T-Run

[2, 5, 4, 1, 4, 2], [3, 6, 6, 5, 6, 3]

$\tilde{\pi}$ = [1, 2, 2, 2, 2, 3]
$\boldsymbol{\delta}$ = [1, 2, 2, 2, 2, 3]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 231/2048, .1127929688

NullSpace of Δ

{2, 3}, {1, 4, 5, 6}

Nullspace of A

[{1, 6},{4, 5}] `,` [{3},{2}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[5] + v[3] v[6] + v[2] v[6]

Degree 3
v[1] v[3] v[4] + v[2] v[5] v[6]

Degree 4
2 v[3] v[4] v[5] v[6] + v[1] v[2] v[4] v[5] + v[2] v[4] v[5] v[6]

Degree 5
2 v[1] v[2] v[3] v[4] v[6]

Degree 6
4 v[1] v[2] v[3] v[4] v[5] v[6]

==

{}

R: [2, 5, 4, 1, 4, 2]
B: [3, 6, 6, 5, 6, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{1}{2048}(1+s)(-33-7s-s^2+s^3)(7+2s+s^2)(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 3

B ranking is 2, "vs", 3

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[2] v[4] v[5]
"B CYCLES", 1 + v[3] v[6]

Eigenvalues

R: [0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, −1], [0, 0, 1, 0, −1, 0]}

NullSpace of B^*

{[1, 0, 0, 0, 0, −1], [0, 0, 1, 0, −1, 0], [0, 1, 0, 0, −1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 0 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{2}{3} & 1 & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 1 & \frac{1}{3} & \frac{2}{3} \\ 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & 1 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 1 \\ 0 & \frac{2}{3} & 1 & \frac{1}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6]$)
degree 2: $\frac{1}{6}$ ($v[1]v[5] + 2v[2]v[4] + 2v[3]v[6] + v[5]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {3, 6}

"RG3" = {2, 4}

"RG4" = {1, 5}

$$\pi 2 = [0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 1]$$

supp $\pi 2$ = {4, 7, 12, 15}

$$u2 = [2, 3, 1, 3, 0, 1, 3, 1, 2, 2, 0, 3, 2, 1, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15}

Action of R on ranges, [[3], [3], [4], [3]]
Action of B on ranges, [[2], [2], [1], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[4, 1, 2, 3]

B−BLOCKS,

[2, 4, 4, 2]

with invariant measure, [1, 2, 1, 2]

N by blocks, N − check: true

$b_1 = \{1, 2, 6\}$
$b_2 = \{2, 3, 5\}$
$b_3 = \{3, 4, 5\}$
$b_4 = \{1, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 ⊕ 10/8

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 4, 5}}, true

Ω_B in Vec(K)? , {{3, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{-5}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{-1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 6}, {3, 4, 5}}

1, "range", [5, 6], [[6, 6, 5, 5, 5, 6], [5, 5, 6, 6, 6, 5]]

2, "range", [3, 6], [[6, 6, 3, 3, 3, 6], [3, 3, 6, 6, 6, 3]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4], [2, 2, 4, 4, 4, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 5], [1, 1, 5, 5, 5, 1]]

2, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [5, 6], [[6, 5, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5]]

2, "range", [3, 6], [[6, 3, 3, 6, 3, 6], [3, 6, 6, 3, 6, 3]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 4], [2, 4, 4, 2, 4, 2]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5], [1, 5, 5, 1, 5, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\,1\,] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}$

{4, 7, 12, 15}

$u2 = \begin{pmatrix} 2 & 3 & 1 & 3 & 0 & 1 & 3 & 1 & 2 & 2 & 0 & 3 & 2 & 1 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15}

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $(1 \ \ 2 \ \ 2 \ \ 2 \ \ 2 \ \ 3)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & 0 & \frac{2}{9} & 0 & \frac{1}{2} \\ \frac{1}{18} & \frac{1}{3} & \frac{2}{9} & 0 & \frac{2}{9} & \frac{1}{6} \\ 0 & \frac{2}{9} & \frac{1}{3} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{3} & \frac{1}{9} & \frac{1}{3} \\ 0 & \frac{2}{9} & \frac{1}{3} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{9} & 0 & \frac{2}{9} & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 1 & 0 & 2 & 0 & \frac{9}{2} \\ \frac{1}{2} & 3 & 2 & 0 & 2 & \frac{3}{2} \\ 0 & 2 & 3 & 1 & 3 & 0 \\ 1 & 0 & 1 & 3 & 1 & 3 \\ 0 & 2 & 3 & 1 & 3 & 0 \\ \frac{3}{2} & 1 & 0 & 2 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 2, -2, 2, 0, -3]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & -s & -t & -s & s+t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 2 & 0 & -3 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & -t \\ 0 & -s+t \\ -s & t \\ 0 & s-t \\ -s & t \\ s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t \\ t & t & s-t \\ t & s+t & -t \\ s & s & -s+t \\ t & s+t & -t \\ s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & 1 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & 1 & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & \frac{2}{3} & 1 & 0 \\ 1 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 3, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ -3 & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{18} & 0 & \frac{1}{9} & 0 & \frac{1}{3} \\ \frac{-1}{18} & 0 & 0 & 0 & 0 & \frac{1}{18} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & 0 & 0 & 0 & 0 & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{-1}{18} & 0 & \frac{-1}{9} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & 3 & \frac{3}{2} \\ 0 & 0 & 3 & 0 & \frac{3}{2} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & 1 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{2}{3} & 1 & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & \frac{1}{3} & 1 & 0 \\ 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{4} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \right)$

$T \left(\frac{1}{3} \;\; 0 \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{1}{3} \;\; \frac{1}{18} \;\; \frac{1}{2} \;\; 0 \;\; \frac{2}{9} \;\; 0 \;\; \frac{1}{9} \;\; \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$NM \left(3 \;\; 0 \;\; 2 \;\; 2 \;\; 3 \;\; \frac{1}{2} \;\; \frac{9}{2} \;\; 0 \;\; 2 \;\; 0 \;\; 1 \;\; \frac{3}{2} \right)$

"IS MN in Vec(K)?", false

MN $(3 \quad 0 \quad 2 \quad 2 \quad 3 \quad 1 \quad 3 \quad 0 \quad 2 \quad 0 \quad 1 \quad 3)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 23/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {3, 6}

"RG3" = {2, 4}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} \frac{5}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{2} & 2 & -1 & 2 & -1 & \frac{-3}{2} \\ \frac{-1}{2} & -1 & 2 & -1 & -1 & \frac{3}{2} \\ \frac{-1}{2} & 2 & -1 & 2 & -1 & \frac{-3}{2} \\ 1 & -1 & -1 & -1 & 2 & 0 \\ \frac{-3}{4} & \frac{-3}{2} & \frac{3}{2} & \frac{-3}{2} & 0 & \frac{9}{4} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{11}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{11}{72} \\ \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{35}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} & \frac{11}{72} & \frac{35}{72} \\ \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{4}{5} & \frac{-3}{5} \\ \frac{-1}{4} & 1 & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{4} \\ \frac{-1}{4} & 1 & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-3}{4} \\ \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 0 \\ \frac{-1}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} & \frac{-13}{59} & 1 \\ \frac{11}{59} & 1 & \frac{35}{59} & \frac{-13}{59} & \frac{35}{59} & \frac{11}{59} \\ \frac{-13}{59} & \frac{35}{59} & 1 & \frac{11}{59} & 1 & \frac{-13}{59} \\ \frac{35}{59} & \frac{-13}{59} & \frac{11}{59} & 1 & \frac{11}{59} & \frac{35}{59} \\ \frac{-13}{59} & \frac{35}{59} & 1 & \frac{11}{59} & 1 & \frac{-13}{59} \\ 1 & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 1.916666667, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C–scaled

[0., 0., 0., 2.338983051, 2.669106229, 0.9919107206]

NullSpace M_C

{[0, 0, 1, 1, 1, 0], [0, 1, 0, −1, 0, 0], [1, 0, 0, 1, 0, 1]}

NullSpace N_C

{[−1, 1, 0, 1, −1, 0], [0, 0, 1, 0, −1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[−1, 0, 1, 0, 1, −1], [0, −1, 0, 1, 0, 0]}

NullSpace N_0

{[0, 0, −1, 0, 1, 0], [−1, 1, −1, 1, 0, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[−3., 3., −3.432368417, 3.432368417, −1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[0, 0, −1, 0, 1, 0], [−1, 1, −1, 1, 0, 0], [−1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 1 & 3 & 0 \\ 2 & 0 & 1 & 3 & 1 & 2 \\ 3 & 1 & 0 & 2 & 0 & 3 \\ 1 & 3 & 2 & 0 & 2 & 1 \\ 3 & 1 & 0 & 2 & 0 & 3 \\ 0 & 2 & 3 & 1 & 3 & 0 \end{pmatrix}$$

===

{5}

R: [2, 5, 4, 1, 6, 2]
B: [3, 6, 6, 5, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{2048}(-1+s)(3+s)(-7+s)(11-4s+s^2)$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[2] v[5] v[6]
"B CYCLES", (1 + v[3] v[6]) (1 + v[4] v[5])

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [1., −1., 1., −1., 0., 0.]

