T - Run

[4, 3, 1, 2], [3, 4, 4, 3]

$\tilde{\pi}$ = [1, 1, 2, 2]
$\boldsymbol{\delta}$ = [1, 1, 3, 3]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 3/16, .1875000000

NullSpace of Δ

{1, 2, 3, 4}

Nullspace of A

[{1, 3},{2, 4}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[3] + v[2] v[4] + v[1] v[3]

Degree 3
v[2] v[4] v[3] + v[1] v[4] v[3]

Degree 4
2 v[1] v[2] v[4] v[3]

===

{}

R: [4, 3, 1, 2]
B: [3, 4, 4, 3]

TRACE TWO = 1

det AT = $\frac{-1}{4}\,(1+t)^2\,(t)$

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{128}\,(4+s)\,(6+3s+s^2)\,(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 2

B ranking is 1, "vs", 2

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[2] v[4] v[3]
"B CYCLES", 1 + v[4] v[3]

Eigenvalues

R: [−1., 1., 1. I, −1. I]

B: [0., 0., 1., −1.]

NullSpace of R

{}

NullSpace of B

{[1, 0, 0, 0], [0, 1, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[1, 0, 0, −1], [0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + 2v[3] + 2v[4]$)
degree 2: $\frac{1}{3}$ ($v[1]v[2] + 2v[4]v[3]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}}

"PT2" = {{1, 3}, {2, 4}}

"RG1" = {3, 4}

"RG2" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 6}

$$u2 = [3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 5, 6}

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 4, 1, 3]

B–BLOCKS,

[4, 1, 4, 1]

with invariant measure, [2, 1, 1, 2]

N by blocks, N − check: true

$b_1 = \{1, 4\}$
$b_2 = \{1, 3\}$
$b_3 = \{2, 4\}$
$b_4 = \{2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & 0 & 0 \\ h[2] & h[1] & 0 & 0 \\ 0 & 0 & h[1] & h[2] \\ 0 & 0 & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 6, Shape: 3 ⊕ 3/1

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{6} & \frac{1}{6} & \frac{-1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{2} & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{3} & 0 \end{pmatrix}$$

Omega−checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}}

1, "range", [3, 4], [[4, 3, 3, 4], [3, 4, 4, 3]]

2, "range", [1, 2], [[2, 1, 1, 2], [1, 2, 2, 1]]

2, "partition", {{1, 3}, {2, 4}}

1, "range", [3, 4], [[4, 3, 4, 3], [3, 4, 3, 4]]

2, "range", [1, 2], [[2, 1, 2, 1], [1, 2, 1, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n–choose–rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [2, 3]}, {5, [2, 4]}, {6, [3, 4]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 6}

$u2 = \begin{pmatrix} 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 5, 6}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{9} & \frac{4}{9} \\ 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{3} & 0 \\ \frac{2}{9} & \frac{1}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{4}{3} & 0 & \frac{8}{9} & \frac{16}{9} \\ 0 & \frac{4}{3} & \frac{16}{9} & \frac{8}{9} \\ \frac{4}{9} & \frac{8}{9} & \frac{8}{3} & 0 \\ \frac{8}{9} & \frac{4}{9} & 0 & \frac{8}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, -1]$

$\ker N_c = \begin{pmatrix} -1 & -1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} s & s & -s & -s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & t-s \\ 0 & s-t \\ s & -t \\ -s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s-t & t & t \\ t-s & s & s \\ t & 0 & s \\ -t & s+t & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^\dagger = \begin{pmatrix} 0 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 2, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & \frac{2}{9} & \frac{1}{9} \\ \frac{-1}{9} & \frac{-2}{9} & 0 & 0 \\ \frac{-2}{9} & \frac{-1}{9} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 0\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$T \begin{pmatrix} \frac{2}{3} & \frac{1}{9} & \frac{4}{9} & \frac{2}{9} & 0 & \frac{1}{3} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \begin{pmatrix} \frac{8}{3} & \frac{4}{9} & \frac{16}{9} & \frac{8}{9} & 0 & \frac{4}{3} \end{pmatrix}$$

