T - Run

[3, 1, 1, 1, 2, 3], [5, 4, 4, 6, 4, 4]

$\tilde{\pi}$ = [4, 1, 3, 4, 2, 2]
$\boldsymbol{\delta}$ = [3, 1, 2, 4, 1, 1]

POSSIBLE RANKS

1 x 16
2 x 8
4 x 4

BASE DETERMINANT 2831/16384, .1727905273

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

[{1, 2, 3},{4, 5, 6}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[6] + v[1] v[3]

Degree 3
v[1] v[2] v[5] + v[1] v[3] v[4] + v[3] v[4] v[6] + v[1] v[4] v[5]

Degree 4
v[1] v[3] v[4] v[6] + v[1] v[2] v[4] v[5]

Degree 5
v[1] v[2] v[4] v[5] v[6] + v[1] v[3] v[4] v[5] v[6]

Degree 6
2 v[1] v[2] v[3] v[4] v[5] v[6]

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R: [3, 1, 1, 1, 2, 3]
B: [5, 4, 4, 6, 4, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{16384}(-2831 + 614s + 175s^2 - 28s^3 + 31s^4 - 10s^5 + s^6)(-1 + s)(1 + s)$

RANK of R is 3

R ranking is 2, "vs", 3

RBAR ranking 1, "vs", 2

RANK of B is 3

B ranking is 2, "vs", 3

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[3]
"B CYCLES", 1 + v[4] v[6]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 17 & 51 & 0 & 0 & 0 \\ 17 & 0 & 0 & 0 & 0 & 0 \\ 51 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 34 & 34 \\ 0 & 0 & 0 & 34 & 0 & 0 \\ 0 & 0 & 0 & 34 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{4}$ ($4v[1] + v[2] + 3v[3] + 4v[4] + 2v[5] + 2v[6]$)

degree 2: $\frac{1}{8}$ ($v[1]v[2] + 3v[1]v[3] + 2v[4]v[5] + 2v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 5, 6}, {2, 3, 4}}

"PT2" = {{1, 4}, {2, 3, 5, 6}}

"RG1" = {4, 6}

"RG2" = {4, 5}

"RG3" = {1, 3}

"RG4" = {1, 2}

$$\pi 2 = [1, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0]$$

supp $\pi 2$ = {1, 2, 13, 14}

$$u2 = [2, 2, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 2, 2, 0]$$

supp u2 = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14}

Action of R on ranges, [[3], [4], [3], [3]]
Action of B on ranges, [[1], [1], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 3, 4, 3]

B-BLOCKS,

[2, 1, 1, 2]

with invariant measure, [1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{1, 4\}$
$b_2 = \{2, 3, 5, 6\}$
$b_3 = \{1, 5, 6\}$
$b_4 = \{2, 3, 4\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 ⊕ 10/8

$$\mathrm{CLB} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}}, true

Ω_B in Vec(K)? , {{4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{20} & \frac{-17}{80} & \frac{9}{16} & \frac{-1}{20} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{20} & \frac{7}{80} & \frac{1}{16} & \frac{-9}{20} & \frac{7}{40} & \frac{-9}{40} \\ \frac{7}{20} & \frac{7}{80} & \frac{1}{16} & \frac{-9}{20} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{20} & \frac{9}{80} & \frac{-1}{16} & \frac{-3}{20} & \frac{9}{40} & \frac{-23}{40} \\ \frac{1}{20} & \frac{41}{80} & \frac{-1}{16} & \frac{-7}{20} & \frac{1}{40} & \frac{-7}{40} \\ \frac{1}{20} & \frac{1}{80} & \frac{7}{16} & \frac{-7}{20} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega−checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 5, 6}, {2, 3, 4}}

1, "range", [4, 6], [[6, 4, 4, 4, 6, 6], [4, 6, 6, 6, 4, 4]]

2, "range", [4, 5], [[5, 4, 4, 4, 5, 5], [4, 5, 5, 5, 4, 4]]

3, "range", [1, 3], [[3, 1, 1, 1, 3, 3], [1, 3, 3, 3, 1, 1]]

