

T-Run

[3, 5, 4, 3, 1, 2], [6, 6, 5, 5, 6, 4]

$$\tilde{\pi} = [3, 3, 4, 5, 6, 6]$$

$$\delta = [1, 1, 2, 2, 3, 3]$$

POSSIBLE RANKS

$$1 \times 27$$

$$3 \times 9$$

BASE DETERMINANT 161757/1048576, .1542634964

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

$\det(A) = 1/32$

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
 $v[2] v[6] + v[3] v[4]$

Degree 3
 $v[2] v[5] v[6] + v[4] v[5] v[6] + v[1] v[3] v[5]$

Degree 4
 $v[3] v[4] v[5] v[6] + v[1] v[4] v[5] v[6] + v[2] v[3] v[4] v[6] + v[1] v[3] v[4] v[5] + v[1] v[2] v[5] v[6]$

Degree 5
 $v[1] v[3] v[4] v[5] v[6] + v[1] v[2] v[3] v[5] v[6] + v[2] v[3] v[4] v[5] v[6]$

Degree 6
 $2 v[1] v[2] v[3] v[4] v[5] v[6]$

=====

20, [1, -1, 1, -1, -1, 1]

=====

{2, 3, 4, 5, 6}

R: [3, 6, 5, 5, 6, 4]

B: [6, 5, 4, 3, 1, 2]

TRACE TWO = 1

$$\det AT = \frac{-3}{32} (-1 + t)^3 (1 + 3t^2)$$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

$$\text{Level 2 det} = \frac{9}{33554432} (-5176224 - 2926950s - 216943s^2 + 274969s^3 + 75916s^4 + 40880s^5 - 8842s^6 - 8842s^7 - 1972s^8 - 738s^9 - 63s^{10} + 9s^{11}) (-1 + s)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 3

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", $1 + v[4] v[5] v[6]$

"B CYCLES", $(1 + v[3] v[4]) (1 + v[1] v[2] v[5] v[6])$

Eigenvalues

R: [0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

B: [1. I, -1. I, 1., -1., 1., -1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[0, 0, -1, 1, 0, 0], [0, -1, 0, 0, 1, 0]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 12 & 8 & 4 & 0 & 0 \\ 12 & 0 & 8 & 4 & 0 & 0 \\ 8 & 8 & 0 & 0 & 8 & 8 \\ 4 & 4 & 0 & 0 & 16 & 16 \\ 0 & 0 & 8 & 16 & 0 & 24 \\ 0 & 0 & 8 & 16 & 24 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & 1 & 1 & \frac{1}{3} & \frac{2}{3} \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 1 & 1 & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{4}{27} (3v[1] + 3v[2] + 4v[3] + 5v[4] + 6v[5] + 6v[6])$

degree 2: $\frac{2}{27} (3v[1]v[2] + 2v[1]v[3] + v[1]v[4] + 2v[2]v[3] + v[2]v[4] + 2v[3]v[5] + 2v[3]v[6] + 4v[4]v[5] + 4v[4]v[6] + 6v[5]v[6])$

degree 3 : $\frac{2}{9} (2v[1]v[2]v[3] + v[1]v[2]v[4] + 2v[3]v[5]v[6] + 4v[4]v[5]v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"PT2" = {{1, 5}, {2, 6}, {3, 4}}

"RG1" = {4, 5, 6}

"RG2" = {3, 5, 6}

"RG3" = {1, 2, 4}

"RG4" = {1, 2, 3}

$$\pi_3 = [2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 4]$$

supp $\pi_3 = \{1, 2, 19, 20\}$

$$u_3 = [3, 3, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 3, 3]$$

supp $u_3 = \{1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20\}$

Action of R on ranges, [[1], [1], [2], [2]]