NullSpace of R

{[0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, −1]}

NullSpace of B^*

{[1, 0, 0, 0, 0, −1], [0, −1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 2 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 2 & 2 & 4 \\ 1 & 2 & 2 & 0 & 0 & 3 \\ 1 & 2 & 2 & 0 & 0 & 3 \\ 0 & 2 & 4 & 3 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}\left(v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6] \right)$
degree 2: $\frac{1}{12}\left(2v[1]v[2] + v[1]v[4] + v[1]v[5] + 2v[2]v[4] + 2v[2]v[5] + 2v[2]v[6] + 2v[3]v[4] + 2v[3]v[5] \right.$
$\left. + 4v[3]v[6] + 3v[4]v[6] + 3v[5]v[6] \right)$
degree 3 : $\frac{1}{8}\left(v[1]v[2] + v[2]v[6] + 2v[3]v[6] \right)\left(v[4] + v[5] \right)$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 2, 5}

"RG6" = {1, 2, 4}

$$\pi 3 = [0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 2, 2, 0]$$

supp π3 = {2, 3, 15, 16, 18, 19}

$$u3 = [0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]$$

supp u3 = {2, 3, 5, 6, 15, 16, 18, 19}

Action of R on ranges, [[4], [6], [3], [5], [3], [5]]
Action of B on ranges, [[2], [1], [2], [1], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 1, 2]

B–BLOCKS,

[1, 3, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4, 5\}$
$b_2 = \{1, 6\}$
$b_3 = \{2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: $3 \oplus 14/12$

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 5, 6}}, true

Ω_B in Vec(K)? , {{4, 5}, {3, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{-5}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 6}, {2, 3}}

1, "range", [3, 5, 6], [[6, 5, 5, 3, 3, 6], [6, 3, 3, 5, 5, 6], [5, 6, 6, 3, 3, 5], [5, 3, 3, 6, 6, 5], [3, 6, 6, 5, 5, 3], [3, 5, 5, 6, 6, 3]]

2, "range", [3, 4, 6], [[6, 4, 4, 3, 3, 6], [6, 3, 3, 4, 4, 6], [4, 6, 6, 3, 3, 4], [4, 3, 3, 6, 6, 4], [3, 6, 6, 4, 4, 3], [3, 4, 4, 6, 6, 3]]

3, "range", [2, 5, 6], [[6, 5, 5, 2, 2, 6], [6, 2, 2, 5, 5, 6], [5, 6, 6, 2, 2, 5], [5, 2, 2, 6, 6, 5], [2, 6, 6, 5, 5, 2], [2, 5, 5, 6, 6, 2]]

4, "range", [2, 4, 6], [[6, 4, 4, 2, 2, 6], [6, 2, 2, 4, 4, 6], [4, 6, 6, 2, 2, 4], [4, 2, 2, 6, 6, 4], [2, 6, 6, 4, 4, 2], [2, 4, 4, 6, 6, 2]]

5, "range", [1, 2, 5], [[5, 2, 2, 1, 1, 5], [5, 1, 1, 2, 2, 5], [2, 5, 5, 1, 1, 2], [2, 1, 1, 5, 5, 2], [1, 5, 5, 2, 2, 1], [1, 2, 2, 5, 5, 1]]

6, "range", [1, 2, 4], [[4, 2, 2, 1, 1, 4], [4, 1, 1, 2, 2, 4], [2, 4, 4, 1, 1, 2], [2, 1, 1, 4, 4, 2], [1, 4, 4, 2, 2, 1], [1, 2, 2, 4, 4, 1]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1] - h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {[1, 2, 4], 2}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 2 & 0 \end{pmatrix}$

{2, 3, 15, 16, 18, 19}

$u3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$

{2, 3, 5, 6, 15, 16, 18, 19}

picheck $\begin{pmatrix} 2 & 4 & 4 & 4 & 4 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 2 & 0 & 1 & 1 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 4 & 0 & 3 & 3 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 6 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 2 & 2 & 2 & 2 & 2 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 2, -2, 0, 0, -1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s & s & 0 \\ s & 0 & 0 & -t & t & -s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 2 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ -s+t & -s \\ -s+t & -s \\ s-t & -t \\ s-t & -t \\ 0 & s+t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s+t \\ s & t & 0 \\ s & t & 0 \\ t & s & 0 \\ t & s & 0 \\ 0 & 0 & s+t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 4, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 & 2 \\ \frac{1}{2} & 1 & 1 & 2 & 0 & \frac{3}{2} \\ \frac{1}{2} & 1 & 1 & 0 & 2 & \frac{3}{2} \\ 0 & 1 & 2 & \frac{3}{2} & \frac{3}{2} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$\Omega \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{5}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \right)$

$T \left(\frac{1}{2} \ 0 \ 0 \ \frac{1}{4} \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ \frac{3}{4} \ 0 \ 0 \ 0 \ 0 \ \frac{1}{4} \right)$

"IS NM in Vec(K)?", true

NM $(4 \ 2 \ 1 \ 6 \ 2 \ 4 \ 4 \ 1 \ 6 \ 2 \ 2 \ 2 \ 2 \ 2)$

"IS MN in Vec(K)?", false

MN $(4 \ 2 \ 2 \ 8 \ 2 \ 4 \ 4 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 4)$

$$\boldsymbol{\tau} = 12/1 \text{ , rank } = 3 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 11/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 36

dim span idems 4 vs no. of idems 6

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 2, 5}

"RG6" = {1, 2, 4}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & \frac{-3}{4} \\ \frac{1}{2} & 1 & -1 & 0 & 0 & \frac{-1}{2} \\ \frac{-1}{2} & -1 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ \frac{-3}{4} & \frac{-1}{2} & \frac{1}{2} & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{-2}{3} & 0 & 0 & -1 \\ \frac{1}{2} & 1 & -1 & 0 & 0 & \frac{-1}{2} \\ \frac{-1}{2} & -1 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & \frac{-2}{3} & \frac{2}{3} & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2., 2.780776406, 0.719223594]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 2., 3.154700539, 0.845299461]

Eigenvalues N_C–scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, 1, 1, 0, 0, 0], [0, 0, 0, 1, 1, 0], [1, 0, 0, 0, 0, 1]}

NullSpace N_C

{[0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 2., 6.672823700, 0.6791154995, 2.648060798]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, −1, −1, 1, 1, 0], [1, −1, −1, 0, 0, 1]}

NullSpace N_0

{[0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0]}

Eigenvalues M

[0., −2., 0.9044505452, 4.333539961, −0.8573820427, −2.380608463]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, −1, 1, 0]}

NullSpace N

{[0, −1, 1, 0, 0, 0], [0, 0, 0, 1, −1, 0], [1, 0, 0, 0, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{2, 3}

R: [2, 6, 6, 1, 4, 2]
B: [3, 5, 4, 5, 6, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{2048}(7 + 2s + s^2)(-1 + s)(-33 - 36s - 12s^2 + s^4)$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[6]
"B CYCLES", 1 + v[3] v[4] v[5] v[6]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 6 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 1 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 1 & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 1 & \frac{1}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6]$)
degree 2: $\frac{1}{6}$ ($v[1]v[2] + v[2]v[6] + 2v[3]v[5] + 2v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{1, 5, 6}, {2, 3, 4}}

"RG1" = {4, 6}

"RG2" = {2, 6}

"RG3" = {3, 5}

"RG4" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 2, 0]$$

supp $\pi 2$ = {1, 9, 11, 14}

$$u2 = [3, 2, 3, 1, 0, 1, 0, 2, 3, 1, 3, 2, 2, 3, 1]$$

supp u2 = {1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15}

Action of R on ranges, [[4], [2], [1], [2]]
Action of B on ranges, [[3], [3], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[4, 4, 1, 1]