4, "range", [1, 2], [[2, 1, 1, 1, 2, 2], [1, 2, 2, 2, 1, 1]]

2, "partition", {{1, 4}, {2, 3, 5, 6}}

1, "range", [4, 6], [[6, 4, 4, 6, 4, 4], [4, 6, 6, 4, 6, 6]]

2, "range", [4, 5], [[5, 4, 4, 5, 4, 4], [4, 5, 5, 4, 5, 5]]

3, "range", [1, 3], [[3, 1, 1, 3, 1, 1], [1, 3, 3, 1, 3, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 1], [1, 2, 2, 1, 2, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 \end{pmatrix}$

{1, 2, 13, 14}

$u2 = \begin{pmatrix} 2 & 2 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 \end{pmatrix}$

{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14}

picheck $\begin{pmatrix} 4 & 1 & 3 & 4 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{4} & \frac{1}{16} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 1 & 3 & 4 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 1 & 3 & 4 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{8} & \frac{3}{8} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{8} & \frac{3}{8} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{8} & \frac{3}{8} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{8} & \frac{3}{8} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{8} & \frac{3}{8} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{8} & \frac{3}{8} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{8} & \frac{3}{8} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{16} & \frac{3}{16} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{17}{4} & 0 & 0 & \frac{17}{8} & \frac{17}{16} & \frac{17}{16} \\ 0 & \frac{17}{16} & \frac{51}{16} & \frac{17}{8} & \frac{17}{16} & \frac{17}{16} \\ 0 & \frac{17}{16} & \frac{51}{16} & \frac{17}{8} & \frac{17}{16} & \frac{17}{16} \\ \frac{17}{8} & \frac{17}{32} & \frac{51}{32} & \frac{17}{4} & 0 & 0 \\ \frac{17}{8} & \frac{17}{32} & \frac{51}{32} & 0 & \frac{17}{8} & \frac{17}{8} \\ \frac{17}{8} & \frac{17}{32} & \frac{51}{32} & 0 & \frac{17}{8} & \frac{17}{8} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [4, 1, 3, -4, -2, -2]$

$$\ker N_C = \begin{pmatrix} 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & s & 0 & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & s & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 4 & 3 & -2 \end{pmatrix}$

