 & s \\ 0 & s & t & 0 \\ 0 & t & -t & s+t \\ 0 & t & -t & s+t \\ 0 & s & t & 0 \\ t-s & s & 0 & s \\ s-t & t & 0 & t \\ 0 & s & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 0 & 4 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 1&0&1&1&0&1&0&0 \\ 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 0&1&0&0&1&0&1&1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&1&0&0 \\ 0&1&0&0&1&0&0&1 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&1&0&0&1&0&0&1 \\ 1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&1&0&0&1&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 1&0&1&1&0&1&0&0 \\ 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&1 \\ 0&1&0&0&1&0&1&1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&1&0&0 \\ 0&1&0&0&1&0&0&1 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&1&0&0&1&0&0&1 \\ 1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&1&0&0&1&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad Skew\,T = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \\ 0 & \frac{-1}{6} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \end{pmatrix}$$

$$Skew\,Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 \\ 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{8}{3} & \frac{16}{3} & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & \frac{8}{3} & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \left( \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$

$T \left( \frac{1}{6} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left( \frac{8}{3} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{8}{3} \;\; 0 \;\; \frac{16}{3} \;\; \frac{16}{3} \;\; 0 \;\; \frac{8}{3} \right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left( \frac{8}{3} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{8}{3} \;\; 0 \;\; 4 \;\; \frac{16}{3} \;\; 0 \;\; \frac{8}{3} \right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 5, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 10
out of total no. of elements equal to 10

dim span idems 5 vs no. of idems 5

"PT1" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {6, 7}

"RG2" = {4, 8}

"RG3" = {3, 5}

"RG4" = {2, 3}

"RG5" = {1, 7}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{16}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{16}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{16}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{16}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{16}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{16}{9} & \frac{-8}{9} \\ \frac{16}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{16}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{4}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{4}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{1}{2} & 1 & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{4}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{4}{5} & \frac{-2}{5} \\ \frac{1}{2} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{1}{2} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 \\ 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 \\ 1 & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_c$

[9.777777778, 8., 2.666666667, 2.666666667, 0., 0., 0., 0.]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 0., 4., 2.888888889]

Eigenvalues $M_C$–scaled

[2.900000000, 2.700000000, 1.200000000, 1.200000000, 0., 0., 0., 0.]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 0., 4.645161290, 3.354838710]

NullSpace $M_C$

{[0, 1, 0, 0, 1, 0, 1, 1], [0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 1, 0, 0, 0, 1, 1], [1, 0, 0, 0, 0, 1, −1, 0]}

NullSpace $N_C$

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 8., 2.666666667, 8., 2.666666667, 0., 0., 0.]

Eigenvalues $N_0$

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{[1, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 1, 0, 0, 0, −1], [0, 1, −1, 0, 1, 0, 0, 0]}

NullSpace $N_0$

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [1, 0, −1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[5.333333333, −5.333333333, 0., 0., 3.771236166, −3.771236166, 3.771236166, −3.771236166]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

==============================================

{3, 4, 5, 7, 8}

R: [4, 3, 5, 6, 7, 4, 8, 7]
B: [8, 7, 1, 2, 3, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{1}{4096}\left(-672 + 540s + 60s^2 - 13s^3 - 13s^4 - 24s^5 + 2s^6\right)(-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6])
"B CYCLES", (1 + v[2] v[4] v[7]) (1 + v[1] v[3] v[8])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, 0, 0, 0, −1, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $B^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 21 & 21 & 0 & 0 & 0 & 14 & 14 \\ 21 & 0 & 28 & 14 & 0 & 0 & 7 & 0 \\ 21 & 28 & 0 & 42 & 14 & 7 & 0 & 28 \\ 0 & 14 & 42 & 0 & 14 & 0 & 28 & 42 \\ 0 & 0 & 14 & 14 & 0 & 21 & 21 & 0 \\ 0 & 0 & 7 & 0 & 21 & 0 & 28 & 14 \\ 14 & 7 & 0 & 28 & 21 & 28 & 0 & 42 \\ 14 & 0 & 28 & 42 & 0 & 14 & 42 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3    "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}\,(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{20}\,(3v[1]v[2] + 3v[1]v[3] + 2v[1]v[7] + 2v[1]v[8] + 4v[2]v[3] + 2v[2]v[4] + v[2]v[7] + 6v[3]v[4] + 2v[3]v[5] + v[3]v[6] + 4v[3]v[8] + 2v[4]v[5] + 4v[4]v[7] + 6v[4]v[8] + 3v[5]v[6] + 3v[5]v[7] + 4v[6]v[7] + 2v[6]v[8] + 6v[8]v[7])$

degree 3 : $\frac{1}{10}\,(2v[1]v[2]v[3] + v[1]v[2]v[7] + v[1]v[3]v[8] + v[1]v[8]v[7] + 2v[2]v[3]v[4] + v[3]v[4]v[5] + 3v[3]v[4]v[8] + v[3]v[5]v[6] + v[4]v[5]v[7] + 3v[4]v[8]v[7] + 2v[5]v[6]v[7] + 2v[6]v[8]v[7])$

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {5, 6, 7}

"RG3" = {3, 5, 6}

"RG4" = {4, 7, 8}

"RG5" = {3, 4, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 4, 5}

"RG8" = {2, 3, 4}

"RG9" = {1, 7, 8}

"RG10" = {1, 3, 8}

"RG11" = {1, 2, 7}

"RG12" = {1, 2, 3}

$\pi 3$ = [2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 3, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 3, 2, 0, 0, 2]

supp $\pi 3$ = {1, 5, 11, 21, 22, 37, 40, 41, 48, 52, 53, 56}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[4], [4], [6], [1], [2], [1], [2], [3], [4], [6], [5], [7]]
Action of B on ranges, [[5], [5], [10], [8], [12], [8], [12], [11], [5], [10], [4], [9]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{20} & \frac{3}{20} & \frac{3}{20} & \frac{1}{20} & \frac{1}{20} & \frac{1}{10} & \frac{1}{20} & \frac{1}{20} & \frac{1}{20} & \frac{1}{10} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 3, 2]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1 = \{1, 4, 6\}$
$b_2 = \{2, 5, 8\}$
$b_3 = \{3, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 34, Shape: 11 ⊕ 23/21

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}, {2, 4, 7}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{3}{16} & \frac{5}{16} & \frac{5}{16} \end{pmatrix} \text{vs} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} \frac{5}{24} & \frac{1}{8} & \frac{5}{24} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{5}{24} \end{pmatrix} \text{vs} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [6, 7, 8], [[8, 7, 6, 8, 7, 8, 6, 7], [8, 6, 7, 8, 6, 8, 7, 6], [7, 8, 6, 7, 8, 7, 6, 8], [7, 6, 8, 7, 6, 7, 8, 6], [6, 8, 7, 6, 8, 6, 7, 8], [6, 7, 8, 6, 7, 6, 8, 7]]

2, "range", [5, 6, 7], [[7, 6, 5, 7, 6, 7, 5, 6], [7, 5, 6, 7, 5, 7, 6, 5], [6, 7, 5, 6, 7, 6, 5, 7], [6, 5, 7, 6, 5, 6, 7, 5], [5, 7, 6, 5, 7, 5, 6, 7], [5, 6, 7, 5, 6, 5, 7, 6]]

3, "range", [3, 5, 6], [[6, 5, 3, 6, 5, 6, 3, 5], [6, 3, 5, 6, 3, 6, 5, 3], [5, 6, 3, 5, 6, 5, 3, 6], [5, 3, 6, 5, 3, 5, 6, 3], [3, 6, 5, 3, 6, 3, 5, 6], [3, 5, 6, 3, 5, 3, 6, 5]]

4, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

5, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

6, "range", [4, 5, 7], [[7, 5, 4, 7, 5, 7, 4, 5], [7, 4, 5, 7, 4, 7, 5, 4], [5, 7, 4, 5, 7, 5, 4, 7], [5, 4, 7, 5, 4, 5, 7, 4], [4, 7, 5, 4, 7, 4, 5, 7], [4, 5, 7, 4, 5, 4, 7, 5]]