B–BLOCKS,

[3, 1, 4, 2]

with invariant measure, [2, 1, 1, 2]

N by blocks, N – check: true

$b_1 = \{1, 5, 6\}$
$b_2 = \{1, 3, 6\}$
$b_3 = \{2, 4, 5\}$
$b_4 = \{2, 3, 4\}$

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 ⊕ 11/9

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 6}}, true

Ω_B in Vec(K)? , {{3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{5}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{-1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [4, 6], [[6, 4, 6, 4, 4, 6], [4, 6, 4, 6, 6, 4]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 5], [[5, 3, 5, 3, 3, 5], [3, 5, 3, 5, 5, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 2, 1]]

2, "partition", {{1, 5, 6}, {2, 3, 4}}

1, "range", [4, 6], [[6, 4, 4, 4, 6, 6], [4, 6, 6, 6, 4, 4]]

2, "range", [2, 6], [[6, 2, 2, 2, 6, 6], [2, 6, 6, 6, 2, 2]]

3, "range", [3, 5], [[5, 3, 3, 3, 5, 5], [3, 5, 5, 5, 3, 3]]

4, "range", [1, 2], [[2, 1, 1, 1, 2, 2], [1, 2, 2, 2, 1, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 2 & 0 \end{pmatrix}$

{1, 9, 11, 14}

$u2 = \begin{pmatrix} 3 & 2 & 3 & 1 & 0 & 1 & 0 & 2 & 3 & 1 & 3 & 2 & 2 & 3 & 1 \end{pmatrix}$

{1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $(1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 3)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & 0 & \frac{2}{9} & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{2}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{1}{18} & \frac{2}{9} & \frac{1}{3} & \frac{2}{9} & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{9} & \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{9} & 0 & \frac{2}{9} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 1 & 0 & 2 & \frac{9}{2} \\ 0 & 3 & 2 & 3 & 1 & 0 \\ \frac{1}{2} & 2 & 3 & 2 & 0 & \frac{3}{2} \\ 0 & 3 & 2 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 & 3 & 3 \\ \frac{3}{2} & 0 & 1 & 0 & 2 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 2, -2, 0, -2, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & -s & -t & t+s & -t & t+s \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -s & 0 & 0 & 0 & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & 1 & 2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -t & -s \\ t & s \\ 0 & -t+s \\ t & s \\ 0 & -s+t \\ -t & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \quad \begin{pmatrix} t & 0 & s \\ s & t+s & -s \\ s & s & -s+t \\ s & t+s & -s \\ t & t & -t+s \\ t & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 1 \\ 0 & 1 & \frac{1}{3} & 1 & \frac{2}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & 1 & \frac{2}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{2}{3} & 1 & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ -3 & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{18} & 0 & 0 & 0 & 0 & \frac{1}{18} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & 0 & 0 & 0 & 0 & \frac{1}{9} \\ \frac{-1}{3} & 0 & \frac{-1}{18} & 0 & \frac{-1}{9} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & 3 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & \frac{3}{2} & 0 & 3 & 0 & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 1 \\ 0 & 1 & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 1 & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{4} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12}\right)$

$T \left(\frac{2}{9} \ \ \frac{1}{3} \ \ \frac{2}{9} \ \ \frac{1}{18} \ \ \frac{1}{3} \ \ 0 \ \ \frac{1}{2} \ \ \frac{2}{9} \ \ 0 \ \ \frac{1}{9} \ \ 0 \ \ \frac{1}{6}\right)$

"IS NM in Vec(K)?", true

$\text{NM} \left(2 \ \ 3 \ \ 2 \ \ \frac{1}{2} \ \ 3 \ \ 0 \ \ \frac{9}{2} \ \ 2 \ \ 0 \ \ 1 \ \ 0 \ \ \frac{3}{2}\right)$

"IS MN in Vec(K)?", false

MN $(2\ \ 3\ \ 2\ \ 1\ \ 3\ \ 0\ \ 3\ \ 2\ \ 0\ \ 1\ \ 0\ \ 3)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 23/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{1, 5, 6}, {2, 3, 4}}

"RG1" = {4, 6}

"RG2" = {2, 6}

"RG3" = {3, 5}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{5}{4} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-3}{4} \\ 1 & 2 & -1 & -1 & -1 & 0 \\ \frac{-1}{2} & -1 & 2 & -1 & 2 & \frac{-3}{2} \\ \frac{-1}{2} & -1 & -1 & 2 & -1 & \frac{3}{2} \\ \frac{-1}{2} & -1 & 2 & -1 & 2 & \frac{-3}{2} \\ \frac{-3}{4} & 0 & \frac{-3}{2} & \frac{3}{2} & \frac{-3}{2} & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} \\ \frac{11}{72} & \frac{35}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{11}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} \\ \frac{35}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} & \frac{35}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{4}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-3}{5} \\ \frac{1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{3} & 0 & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} & 1 \\ \frac{-13}{59} & 1 & \frac{35}{59} & 1 & \frac{11}{59} & \frac{-13}{59} \\ \frac{11}{59} & \frac{35}{59} & 1 & \frac{35}{59} & \frac{-13}{59} & \frac{11}{59} \\ \frac{-13}{59} & 1 & \frac{35}{59} & 1 & \frac{11}{59} & \frac{-13}{59} \\ \frac{35}{59} & \frac{11}{59} & \frac{-13}{59} & \frac{11}{59} & 1 & \frac{35}{59} \\ 1 & \frac{-13}{59} & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 1.916666667, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C–scaled

[0., 0., 0., 2.338983051, 2.669106229, 0.9919107206]

NullSpace M_C

{[0, 1, 0, 1, 1, 0], [1, −1, 0, −1, 0, 1], [0, 1, 1, 1, 0, 0]}

NullSpace N_C

{[−1, 0, 0, 0, 0, 1], [−1, −1, 1, 0, 1, 0], [0, −1, 0, 1, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, 0, −1, 0, 1, 0], [1, −1, 0, −1, 0, 1]}

NullSpace N_0

{[1, 1, −1, 0, −1, 0], [1, 0, −1, 1, −1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[−3., 3., −3.432368417, 3.432368417, −1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 1], [−1, −1, 1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 3 & 1 & 0 \\ 3 & 0 & 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & 1 & 3 & 2 \\ 3 & 0 & 1 & 0 & 2 & 3 \\ 1 & 2 & 3 & 2 & 0 & 1 \\ 0 & 3 & 2 & 3 & 1 & 0 \end{pmatrix}$$

===

{2, 4}

R: [2, 6, 4, 5, 4, 2]
B: [3, 5, 6, 1, 6, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{2048} (-3 + s) (11 + 9s + 3s^2 + s^3) (-1 + s) (-7 + s^2)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[4] v[5]) (1 + v[2] v[6])
"B CYCLES", 1 + v[3] v[6]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, −1], [0, 0, 1, 0, −1, 0]}

NullSpace of B^*

{[0, 0, −1, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 0 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(\,v[1] + 2\,v[2] + 2\,v[3] + 2\,v[4] + 2\,v[5] + 3\,v[6]\,)$
degree 2: $\frac{1}{6}\,(\,v[1]v[5] + 2\,v[2]v[4] + 2\,v[3]v[6] + v[5]v[6]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"RG1" = {5, 6}

"RG2" = {3, 6}

"RG3" = {2, 4}

"RG4" = {1, 5}

$$\pi 2 = [0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 1]$$

supp $\pi 2$ = {4, 7, 12, 15}

$$u2 = [0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1]$$

supp u2 = {2, 3, 4, 6, 7, 8, 12, 14, 15}

Action of R on ranges, [[3], [3], [1], [3]]
Action of B on ranges, [[2], [2], [4], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N - check: true

b_1 = {1, 2, 6}
b_2 = {3, 4, 5}

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 9, Shape: 0 ⊕ 9/7

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5}, {2, 6}}, true

Ω_B in Vec(K)? , {{3, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{5}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 6}, {3, 4, 5}}

1, "range", [5, 6], [[6, 6, 5, 5, 5, 6], [5, 5, 6, 6, 6, 5]]