M0 is invertible. det= 87723/4096

$$\ker M_C = \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}s+t\\s+t\\s+t\\s+t\\s+t\\s+t\end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = (6)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1&0&0&0&1&1\\0&1&1&1&0&0\\0&1&1&1&0&0\\0&1&1&1&0&0\\1&0&0&0&1&1\\1&0&0&0&1&1\end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix}1&0&0&0&0&1\\0&1&1&1&0&0\\0&1&1&1&0&0\\0&1&1&1&0&0\\0&0&0&0&1&0\\1&0&0&0&0&1\end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix}1&0&0&1&0&0\\0&1&1&0&1&1\\0&1&1&0&1&1\\1&0&0&1&0&0\\0&1&1&0&1&1\\0&1&1&0&1&1\end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix}1&0&0&0&0&0\\0&1&1&0&1&1\\0&1&1&0&1&1\\0&0&0&1&0&0\\0&1&1&0&1&1\\0&1&1&0&1&1\end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 3, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix}0&\frac{-51}{16}&\frac{-17}{16}&0&\frac{-9}{4}&\frac{-9}{4}\\ \frac{51}{16}&0&\frac{19}{8}&\frac{27}{8}&\frac{9}{8}&\frac{9}{8}\\ \frac{17}{16}&\frac{-19}{8}&0&\frac{9}{8}&\frac{-9}{8}&\frac{-9}{8}\\ 0&\frac{-27}{8}&\frac{-9}{8}&0&\frac{-17}{8}&\frac{-17}{8}\\ \frac{9}{4}&\frac{-9}{8}&\frac{9}{8}&\frac{17}{8}&0&0\\ \frac{9}{4}&\frac{-9}{8}&\frac{9}{8}&\frac{17}{8}&0&0\end{pmatrix} \quad \text{Skew T} = \begin{pmatrix}0&0&0&0&\frac{-1}{8}&\frac{-1}{8}\\ 0&0&\frac{1}{4}&\frac{3}{16}&\frac{1}{16}&\frac{1}{16}\\ 0&\frac{-1}{4}&0&\frac{1}{16}&\frac{-1}{16}&\frac{-1}{16}\\ 0&\frac{-3}{16}&\frac{-1}{16}&0&0&0\\ \frac{1}{8}&\frac{-1}{16}&\frac{1}{16}&0&0&0\\ \frac{1}{8}&\frac{-1}{16}&\frac{1}{16}&0&0&0\end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix}0&\frac{-3}{16}&\frac{-1}{16}&0&\frac{-1}{8}&\frac{-1}{8}\\ \frac{3}{16}&0&\frac{1}{8}&\frac{3}{16}&\frac{1}{16}&\frac{1}{16}\\ \frac{1}{16}&\frac{-1}{8}&0&\frac{1}{16}&\frac{-1}{16}&\frac{-1}{16}\\ 0&\frac{-3}{16}&\frac{-1}{16}&0&\frac{-1}{8}&\frac{-1}{8}\\ \frac{1}{8}&\frac{-1}{16}&\frac{1}{16}&\frac{1}{8}&0&0\\ \frac{1}{8}&\frac{-1}{16}&\frac{1}{16}&\frac{1}{8}&0&0\end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{19}{4} & \frac{17}{16} & \frac{51}{16} & 0 & 0 & 0 \\ \frac{17}{16} & \frac{19}{16} & 0 & 0 & 0 & 0 \\ \frac{51}{16} & 0 & \frac{57}{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{19}{4} & \frac{17}{8} & \frac{17}{8} \\ 0 & 0 & 0 & \frac{17}{8} & \frac{19}{8} & 0 \\ 0 & 0 & 0 & \frac{17}{8} & 0 & \frac{19}{8} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{17}{2} T + 0\Omega$$

$\Omega \left(0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{8} \;\; \frac{1}{16} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{4} \;\; \frac{3}{16} \;\; \frac{1}{16} \;\; \frac{1}{4}\right)$

$T \left(\frac{-1}{8} \;\; \frac{1}{2} \;\; \frac{1}{16} \;\; \frac{1}{4} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; \frac{1}{2}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(\frac{-17}{16} \;\; \frac{17}{4} \;\; \frac{17}{32} \;\; \frac{17}{8} \;\; \frac{17}{16} \;\; \frac{17}{16} \;\; \frac{17}{16} \;\; \frac{17}{16} \;\; \frac{17}{8} \;\; 0 \;\; 0 \;\; \frac{17}{4}\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left(\frac{-17}{18} \;\; \frac{595}{144} \;\; \frac{17}{18} \;\; \frac{323}{144} \;\; \frac{323}{288} \;\; \frac{595}{288} \;\; \frac{323}{360} \;\; \frac{323}{360} \;\; \frac{323}{144} \;\; \frac{-17}{360} \;\; \frac{-17}{360} \;\; \frac{595}{144}\right)$

$$\boldsymbol{\tau} = 19/1 \text{ , rank} = 2 \text{ , ratio} = 19/2 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 17/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 19/36$$

$$p^2 = 25/128 \text{ , min } \boldsymbol{\tau} = 225/32 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 383/32$$

$$\text{max r} = 128/25 \text{ , } r\text{-check is positive? } 39/64$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 1T + 17\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 5, 6}, {2, 3, 4}}