7, "range", [3, 4, 5], [[5, 4, 3, 5, 4, 5, 3, 4], [5, 3, 4, 5, 3, 5, 4, 3], [4, 5, 3, 4, 5, 4, 3, 5], [4, 3, 5, 4, 3, 4, 5, 3], [3, 5, 4, 3, 5, 3, 4, 5], [3, 4, 5, 3, 4, 3, 5, 4]]

8, "range", [2, 3, 4], [[4, 3, 2, 4, 3, 4, 2, 3], [4, 2, 3, 4, 2, 4, 3, 2], [3, 4, 2, 3, 4, 3, 2, 4], [3, 2, 4, 3, 2, 3, 4, 2], [2, 4, 3, 2, 4, 2, 3, 4], [2, 3, 4, 2, 3, 2, 4, 3]]

9, "range", [1, 7, 8], [[8, 7, 1, 8, 7, 8, 1, 7], [8, 1, 7, 8, 1, 8, 7, 1], [7, 8, 1, 7, 8, 7, 1, 8], [7, 1, 8, 7, 1, 7, 8, 1], [1, 8, 7, 1, 8, 1, 7, 8], [1, 7, 8, 1, 7, 1, 8, 7]]

10, "range", [1, 3, 8], [[8, 3, 1, 8, 3, 8, 1, 3], [8, 1, 3, 8, 1, 8, 3, 1], [3, 8, 1, 3, 8, 3, 1, 8], [3, 1, 8, 3, 1, 3, 8, 1], [1, 8, 3, 1, 8, 1, 3, 8], [1, 3, 8, 1, 3, 1, 8, 3]]

11, "range", [1, 2, 7], [[7, 2, 1, 7, 2, 7, 1, 2], [7, 1, 2, 7, 1, 7, 2, 1], [2, 7, 1, 2, 7, 2, 1, 7], [2, 1, 7, 2, 1, 2, 7, 1], [1, 7, 2, 1, 7, 1, 2, 7], [1, 2, 7, 1, 2, 1, 7, 2]]

12, "range", [1, 2, 3], [[3, 2, 1, 3, 2, 3, 1, 2], [3, 1, 2, 3, 1, 3, 2, 1], [2, 3, 1, 2, 3, 2, 1, 3], [2, 1, 3, 2, 1, 2, 3, 1], [1, 3, 2, 1, 3, 1, 2, 3], [1, 2, 3, 1, 2, 1, 3, 2]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1$ = [[1, 3, 2]]

$g_2$ = [[1, 3]]

$g_3$ = [[1, 2]]

$g_4$ = [[1, 2, 3]]

$g_5$ = []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & h[2] & 2\,h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[ 1 ] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3$ =
$(2\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

{1, 5, 11, 21, 22, 37, 40, 41, 48, 52, 53, 56}

$u3$ =
$(1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

picheck $\begin{pmatrix} 5 & 5 & 10 & 10 & 5 & 5 & 10 & 10 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 =$
$$\begin{pmatrix}3 & 3 & 0 & 0 & 0 & 2 & 2 & 4 & 2 & 0 & 0 & 1 & 0 & 6 & 2 & 1 & 0 & 4 & 2 & 0 & 4 & 6 & 3 & 3 & 0 & 4 & 2 & 6\end{pmatrix}$$

$u2 =$
$$\left(\frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3} \; 0 \; \frac{1}{3} \; \frac{1}{3} \; \frac{1}{3}\right.$$

picheck $\begin{pmatrix}10 & 10 & 20 & 20 & 10 & 10 & 20 & 20\end{pmatrix}$

$\pi 1 = \begin{pmatrix}10 & 10 & 20 & 20 & 10 & 10 & 20 & 20\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9}\end{pmatrix}$

picheck $\begin{pmatrix}10 & 10 & 20 & 20 & 10 & 10 & 20 & 20\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -1, 0, 1, 1, 1, 0]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & -s & 0 & s & 0 & -t \\ 0 & 0 & -s+t & 0 & 0 & 0 & -t+s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & t & -s & 0 & 0 & s \\ 0 & 0 & -s+t & 0 & 0 & 0 & -t+s & 0 \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \end{pmatrix}$

M0 is invertible. det= 1065228727/164025000

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&1&0&0&0&1&0\\1&0&0&1&0&1&0&0\\0&1&0&0&1&0&0&1\\1&0&0&1&0&1&0&0\\0&0&1&0&0&0&1&0\\0&1&0&0&1&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&1\\1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \text{ Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{21}{20} & \frac{21}{20} & 0 & 0 & 0 & \frac{7}{10} & \frac{7}{10} \\ \frac{21}{20} & \frac{11}{6} & \frac{7}{5} & \frac{7}{10} & 0 & 0 & \frac{7}{20} & 0 \\ \frac{21}{20} & \frac{7}{5} & \frac{11}{3} & \frac{21}{10} & \frac{7}{10} & \frac{7}{20} & 0 & \frac{7}{5} \\ 0 & \frac{7}{10} & \frac{21}{10} & \frac{11}{3} & \frac{7}{10} & 0 & \frac{7}{5} & \frac{21}{10} \\ 0 & 0 & \frac{7}{10} & \frac{7}{10} & \frac{11}{6} & \frac{21}{20} & \frac{21}{20} & 0 \\ 0 & 0 & \frac{7}{20} & 0 & \frac{21}{20} & \frac{11}{6} & \frac{7}{5} & \frac{7}{10} \\ \frac{7}{10} & \frac{7}{20} & 0 & \frac{7}{5} & \frac{21}{20} & \frac{7}{5} & \frac{11}{3} & \frac{21}{10} \\ \frac{7}{10} & 0 & \frac{7}{5} & \frac{21}{10} & 0 & \frac{7}{10} & \frac{21}{10} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ 0\ \frac{1}{2}\ 0\ 0\ 0\ 0\ \frac{1}{4}\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{21}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{322}{29}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{80}{17}\right.$$

$\boldsymbol{\tau}$ = 22/1 , rank = 3 , ratio = 22/3 , $n^2 / r$ = 64/3

$\boldsymbol{\tau}'$ = 42/1 , $r'$ = 2/3 , $\boldsymbol{\tau} / n^2$ = 11/32

$p^2$ = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$–check is positive? 118/9

max r = 36/5 , $r$–check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 12, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 72

dim span idems 6 vs no. of idems 12

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {5, 6, 7}

"RG3" = {3, 5, 6}

"RG4" = {4, 7, 8}

"RG5" = {3, 4, 8}

"RG6" = {4, 5, 7}

"RG7" = {3, 4, 5}

"RG8" = {2, 3, 4}

"RG9" = {1, 7, 8}

"RG10" = {1, 3, 8}

"RG11" = {1, 2, 7}

"RG12" = {1, 2, 3}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{109}{180} & \frac{29}{180} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-17}{90} & \frac{-17}{90} \\ \frac{109}{180} & \frac{25}{18} & \frac{23}{45} & \frac{-17}{90} & \frac{-4}{9} & \frac{-4}{9} & \frac{-97}{180} & \frac{-8}{9} \\ \frac{29}{180} & \frac{23}{45} & \frac{17}{9} & \frac{29}{90} & \frac{-17}{90} & \frac{-97}{180} & \frac{-16}{9} & \frac{-17}{45} \\ \frac{-8}{9} & \frac{-17}{90} & \frac{29}{90} & \frac{17}{9} & \frac{-17}{90} & \frac{-8}{9} & \frac{-17}{45} & \frac{29}{90} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-17}{90} & \frac{-17}{90} & \frac{25}{18} & \frac{109}{180} & \frac{29}{180} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-97}{180} & \frac{-8}{9} & \frac{109}{180} & \frac{25}{18} & \frac{23}{45} & \frac{-17}{90} \\ \frac{-17}{90} & \frac{-97}{180} & \frac{-16}{9} & \frac{-17}{45} & \frac{29}{180} & \frac{23}{45} & \frac{17}{9} & \frac{29}{90} \\ \frac{-17}{90} & \frac{-8}{9} & \frac{-17}{45} & \frac{29}{90} & \frac{-8}{9} & \frac{-17}{90} & \frac{29}{90} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