2, "range", [3, 6], [[6, 6, 3, 3, 3, 6], [3, 3, 6, 6, 6, 3]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4], [2, 2, 4, 4, 4, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 5], [1, 1, 5, 5, 5, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}$

{4, 7, 12, 15}

$u2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$

{2, 3, 4, 6, 7, 8, 12, 14, 15}

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 3 & 0 & 0 & 0 & \frac{9}{2} \\ \frac{3}{2} & 3 & 0 & 0 & 0 & \frac{9}{2} \\ 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 & 3 & 0 \\ \frac{3}{2} & 3 & 0 & 0 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 2, -2, 2, 0, -1]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -s & -t & 0 & t & s \\ 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & 0 & -s & s & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 2 & -2 & 2 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & -t \\ 0 & -t+s \\ -s & t \\ 0 & t-s \\ -s & t \\ s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s & s+t & -s \\ s & t & 0 \\ t & 0 & s \\ t & s & 0 \\ t & 0 & s \\ s & s+t & -s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ -3 & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{-1}{6} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & 3 & \frac{3}{2} \\ 0 & 0 & 3 & 0 & \frac{3}{2} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{4} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12}\right)$

$T \left(\frac{1}{3} \ \ 0 \ \ \frac{1}{2} \ \ 0 \ \ 0 \ \ 0 \ \ \frac{1}{3} \ \ \frac{1}{6}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(3 \ \ 0 \ \ \frac{9}{2} \ \ 0 \ \ 0 \ \ 0 \ \ 3 \ \ \frac{3}{2}\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left(3 \ \ 0 \ \ 3 \ \ 0 \ \ 0 \ \ 0 \ \ 3 \ \ 3\right)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 23/2$$

$$\text{max r} = 72/13 \text{ , } r\text{–check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"RG1" = {5, 6}

"RG2" = {3, 6}

"RG3" = {2, 4}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{5}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{2} & 2 & -1 & 2 & -1 & \frac{-3}{2} \\ \frac{-1}{2} & -1 & 2 & -1 & -1 & \frac{3}{2} \\ \frac{-1}{2} & 2 & -1 & 2 & -1 & \frac{-3}{2} \\ 1 & -1 & -1 & -1 & 2 & 0 \\ \frac{-3}{4} & \frac{-3}{2} & \frac{3}{2} & \frac{-3}{2} & 0 & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{4}{5} & \frac{-3}{5} \\ \frac{-1}{4} & 1 & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{4} \\ \frac{-1}{4} & 1 & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-3}{4} \\ \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 0 \\ \frac{-1}{3} & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 0 & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & 1 & \frac{-13}{59} \\ 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 0., 3., 1.916666667]

Eigenvalues M_C-scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C-scaled

[0., 0., 0., 0., 3.661016949, 2.338983051]

NullSpace M_C

{[1, 1, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0], [0, 1, 1, 0, 1, 0]}

NullSpace N_C

{[0, 0, 0, 1, −1, 0], [1, 0, 0, 0, 0, −1], [0, 1, 0, 0, 0, −1], [0, 0, 1, 0, −1, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[1, 0, −1, 0, −1, 1], [0, −1, 0, 1, 0, 0]}

NullSpace N_0

{[0, −1, 0, 0, 0, 1], [0, 0, 0, 1, −1, 0], [0, 0, 1, 0, −1, 0], [1, −1, 0, 0, 0, 0]}

Eigenvalues M

[−3., 3., −3.432368417, 3.432368417, −1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{}

NullSpace N

{[−1, 1, 0, 0, 0, 0], [0, 0, 1, 0, −1, 0], [0, 0, 0, 1, −1, 0], [−1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{2, 5}

R: [2, 6, 4, 1, 6, 2]
B: [3, 5, 6, 5, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-1}{2048}(-1+s)(-33-20s+4s^2+s^4)(-7+s^2)$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[6]
"B CYCLES", (1 + v[3] v[6]) (1 + v[4] v[5])

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [1., −1., 1., −1., 0., 0.]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace of B^*

{[1, 0, 0, 0, 0, −1], [0, 1, 0, −1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 6 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6]$)
degree 2: $\frac{1}{6}$ ($v[1]v[2] + v[2]v[6] + 2v[3]v[5] + 2v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"RG1" = {4, 6}

"RG2" = {2, 6}

"RG3" = {3, 5}

"RG4" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 2, 0]$$

supp $\pi 2$ = {1, 9, 11, 14}

$$u2 = [1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]$$

supp u2 = {1, 3, 4, 6, 9, 10, 11, 14, 15}

Action of R on ranges, [[4], [2], [1], [2]]
Action of B on ranges, [[3], [3], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1]

B−BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N − check: true

$b_1 = \{1, 3, 6\}$
$b_2 = \{2, 4, 5\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: 0 ⊕ 10/8

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 6}}, true

Ω_B in Vec(K)? , {{4, 5}, {3, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{5}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [4, 6], [[6, 4, 6, 4, 4, 6], [4, 6, 4, 6, 6, 4]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 5], [[5, 3, 5, 3, 3, 5], [3, 5, 3, 5, 5, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 2, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 2 & 0 \end{pmatrix}$$

{1, 9, 11, 14}

$$u2 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

{1, 3, 4, 6, 9, 10, 11, 14, 15}

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 3 & 0 & 0 & \frac{9}{2} \\ 0 & 3 & 0 & 3 & 3 & 0 \\ \frac{3}{2} & 0 & 3 & 0 & 0 & \frac{9}{2} \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ \frac{3}{2} & 0 & 3 & 0 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 2, -2, 0, -2, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t & -t & 0 \\ s & 0 & 0 & 0 & 0 & -s \\ 0 & -s & -t & s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -2 & 0 & -2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -s & -t \\ s & t \\ -s+t & 0 \\ s & t \\ -t+s & 0 \\ -s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & s \\ t+s & -t & t \\ t & 0 & s \\ t+s & -t & t \\ s & 0 & t \\ 0 & t & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 0 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ -3 & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{6} & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & 3 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & \frac{3}{2} & 0 & 3 & 0 & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{4} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \right)$$

$T\left(\frac{1}{3}\ \ 0\ \ \frac{1}{2}\ \ 0\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{6}\right)$

"IS NM in Vec(K)?", true

$NM\left(3\ \ 0\ \ \frac{9}{2}\ \ 0\ \ 0\ \ 3\ \ 0\ \ \frac{3}{2}\right)$

"IS MN in Vec(K)?", false

$MN\left(3\ \ 0\ \ 3\ \ 0\ \ 0\ \ 3\ \ 0\ \ 3\right)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank } = 2 \text{ , ratio } = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 23/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"RG1" = {4, 6}

"RG2" = {2, 6}

"RG3" = {3, 5}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{5}{4} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-3}{4} \\ 1 & 2 & -1 & -1 & -1 & 0 \\ \frac{-1}{2} & -1 & 2 & -1 & 2 & \frac{-3}{2} \\ \frac{-1}{2} & -1 & -1 & 2 & -1 & \frac{3}{2} \\ \frac{-1}{2} & -1 & 2 & -1 & 2 & \frac{-3}{2} \\ \frac{-3}{4} & 0 & \frac{-3}{2} & \frac{3}{2} & \frac{-3}{2} & \frac{9}{4} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{4}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-3}{5} \\ \frac{1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{3}{4} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{2} & 1 & \frac{-3}{4} \\ \frac{-1}{3} & 0 & \frac{-2}{3} & \frac{2}{3} & \frac{-2}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 0., 3., 1.916666667]

Eigenvalues M_C–scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.661016949, 2.338983051]

NullSpace M_C

{[0, 1, 1, 1, 0, 0], [0, 0, −1, 0, 1, 0], [1, 0, 1, 0, 0, 1]}

NullSpace N_C

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0], [0, −1, 0, 1, 0, 0], [−1, 0, 1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[0, 0, −1, 0, 1, 0], [−1, 1, 0, 1, 0, −1]}

NullSpace N_0

{[0, 0, 0, −1, 1, 0], [1, 0, 0, 0, 0, −1], [0, 1, 0, −1, 0, 0], [0, 0, 1, 0, 0, −1]}

Eigenvalues M

[−3., 3., −3.432368417, 3.432368417, −1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{}

NullSpace N

{[−1, 0, 1, 0, 0, 0], [0, −1, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==

{5, 6}

R: [2, 5, 4, 1, 6, 3]
B: [3, 6, 6, 5, 4, 2]

TRACE TWO = 1

det AT = $\frac{-1}{4}(t)^2(1+t)^2$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{-1}{8192}\,(-1+s)\,(1848 - 410s + 128s^2 - 143s^3 - 105s^4 - 17s^5 - 7s^6 + 2s^7)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 4, "vs", 6

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]
"B CYCLES", (1 + v[4] v[5]) (1 + v[2] v[6])

Eigenvalues

R: [−1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

B: [1., −1., 1., −1., 0., 0.]