"IS MN in Vec(K)?", false

$$\mathrm{MN} \begin{pmatrix} \frac{7}{3} & \frac{5}{9} & \frac{11}{9} & \frac{7}{9} & \frac{1}{3} & \frac{5}{3} \end{pmatrix}$$

$\boldsymbol{\tau} = 8/1$, rank = 2 , ratio = 4/1 , $n^2 / r = 8/1$

$\boldsymbol{\tau}' = 8/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 5/18$, min $\boldsymbol{\tau} = 40/9$, $\boldsymbol{\tau}$–check is positive? 32/9

max r = 18/5 , r–check is positive? 4/9

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 4}, {2, 3}}

"PT2" = {{1, 3}, {2, 4}}

"RG1" = {3, 4}

"RG2" = {1, 2}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{13}{18} & \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{-5}{18} & \frac{13}{18} & \frac{7}{18} & \frac{1}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{13}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{1}{18} & \frac{-5}{18} & \frac{13}{18} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-5}{13} & \frac{1}{13} & \frac{7}{13} \\ \frac{-5}{13} & 1 & \frac{7}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{7}{13} & 1 & \frac{-5}{13} \\ \frac{7}{13} & \frac{1}{13} & \frac{-5}{13} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 3.555555556]

Eigenvalues N_C

[0., 0.6666666667, 1.333333333, 0.8888888889]

Eigenvalues M_C−scaled

[0., 0., 0., 4.]

Eigenvalues N_C−scaled

[0., 0.9230769231, 1.230769231, 1.846153846]

NullSpace M_C

{[−1, 1, 0, 0], [1, 0, 1, 0], [1, 0, 0, 1]}

NullSpace N_C

{[−1, −1, 1, 1]}

Eigenvalues M_0

[0., 0., 5.333333333, 2.666666667]

Eigenvalues N_0

[0., 2., 0.6666666667, 1.333333333]

NullSpace M_0

{[0, 0, −1, 1], [1, −1, 0, 0]}

NullSpace N_0

{[1, 1, −1, −1]}

Eigenvalues M

[1.333333333, −1.333333333, −2.666666667, 2.666666667]

Eigenvalues N

[0., 2., −0.6666666667, −1.333333333]

NullSpace M

{}

NullSpace N

{[1, 1, −1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$

===

{2, 4}

R: [4, 4, 1, 3]
B: [3, 3, 4, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 3

Level 2 det = $\frac{-3}{32}(2+s)(-1+s)(1+s)$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 3

B ranking is 1, "vs", 3

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[1] v[4] v[3]
"B CYCLES", 1 + v[2] v[4] v[3]

Eigenvalues

R: [0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0]}

NullSpace of B^*

{[−1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 10 \\ 5 & 5 & 10 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}\,(\,v[1] + v[2] + 2\,v[3] + 2\,v[4]\,)$
degree 2: $\frac{1}{6}\,(\,v[1]v[3] + v[1]v[4] + v[2]v[3] + v[2]v[4] + 2\,v[4]v[3]\,)$
degree 3 : $\frac{1}{2}\,(\,v[1] + v[2]\,)\,(\,v[3]\,)\,(\,v[4]\,)$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 2}, {3}, {4}}

"RG1" = {2, 3, 4}

"RG2" = {1, 3, 4}

$$\pi 3 = [0, 0, 1, 1]$$

supp $\pi 3$ = {3, 4}

$$u3 = [0, 0, 1, 1]$$

supp u3 = {3, 4}

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

b_1 = {1, 2}
b_2 = {3}
b_3 = {4}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 5, Shape: 0 ⊕ 5/3

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4}}, true

Ω_B in Vec(K)? , {{2, 3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{6} & \frac{1}{6} & \frac{-1}{3} & \frac{1}{3} \\ \frac{-1}{6} & \frac{1}{6} & \frac{-1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{3} & 0 \end{pmatrix}$$

Omega−checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2}, {3}, {4}}

1, "range", [2, 3, 4], [[4, 4, 2, 3], [3, 3, 4, 2], [2, 2, 3, 4]]

2, "range", [1, 3, 4], [[4, 4, 1, 3], [3, 3, 4, 1], [1, 1, 3, 4]]