 

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{109}{250} & \frac{29}{250} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-17}{125} & \frac{-17}{125} \\ \frac{109}{250} & 1 & \frac{46}{125} & \frac{-17}{125} & \frac{-8}{25} & \frac{-8}{25} & \frac{-97}{250} & \frac{-16}{25} \\ \frac{29}{340} & \frac{23}{85} & 1 & \frac{29}{170} & \frac{-1}{10} & \frac{-97}{340} & \frac{-16}{17} & \frac{-1}{5} \\ \frac{-8}{17} & \frac{-1}{10} & \frac{29}{170} & 1 & \frac{-1}{10} & \frac{-8}{17} & \frac{-1}{5} & \frac{29}{170} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-17}{125} & \frac{-17}{125} & 1 & \frac{109}{250} & \frac{29}{250} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-97}{250} & \frac{-16}{25} & \frac{109}{250} & 1 & \frac{46}{125} & \frac{-17}{125} \\ \frac{-1}{10} & \frac{-97}{340} & \frac{-16}{17} & \frac{-1}{5} & \frac{29}{340} & \frac{23}{85} & 1 & \frac{29}{170} \\ \frac{-1}{10} & \frac{-8}{17} & \frac{-1}{5} & \frac{29}{170} & \frac{-8}{17} & \frac{-1}{10} & \frac{29}{170} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_C N_C =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.7833333333, 3.279848639, 0.147929139, 0.1102985171, 1.643590432, 2.580809521, 4.565301530]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.5640000000, 1.942818529, 0.0825932351, 0.07125610164, 0.9630711489, 1.672762708, 2.703498278]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

Eigenvalues $M_0$

[0.7833333333, 9.217301912, 0.142412942, 2.956951814, 0.1102985171, 1.643590432, 2.580809521, 4.565301530]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[−1.050000000, 5.836625774, −3.152638807, 0.466013033, 0.1054473612, 1.736215285, −1.192665167, −2.748997478]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

{3, 4, 6, 7, 8}

R: [4, 3, 5, 6, 3, 8, 8, 7]
B: [8, 7, 1, 2, 7, 4, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{1}{4096}(-672 + 540s + 60s^2 - 13s^3 - 13s^4 - 24s^5 + 2s^6)(-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[8] v[7]) (1 + v[3] v[5])
"B CYCLES", (1 + v[2] v[4] v[7]) (1 + v[1] v[3] v[8])

Eigenvalues

R:   [1., −1., 1., −1., 0., 0., 0., 0.]

B:   [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[0, 0, 0, 0, 0, −1, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 21 & 14 & 28 & 0 & 0 & 0 & 7 \\ 21 & 0 & 0 & 21 & 0 & 0 & 14 & 14 \\ 14 & 0 & 0 & 42 & 0 & 14 & 42 & 28 \\ 28 & 21 & 42 & 0 & 7 & 14 & 28 & 0 \\ 0 & 0 & 0 & 7 & 0 & 21 & 14 & 28 \\ 0 & 0 & 14 & 14 & 21 & 0 & 0 & 21 \\ 0 & 14 & 42 & 28 & 14 & 0 & 0 & 42 \\ 7 & 14 & 28 & 0 & 28 & 21 & 42 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3    "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )

degree 2: $\frac{1}{20}$ ( $3v[1]v[2] + 2v[1]v[3] + 4v[1]v[4] + v[1]v[8] + 3v[2]v[4] + 2v[2]v[7]$
$+ 2v[2]v[8] + 6v[3]v[4] + 2v[3]v[6] + 6v[3]v[7] + 4v[3]v[8] + v[4]v[5] + 2v[4]v[6] +$

$4v[4]v[7] + 3v[5]v[6] + 2v[5]v[7] + 4v[5]v[8] + 3v[6]v[8] + 6v[8]v[7])$

degree 3 : $\frac{1}{10}(2v[1]v[2]v[4] + v[1]v[2]v[8] + 2v[1]v[3]v[4] + v[2]v[4]v[7] + v[2]v[8]v[7] + v[3]v[4]v[6] + 3v[3]v[4]v[7] + v[3]v[6]v[8] + 3v[3]v[8]v[7] + v[4]v[5]v[6] + 2v[5]v[6]v[8] + 2v[5]v[8]v[7])$

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 7, 8}

"RG5" = {2, 4, 7}

"RG6" = {5, 6, 8}

"RG7" = {4, 5, 6}

"RG8" = {3, 6, 8}

"RG9" = {3, 4, 6}

"RG10" = {1, 3, 4}

"RG11" = {1, 2, 8}

"RG12" = {1, 2, 4}

$\pi 3$ = [0, 2, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 3, 0, 0, 0, 0, 0, 1, 3, 1, 0, 0, 0, 0, 0, 0, 2, 2, 0]

supp $\pi 3$ = {2, 6, 7, 29, 36, 38, 39, 45, 46, 47, 54, 55}

$u3$ = [0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0]

supp u3 = {2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

Action of R on ranges, [[2], [1], [6], [2], [8], [2], [8], [1], [6], [7], [3], [9]]
Action of B on ranges, [[3], [10], [12], [3], [5], [3], [5], [10], [12], [11], [2], [4]]

$\boldsymbol{\beta} = \left(\frac{1}{10} \quad \frac{3}{20} \quad \frac{3}{20} \quad \frac{1}{20} \quad \frac{1}{20} \quad \frac{1}{10} \quad \frac{1}{20} \quad \frac{1}{20} \quad \frac{1}{20} \quad \frac{1}{10} \quad \frac{1}{20} \quad \frac{1}{10}\right)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1, 3]

B–BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4, 8\}$
$b_2 = \{1, 6, 7\}$
$b_3 = \{2, 3, 5\}$

dim(span of partition vectors), rank($N_0$), rank($N$):    3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 34, Shape: 11 ⊕ 23/21

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{7, 8}, {3, 5}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}, {2, 4, 7}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{3}{16} & 0 & \frac{3}{16} & 0 & \frac{5}{16} & \frac{5}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{5}{24} & \frac{1}{8} & \frac{5}{24} & 0 & 0 & \frac{5}{24} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{4, 8}, {1, 6, 7}, {2, 3, 5}}

1, "range", [5, 7, 8], [[8, 7, 7, 5, 7, 8, 8, 5], [8, 5, 5, 7, 5, 8, 8, 7], [7, 8, 8, 5, 8, 7, 7, 5], [7, 5, 5, 8, 5, 7, 7, 8], [5, 8, 8, 7, 8, 5, 5, 7], [5, 7, 7, 8, 7, 5, 5, 8]]

2, "range", [3, 7, 8], [[8, 7, 7, 3, 7, 8, 8, 3], [8, 3, 3, 7, 3, 8, 8, 7], [7, 8, 8, 3, 8, 7, 7, 3], [7, 3, 3, 8, 3, 7, 7, 8], [3, 8, 8, 7, 8, 3, 3, 7], [3, 7, 7, 8, 7, 3, 3, 8]]

3, "range", [3, 4, 7], [[7, 4, 4, 3, 4, 7, 7, 3], [7, 3, 3, 4, 3, 7, 7, 4], [4, 7, 7, 3, 7, 4, 4, 3], [4, 3, 3, 7, 3, 4, 4, 7], [3, 7, 7, 4, 7, 3, 3, 4], [3, 4, 4, 7, 4, 3, 3, 7]]

4, "range", [2, 7, 8], [[8, 7, 7, 2, 7, 8, 8, 2], [8, 2, 2, 7, 2, 8, 8, 7], [7, 8, 8, 2, 8, 7, 7, 2], [7, 2, 2, 8, 2, 7, 7, 8], [2, 8, 8, 7, 8, 2, 2, 7], [2, 7, 7, 8, 7, 2, 2, 8]]