Action of B on ranges, [[4], [3], [2], [1]]

$$\beta = \left(\frac{4}{9} \quad \frac{2}{9} \quad \frac{1}{9} \quad \frac{2}{9} \right)$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \left(\frac{2}{3} \quad \frac{1}{3} \right)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[5, 4, 4, 5, 2]

B-BLOCKS,

[4, 1, 2, 3, 5]

with invariant measure, [1, 2, 1, 2, 3]

N by blocks, N - check: true

$$b_1 = \{1, 5\}$$

$$b_2 = \{1, 6\}$$

$$b_3 = \{2, 6\}$$

$$b_4 = \{2, 5\}$$

$$b_5 = \{3, 4\}$$

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: $8 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 5, 6}, {3, 4}}, true

$$V = \begin{pmatrix} \frac{2}{27} & \frac{-7}{27} & \frac{44}{81} & \frac{1}{81} & \frac{4}{27} & \frac{-14}{27} \\ \frac{-5}{27} & \frac{4}{27} & \frac{-2}{81} & \frac{11}{81} & \frac{-10}{27} & \frac{8}{27} \\ \frac{1}{9} & \frac{1}{9} & \frac{-5}{27} & \frac{-13}{27} & \frac{2}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-14}{27} & \frac{-4}{27} & \frac{2}{9} & \frac{2}{9} \\ \frac{-13}{27} & \frac{5}{27} & \frac{-16}{81} & \frac{7}{81} & \frac{1}{27} & \frac{10}{27} \\ \frac{1}{27} & \frac{-17}{27} & \frac{22}{81} & \frac{41}{81} & \frac{2}{27} & \frac{-7}{27} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$$\pi_R = \left(0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right) \text{ vs } \left(0 \ 0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3, 4}}

1, "range", [4, 5, 6], [[6, 5, 4, 4, 5, 6], [6, 4, 5, 5, 4, 6], [5, 6, 4, 4, 6, 5], [5, 4, 6, 6, 4, 5], [4, 6, 5, 5, 6, 4], [4, 5, 6, 6, 5, 4]]

2, "range", [3, 5, 6], [[6, 5, 3, 3, 5, 6], [6, 3, 5, 5, 3, 6], [5, 6, 3, 3, 6, 5], [5, 3, 6, 6, 3, 5], [3, 6, 5, 5, 6, 3], [3, 5, 6, 6, 5, 3]]

3, "range", [1, 2, 4], [[4, 2, 1, 1, 2, 4], [4, 1, 2, 2, 1, 4], [2, 4, 1, 1, 4, 2], [2, 1, 4, 4, 1, 2], [1, 4, 2, 2, 4, 1], [1, 2, 4, 4, 2, 1]]

4, "range", [1, 2, 3], [[3, 2, 1, 1, 2, 3], [3, 1, 2, 2, 1, 3], [2, 3, 1, 1, 3, 2], [2, 1, 3, 3, 1, 2], [1, 3, 2, 2, 3, 1], [1, 2, 3, 3, 2, 1]]

2, "partition", {{1, 5}, {2, 6}, {3, 4}}

1, "range", [4, 5, 6], [[6, 5, 4, 4, 6, 5], [6, 4, 5, 5, 6, 4], [5, 6, 4, 4, 5, 6], [5, 4, 6, 6, 5, 4], [4, 6, 5, 5, 4, 6], [4, 5, 6, 6, 4, 5]]

2, "range", [3, 5, 6], [[6, 5, 3, 3, 6, 5], [6, 3, 5, 5, 6, 3], [5, 6, 3, 3, 5, 6], [5, 3, 6, 6, 5, 3], [3, 6, 5, 5, 3, 6], [3, 5, 6, 6, 3, 5]]

3, "range", [1, 2, 4], [[4, 2, 1, 1, 4, 2], [4, 1, 2, 2, 4, 1], [2, 4, 1, 1, 2, 4], [2, 1, 4, 4, 2, 1], [1, 4, 2, 2, 1, 4], [1, 2, 4, 4, 1, 2]]