"PT2" = {{1, 4}, {2, 3, 5, 6}}

"RG1" = {4, 6}

"RG2" = {4, 5}

"RG3" = {1, 3}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{5}{2} & \frac{1}{2} & \frac{3}{2} & \frac{-9}{4} & \frac{-9}{8} & \frac{-9}{8} \\ \frac{1}{2} & \frac{67}{64} & \frac{-27}{64} & \frac{-9}{16} & \frac{-9}{32} & \frac{-9}{32} \\ \frac{3}{2} & \frac{-27}{64} & \frac{147}{64} & \frac{-27}{16} & \frac{-27}{32} & \frac{-27}{32} \\ \frac{-9}{4} & \frac{-9}{16} & \frac{-27}{16} & \frac{5}{2} & 1 & 1 \\ \frac{-9}{8} & \frac{-9}{32} & \frac{-27}{32} & 1 & \frac{29}{16} & \frac{-9}{16} \\ \frac{-9}{8} & \frac{-9}{32} & \frac{-27}{32} & 1 & \frac{-9}{16} & \frac{29}{16} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{39}{128} & \frac{39}{128} & \frac{39}{128} \\ \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} & \frac{39}{128} & \frac{39}{128} & \frac{39}{128} \\ \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} & \frac{39}{128} & \frac{39}{128} & \frac{39}{128} \\ \frac{39}{128} & \frac{39}{128} & \frac{39}{128} & \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} \\ \frac{39}{128} & \frac{39}{128} & \frac{39}{128} & \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} \\ \frac{39}{128} & \frac{39}{128} & \frac{39}{128} & \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{1}{5} & \frac{3}{5} & \frac{-9}{10} & \frac{-9}{20} & \frac{-9}{20} \\ \frac{32}{67} & 1 & \frac{-27}{67} & \frac{-36}{67} & \frac{-18}{67} & \frac{-18}{67} \\ \frac{32}{49} & \frac{-9}{49} & 1 & \frac{-36}{49} & \frac{-18}{49} & \frac{-18}{49} \\ \frac{-9}{10} & \frac{-9}{40} & \frac{-27}{40} & 1 & \frac{2}{5} & \frac{2}{5} \\ \frac{-18}{29} & \frac{-9}{58} & \frac{-27}{58} & \frac{16}{29} & 1 & \frac{-9}{29} \\ \frac{-18}{29} & \frac{-9}{58} & \frac{-27}{58} & \frac{16}{29} & \frac{-9}{29} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-25}{103} & \frac{-25}{103} & \frac{39}{103} & \frac{39}{103} & \frac{39}{103} \\ \frac{-25}{103} & 1 & 1 & \frac{39}{103} & \frac{39}{103} & \frac{39}{103} \\ \frac{-25}{103} & 1 & 1 & \frac{39}{103} & \frac{39}{103} & \frac{39}{103} \\ \frac{39}{103} & \frac{39}{103} & \frac{39}{103} & 1 & \frac{-25}{103} & \frac{-25}{103} \\ \frac{39}{103} & \frac{39}{103} & \frac{39}{103} & \frac{-25}{103} & 1 & 1 \\ \frac{39}{103} & \frac{39}{103} & \frac{39}{103} & \frac{-25}{103} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{16} & \frac{-3}{16} & 0 & 0 & 0 \\ \frac{-1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & 0 & 0 \\ \frac{-1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{-1}{8} & \frac{-1}{8} \\ 0 & 0 & 0 & \frac{-1}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & \frac{-1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 & 0 & 0 \\ \frac{-1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 \\ \frac{-3}{16} & \frac{3}{16} & \frac{3}{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ 0 & 0 & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & \frac{-3}{16} & \frac{-1}{16} & 0 & 0 & 0 \\ \frac{3}{16} & 0 & \frac{-1}{8} & 0 & 0 & 0 \\ \frac{1}{16} & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{8} & \frac{-1}{8} \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 2.375000000, 0.3243492811, 0.3714928922, 1.743391560, 7.154516266]

Eigenvalues N_C

[0., 0., 0., 1.500000000, 2.239913178, 1.088211822]

Eigenvalues M_C–scaled

[0., 1.310344828, 0.1652020592, 0.1733495997, 1.232315122, 3.118788391]

Eigenvalues N_C–scaled

[0., 0., 0., 1.864077670, 2.783581426, 1.352340904]

NullSpace M_C

{[1, 1, 1, 1, 1, 1]}

NullSpace N_C

{[−1, −1, 0, 1, 0, 1], [0, −1, 1, 0, 0, 0], [−1, −1, 0, 1, 1, 0]}

Eigenvalues M_0

[2.375000000, 6.793816489, 0.331183511, 7.505463618, 0.3186240680, 1.675912312]