NullSpace of R

{}

NullSpace of B

{[1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO−M} = \begin{pmatrix} 0 & 2 & 7 & 4 & 5 & 0 \\ 2 & 0 & 0 & 10 & 8 & 16 \\ 7 & 0 & 0 & 8 & 10 & 11 \\ 4 & 10 & 8 & 0 & 0 & 14 \\ 5 & 8 & 10 & 0 & 0 & 13 \\ 0 & 16 & 11 & 14 & 13 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}\,(v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6])$

degree 2: $\frac{1}{54}$ ($2v[1]v[2] + 7v[1]v[3] + 4v[1]v[4] + 5v[1]v[5] + 10v[2]v[4] + 8v[2]v[5] + 16v[2]v[6] + 8v[3]v[4] + 10v[3]v[5] + 11v[3]v[6] + 14v[4]v[6] + 13v[5]v[6]$)
degree 3 : $\frac{1}{27}$ ($4v[1]v[2]v[4] + 2v[1]v[2]v[5] + 8v[1]v[3]v[4] + 13v[1]v[3]v[5] + 26v[2]v[4]v[6] + 22v[2]v[5]v[6] + 16v[3]v[4]v[6] + 17v[3]v[5]v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 3, 4}

"RG7" = {1, 2, 5}

"RG8" = {1, 2, 4}

$$\pi 3 = [0, 4, 2, 0, 8, 13, 0, 0, 0, 0, 0, 0, 0, 0, 26, 22, 0, 16, 17, 0]$$

supp π3 = {2, 3, 5, 6, 15, 16, 18, 19}

$$u3 = [0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]$$

supp u3 = {2, 3, 5, 6, 15, 16, 18, 19}

Action of R on ranges, [[2], [6], [1], [5], [4], [8], [3], [7]]
Action of B on ranges, [[4], [3], [4], [3], [2], [1], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{17}{108} & \frac{4}{27} & \frac{11}{54} & \frac{13}{54} & \frac{13}{108} & \frac{2}{27} & \frac{1}{54} & \frac{1}{27} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 2]

B-BLOCKS,

[1, 3, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1 = \{4, 5\}$
$b_2 = \{1, 6\}$
$b_3 = \{2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 11 ⊕ 11/9

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

Ω_B in Vec(K)? , {{4, 5}, {2, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{-5}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{1}{12} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 6}, {2, 3}}

1, "range", [3, 5, 6], [[6, 5, 5, 3, 3, 6], [6, 3, 3, 5, 5, 6], [5, 6, 6, 3, 3, 5], [5, 3, 3, 6, 6, 5], [3, 6, 6, 5, 5, 3], [3, 5, 5, 6, 6, 3]]

2, "range", [3, 4, 6], [[6, 4, 4, 3, 3, 6], [6, 3, 3, 4, 4, 6], [4, 6, 6, 3, 3, 4], [4, 3, 3, 6, 6, 4], [3, 6, 6, 4, 4, 3], [3, 4, 4, 6, 6, 3]]

3, "range", [2, 5, 6], [[6, 5, 5, 2, 2, 6], [6, 2, 2, 5, 5, 6], [5, 6, 6, 2, 2, 5], [5, 2, 2, 6, 6, 5], [2, 6, 6, 5, 5, 2], [2, 5, 5, 6, 6, 2]]

4, "range", [2, 4, 6], [[6, 4, 4, 2, 2, 6], [6, 2, 2, 4, 4, 6], [4, 6, 6, 2, 2, 4], [4, 2, 2, 6, 6, 4], [2, 6, 6, 4, 4, 2], [2, 4, 4, 6, 6, 2]]

5, "range", [1, 3, 5], [[5, 3, 3, 1, 1, 5], [5, 1, 1, 3, 3, 5], [3, 5, 5, 1, 1, 3], [3, 1, 1, 5, 5, 3], [1, 5, 5, 3, 3, 1], [1, 3, 3, 5, 5, 1]]

6, "range", [1, 3, 4], [[4, 3, 3, 1, 1, 4], [4, 1, 1, 3, 3, 4], [3, 4, 4, 1, 1, 3], [3, 1, 1, 4, 4, 3], [1, 4, 4, 3, 3, 1], [1, 3, 3, 4, 4, 1]]

7, "range", [1, 2, 5], [[5, 2, 2, 1, 1, 5], [5, 1, 1, 2, 2, 5], [2, 5, 5, 1, 1, 2], [2, 1, 1, 5, 5, 2], [1, 5, 5, 2, 2, 1], [1, 2, 2, 5, 5, 1]]

8, "range", [1, 2, 4], [[4, 2, 2, 1, 1, 4], [4, 1, 1, 2, 2, 4], [2, 4, 4, 1, 1, 2], [2, 1, 1, 4, 4, 2], [1, 4, 4, 2, 2, 1], [1, 2, 2, 4, 4, 1]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1]-h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi 3 = \begin{pmatrix} 0 & 4 & 2 & 0 & 8 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 26 & 22 & 0 & 16 & 17 & 0 \end{pmatrix}$$

{2, 3, 5, 6, 15, 16, 18, 19}

$$u3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

{2, 3, 5, 6, 15, 16, 18, 19}

picheck $\begin{pmatrix} 27 & 54 & 54 & 54 & 54 & 81 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$$

$\pi 2 = \begin{pmatrix} 6 & 21 & 12 & 15 & 0 & 0 & 30 & 24 & 48 & 24 & 30 & 33 & 0 & 42 & 39 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 54 & 108 & 108 & 108 & 108 & 162 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 54 & 108 & 108 & 108 & 108 & 162 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 54 & 108 & 108 & 108 & 108 & 162 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 6 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 2 & 2 & 2 & 2 & 2 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 1, 0, 0, -1]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & s & -s & 0 \\ s & 0 & 0 & -t & t & -s \\ 0 & -s+t & -t+s & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s-t & -s-t \\ t & s \\ t & s \\ s & t \\ s & t \\ -s-t & -s-t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t & 0 \\ t & 0 & s \\ t & 0 & s \\ s & 0 & t \\ s & 0 & t \\ 0 & s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \ BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 5, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{2}{9} & \frac{7}{9} & \frac{4}{9} & \frac{5}{9} & 0 \\ \frac{2}{9} & 2 & 0 & \frac{10}{9} & \frac{8}{9} & \frac{16}{9} \\ \frac{7}{9} & 0 & 2 & \frac{8}{9} & \frac{10}{9} & \frac{11}{9} \\ \frac{4}{9} & \frac{10}{9} & \frac{8}{9} & 2 & 0 & \frac{14}{9} \\ \frac{5}{9} & \frac{8}{9} & \frac{10}{9} & 0 & 2 & \frac{13}{9} \\ 0 & \frac{16}{9} & \frac{11}{9} & \frac{14}{9} & \frac{13}{9} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{5}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \right)$$

$T\left(\frac{1}{2}\ \ 0\ \ 0\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 0\ \ \frac{3}{4}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$NM\left(4\ \ 2\ \ 1\ \ 6\ \ 2\ \ 4\ \ 4\ \ 1\ \ 6\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\right)$

"IS MN in Vec(K)?", false

$MN\left(4\ \ 2\ \ 2\ \ 8\ \ 2\ \ 4\ \ 4\ \ 2\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 4\right)$

$$\boldsymbol{\tau} = 12/1\ ,\ \text{rank} = 3\ ,\ \text{ratio} = 4/1\ ,\ n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1\ ,\ r' = 2/3\ ,\ \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 13/72\ ,\ \text{min}\ \boldsymbol{\tau} = 13/2\ ,\ \boldsymbol{\tau}\text{-check is positive? } 11/2$$

$$\text{max r} = 72/13\ ,\ r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 3, 4}