5, "range", [2, 4, 7], [[7, 4, 4, 2, 4, 7, 7, 2], [7, 2, 2, 4, 2, 7, 7, 4], [4, 7, 7, 2, 7, 4, 4, 2], [4, 2, 2, 7, 2, 4, 4, 7], [2, 7, 7, 4, 7, 2, 2, 4], [2, 4, 4, 7, 4, 2, 2, 7]]

6, "range", [5, 6, 8], [[8, 6, 6, 5, 6, 8, 8, 5], [8, 5, 5, 6, 5, 8, 8, 6], [6, 8, 8, 5, 8, 6, 6, 5], [6, 5, 5, 8, 5, 6, 6, 8], [5, 8, 8, 6, 8, 5, 5, 6], [5, 6, 6, 8, 6, 5, 5, 8]]

7, "range", [4, 5, 6], [[6, 5, 5, 4, 5, 6, 6, 4], [6, 4, 4, 5, 4, 6, 6, 5], [5, 6, 6, 4, 6, 5, 5, 4], [5, 4, 4, 6, 4, 5, 5, 6], [4, 6, 6, 5, 6, 4, 4, 5], [4, 5, 5, 6, 5, 4, 4, 6]]

8, "range", [3, 6, 8], [[8, 6, 6, 3, 6, 8, 8, 3], [8, 3, 3, 6, 3, 8, 8, 6], [6, 8, 8, 3, 8, 6, 6, 3], [6, 3, 3, 8, 3, 6, 6, 8], [3, 8, 8, 6, 8, 3, 3, 6], [3, 6, 6, 8, 6, 3, 3, 8]]

9, "range", [3, 4, 6], [[6, 4, 4, 3, 4, 6, 6, 3], [6, 3, 3, 4, 3, 6, 6, 4], [4, 6, 6, 3, 6, 4, 4, 3], [4, 3, 3, 6, 3, 4, 4, 6], [3, 6, 6, 4, 6, 3, 3, 4], [3, 4, 4, 6, 4, 3, 3, 6]]

10, "range", [1, 3, 4], [[4, 3, 3, 1, 3, 4, 4, 1], [4, 1, 1, 3, 1, 4, 4, 3], [3, 4, 4, 1, 4, 3, 3, 1], [3, 1, 1, 4, 1, 3, 3, 4], [1, 4, 4, 3, 4, 1, 1, 3], [1, 3, 3, 4, 3, 1, 1, 4]]

11, "range", [1, 2, 8], [[8, 2, 2, 1, 2, 8, 8, 1], [8, 1, 1, 2, 1, 8, 8, 2], [2, 8, 8, 1, 8, 2, 2, 1], [2, 1, 1, 8, 1, 2, 2, 8], [1, 8, 8, 2, 8, 1, 1, 2], [1, 2, 2, 8, 2, 1, 1, 8]]

12, "range", [1, 2, 4], [[4, 2, 2, 1, 2, 4, 4, 1], [4, 1, 1, 2, 1, 4, 4, 2], [2, 4, 4, 1, 4, 2, 2, 1], [2, 1, 1, 4, 1, 2, 2, 4], [1, 4, 4, 2, 4, 1, 1, 2], [1, 2, 2, 4, 2, 1, 1, 4]]

"group has", 6, "elements"    Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1$ =    [[1, 2, 3]]

$g_2$ =    [[2, 3]]

$g_3$ =    [[1, 3]]

$g_4$ =    []

$g_5$ =    [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2\,h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[ 1 ] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(0\ \ 2\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1$

{2, 6, 7, 29, 36, 38, 39, 45, 46, 47, 54, 55}

$u3 =$
$(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1$

{2, 6, 7, 11, 12, 18, 28, 29, 35, 36, 38, 39, 45, 46, 47, 48, 54, 55}

picheck $\begin{pmatrix} 5 & 5 & 10 & 10 & 5 & 5 & 10 & 10 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 =$
$(3\ 2\ 4\ 0\ 0\ 0\ 1\ 0\ 3\ 0\ 0\ 2\ 2\ 6\ 0\ 2\ 6\ 4\ 1\ 2\ 4\ 0\ 3\ 2\ 4\ 0\ 3\ 6)$

$u2 =$
$\left(\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ 0\ \frac{1}{3}\ 0\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\right.$

picheck $(10\ 10\ 20\ 20\ 10\ 10\ 20\ 20)$

$\pi 1 = (10\ 10\ 20\ 20\ 10\ 10\ 20\ 20)$

$u1 = \left(\frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\ \frac{2}{9}\right)$

picheck $(10\ 10\ 20\ 20\ 10\ 10\ 20\ 20)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, −1, 0, −1, 1, 1, 0, 1]

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & -s & 0 & s & 0 & -t & 0 \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \\ 0 & 0 & 0 & -s+t & 0 & 0 & 0 & -t+s \\ 0 & -t & t & 0 & 0 & -s & s & 0 \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

M0 is invertible. det= 1065228727/164025000

$$\ker M_c = \begin{pmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&1&0\\0&0&0&0&0&1&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&0&0&1&1&0\\0&1&1&0&1&0&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&0&1\\0&1&1&0&1&0&0&0\\1&0&0&0&0&1&1&0\\1&0&0&0&0&1&1&0\\0&0&0&1&0&0&0&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&1&0&0&1&0&0&0\\0&0&0&0&0&1&1&0\\0&0&0&0&0&1&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ 0 & 0 & \frac{11}{6} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{11}{6} & \frac{7}{4} \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{21}{20} & \frac{7}{10} & \frac{7}{5} & 0 & 0 & 0 & \frac{7}{20} \\ \frac{21}{20} & \frac{11}{6} & 0 & \frac{21}{20} & 0 & 0 & \frac{7}{10} & \frac{7}{10} \\ \frac{7}{10} & 0 & \frac{11}{3} & \frac{21}{10} & 0 & \frac{7}{10} & \frac{21}{10} & \frac{7}{5} \\ \frac{7}{5} & \frac{21}{20} & \frac{21}{10} & \frac{11}{3} & \frac{7}{20} & \frac{7}{10} & \frac{7}{5} & 0 \\ 0 & 0 & 0 & \frac{7}{20} & \frac{11}{6} & \frac{21}{20} & \frac{7}{10} & \frac{7}{5} \\ 0 & 0 & \frac{7}{10} & \frac{7}{10} & \frac{21}{20} & \frac{11}{6} & 0 & \frac{21}{20} \\ 0 & \frac{7}{10} & \frac{21}{10} & \frac{7}{5} & \frac{7}{10} & 0 & \frac{11}{3} & \frac{21}{10} \\ \frac{7}{20} & \frac{7}{10} & \frac{7}{5} & 0 & \frac{7}{5} & \frac{21}{20} & \frac{21}{10} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{2}\ 0\ 0\ 0\ 1\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ \frac{1}{2}\ \frac{1}{4}\ 0\ 0\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{4}\ \ \frac{35}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{4}\ \ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \ \frac{413}{58}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{63}{29}\ \ \frac{35}{2}\ \ \frac{133}{58}\ \ \frac{805}{174}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\ \ \frac{805}{174}\ \ \frac{805}{174}\ \ \frac{133}{58}\ \ \frac{189}{58}\right.$$

$\boldsymbol{\tau} = 22/1$ , rank = 3 , ratio = 22/3 , $n^2 / r = 64/3$

$\boldsymbol{\tau}' = 42/1$ , $r' = 2/3$ , $\boldsymbol{\tau} / n^2 = 11/32$

$p^2 = 5/36$ , min $\boldsymbol{\tau} = 80/9$ , $\boldsymbol{\tau}$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21 \Omega$$