Eigenvalues N_0

[0., 0., 0., 1.500000000, 3.280776406, 1.219223594]

NullSpace M_0

{}

NullSpace N_0

{[−1, 0, −1, 1, 1, 0], [−1, 0, −1, 1, 0, 1], [0, 1, −1, 0, 0, 0]}

Eigenvalues M

[0., 0., 3.005203819, −3.005203819, 3.359920014, −3.359920014]

Eigenvalues N

[0., 0., 0., −1.500000000, 2.886000936, −1.386000936]

NullSpace M

{[0, 0, 0, 0, −1, 1], [0, −3, 1, 0, 0, 0]}

NullSpace N

{[0, −1, 1, 0, 0, 0], [−1, −1, 0, 1, 1, 0], [−1, −1, 0, 1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 2 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 \end{pmatrix}$$

==

20, [1, −1, 1, −1, −1, 1]

===

{4, 5, 6}

R: [3, 1, 1, 6, 4, 4]
B: [5, 4, 4, 1, 2, 3]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 5

Level 2 det = $\frac{-1}{4096} (-1 + s)^2 (-2831 - 1475 s - 650 s^2 - 94 s^3 - 75 s^4 + s^5 + 4 s^6)$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[1] v[3]) (1 + v[4] v[6])
"B CYCLES", 1 + v[1] v[2] v[4] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, 0, 0, 0, 1, −1], [0, 1, −1, 0, 0, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 24 & 36 & 18 & 18 \\ 8 & 0 & 0 & 9 & 9 & 0 \\ 24 & 0 & 0 & 27 & 9 & 18 \\ 36 & 9 & 27 & 0 & 16 & 16 \\ 18 & 9 & 9 & 16 & 0 & 0 \\ 18 & 0 & 18 & 16 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 6

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{4}\left(4v[1] + v[2] + 3v[3] + 4v[4] + 2v[5] + 2v[6]\right)$

degree 2: $\frac{1}{8}\left(2v[1]v[2] + 6v[1]v[3] + 4v[1]v[4] + 2v[1]v[5] + 2v[1]v[6] + v[2]v[4] + v[2]v[5] + 3v[3]v[4] + v[3]v[5] + 2v[3]v[6] + 4v[4]v[5] + 4v[4]v[6]\right)$

degree 3 : $\frac{1}{16}\left(v[1]v[2]v[4] + v[1]v[2]v[5] + 3v[1]v[3]v[4] + v[1]v[3]v[5] + 2v[1]v[3]v[6] + 2v[1]v[4]v[5] + 2v[1]v[4]v[6] + v[2]v[4]v[5] + v[3]v[4]v[5] + 2v[3]v[4]v[6]\right)$

degree 4 : $\frac{1}{4}\left(v[2]v[5] + v[3]v[5] + 2v[3]v[6]\right)\left(v[4]\right)\left(v[1]\right)$

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {2, 3}, {5, 6}}

"RG1" = {1, 3, 4, 6}

"RG2" = {1, 3, 4, 5}

"RG3" = {1, 2, 4, 5}

$$\pi 4 = [0, 0, 0, 1, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0]$$

supp $\pi 4$ = {4, 7, 8}

$$u4 = [0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0]$$

supp u4 = {4, 5, 7, 8}

Action of R on ranges, [[1], [1], [1]]
Action of B on ranges, [[2], [3], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 3, 2, 1]

B-BLOCKS,

[3, 1, 4, 2]

with invariant measure, [1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{2, 3\}$
$b_4 = \{5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $0 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}, {4, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 4, 5}}, true

$$V = \begin{pmatrix} \frac{3}{20} & \frac{-17}{80} & \frac{9}{16} & \frac{-1}{20} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{20} & \frac{7}{80} & \frac{1}{16} & \frac{-9}{20} & \frac{7}{40} & \frac{-9}{40} \\ \frac{7}{20} & \frac{7}{80} & \frac{1}{16} & \frac{-9}{20} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{20} & \frac{-9}{80} & \frac{1}{16} & \frac{3}{20} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-1}{20} & \frac{-41}{80} & \frac{1}{16} & \frac{7}{20} & \frac{-1}{40} & \frac{7}{40} \\ \frac{-1}{20} & \frac{-1}{80} & \frac{-7}{16} & \frac{7}{20} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {2, 3}, {5, 6}}