"RG7" = {1, 2, 5}

"RG8" = {1, 2, 4}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{-5}{18} & \frac{5}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-3}{4} \\ \frac{-5}{18} & 1 & -1 & \frac{1}{9} & \frac{-1}{9} & \frac{5}{18} \\ \frac{5}{18} & -1 & 1 & \frac{-1}{9} & \frac{1}{9} & \frac{-5}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{9} & 1 & -1 & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{9} & \frac{1}{9} & -1 & 1 & \frac{-1}{18} \\ \frac{-3}{4} & \frac{5}{18} & \frac{-5}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{3}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-10}{27} & \frac{10}{27} & \frac{-2}{27} & \frac{2}{27} & -1 \\ \frac{-5}{18} & 1 & -1 & \frac{1}{9} & \frac{-1}{9} & \frac{5}{18} \\ \frac{5}{18} & -1 & 1 & \frac{-1}{9} & \frac{1}{9} & \frac{-5}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{9} & 1 & -1 & \frac{1}{18} \\ \frac{1}{18} & \frac{-1}{9} & \frac{1}{9} & -1 & 1 & \frac{-1}{18} \\ -1 & \frac{10}{27} & \frac{-10}{27} & \frac{2}{27} & \frac{-2}{27} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2.485072161, 1.139862126, 1.875065712]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 2.726443974, 1.351163554, 1.922392474]

Eigenvalues N_C–scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[1, 0, 0, 0, 0, 1], [0, 1, 1, 0, 0, 0], [0, 0, 0, 1, 1, 0]}

NullSpace N_C

{[0, −1, 1, 0, 0, 0], [0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 1.062640726, 1.860175760, 2.422740712, 6.654442803]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[−1, 1, 1, 0, 0, −1], [−1, 0, 0, 1, 1, −1]}

NullSpace N_0

{[0, 0, 0, −1, 1, 0], [0, −1, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[−2., 0.5907398838, 4.322012694, −0.07291006400, −0.5049609398, −2.334881574]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1], [0, 1, −1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

===

{2, 3, 4}

R: [2, 6, 6, 5, 4, 2]
B: [3, 5, 4, 1, 6, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{2048}(3+s)(11-4s+s^2)(-7+s)(-1+s)$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[4] v[5]) (1 + v[2] v[6])
"B CYCLES", 1 + v[1] v[3] v[4]

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, -1], [0, 1, -1, 0, 0, 0]}

NullSpace of B^*

{[-1, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & 2 & 4 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 1 & 2 & 2 & 0 & 0 & 3 \\ 1 & 2 & 2 & 0 & 0 & 3 \\ 0 & 4 & 2 & 3 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}$ $(v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6])$
degree 2: $\frac{1}{12}$ $(2v[1]v[3] + v[1]v[4] + v[1]v[5] + 2v[2]v[4] + 2v[2]v[5] + 4v[2]v[6] + 2v[3]v[4] + 2v[3]v[5]$ $+ 2v[3]v[6] + 3v[4]v[6] + 3v[5]v[6])$
degree 3 : $\frac{1}{8}$ $(v[1]v[3] + 2v[2]v[6] + v[3]v[6]) (v[4] + v[5])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 1, 1, 0]$$

supp $\pi 3$ = {5, 6, 15, 16, 18, 19}

$$u3 = [0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]$$

supp u3 = {2, 3, 5, 6, 15, 16, 18, 19}

Action of R on ranges, [[4], [3], [4], [3], [4], [3]]
Action of B on ranges, [[2], [6], [1], [5], [2], [6]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 3, 2]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4, 5\}$
$b_2 = \{1, 6\}$
$b_3 = \{2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5}, {2, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{5}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 6}, {2, 3}}

1, "range", [3, 5, 6], [[6, 5, 5, 3, 3, 6], [6, 3, 3, 5, 5, 6], [5, 6, 6, 3, 3, 5], [5, 3, 3, 6, 6, 5], [3, 6, 6, 5, 5, 3], [3, 5, 5, 6, 6, 3]]

2, "range", [3, 4, 6], [[6, 4, 4, 3, 3, 6], [6, 3, 3, 4, 4, 6], [4, 6, 6, 3, 3, 4], [4, 3, 3, 6, 6, 4], [3, 6, 6, 4, 4, 3], [3, 4, 4, 6, 6, 3]]

3, "range", [2, 5, 6], [[6, 5, 5, 2, 2, 6], [6, 2, 2, 5, 5, 6], [5, 6, 6, 2, 2, 5], [5, 2, 2, 6, 6, 5], [2, 6, 6, 5, 5, 2], [2, 5, 5, 6, 6, 2]]

4, "range", [2, 4, 6], [[6, 4, 4, 2, 2, 6], [6, 2, 2, 4, 4, 6], [4, 6, 6, 2, 2, 4], [4, 2, 2, 6, 6, 4], [2, 6, 6, 4, 4, 2], [2, 4, 4, 6, 6, 2]]

5, "range", [1, 3, 5], [[5, 3, 3, 1, 1, 5], [5, 1, 1, 3, 3, 5], [3, 5, 5, 1, 1, 3], [3, 1, 1, 5, 5, 3], [1, 5, 5, 3, 3, 1], [1, 3, 3, 5, 5, 1]]

6, "range", [1, 3, 4], [[4, 3, 3, 1, 1, 4], [4, 1, 1, 3, 3, 4], [3, 4, 4, 1, 1, 3], [3, 1, 1, 4, 4, 3], [1, 4, 4, 3, 3, 1], [1, 3, 3, 4, 4, 1]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1]-h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 1 & 1 & 0 \end{pmatrix}$

{5, 6, 15, 16, 18, 19}

$u3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$

{2, 3, 5, 6, 15, 16, 18, 19}

picheck $\begin{pmatrix} 2 & 4 & 4 & 4 & 4 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 2 & 1 & 1 & 0 & 0 & 2 & 2 & 4 & 2 & 2 & 2 & 0 & 3 & 3 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 8 & 8 & 8 & 8 & 12 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 6 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 2 & 2 & 2 & 2 & 2 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [−1, 2, −2, 0, 0, 1]

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & 0 & -s & s & -t \\ 0 & 0 & 0 & t & -t & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & -2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s-t & -s-t \\ s & t \\ s & t \\ t & s \\ t & s \\ -s-t & -s-t \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s+t \\ s & t & 0 \\ s & t & 0 \\ t & s & 0 \\ t & s & 0 \\ 0 & 0 & s+t \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 5, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 2 & 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 & 1 & 1 \\ \frac{1}{2} & 1 & 1 & 2 & 0 & \frac{3}{2} \\ \frac{1}{2} & 1 & 1 & 0 & 2 & \frac{3}{2} \\ 0 & 2 & 1 & \frac{3}{2} & \frac{3}{2} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\text{NM} \begin{pmatrix} 4 & 2 & 1 & 6 & 2 & 4 & 4 & 1 & 6 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

"IS MN in Vec(K)?", false

$\text{MN} \begin{pmatrix} 4 & 2 & 2 & 8 & 2 & 4 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 11/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 36

dim span idems 4 vs no. of idems 6

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 3, 4}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{-1}{2} & \frac{1}{2} & 0 & 0 & \frac{-3}{4} \\ \frac{-1}{2} & 1 & -1 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & 1 & 0 & 0 & \frac{-1}{2} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ \frac{-3}{4} & \frac{1}{2} & \frac{-1}{2} & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{3} & \frac{2}{3} & 0 & 0 & -1 \\ \frac{-1}{2} & 1 & -1 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & 1 & 0 & 0 & \frac{-1}{2} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & \frac{2}{3} & \frac{-2}{3} & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2., 2.780776406, 0.719223594]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 2., 3.154700539, 0.845299461]

Eigenvalues N_C–scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[1, 0, 0, 0, 0, 1], [0, 0, 0, 1, 1, 0], [0, 1, 1, 0, 0, 0]}

NullSpace N_C

{[0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 2., 6.672823700, 0.6791154995, 2.648060798]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, −1, −1, 0, 0, 1], [0, −1, −1, 1, 1, 0]}

NullSpace N_0

{[0, −1, 1, 0, 0, 0], [0, 0, 0, −1, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[0., −2., 0.9044505452, 4.333539961, −0.8573820427, −2.380608463]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, 1, −1, 0]}

NullSpace N

{[0, 0, 0, 1, −1, 0], [0, 1, −1, 0, 0, 0], [1, 0, 0, 0, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

===

20, [1, −1, 1, −1, −1, 1]