There are, 1, partitions and, 12, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 72

dim span idems 6 vs no. of idems 12

"PT1" = {{4, 8}, {1, 6, 7}, {2, 3, 5}}

"RG1" = {5, 7, 8}

"RG2" = {3, 7, 8}

"RG3" = {3, 4, 7}

"RG4" = {2, 7, 8}

"RG5" = {2, 4, 7}

"RG6" = {5, 6, 8}

"RG7" = {4, 5, 6}

"RG8" = {3, 6, 8}

"RG9" = {3, 4, 6}

"RG10" = {1, 3, 4}

"RG11" = {1, 2, 8}

"RG12" = {1, 2, 4}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{109}{180} & \frac{-17}{90} & \frac{23}{45} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-97}{180} \\ \frac{109}{180} & \frac{25}{18} & \frac{-8}{9} & \frac{29}{180} & \frac{-4}{9} & \frac{-4}{9} & \frac{-17}{90} & \frac{-17}{90} \\ \frac{-17}{90} & \frac{-8}{9} & \frac{17}{9} & \frac{29}{90} & \frac{-8}{9} & \frac{-17}{90} & \frac{29}{90} & \frac{-17}{45} \\ \frac{23}{45} & \frac{29}{180} & \frac{29}{90} & \frac{17}{9} & \frac{-97}{180} & \frac{-17}{90} & \frac{-17}{45} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-97}{180} & \frac{25}{18} & \frac{109}{180} & \frac{-17}{90} & \frac{23}{45} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-17}{90} & \frac{-17}{90} & \frac{109}{180} & \frac{25}{18} & \frac{-8}{9} & \frac{29}{180} \\ \frac{-8}{9} & \frac{-17}{90} & \frac{29}{90} & \frac{-17}{45} & \frac{-17}{90} & \frac{-8}{9} & \frac{17}{9} & \frac{29}{90} \\ \frac{-97}{180} & \frac{-17}{90} & \frac{-17}{45} & \frac{-16}{9} & \frac{23}{45} & \frac{29}{180} & \frac{29}{90} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$
M_C\text{-scaled} = \begin{pmatrix}
1 & \frac{109}{250} & \frac{-17}{125} & \frac{46}{125} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-97}{250} \\
\frac{109}{250} & 1 & \frac{-16}{25} & \frac{29}{250} & \frac{-8}{25} & \frac{-8}{25} & \frac{-17}{125} & \frac{-17}{125} \\
\frac{-1}{10} & \frac{-8}{17} & 1 & \frac{29}{170} & \frac{-8}{17} & \frac{-1}{10} & \frac{29}{170} & \frac{-1}{5} \\
\frac{23}{85} & \frac{29}{340} & \frac{29}{170} & 1 & \frac{-97}{340} & \frac{-1}{10} & \frac{-1}{5} & \frac{-16}{17} \\
\frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-97}{250} & 1 & \frac{109}{250} & \frac{-17}{125} & \frac{46}{125} \\
\frac{-8}{25} & \frac{-8}{25} & \frac{-17}{125} & \frac{-17}{125} & \frac{109}{250} & 1 & \frac{-16}{25} & \frac{29}{250} \\
\frac{-8}{17} & \frac{-1}{10} & \frac{29}{170} & \frac{-1}{5} & \frac{-1}{10} & \frac{-8}{17} & 1 & \frac{29}{170} \\
\frac{-97}{340} & \frac{-1}{10} & \frac{-1}{5} & \frac{-16}{17} & \frac{23}{85} & \frac{29}{340} & \frac{29}{170} & 1
\end{pmatrix}
\quad N_C\text{-scaled} =
$$

$$
\begin{pmatrix}
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \\
\frac{-5}{31} & 1 & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\
\frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1
\end{pmatrix}
$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ 0 & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.7833333333, 3.279848639, 0.147929139, 0.1102985171, 1.643590432, 2.580809521, 4.565301530]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.5640000000, 1.942818529, 0.0825932351, 0.07125610164, 0.9630711489, 1.672762708, 2.703498278]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 1, 0, −1, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, 0, −1, 0]}

Eigenvalues $M_0$

[0.7833333333, 9.217301912, 0.142412942, 2.956951814, 0.1102985171, 1.643590432, 2.580809521, 4.565301530]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0]}

Eigenvalues M

[−1.050000000, 5.836625774, −3.152638807, 0.466013033, 0.1054473612, 1.736215285, −1.192665167, −2.748997478]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

=========================================================

120, [1, 1, 1, −1, −1, −1, −1, −1]

=========================================================

{2, 3, 4, 5, 7, 8}

R: [4, 7, 5, 6, 7, 4, 8, 7]
B: [8, 3, 1, 2, 3, 8, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{1024}(-84 - 10s - s^2 - 2s^3 + s^4)(-1 + s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[8] v[7]) (1 + v[4] v[6])
"B CYCLES", 1 + v[1] v[3] v[8]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of $R^*$

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of $B^*$

{[0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 7 & 14 & 0 & 0 & 0 & 0 & 7 \\ 7 & 0 & 14 & 7 & 0 & 0 & 0 & 0 \\ 14 & 14 & 0 & 14 & 0 & 0 & 0 & 14 \\ 0 & 7 & 14 & 0 & 7 & 0 & 14 & 14 \\ 0 & 0 & 0 & 7 & 0 & 7 & 14 & 0 \\ 0 & 0 & 0 & 0 & 7 & 0 & 14 & 7 \\ 0 & 0 & 0 & 14 & 14 & 14 & 0 & 14 \\ 7 & 0 & 14 & 14 & 0 & 7 & 14 & 0 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]\right)$

degree 2: $\frac{1}{24}$ ( $v[1]v[2] + 2v[1]v[3] + v[1]v[8] + 2v[2]v[3] + v[2]v[4] + 2v[3]v[4] + 2v[3]v[8] + v[4]v[5] + 2v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[5]v[7] + 2v[6]v[7] + v[6]v[8] + 2v[8]v[7]$ )

degree 3 : $\frac{1}{8}$ ( $v[1]v[2]v[3] + v[1]v[3]v[8] + v[2]v[3]v[4] + v[3]v[4]v[8] + v[4]v[5]v[7] + v[4]v[8]v[7] + v[5]v[6]v[7] + v[6]v[8]v[7]$ )

Group spectrum $1 + t + t^2 + t^3$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {5, 6, 7}

"RG3" = {4, 7, 8}

"RG4" = {3, 4, 8}

"RG5" = {4, 5, 7}

"RG6" = {2, 3, 4}

"RG7" = {1, 3, 8}

"RG8" = {1, 2, 3}

$\pi3$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp $\pi3$ = {1, 11, 22, 40, 48, 52, 53, 56}

$u3$ = [1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1]

supp u3 = {1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

Action of R on ranges, [[3], [3], [1], [2], [1], [2], [5], [5]]
Action of B on ranges, [[4], [4], [6], [8], [6], [8], [7], [7]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[1, 3, 2]

B−BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 4, 6\}$
$b_2 = \{2, 5, 8\}$
$b_3 = \{3, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 3, 3, 3

## LIE STRUCTURE

Dimension of Lie algebra: 23, Shape: 3 ⊕ 20/18

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{4, 6}, {7, 8}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{3}{16} & \frac{5}{16} & \frac{5}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{3}\right)$ vs $\left(\frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{3}\right)$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 6}, {2, 5, 8}, {3, 7}}

1, "range", [6, 7, 8], [[8, 7, 6, 8, 7, 8, 6, 7], [8, 6, 7, 8, 6, 8, 7, 6], [7, 8, 6, 7, 8, 7, 6, 8], [7, 6, 8, 7, 6, 7, 8, 6], [6, 8, 7, 6, 8, 6, 7, 8], [6, 7, 8, 6, 7, 6, 8, 7]]

2, "range", [5, 6, 7], [[7, 6, 5, 7, 6, 7, 5, 6], [7, 5, 6, 7, 5, 7, 6, 5], [6, 7, 5, 6, 7, 6, 5, 7], [6, 5, 7, 6, 5, 6, 7, 5], [5, 7, 6, 5, 7, 5, 6, 7], [5, 6, 7, 5, 6, 5, 7, 6]]

3, "range", [4, 7, 8], [[8, 7, 4, 8, 7, 8, 4, 7], [8, 4, 7, 8, 4, 8, 7, 4], [7, 8, 4, 7, 8, 7, 4, 8], [7, 4, 8, 7, 4, 7, 8, 4], [4, 8, 7, 4, 8, 4, 7, 8], [4, 7, 8, 4, 7, 4, 8, 7]]