4, "range", [1, 2, 3], [[3, 2, 1, 1, 3, 2], [3, 1, 2, 2, 3, 1], [2, 3, 1, 1, 2, 3], [2, 1, 3, 3, 2, 1], [1, 3, 2, 2, 1, 3], [1, 2, 3, 3, 1, 2]]

"group has", 6, "elements" Group element 1,1 =
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$g_1 = []$

$g_2 = [[1, 2]]$

$g_3 = [[2, 3]]$

$g_4 = [[1, 3, 2]]$

$g_5 = [[1, 2, 3]]$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$(2h[1] \ 0 \ 0 \ h[2] \ h[2])$

"Basis for Z(G)"

1, "coeff", 2

$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (2 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 4)$$

{1, 2, 19, 20}

$$\mu_3 = (3 \ 3 \ 0 \ 0 \ 0 \ 2 \ 1 \ 2 \ 1 \ 0 \ 0 \ 1 \ 2 \ 1 \ 2 \ 0 \ 0 \ 0 \ 3 \ 3)$$

{1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20}

picheck (3 3 4 5 6 6)

$$\pi = \left(\frac{1}{9} \ \frac{1}{9} \ \frac{4}{27} \ \frac{5}{27} \ \frac{2}{9} \ \frac{2}{9} \right)$$

$$\pi_2 = (3 \ 2 \ 1 \ 0 \ 0 \ 2 \ 1 \ 0 \ 0 \ 0 \ 2 \ 2 \ 4 \ 4 \ 6)$$

$$\mu_2 = \left(1 \ 1 \ 1 \ \frac{2}{3} \ \frac{1}{3} \ 1 \ 1 \ \frac{1}{3} \ \frac{2}{3} \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \right)$$

picheck (6 6 8 10 12 12)

$$\pi_1 = (6 \ 6 \ 8 \ 10 \ 12 \ 12)$$

$$\mu_1 = \left(\frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \right)$$

picheck (6 6 8 10 12 12)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & \frac{4}{9} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{4}{9} & \frac{2}{9} \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 \\ \frac{1}{9} & \frac{2}{9} & 0 & 0 & \frac{2}{3} & 0 \\ \frac{2}{9} & \frac{1}{9} & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & \frac{32}{9} & \frac{40}{9} \\ \frac{4}{3} & \frac{8}{3} & \frac{16}{9} & \frac{20}{9} & \frac{40}{9} & \frac{32}{9} \\ \frac{4}{3} & \frac{4}{3} & \frac{32}{9} & \frac{40}{9} & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{32}{9} & \frac{40}{9} & \frac{8}{3} & \frac{8}{3} \\ \frac{16}{9} & \frac{20}{9} & \frac{16}{9} & \frac{20}{9} & \frac{16}{3} & \frac{8}{3} \\ \frac{20}{9} & \frac{16}{9} & \frac{16}{9} & \frac{20}{9} & \frac{8}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-3, -3, -1, 1, 3, 3]$$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -t & t & 0 & 0 \\ -t & -t & s & -s & t & t \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker NC \begin{pmatrix} -1 & -3 \end{pmatrix}$$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} -s+t & -s+t & -t \\ s & s & -s+t \\ -t & -t & s \\ -t & -t & s \\ t+s & s & -s \\ -s & -s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & t \\ 0 & 0 & t & s \\ 0 & t & s & 0 \\ 0 & t & s & 0 \\ -t & 0 & t & t+s \\ t & s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^\dagger = (0 \ 2 \ 2 \ 2)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 1 \end{pmatrix}, \quad BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 6, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 0 & \frac{4}{9} & \frac{8}{9} & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{9} & \frac{8}{9} & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{9} & \frac{-4}{9} & 0 & \frac{4}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & 0 & \frac{4}{9} & \frac{4}{9} \\ \frac{-4}{3} & \frac{-4}{3} & \frac{-8}{9} & \frac{-4}{9} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & \frac{-8}{9} & \frac{-4}{9} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 0 & \frac{2}{9} & \frac{1}{9} \\ 0 & 0 & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & \frac{-1}{9} & 0 & 0 & 0 \\ \frac{-1}{9} & \frac{-2}{9} & 0 & 0 & 0 & 0 \\ \frac{-2}{9} & \frac{-1}{9} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{27} & \frac{2}{27} & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{27} & \frac{2}{27} & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-1}{27} & 0 & \frac{1}{27} & \frac{2}{27} & \frac{2}{27} \\ \frac{-2}{27} & \frac{-2}{27} & \frac{-1}{27} & 0 & \frac{1}{27} & \frac{1}{27} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{27} & \frac{-1}{27} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{27} & \frac{-1}{27} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & \frac{8}{9} & \frac{4}{9} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & \frac{8}{9} & \frac{4}{9} & 0 & 0 \\ \frac{8}{9} & \frac{8}{9} & \frac{16}{9} & 0 & \frac{8}{9} & \frac{8}{9} \\ \frac{4}{9} & \frac{4}{9} & 0 & \frac{20}{9} & \frac{16}{9} & \frac{16}{9} \\ 0 & 0 & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \left(\frac{2}{9} \frac{5}{27} \frac{4}{27} \frac{1}{9} \frac{1}{9} \frac{4}{27} \frac{-1}{9} \frac{1}{3} \frac{5}{27} \frac{4}{27} \frac{1}{9} \frac{1}{9} \frac{2}{9} \frac{2}{9} \frac{5}{27} \frac{4}{27} \frac{1}{9} \frac{1}{9} \right)$$