==

{3, 4, 5}

R: [2, 5, 6, 5, 6, 2]
B: [3, 6, 4, 1, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{1}{2048}(33+15s-s^2+s^3)(-1+s)(-7+s)(1+s)^2$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[5] v[6]
"B CYCLES", 1 + v[1] v[3] v[4]

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0], [0, −1, 0, 1, 0, 0]}

NullSpace of B^*

{[0, 0, −1, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 1 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 1 & 1 & 2 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12}(v[1]+2v[2]+2v[3]+2v[4]+2v[5]+3v[6])$

degree 2: $\frac{1}{12}$ ($v[1]v[3] + v[1]v[4] + 2\,v[2]v[5] + 2\,v[2]v[6] + 2\,v[3]v[4] + v[3]v[6] + v[4]v[6] + 2\,v[5]v[6]$)
degree 3 : $\frac{1}{4}$ ($v[1]v[3]v[4] + 2\,v[2]v[5]v[6] + v[3]v[4]v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{3, 5}, {1, 6}, {2, 4}}

"RG1" = {3, 4, 6}

"RG2" = {2, 5, 6}

"RG3" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0]$$

supp $\pi 3$ = {5, 16, 18}

$$u3 = [1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1]$$

supp u3 = {1, 3, 5, 8, 13, 16, 18, 20}

Action of R on ranges, [[2], [2], [2]]
Action of B on ranges, [[3], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 1, 2]

B–BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

b_1 = {3, 5}
b_2 = {1, 6}
b_3 = {2, 4}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 8, Shape: $0 \oplus 8/6$

$$CLB = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 5, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{-5}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5}, {1, 6}, {2, 4}}

1, "range", [3, 4, 6], [[6, 4, 3, 4, 3, 6], [4, 3, 6, 3, 6, 4], [3, 6, 4, 6, 4, 3]]

2, "range", [2, 5, 6], [[6, 2, 5, 2, 5, 6], [5, 6, 2, 6, 2, 5], [2, 5, 6, 5, 6, 2]]

3, "range", [1, 3, 4], [[4, 3, 1, 3, 1, 4], [3, 1, 4, 1, 4, 3], [1, 4, 3, 4, 3, 1]]

"group has", 3, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 3, 2]]

$g_3 =$ [[1, 2, 3]]

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[3] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

Z[3] = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \end{pmatrix}$

{5, 16, 18}

$u3 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{1, 3, 5, 8, 13, 16, 18, 20}

picheck $\begin{pmatrix} 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 1 & 2 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 4 & 4 & 4 & 4 & 6 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 4 & 4 & 4 & 4 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 4 & 4 & 4 & 4 & 6 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 6 \\ 1 & 4 & 2 & 4 & 2 & 3 \\ 1 & 2 & 4 & 2 & 4 & 3 \\ 1 & 4 & 2 & 4 & 2 & 3 \\ 1 & 2 & 4 & 2 & 4 & 3 \\ 2 & 2 & 2 & 2 & 2 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [-1, 2, -2, -2, 2, 1]

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & 0 & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -2 & 2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t \\ -s+t & -s & 0 \\ -t+s & 0 & -t \\ -s+t & -s & 0 \\ -t+s & 0 & -t \\ 0 & s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & 0 \\ s & -s & s & t \\ t & 0 & 0 & s \\ s & -s & s & t \\ t & 0 & 0 & s \\ 0 & s & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 0 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & -1 & -1 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 & 2 \\ 1 & 0 & 2 & 2 & 0 & 1 \\ 1 & 0 & 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 2 & 1 & 1 & 2 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$\Omega \left(\frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{4} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \right)$

$T \left(0 \ \ \frac{1}{2} \ \ 0 \ \ \frac{3}{4} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{1}{4} \right)$

"IS NM in Vec(K)?", true

NM $(1 \ \ 4 \ \ 1 \ \ 6 \ \ 2 \ \ 2 \ \ 2 \ \ 2 \ \ 2)$

"IS MN in Vec(K)?", false

MN $(2 \ \ 4 \ \ 2 \ \ 4 \ \ 2 \ \ 2 \ \ 2 \ \ 2 \ \ 4)$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 11/2$$

max r = 72/13 , r−check is positive? 11/24

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 9

dim span idems 3 vs no. of idems 3

"PT1" = {{3, 5}, {1, 6}, {2, 4}}

"RG1" = {3, 4, 6}

"RG2" = {2, 5, 6}

"RG3" = {1, 3, 4}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-3}{4} \\ \frac{-1}{2} & 1 & -1 & -1 & 1 & \frac{1}{2} \\ \frac{1}{2} & -1 & 1 & 1 & -1 & \frac{-1}{2} \\ \frac{1}{2} & -1 & 1 & 1 & -1 & \frac{-1}{2} \\ \frac{-1}{2} & 1 & -1 & -1 & 1 & \frac{1}{2} \\ \frac{-3}{4} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{3}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{−scaled} = \begin{pmatrix} 1 & \frac{-2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{-2}{3} & -1 \\ \frac{-1}{2} & 1 & -1 & -1 & 1 & \frac{1}{2} \\ \frac{1}{2} & -1 & 1 & 1 & -1 & \frac{-1}{2} \\ \frac{1}{2} & -1 & 1 & 1 & -1 & \frac{-1}{2} \\ \frac{-1}{2} & 1 & -1 & -1 & 1 & \frac{1}{2} \\ -1 & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} & \frac{2}{3} & 1 \end{pmatrix} \quad N_C\text{−scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & \frac{-13}{59} \\ \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 4.637458609, 0.862541391]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 4.914854215, 1.085145785]

Eigenvalues N_C–scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, 1, 0, 1, 0, 0], [0, 0, 0, 1, 1, 0], [1, 0, 0, 0, 0, 1], [0, 0, 1, −1, 0, 0]}

NullSpace N_C

{[0, 0, −1, 0, 1, 0], [0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 0., 6.746568247, 0.798527660, 4.454904092]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, 0, 1, −1, 0, 0], [0, 1, 0, 0, −1, 0], [1, 0, 0, −1, −1, 1]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0], [0, 0, 1, 0, −1, 0]}

Eigenvalues M

[−2.387054170, 4.387054170, −0.589925799, 2.589925799, −2., −2.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{2, 3, 4, 6}

R: [2, 6, 6, 5, 4, 3]
B: [3, 5, 4, 1, 6, 2]

TRACE TWO = 1

det AT = $\frac{1}{4}(t)^2(-1+t)^2$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

Level 2 det = $\frac{1}{8192}(-1848+1798s+88s^2-187s^3-93s^4-5s^5+5s^6+2s^7)(-1+s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[4] v[5]) (1 + v[3] v[6])
"B CYCLES", (1 + v[1] v[3] v[4]) (1 + v[2] v[5] v[6])

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[0, 1, −1, 0, 0, 0]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 1 & 2 & 3 & 0 \\ 4 & 0 & 0 & 6 & 4 & 6 \\ 1 & 0 & 0 & 4 & 6 & 9 \\ 2 & 6 & 4 & 0 & 0 & 8 \\ 3 & 4 & 6 & 0 & 0 & 7 \\ 0 & 6 & 9 & 8 & 7 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12}$ ($v[1] + 2v[2] + 2v[3] + 2v[4] + 2v[5] + 3v[6]$)
degree 2: $\frac{1}{15}$ ($4v[1]v[2] + v[1]v[3] + 2v[1]v[4] + 3v[1]v[5] + 6v[2]v[4] + 4v[2]v[5] + 6v[2]v[6] + 4v[3]v[4] + 6v[3]v[5] + 9v[3]v[6] + 8v[4]v[6] + 7v[5]v[6]$)
degree 3 : $\frac{1}{10}$ ($2v[1]v[2]v[4] + 2v[1]v[2]v[5] + v[1]v[3]v[5] + 4v[2]v[4]v[6] + 2v[2]v[5]v[6] + 4v[3]v[4]v[6] + 5v[3]v[5]v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 2, 5}

"RG7" = {1, 2, 4}

$$\pi 3 = [0, 2, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 4, 2, 0, 4, 5, 0]$$

supp $\pi 3$ = {2, 3, 6, 15, 16, 18, 19}

$$u3 = [0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]$$

supp u3 = {2, 3, 5, 6, 15, 16, 18, 19}

Action of R on ranges, [[2], [1], [2], [1], [4], [4], [3]]
Action of B on ranges, [[4], [7], [3], [6], [2], [1], [5]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{5} & \frac{1}{10} & \frac{1}{5} & \frac{1}{20} & \frac{1}{10} & \frac{1}{10} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 3, 2]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

b_1 = {4, 5}
b_2 = {1, 6}
b_3 = {2, 3}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 11 ⊕ 11/9