4, "range", [3, 4, 8], [[8, 4, 3, 8, 4, 8, 3, 4], [8, 3, 4, 8, 3, 8, 4, 3], [4, 8, 3, 4, 8, 4, 3, 8], [4, 3, 8, 4, 3, 4, 8, 3], [3, 8, 4, 3, 8, 3, 4, 8], [3, 4, 8, 3, 4, 3, 8, 4]]

5, "range", [4, 5, 7], [[7, 5, 4, 7, 5, 7, 4, 5], [7, 4, 5, 7, 4, 7, 5, 4], [5, 7, 4, 5, 7, 5, 4, 7], [5, 4, 7, 5, 4, 5, 7, 4], [4, 7, 5, 4, 7, 4, 5, 7], [4, 5, 7, 4, 5, 4, 7, 5]]

6, "range", [2, 3, 4], [[4, 3, 2, 4, 3, 4, 2, 3], [4, 2, 3, 4, 2, 4, 3, 2], [3, 4, 2, 3, 4, 3, 2, 4], [3, 2, 4, 3, 2, 3, 4, 2], [2, 4, 3, 2, 4, 2, 3, 4], [2, 3, 4, 2, 3, 2, 4, 3]]

7, "range", [1, 3, 8], [[8, 3, 1, 8, 3, 8, 1, 3], [8, 1, 3, 8, 1, 8, 3, 1], [3, 8, 1, 3, 8, 3, 1, 8], [3, 1, 8, 3, 1, 3, 8, 1], [1, 8, 3, 1, 8, 1, 3, 8], [1, 3, 8, 1, 3, 1, 8, 3]]

8, "range", [1, 2, 3], [[3, 2, 1, 3, 2, 3, 1, 2], [3, 1, 2, 3, 1, 3, 2, 1], [2, 3, 1, 2, 3, 2, 1, 3], [2, 1, 3, 2, 1, 2, 3, 1], [1, 3, 2, 1, 3, 1, 2, 3], [1, 2, 3, 1, 2, 1, 3, 2]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 3]]

$g_3 =$ [[1, 2]]

$g_4 =$ [[1, 2, 3]]

$g_5 =$ []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & h[2] & 2h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 3, 4]}, {8, [1, 3, 5]}, {9, [1, 3, 6]}, {10, [1, 3, 7]}, {11, [1, 3, 8]}, {12, [1, 4, 5]}, {13, [1, 4, 6]}, {14, [1, 4, 7]}, {15, [1, 4, 8]}, {16, [1, 5, 6]}, {17, [1, 5, 7]}, {18, [1, 5, 8]}, {19, [1, 6, 7]}, {20, [1, 6, 8]}, {21, [1, 7, 8]}, {22, [2, 3, 4]}, {23, [2, 3, 5]}, {24, [2, 3, 6]}, {25, [2, 3, 7]}, {26, [2, 3, 8]}, {27, [2, 4, 5]}, {28, [2, 4, 6]}, {29, [2, 4, 7]}, {30, [2, 4, 8]}, {31, [2, 5, 6]}, {32, [2, 5, 7]}, {33, [2, 5, 8]}, {34, [2, 6, 7]}, {35, [2, 6, 8]}, {36, [2, 7, 8]}, {37, [3, 4, 5]}, {38, [3, 4, 6]}, {39, [3, 4, 7]}, {40, [3, 4, 8]}, {41, [3, 5, 6]}, {42, [3, 5, 7]}, {43, [3, 5, 8]}, {44, [3, 6, 7]}, {45, [3, 6, 8]}, {46, [3, 7, 8]}, {47, [4, 5, 6]}, {48, [4, 5, 7]}, {49, [4, 5, 8]}, {50, [4, 6, 7]}, {51, [4, 6, 8]}, {52, [4, 7, 8]}, {53, [5, 6, 7]}, {54, [5, 6, 8]}, {55, [5, 7, 8]}, {56, [6, 7, 8]}

## KERNEL HIERARCHY

$\pi 3 =$
$(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

{1, 11, 22, 40, 48, 52, 53, 56}

$u3 =$
$(1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1$

{1, 5, 8, 11, 17, 21, 22, 24, 29, 34, 37, 40, 41, 45, 48, 52, 53, 56}

picheck $(2\ \ 2\ \ 4\ \ 4\ \ 2\ \ 2\ \ 4\ \ 4)$

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 2 =$
$(1\ 2\ 0\ 0\ 0\ 0\ 1\ 2\ 1\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 2\ 1\ 0\ 2\ 2\ 1\ 2\ 0\ 2\ 1\ 2)$

$u2 =$
$\left(\frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ 0\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\right.$

picheck $(4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

$\pi 1 = (4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

$u1 = \left(\frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\ \ \frac{2}{9}\right)$

picheck $(4\ \ 4\ \ 8\ \ 8\ \ 4\ \ 4\ \ 8\ \ 8)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \\ \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 & \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} & \frac{7}{4} & \frac{7}{4} & 7 & \frac{7}{2} \\ \frac{7}{4} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{4} & \frac{7}{2} & 7 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -2, 0, 1, 1, 2, 0]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & -s & 0 & s & 0 & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & t & -s & 0 & 0 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \end{pmatrix}$

M0 is invertible. det= 381271/104976

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (8)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-11}{6} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-11}{6} & 0 & 0 \\ 0 & 0 & \frac{7}{4} & \frac{7}{4} & 0 & 0 & \frac{7}{4} & \frac{11}{6} \\ 0 & 0 & \frac{7}{4} & \frac{11}{6} & 0 & 0 & \frac{7}{4} & \frac{7}{4} \\ \frac{-7}{4} & \frac{-7}{4} & 0 & 0 & \frac{-7}{4} & \frac{-7}{4} & 0 & 0 \\ \frac{-7}{4} & \frac{-11}{6} & 0 & 0 & \frac{-11}{6} & \frac{-7}{4} & 0 & 0 \end{pmatrix} \text{ Skew T} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & 0 & 0 & 0 & \frac{-1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & \frac{-1}{4} & 0 & 0 & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{11}{6} & \frac{7}{8} & \frac{7}{4} & 0 & 0 & 0 & 0 & \frac{7}{8} \\ \frac{7}{8} & \frac{11}{6} & \frac{7}{4} & \frac{7}{8} & 0 & 0 & 0 & 0 \\ \frac{7}{4} & \frac{7}{4} & \frac{11}{3} & \frac{7}{4} & 0 & 0 & 0 & \frac{7}{4} \\ 0 & \frac{7}{8} & \frac{7}{4} & \frac{11}{3} & \frac{7}{8} & 0 & \frac{7}{4} & \frac{7}{4} \\ 0 & 0 & 0 & \frac{7}{8} & \frac{11}{6} & \frac{7}{8} & \frac{7}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{7}{8} & \frac{11}{6} & \frac{7}{4} & \frac{7}{8} \\ 0 & 0 & 0 & \frac{7}{4} & \frac{7}{4} & \frac{7}{4} & \frac{11}{3} & \frac{7}{4} \\ \frac{7}{8} & 0 & \frac{7}{4} & \frac{7}{4} & 0 & \frac{7}{8} & \frac{7}{4} & \frac{11}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 7T + 21\Omega$$

$$\Omega\left(\frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ 0\ \frac{1}{2}\ 0\ 0\ 0\ 0\ \frac{1}{4}\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ \frac{1}{4}\ 0\ \frac{1}{2}\ 0\ 0\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{4}\ \frac{7}{2}\ 7\ \frac{7}{4}\ \frac{7}{4}\ \frac{21}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{4}\ 7\ \frac{7}{2}\ \frac{7}{4}\ \frac{7}{2}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{63}{29}\ \frac{63}{29}\ \frac{413}{58}\ \frac{63}{29}\ \frac{63}{29}\ \frac{322}{29}\ \frac{133}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{133}{58}\ \frac{189}{58}\ \frac{805}{174}\ \frac{133}{58}\ \frac{80}{17}\right.$$