1, "range", [1, 3, 4, 6], [[6, 4, 4, 1, 3, 3], [4, 6, 6, 3, 1, 1], [3, 1, 1, 6, 4, 4], [1, 3, 3, 4, 6, 6]]

2, "range", [1, 3, 4, 5], [[5, 4, 4, 1, 3, 3], [4, 5, 5, 3, 1, 1], [3, 1, 1, 5, 4, 4], [1, 3, 3, 4, 5, 5]]

3, "range", [1, 2, 4, 5], [[5, 4, 4, 1, 2, 2], [4, 5, 5, 2, 1, 1], [2, 1, 1, 5, 4, 4], [1, 2, 2, 4, 5, 5]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 4, 2, 3]]

g_2 = [[1, 3, 2, 4]]

g_3 = [[1, 2], [3, 4]]

g_4 = []

linear dimension, 4

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[4] & h[3] & h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ -1. & 1. & 1.I & -1.I \\ -1. & 1. & 1.I & -1.I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 14t^5 + 22t^6 + 30t^7 + 43t^8 + 55t^9 + 73t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 4, 5]}, {5, [1, 2, 4, 6]}, {6, [1, 2, 5, 6]}, {7, [1, 3, 4, 5]}, {8, [1, 3, 4, 6]}, {9, [1, 3, 5, 6]}, {10, [1, 4, 5, 6]}, {11, [2, 3, 4, 5]}, {12, [2, 3, 4, 6]}, {13, [2, 3, 5, 6]}, {14, [2, 4, 5, 6]}, {15, [3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 7, 8}

$u4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 5, 7, 8}

picheck $\begin{pmatrix} 4 & 1 & 3 & 4 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{4} & \frac{1}{16} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 3 & 1 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 \end{pmatrix}$

$u3 = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 12 & 3 & 9 & 12 & 6 & 6 \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 2 & 6 & 8 & 4 & 4 & 0 & 2 & 2 & 0 & 6 & 2 & 4 & 4 & 4 & 0 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 24 & 6 & 18 & 24 & 12 & 12 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 24 & 6 & 18 & 24 & 12 & 12 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} \end{pmatrix}$

picheck $\begin{pmatrix} 24 & 6 & 18 & 24 & 12 & 12 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ idem-checks N0-checks

$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ idem-checks N0-checks

Row Projections

$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ idem-checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{2} & \frac{9}{8} & \frac{27}{8} & \frac{17}{4} & \frac{17}{8} & \frac{17}{8} \\ \frac{9}{2} & \frac{13}{8} & \frac{39}{8} & \frac{17}{4} & \frac{17}{8} & \frac{17}{8} \\ \frac{9}{2} & \frac{13}{8} & \frac{39}{8} & \frac{17}{4} & \frac{17}{8} & \frac{17}{8} \\ \frac{17}{4} & \frac{17}{16} & \frac{51}{16} & \frac{13}{2} & \frac{9}{4} & \frac{9}{4} \\ \frac{17}{4} & \frac{17}{16} & \frac{51}{16} & \frac{9}{2} & \frac{13}{4} & \frac{13}{4} \\ \frac{17}{4} & \frac{17}{16} & \frac{51}{16} & \frac{9}{2} & \frac{13}{4} & \frac{13}{4} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 0, -2, 2]$

$\ker N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & -t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & -1 \end{pmatrix}$

$\ker M_0 = \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -s+t \\ -s+t \\ -s+t \\ -t+s \\ -t+s \\ -t+s \end{pmatrix}$ RB checks