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5}, {3, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 4}, {2, 5, 6}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{2} & \frac{5}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{-1}{6} & \frac{7}{12} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-7}{12} \\ \frac{1}{12} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} \end{pmatrix}$$

Omega-checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 6}, {2, 3}}

1, "range", [3, 5, 6], [[6, 5, 5, 3, 3, 6], [6, 3, 3, 5, 5, 6], [5, 6, 6, 3, 3, 5], [5, 3, 3, 6, 6, 5], [3, 6, 6, 5, 5, 3], [3, 5, 5, 6, 6, 3]]

2, "range", [3, 4, 6], [[6, 4, 4, 3, 3, 6], [6, 3, 3, 4, 4, 6], [4, 6, 6, 3, 3, 4], [4, 3, 3, 6, 6, 4], [3, 6, 6, 4, 4, 3], [3, 4, 4, 6, 6, 3]]

3, "range", [2, 5, 6], [[6, 5, 5, 2, 2, 6], [6, 2, 2, 5, 5, 6], [5, 6, 6, 2, 2, 5], [5, 2, 2, 6, 6, 5], [2, 6, 6, 5, 5, 2], [2, 5, 5, 6, 6, 2]]

4, "range", [2, 4, 6], [[6, 4, 4, 2, 2, 6], [6, 2, 2, 4, 4, 6], [4, 6, 6, 2, 2, 4], [4, 2, 2, 6, 6, 4], [2, 6, 6, 4, 4, 2], [2, 4, 4, 6, 6, 2]]

5, "range", [1, 3, 5], [[5, 3, 3, 1, 1, 5], [5, 1, 1, 3, 3, 5], [3, 5, 5, 1, 1, 3], [3, 1, 1, 5, 5, 3], [1, 5, 5, 3, 3, 1], [1, 3, 3, 5, 5, 1]]

6, "range", [1, 2, 5], [[5, 2, 2, 1, 1, 5], [5, 1, 1, 2, 2, 5], [2, 5, 5, 1, 1, 2], [2, 1, 1, 5, 5, 2], [1, 5, 5, 2, 2, 1], [1, 2, 2, 5, 5, 1]]

7, "range", [1, 2, 4], [[4, 2, 2, 1, 1, 4], [4, 1, 1, 2, 2, 4], [2, 4, 4, 1, 1, 2], [2, 1, 1, 4, 4, 2], [1, 4, 4, 2, 2, 1], [1, 2, 2, 4, 4, 1]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1] - h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 2 & 0 & 4 & 5 & 0 \end{pmatrix}$

{2, 3, 6, 15, 16, 18, 19}

$u3 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$

{2, 3, 5, 6, 15, 16, 18, 19}

picheck $\begin{pmatrix} 5 & 10 & 10 & 10 & 10 & 15 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 4 & 1 & 2 & 3 & 0 & 0 & 6 & 4 & 6 & 4 & 6 & 9 & 0 & 8 & 7 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 10 & 20 & 20 & 20 & 20 & 30 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 10 & 20 & 20 & 20 & 20 & 30 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 10 & 20 & 20 & 20 & 20 & 30 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 6 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 4 & 4 & 2 & 2 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 1 & 2 & 2 & 4 & 4 & 3 \\ 2 & 2 & 2 & 2 & 2 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 0, 0, 1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & -s & s & -t \\ 0 & 0 & 0 & -t & t & 0 \\ 0 & s-t & t-s & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & -1 \end{pmatrix}$

$\ker M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ s-t & -t \\ s-t & -t \\ t-s & -s \\ t-s & -s \\ 0 & s+t \end{pmatrix}$ RB checks

$\ker M_c = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s+t \\ t & s & 0 \\ t & s & 0 \\ s & t & 0 \\ s & t & 0 \\ 0 & 0 & s+t \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 6, "vs", 3

$\mathrm{CNM} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 2 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & -1 & -1 & 0 \end{pmatrix}$ Skew T = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ Skew Omega =

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{4}{5} & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & 0 \\ \frac{4}{5} & 2 & 0 & \frac{6}{5} & \frac{4}{5} & \frac{6}{5} \\ \frac{1}{5} & 0 & 2 & \frac{4}{5} & \frac{6}{5} & \frac{9}{5} \\ \frac{2}{5} & \frac{6}{5} & \frac{4}{5} & 2 & 0 & \frac{8}{5} \\ \frac{3}{5} & \frac{4}{5} & \frac{6}{5} & 0 & 2 & \frac{7}{5} \\ 0 & \frac{6}{5} & \frac{9}{5} & \frac{8}{5} & \frac{7}{5} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$\Omega \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{5}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{12} \right)$

$T \left(\frac{1}{2} \ 0 \ 0 \ \frac{1}{4} \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ \frac{3}{4} \ 0 \ 0 \ 0 \ 0 \ \frac{1}{4} \right)$

"IS NM in Vec(K)?", true

NM $(4 \ 2 \ 1 \ 6 \ 2 \ 4 \ 4 \ 1 \ 6 \ 2 \ 2 \ 2 \ 2 \ 2)$

"IS MN in Vec(K)?", false

MN $(4 \ 2 \ 2 \ 8 \ 2 \ 4 \ 4 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 4)$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 13/72 \text{ , min } \boldsymbol{\tau} = 13/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 11/2$$

$$\text{max r} = 72/13 \text{ , } r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 7, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 42

dim span idems 4 vs no. of idems 7

"PT1" = {{4, 5}, {1, 6}, {2, 3}}

"RG1" = {3, 5, 6}

"RG2" = {3, 4, 6}

"RG3" = {2, 5, 6}

"RG4" = {2, 4, 6}

"RG5" = {1, 3, 5}

"RG6" = {1, 2, 5}

"RG7" = {1, 2, 4}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{3}{10} & \frac{-3}{10} & \frac{-1}{10} & \frac{1}{10} & \frac{-3}{4} \\ \frac{3}{10} & 1 & -1 & \frac{1}{5} & \frac{-1}{5} & \frac{-3}{10} \\ \frac{-3}{10} & -1 & 1 & \frac{-1}{5} & \frac{1}{5} & \frac{3}{10} \\ \frac{-1}{10} & \frac{1}{5} & \frac{-1}{5} & 1 & -1 & \frac{1}{10} \\ \frac{1}{10} & \frac{-1}{5} & \frac{1}{5} & -1 & 1 & \frac{-1}{10} \\ \frac{-3}{4} & \frac{-3}{10} & \frac{3}{10} & \frac{1}{10} & \frac{-1}{10} & \frac{3}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{15} & \frac{2}{15} & -1 \\ \frac{3}{10} & 1 & -1 & \frac{1}{5} & \frac{-1}{5} & \frac{-3}{10} \\ \frac{-3}{10} & -1 & 1 & \frac{-1}{5} & \frac{1}{5} & \frac{3}{10} \\ \frac{-1}{10} & \frac{1}{5} & \frac{-1}{5} & 1 & -1 & \frac{1}{10} \\ \frac{1}{10} & \frac{-1}{5} & \frac{1}{5} & -1 & 1 & \frac{-1}{10} \\ -1 & \frac{-2}{5} & \frac{2}{5} & \frac{2}{15} & \frac{-2}{15} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2.519209951, 0.9514021828, 2.029387863]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 2.717609456, 1.087019456, 2.195371088]

Eigenvalues N_C–scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[1, 0, 0, 0, 0, 1], [0, 1, 1, 0, 0, 0], [0, 0, 0, 1, 1, 0]}

NullSpace N_C

{[0, 0, 0, −1, 1, 0], [0, −1, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 0.9007139947, 1.961107586, 2.482271186, 6.655907233]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, 0, 0, −1, −1, 1], [0, 1, 1, −1, −1, 0]}

NullSpace N_0

{[1, 0, 0, 0, 0, −1], [0, 0, 0, 1, −1, 0], [0, 1, −1, 0, 0, 0]}

Eigenvalues M

[−2., 0.1707147561, 0.5956019496, 4.322894958, −0.7455189694, −2.343692694]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0], [0, 0, 0, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$