$$T \left(\frac{2}{3} 0 0 \frac{2}{9} \frac{1}{9} \frac{4}{9} \frac{-1}{3} 0 0 0 \frac{1}{3} 0 \frac{4}{9} \frac{2}{9} 0 0 0 \frac{1}{3} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{16}{3} \frac{20}{9} \frac{16}{9} \frac{20}{9} \frac{16}{9} \frac{32}{9} \frac{-8}{3} 4 \frac{20}{9} \frac{16}{9} \frac{8}{3} \frac{4}{3} \frac{40}{9} \frac{32}{9} \frac{20}{9} \frac{16}{9} \frac{4}{3} \frac{8}{3} \right)$$

"IS MN in Vec(K)?", false

$$MN \left(\frac{13}{3} \ 2 \ 2 \ \frac{31}{9} \ \frac{23}{9} \ 4 \ -4 \ 4 \ 2 \ 2 \ \frac{11}{3} \ \frac{7}{3} \ \frac{29}{9} \ \frac{25}{9} \ 2 \ 2 \ \frac{7}{3} \ \frac{11}{3} \right)$$

$$\tau = 12/1, \text{ rank} = 3, \text{ ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 131/729, \text{ min } \tau = 524/81, \tau\text{-check is positive? } 448/81$$

$$\text{max } r = 729/131, r\text{-check is positive? } 112/243$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 5 vs no. of idems 8