"group has", 3, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 2, 3]]

$g_3 =$ []

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[3] & h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 3, 4]}, {4, [2, 3, 4]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}$

{3, 4}

$u3 = \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}$

{3, 4}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \end{pmatrix}$

$u2 = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

Row Projections

$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{5}{3} & \frac{5}{3} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & \frac{10}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{10}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$ RB checks

M0 is invertible. det= 8/27

$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \end{pmatrix}$ RB checks

$n\pi x^\dagger = (4)$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 3, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = \frac{5}{3}T + 5\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\text{NM} \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{3} & \frac{5}{3} \end{pmatrix}$

"IS MN in Vec(K)?", false

$\text{MN} \begin{pmatrix} \frac{5}{7} & \frac{5}{7} & \frac{10}{7} & \frac{10}{7} & \frac{5}{3} & \frac{5}{3} \end{pmatrix}$

$\boldsymbol{\tau} = 6/1$, rank = 3 , ratio = 2/1 , $n^2 / r = 16/3$

$\boldsymbol{\tau}' = 10/1$, $r' = 2/3$, $\boldsymbol{\tau} / n^2 = 3/8$

$p^2 = 5/18$, min $\boldsymbol{\tau} = 40/9$, $\boldsymbol{\tau}$–check is positive? 14/9

max r = 18/5 , r–check is positive? 1/6

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3}T + 5\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 2}, {3}, {4}}

"RG1" = {2, 3, 4}

"RG2" = {1, 3, 4}

$$M_C = \begin{pmatrix} \frac{5}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{13}{18} & \frac{13}{18} & \frac{-5}{18} & \frac{-5}{18} \\ \frac{13}{18} & \frac{13}{18} & \frac{-5}{18} & \frac{-5}{18} \\ \frac{-5}{18} & \frac{-5}{18} & \frac{13}{18} & \frac{-5}{18} \\ \frac{-5}{18} & \frac{-5}{18} & \frac{-5}{18} & \frac{13}{18} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-5}{13} & \frac{-5}{13} \\ 1 & 1 & \frac{-5}{13} & \frac{-5}{13} \\ \frac{-5}{13} & \frac{-5}{13} & 1 & \frac{-5}{13} \\ \frac{-5}{13} & \frac{-5}{13} & \frac{-5}{13} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 1., 0.3333333333, 0.2222222222]

Eigenvalues N_C

[0., 1., 1.691868003, 0.1970208860]

Eigenvalues M_C–scaled

[0., 1.500000000, 1.800000000, 0.7000000000]

Eigenvalues N_C–scaled

[0., 1.384615385, 2.342586466, 0.272798150]

NullSpace M_C

{[1, 1, 1, 1]}

NullSpace N_C

{[−1, 1, 0, 0]}

Eigenvalues M_0

[1., 0.3333333333, 4.467708078, 0.1989585 88]

Eigenvalues N_0

[0., 2., 1., 1.]

NullSpace M_0

{}

NullSpace N_0

{[1, −1, 0, 0]}

Eigenvalues M

[0., −1.666666667, 2.696723314, −1.030056648]

Eigenvalues N

[0., −1., 2.561552813, −1.561552813]

NullSpace M

{[−1, 1, 0, 0]}

NullSpace N

{[1, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{3, 4}

R: [4, 3, 4, 3]
B: [3, 4, 1, 2]

TRACE TWO = 1

det AT = $\frac{-1}{4}\,(t)\,(-1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{128}\,(-1+s)\,(-4+s)\,(-3+s)\,(2+s)$

RANK of R is 2

R ranking is 1, "vs", 2

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[4] v[3]
"B CYCLES", (1 + v[2] v[4]) (1 + v[1] v[3])

Eigenvalues

R: [0., 0., 1., −1.]

B: [1., −1., 1., −1.]