$\ker M_c = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & t \\ s & t \\ s & t \\ t & s \\ t & s \\ t & s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \; BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 5, "vs", 4

$$CNM = \begin{pmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} & 0 & \frac{-9}{8} & \frac{-9}{8} \\ \frac{3}{2} & 0 & \frac{5}{4} & \frac{27}{16} & \frac{9}{16} & \frac{9}{16} \\ \frac{1}{2} & \frac{-5}{4} & 0 & \frac{9}{16} & \frac{-9}{16} & \frac{-9}{16} \\ 0 & \frac{-27}{16} & \frac{-9}{16} & 0 & -1 & -1 \\ \frac{9}{8} & \frac{-9}{16} & \frac{9}{16} & 1 & 0 & 0 \\ \frac{9}{8} & \frac{-9}{16} & \frac{9}{16} & 1 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{-3}{16} & \frac{-1}{16} & 0 & \frac{-1}{8} & \frac{-1}{8} \\ \frac{3}{16} & 0 & \frac{1}{8} & \frac{3}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{-1}{8} & 0 & \frac{1}{16} & \frac{-1}{16} & \frac{-1}{16} \\ 0 & \frac{-3}{16} & \frac{-1}{16} & 0 & \frac{-1}{8} & \frac{-1}{8} \\ \frac{1}{8} & \frac{-1}{16} & \frac{1}{16} & \frac{1}{8} & 0 & 0 \\ \frac{1}{8} & \frac{-1}{16} & \frac{1}{16} & \frac{1}{8} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{5}{2} & \frac{1}{2} & \frac{3}{2} & \frac{9}{4} & \frac{9}{8} & \frac{9}{8} \\ \frac{1}{2} & \frac{5}{8} & 0 & \frac{9}{16} & \frac{9}{16} & 0 \\ \frac{3}{2} & 0 & \frac{15}{8} & \frac{27}{16} & \frac{9}{16} & \frac{9}{8} \\ \frac{9}{4} & \frac{9}{16} & \frac{27}{16} & \frac{5}{2} & 1 & 1 \\ \frac{9}{8} & \frac{9}{16} & \frac{9}{16} & 1 & \frac{5}{4} & 0 \\ \frac{9}{8} & 0 & \frac{9}{8} & 1 & 0 & \frac{5}{4} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{3}{16} & \frac{1}{16} & \frac{1}{4} \end{pmatrix}$$

$T\left(\frac{1}{2}\ 0\ 0\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ 0\ 0\ 1\right)$

"IS NM in Vec(K)?", true

$NM\left(\frac{13}{4}\ \frac{17}{16}\ \frac{9}{4}\ \frac{17}{16}\ \frac{17}{8}\ \frac{13}{8}\ \frac{17}{8}\ \frac{17}{8}\ \frac{17}{4}\ \frac{27}{8}\ \frac{9}{8}\ \frac{13}{2}\right)$

"IS MN in Vec(K)?", false

$MN\left(\frac{13}{4}\ \frac{17}{9}\ \frac{9}{5}\ \frac{17}{9}\ \frac{17}{9}\ \frac{13}{4}\ \frac{17}{9}\ \frac{17}{9}\ \frac{34}{9}\ \frac{9}{5}\ \frac{9}{5}\ \frac{13}{2}\right)$

$$\boldsymbol{\tau} = 10/1\text{ , rank} = 4\text{ , ratio} = 5/2\text{ , } n^2 / r = 9/1$$

$$\boldsymbol{\tau}' = 26/1\text{ , } r' = 3/4\text{ , } \boldsymbol{\tau} / n^2 = 5/18$$

$$p^2 = 25/128\text{ , min } \boldsymbol{\tau} = 225/32\text{ , } \boldsymbol{\tau}\text{-check is positive? } 95/32$$

$$\text{max r} = 128/25\text{ , } r\text{-check is positive? } 7/32$$

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{157}{464} T + \frac{1003}{116} \Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 12