$\tau = 22/1$ , rank = 3 , ratio = 22/3 , $n^2 / r = 64/3$

$\tau' = 42/1$ , $r' = 2/3$ , $\tau / n^2 = 11/32$

$p^2 = 5/36$ , min $\tau = 80/9$ , $\tau$-check is positive? 118/9

max r = 36/5 , $r$-check is positive? 7/12

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 21\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 6 vs no. of idems 8

"PT1" = {{1, 4, 6}, {2, 5, 8}, {3, 7}}

"RG1" = {6, 7, 8}

"RG2" = {5, 6, 7}

"RG3" = {4, 7, 8}

"RG4" = {3, 4, 8}

"RG5" = {4, 5, 7}

"RG6" = {2, 3, 4}

"RG7" = {1, 3, 8}

"RG8" = {1, 2, 3}

$$M_c = \begin{pmatrix} \frac{25}{18} & \frac{31}{72} & \frac{31}{36} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-1}{72} \\ \frac{31}{72} & \frac{25}{18} & \frac{31}{36} & \frac{-1}{72} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{31}{36} & \frac{31}{36} & \frac{17}{9} & \frac{-1}{36} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-1}{36} \\ \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{17}{9} & \frac{-1}{72} & \frac{-8}{9} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-1}{72} & \frac{25}{18} & \frac{31}{72} & \frac{31}{36} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{31}{72} & \frac{25}{18} & \frac{31}{36} & \frac{-1}{72} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-1}{36} & \frac{31}{36} & \frac{31}{36} & \frac{17}{9} & \frac{-1}{36} \\ \frac{-1}{72} & \frac{-8}{9} & \frac{-1}{36} & \frac{-1}{36} & \frac{-8}{9} & \frac{-1}{72} & \frac{-1}{36} & \frac{17}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{100} & \frac{31}{50} & \frac{-16}{25} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-1}{100} \\ \frac{31}{100} & 1 & \frac{31}{50} & \frac{-1}{100} & \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-16}{25} \\ \frac{31}{68} & \frac{31}{68} & 1 & \frac{-1}{68} & \frac{-8}{17} & \frac{-8}{17} & \frac{-16}{17} & \frac{-1}{68} \\ \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & 1 & \frac{-1}{136} & \frac{-8}{17} & \frac{-1}{68} & \frac{-1}{68} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-1}{100} & 1 & \frac{31}{100} & \frac{31}{50} & \frac{-16}{25} \\ \frac{-8}{25} & \frac{-8}{25} & \frac{-16}{25} & \frac{-16}{25} & \frac{31}{100} & 1 & \frac{31}{50} & \frac{-1}{100} \\ \frac{-8}{17} & \frac{-8}{17} & \frac{-16}{17} & \frac{-1}{68} & \frac{31}{68} & \frac{31}{68} & 1 & \frac{-1}{68} \\ \frac{-1}{136} & \frac{-8}{17} & \frac{-1}{68} & \frac{-1}{68} & \frac{-8}{17} & \frac{-1}{136} & \frac{-1}{68} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{1}{18} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{36} & \frac{1}{36} & 0 & 0 & \frac{1}{36} & \frac{-1}{18} \\ 0 & 0 & \frac{1}{36} & \frac{-1}{18} & 0 & 0 & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{36} & 0 & 0 & \frac{-1}{36} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 2.754892744, 0.147885034]

Eigenvalues $N_C$

[0., 0., 0., 0., 0., 3., 2.474410667, 1.414478221]

Eigenvalues $M_C$–scaled

[0., 0.6900000000, 1.633400282, 0.0713056009, 3.473569042, 0.417607428, 1.631532414, 0.0825852329]

Eigenvalues $N_C$–scaled

[0., 0., 0., 0., 0., 3.483870968, 2.873509162, 1.642619870]

NullSpace $M_C$

{[1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace $N_C$

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Eigenvalues $M_0$

[0.6666666667, 0.9583333333, 5.708333333, 2.764470495, 0.110529505, 9.145704965, 0.142342103, 2.503619601]

Eigenvalues $N_0$

[2., 3., 3., 0., 0., 0., 0., 0.]

NullSpace $M_0$

{}

NullSpace $N_0$

{[0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

Eigenvalues M

[0., 0., −0.8750000000, −2.625000000, 2.950746158, −2.075746158, 5.795540962, −3.170540962]

Eigenvalues N

[0., 0., 0., 0., 0., −3., 5.274917218, −2.274917218]

NullSpace M

{[1, 0, 0, −1, 0, 1, 0, 0], [0, −1, 0, 0, −1, 0, 0, 1]}

NullSpace N

{[1, 0, 0, −1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==========================================================

{3, 4, 5, 6, 7, 8}

R: [4, 3, 5, 6, 7, 8, 8, 7]
B: [8, 7, 1, 2, 3, 4, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{4096}(4+s)(-14+2s+s^2)(-3+s)(2+s+s^2)(-1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", 1 + v[8] v[7]
"B CYCLES", (1 + v[2] v[4] v[7]) (1 + v[1] v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of $R^*$

{[0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 0, 1, −1, 0]}

NullSpace of $B^*$

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 0, 1, 0, 0, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 1 & 1 & 0 \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 1 & 0 & 0 & 1 \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 1 & 0 & 0 & 1 \\ \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ( $v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$ )

degree 2: $\frac{1}{6}$ ( $v[1]v[2] + 2v[3]v[4] + v[5]v[6] + 2v[8]v[7]$ )

Group spectrum $1 + t + t^2$

## KERNEL STRUCTURE

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [4, 2, 2, 3, 1, 1, 3, 2, 2, 1, 3, 3, 1, 4, 3, 1, 1, 3, 1, 3, 3, 1, 4, 4, 0, 0, 4, 4]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28}

Action of R on ranges, [[1], [1], [2], [3]]
Action of B on ranges, [[3], [3], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [3, 1, 3]

BPARTS [3, 1, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 5, 6, 1, 1, 4]

B-BLOCKS,

[3, 6, 5, 2, 1, 4]

with invariant measure, [2, 1, 1, 2, 1, 1]

N by blocks, N - check: true

$b_1 = \{1, 3, 6, 7\}$
$b_2 = \{1, 4, 6, 7\}$
$b_3 = \{2, 3, 5, 8\}$
$b_4 = \{2, 4, 5, 8\}$
$b_5 = \{1, 4, 5, 8\}$
$b_6 = \{2, 3, 6, 7\}$

dim(span of partition vectors), rank($N_0$), rank($N$): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

## LIE STRUCTURE

Dimension of Lie algebra: 21, Shape: $11 \oplus 10/8$

$$\mathrm{CLB} = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

**R and B Cycles. V.**

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$\Omega_R$ in Vec(K)? , {{7, 8}}, true

$\Omega_B$ in Vec(K)? , {{1, 3, 8}, {2, 4, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{7}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

## LOCAL GROUPS

1, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 8, 7, 7, 8], [7, 8, 8, 7, 7, 8, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 6, 5, 5, 6], [5, 6, 6, 5, 5, 6, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3, 3, 4], [3, 4, 4, 3, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 2, 1, 1, 2], [1, 2, 2, 1, 1, 2, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

3, "partition", {{1, 3, 6, 7}, {2, 4, 5, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 8, 7], [7, 8, 7, 8, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 6, 5], [5, 6, 5, 6, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 4, 3], [3, 4, 3, 4, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 2, 1], [1, 2, 1, 2, 2, 1, 1, 2]]

"group has", 2, "elements"    Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[ 1 ] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[ 2 ] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

## KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2$ =
$\begin{pmatrix} 4 & 2 & 2 & 3 & 1 & 1 & 3 & 2 & 2 & 1 & 3 & 3 & 1 & 4 & 3 & 1 & 1 & 3 & 1 & 3 & 3 & 1 & 4 & 4 & 0 & 0 & 4 & 4 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{24} & \frac{1}{8} & \frac{1}{4} & \frac{1}{12} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{8} & \frac{1}{24} & \frac{1}{12} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{3} & 0 & \frac{1}{24} & \frac{1}{8} & \frac{1}{4} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{3} & \frac{1}{8} & \frac{1}{24} & \frac{1}{12} & \frac{1}{4} \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{8} & \frac{1}{24} & \frac{1}{4} & \frac{1}{12} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{8} & \frac{1}{24} & \frac{1}{4} & \frac{1}{12} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{8}{3} & \frac{8}{3} & \frac{2}{3} & 2 & 4 & \frac{4}{3} \\ 0 & \frac{8}{3} & \frac{8}{3} & \frac{8}{3} & 2 & \frac{2}{3} & \frac{4}{3} & 4 \\ \frac{4}{3} & \frac{4}{3} & \frac{16}{3} & 0 & \frac{2}{3} & 2 & 4 & \frac{4}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & \frac{16}{3} & 2 & \frac{2}{3} & \frac{4}{3} & 4 \\ \frac{2}{3} & 2 & \frac{4}{3} & 4 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \\ 2 & \frac{2}{3} & 4 & \frac{4}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ 2 & \frac{2}{3} & 4 & \frac{4}{3} & 0 & \frac{8}{3} & \frac{16}{3} & 0 \\ \frac{2}{3} & 2 & \frac{4}{3} & 4 & \frac{8}{3} & 0 & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -1, -1, 1, 1, 1, 1]$

$$\ker N_c = \begin{pmatrix} -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} t & t & -s & -s & s & s & -t & -t \\ 0 & 0 & -s+t & -s+t & 0 & 0 & -t+s & -t+s \\ 0 & 0 & -s+t & -s+t & 0 & 0 & -t+s & -t+s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -t & 0 & s \\ 0 & t & 0 & -s \\ -s & 0 & -t & 0 \\ s & 0 & t & 0 \\ 0 & s & 0 & -t \\ 0 & -s & 0 & t \\ 0 & -s & 0 & t \\ 0 & s & 0 & -t \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & s & 0 & 0 \\ s & 0 & -s & 0 & s+t \\ s+t & -t & 0 & -s & s+t \\ 0 & t & 0 & s & 0 \\ t & 0 & -t & 0 & s+t \\ s & 0 & t & 0 & 0 \\ s & 0 & t & 0 & 0 \\ t & 0 & -t & 0 & s+t \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$

$$
RN_0R^* = \begin{pmatrix} 1 & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} & 1 & 0 & 0 & 1 & 1 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 1 & 1 & 0 & 0 & 1 \\ \frac{1}{4} & \frac{3}{4} & 0 & 1 & 1 & 0 & 0 & 1 \\ \frac{3}{4} & \frac{1}{4} & 1 & 0 & 0 & 1 & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & 1 & 0 & 0 & 1 & 1 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
$$

$$
BN_0B^* = \begin{pmatrix} 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}
$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{8} & \frac{1}{24} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{24} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{24} & \frac{-1}{8} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{8} & \frac{-1}{24} & 0 & 0 \\ 0 & 0 & \frac{1}{24} & \frac{1}{8} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{8} & \frac{1}{24} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{-1}{8} & \frac{-1}{24} & 0 & 0 & 0 & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{24} & \frac{-1}{8} & 0 & 0 & \frac{-1}{6} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 1 & 0 & 0 & 1 \\ \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 1 & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 1 & 1 & 0 \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{12}\right)$$

$$T\left(\frac{1}{6} \ \frac{1}{12} \ \frac{1}{4} \ \frac{1}{8} \ \frac{1}{4} \ \frac{1}{24} \ \frac{1}{3} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{4} \ \frac{1}{8} \ \frac{1}{24} \ \frac{1}{6} \ \frac{1}{6} \ 0 \ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{8}{3}\ \ \frac{4}{3}\ \ 4\ \ 2\ \ 4\ \ \frac{2}{3}\ \ \frac{16}{3}\ \ \frac{4}{3}\ \ \frac{4}{3}\ \ 4\ \ 2\ \ \frac{2}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(2\ \ 2\ \ \frac{11}{3}\ \ \frac{7}{3}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{3}\ \ 2\ \ \frac{4}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{4}{3}\ \ 2\ \ 2\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 3, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 24

dim span idems 12 vs no. of idems 12

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{13}{36} & \frac{13}{36} & \frac{1}{9} & \frac{11}{18} & \frac{11}{18} & \frac{1}{9} \\ \frac{-5}{36} & \frac{31}{36} & \frac{13}{36} & \frac{13}{36} & \frac{11}{18} & \frac{1}{9} & \frac{1}{9} & \frac{11}{18} \\ \frac{13}{36} & \frac{13}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{1}{9} & \frac{11}{18} & \frac{11}{18} & \frac{1}{9} \\ \frac{13}{36} & \frac{13}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{11}{18} & \frac{1}{9} & \frac{1}{9} & \frac{11}{18} \\ \frac{1}{9} & \frac{11}{18} & \frac{1}{9} & \frac{11}{18} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{11}{18} & \frac{1}{9} & \frac{11}{18} & \frac{1}{9} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{11}{18} & \frac{1}{9} & \frac{11}{18} & \frac{1}{9} & \frac{-5}{36} & \frac{31}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{1}{9} & \frac{11}{18} & \frac{1}{9} & \frac{11}{18} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{13}{31} & \frac{13}{31} & \frac{4}{31} & \frac{22}{31} & \frac{22}{31} & \frac{4}{31} \\ \frac{-5}{31} & 1 & \frac{13}{31} & \frac{13}{31} & \frac{22}{31} & \frac{4}{31} & \frac{4}{31} & \frac{22}{31} \\ \frac{13}{31} & \frac{13}{31} & 1 & \frac{-5}{31} & \frac{4}{31} & \frac{22}{31} & \frac{22}{31} & \frac{4}{31} \\ \frac{13}{31} & \frac{13}{31} & \frac{-5}{31} & 1 & \frac{22}{31} & \frac{4}{31} & \frac{4}{31} & \frac{22}{31} \\ \frac{4}{31} & \frac{22}{31} & \frac{4}{31} & \frac{22}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \\ \frac{22}{31} & \frac{4}{31} & \frac{22}{31} & \frac{4}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{22}{31} & \frac{4}{31} & \frac{22}{31} & \frac{4}{31} & \frac{-5}{31} & 1 & 1 & \frac{-5}{31} \\ \frac{4}{31} & \frac{22}{31} & \frac{4}{31} & \frac{22}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues $M_C$

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues $N_C$

[0., 0., 0., 0., 1., 2.888888889, 2.618033988, 0.381966012]

Eigenvalues $M_C$−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues $N_C$−scaled

[0., 0., 0., 0., 1.161290323, 3.354838710, 3.040297535, 0.443573433]

NullSpace $M_C$

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [1, 0, 1, 0, 1, 0, 1, 0], [0, 0, −1, 1, 0, 0, 0, 0], [1, 0, 1, 0, 1, 0, 0, 1] }

NullSpace $N_C$

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [−1, −1, 0, 0, 0, 1, 0, 1], [−1, −1, 0, 0, 1, 1, 0, 0]}

Eigenvalues $M_0$

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues $N_0$

[0., 0., 0., 0., 1., 4., 2.618033988, 0.381966012]

NullSpace $M_0$

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace $N_0$

{[0, 0, 0, 0, 0, −1, 1, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 0, 0, 0, 1, 0, 1]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., −1., 4., −0.381966012, −2.618033988]

NullSpace M

{}

NullSpace N

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 0, 0, 1, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 4 & 2 & 2 & 3 & 1 & 1 & 3 \\ 4 & 0 & 2 & 2 & 1 & 3 & 3 & 1 \\ 2 & 2 & 0 & 4 & 3 & 1 & 1 & 3 \\ 2 & 2 & 4 & 0 & 1 & 3 & 3 & 1 \\ 3 & 1 & 3 & 1 & 0 & 4 & 4 & 0 \\ 1 & 3 & 1 & 3 & 4 & 0 & 0 & 4 \\ 1 & 3 & 1 & 3 & 4 & 0 & 0 & 4 \\ 3 & 1 & 3 & 1 & 0 & 4 & 4 & 0 \end{pmatrix}$$