NullSpace of R

{[1, 0, 0, 0], [0, 1, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[1, 0, −1, 0], [0, 1, 0, −1]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + 2\,v[3] + 2\,v[4]$)
degree 2: $\frac{1}{3}$ ($v[1]v[2] + 2\,v[4]v[3]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3}, {2, 4}}

"RG1" = {3, 4}

"RG2" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 6}

$$u2 = [1, 0, 1, 1, 0, 1]$$

supp u2 = {1, 3, 4, 6}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1]

B−BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N − check: true

b_1 = {1, 3}
b_2 = {2, 4}

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 \\ h[1] & h[2] & 0 & 0 \\ 0 & 0 & h[2] & h[1] \\ 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 4, Shape: $0 \oplus 4/2$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4}}, true

Ω_B in Vec(K)? , {{1, 3}, {2, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{6} & \frac{1}{6} & \frac{-1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{2} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{3} & 0 \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3}, {2, 4}}

1, "range", [3, 4], [[4, 3, 4, 3], [3, 4, 3, 4]]

2, "range", [1, 2], [[2, 1, 2, 1], [1, 2, 1, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [2, 3]}, {5, [2, 4]}, {6, [3, 4]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 6}

$u2 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

{1, 3, 4, 6}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ idem-checks N0-checks

Row Projections

$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix}$ idem-checks

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{4}{3} & 0 & \frac{8}{3} & 0 \\ 0 & \frac{4}{3} & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & \frac{8}{3} & 0 \\ 0 & \frac{4}{3} & 0 & \frac{8}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, −1, 1, 1]

ker $N_C = \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & -t \\ t & 0 & -t & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 \end{pmatrix}$

ker $M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -t+s \\ 0 & -s+t \\ -t & s \\ t & -s \end{pmatrix}$ RB checks

ker $M_C = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & t & s \\ 0 & s & t \\ -t & t & t+s \\ t & s & 0 \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 2, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ \frac{-1}{3} & 0 & 0 & 0 \\ 0 & \frac{-1}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{6} & 0 & 0 \end{pmatrix}$$

$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix}$ $N_0 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\mathrm{NM} \begin{pmatrix} 0 & \frac{8}{3} & 0 & \frac{4}{3} \end{pmatrix}$

"IS MN in Vec(K)?", false

$\mathrm{MN} \begin{pmatrix} 0 & 2 & 0 & 2 \end{pmatrix}$

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 2 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 8/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/18 \text{ , min } \boldsymbol{\tau} = 40/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 32/9$$

$$\text{max r} = 18/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 4
out of total no. of elements equal to 4

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 3}, {2, 4}}

"RG1" = {3, 4}

"RG2" = {1, 2}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{13}{18} & \frac{-5}{18} & \frac{13}{18} & \frac{-5}{18} \\ \frac{-5}{18} & \frac{13}{18} & \frac{-5}{18} & \frac{13}{18} \\ \frac{13}{18} & \frac{-5}{18} & \frac{13}{18} & \frac{-5}{18} \\ \frac{-5}{18} & \frac{13}{18} & \frac{-5}{18} & \frac{13}{18} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-5}{13} & 1 & \frac{-5}{13} \\ \frac{-5}{13} & 1 & \frac{-5}{13} & 1 \\ 1 & \frac{-5}{13} & 1 & \frac{-5}{13} \\ \frac{-5}{13} & 1 & \frac{-5}{13} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 3.555555556]

Eigenvalues N_C

[0., 0., 2., 0.8888888889]

Eigenvalues M_C–scaled

[0., 0., 0., 4.]

Eigenvalues N_C–scaled

[0., 0., 2.769230769, 1.230769231]

NullSpace M_C

{[−1, 1, 0, 0], [1, 0, 1, 0], [1, 0, 0, 1]}

NullSpace N_C

{[−1, 0, 1, 0], [0, −1, 0, 1]}

Eigenvalues M_0

[0., 0., 5.333333333, 2.666666667]

Eigenvalues N_0

[2., 2., 0., 0.]

NullSpace M_0

{[−1, 1, 0, 0], [0, 0, −1, 1]}

NullSpace N_0

{[−1, 0, 1, 0], [0, −1, 0, 1]}

Eigenvalues M

[1.333333333, −1.333333333, −2.666666667, 2.666666667]

Eigenvalues N

[0., 0., 2., −2.]

NullSpace M

{}

NullSpace N

{[−1, 0, 1, 0], [0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$