$$\text{"PT1"} = \{\{1, 6\}, \{2, 5\}, \{3, 4\}\}$$

$$\text{"PT2"} = \{\{1, 5\}, \{2, 6\}, \{3, 4\}\}$$

$$\text{"RG1"} = \{4, 5, 6\}$$

$$\text{"RG2"} = \{3, 5, 6\}$$

$$\text{"RG3"} = \{1, 2, 4\}$$

$$\text{"RG4"} = \{1, 2, 3\}$$

$$M_c = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{8}{27} & \frac{-8}{27} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{8}{27} & \frac{-8}{27} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{27} & \frac{8}{27} & \frac{80}{81} & \frac{-80}{81} & \frac{-8}{27} & \frac{-8}{27} \\ \frac{-8}{27} & \frac{-8}{27} & \frac{-80}{81} & \frac{80}{81} & \frac{8}{27} & \frac{8}{27} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-8}{27} & \frac{8}{27} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-8}{27} & \frac{8}{27} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{598}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{112}{729} & \frac{355}{729} \\ \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{355}{729} & \frac{112}{729} \\ \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{-131}{729} \\ \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{-131}{729} \\ \frac{112}{729} & \frac{355}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} \\ \frac{355}{729} & \frac{112}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{1}{3} & \frac{-1}{3} & -1 & -1 \\ 1 & 1 & \frac{1}{3} & \frac{-1}{3} & -1 & -1 \\ \frac{3}{10} & \frac{3}{10} & 1 & -1 & \frac{-3}{10} & \frac{-3}{10} \\ \frac{-3}{10} & \frac{-3}{10} & -1 & 1 & \frac{3}{10} & \frac{3}{10} \\ -1 & -1 & \frac{-1}{3} & \frac{1}{3} & 1 & 1 \\ -1 & -1 & \frac{-1}{3} & \frac{1}{3} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-131}{598} & \frac{-131}{598} & \frac{-131}{598} & \frac{56}{299} & \frac{355}{598} \\ \frac{-131}{598} & 1 & \frac{-131}{598} & \frac{-131}{598} & \frac{355}{598} & \frac{56}{299} \\ \frac{-131}{598} & \frac{-131}{598} & 1 & 1 & \frac{-131}{598} & \frac{-131}{598} \\ \frac{-131}{598} & \frac{-131}{598} & 1 & 1 & \frac{-131}{598} & \frac{-131}{598} \\ \frac{56}{299} & \frac{355}{598} & \frac{-131}{598} & \frac{-131}{598} & 1 & \frac{-131}{598} \\ \frac{355}{598} & \frac{56}{299} & \frac{-131}{598} & \frac{-131}{598} & \frac{-131}{598} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 3.917225066, 1.613639132]

Eigenvalues N_C

[0., 0., 2., 0.6666666667, 1.3333333333, 0.9218106996]

Eigenvalues $M_C\text{-scaled}$

[0., 0., 0., 0., 4.341640786, 1.658359214]

Eigenvalues $N_C\text{-scaled}$

[0., 0., 0.8127090301, 1.123745819, 1.625418060, 2.438127090]

NullSpace M_C

{[1, 0, 0, 0, 1, 0], [0, 0, 1, 1, 0, 0], [1, 0, 0, 0, 0, 1], [-1, 1, 0, 0, 0, 0]}

NullSpace N_C

{[0, 0, 1, -1, 0, 0], [1, 1, 0, 0, -1, -1]}

Eigenvalues M_0

[0., 0., 0., 7.043268479, 1.607968479, 3.348763041]

Eigenvalues N_0

[1.3333333333, 0.6666666667, 0., 0., 2., 2.]

NullSpace M_0

{[-1, 1, 0, 0, 0, 0], [1, 0, -1, -1, 1, 0], [1, 0, -1, -1, 0, 1]}

NullSpace N_0

{[0, 0, -1, 1, 0, 0], [-1, -1, 0, 0, 1, 1]}

Eigenvalues M

[-1.333333333, -2.666666667, 1.833039588, 4.567463350, -0.3230364042, -2.077466534]

Eigenvalues N

[0., 0., -2., -0.666666667, 4., -1.333333333]

NullSpace M

{}

NullSpace N

{[-1, -1, 0, 0, 1, 1], [0, 0, 1, -1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 3 & 3 & 2 & 1 \\ 3 & 0 & 3 & 3 & 1 & 2 \\ 3 & 3 & 0 & 0 & 3 & 3 \\ 3 & 3 & 0 & 0 & 3 & 3 \\ 2 & 1 & 3 & 3 & 0 & 3 \\ 1 & 2 & 3 & 3 & 3 & 0 \end{pmatrix}$$