dim span idems 3 vs no. of idems 3

"PT1" = {{4}, {1}, {2, 3}, {5, 6}}

"RG1" = {1, 3, 4, 6}

"RG2" = {1, 3, 4, 5}

"RG3" = {1, 2, 4, 5}

$$M_c = \begin{pmatrix} \frac{1}{4} & \frac{-1}{16} & \frac{-3}{16} & 0 & 0 & 0 \\ \frac{-1}{16} & \frac{31}{64} & \frac{-27}{64} & 0 & \frac{9}{32} & \frac{-9}{32} \\ \frac{-3}{16} & \frac{-27}{64} & \frac{39}{64} & 0 & \frac{-9}{32} & \frac{9}{32} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{-1}{8} & \frac{-1}{8} \\ 0 & \frac{9}{32} & \frac{-9}{32} & \frac{-1}{8} & \frac{11}{16} & \frac{-9}{16} \\ 0 & \frac{-9}{32} & \frac{9}{32} & \frac{-1}{8} & \frac{-9}{16} & \frac{11}{16} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} \\ \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} \\ \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} \\ \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{103}{128} & \frac{-25}{128} & \frac{-25}{128} \\ \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} \\ \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{-25}{128} & \frac{103}{128} & \frac{103}{128} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{4} & \frac{-3}{4} & 0 & 0 & 0 \\ \frac{-4}{31} & 1 & \frac{-27}{31} & 0 & \frac{18}{31} & \frac{-18}{31} \\ \frac{-4}{13} & \frac{-9}{13} & 1 & 0 & \frac{-6}{13} & \frac{6}{13} \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{9}{22} & \frac{-9}{22} & \frac{-2}{11} & 1 & \frac{-9}{11} \\ 0 & \frac{-9}{22} & \frac{9}{22} & \frac{-2}{11} & \frac{-9}{11} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} \\ \frac{-25}{103} & 1 & 1 & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} \\ \frac{-25}{103} & 1 & 1 & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} \\ \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & 1 & \frac{-25}{103} & \frac{-25}{103} \\ \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & 1 & 1 \\ \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & \frac{-25}{103} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{16} & \frac{-3}{16} & 0 & 0 & 0 \\ \frac{-1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & 0 & 0 \\ \frac{-1}{4} & \frac{1}{16} & \frac{3}{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{-1}{8} & \frac{-1}{8} \\ 0 & 0 & 0 & \frac{-1}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & \frac{-1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & 0 & 0 & 0 \\ \frac{-1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 \\ \frac{-3}{16} & \frac{3}{16} & \frac{3}{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ 0 & 0 & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & \frac{-3}{16} & \frac{-1}{16} & 0 & 0 & 0 \\ \frac{3}{16} & 0 & \frac{-1}{8} & 0 & 0 & 0 \\ \frac{1}{16} & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{8} & \frac{-1}{8} \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0.3750000000, 1.692571331, 0.3344162860, 0.5667623826]

Eigenvalues N_C

[0., 0., 1., 2., 1.544943038, 0.2831819615]

Eigenvalues M_C–scaled

[0., 0., 1.181818182, 2.732342387, 0.8169457694, 1.268893663]

Eigenvalues N_C–scaled

[0., 0., 1.242718447, 2.485436893, 1.919929213, 0.3519154472]

NullSpace M_C

{[0, 0, 0, 1, 1, 1], [1, 1, 1, 0, 0, 0]}

NullSpace N_c

{[0, 0, 0, 0, 1, −1], [0, 1, −1, 0, 0, 0]}

Eigenvalues M_0

[0., 0.2960928226, 0.3639674924, 0.5655326323, 1.666112754, 7.108294299]

Eigenvalues N_0

[2., 1., 2., 1., 0., 0.]

NullSpace M_0

{[1, 1, 1, −1, −1, −1]}

NullSpace N_0

{[0, 0, 0, 0, −1, 1], [0, −1, 1, 0, 0, 0]}

Eigenvalues M

[0.5142366725, 4.980718243, −0.4787806091, −1.193652748, −1.536672240, −2.285849318]

Eigenvalues N

[0., 0., −1., −2., 4.372281324, −1.372281324]

NullSpace M

{}

NullSpace N

{[0, −1, 1, 0, 0, 0], [0, 0, 0, 0, −1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$