

T-Run

[2, 4, 4, 2, 6, 5], [3, 6, 5, 3, 1, 4]

$$\tilde{\pi} = [1, 2, 2, 3, 2, 2]$$

$$\delta = [1, 2, 2, 3, 2, 2]$$

POSSIBLE RANKS

- 1 x 12
- 2 x 6
- 3 x 4

BASE DETERMINANT 231/2048, .1127929688

NullSpace of Δ

{1, 4, 5, 6}, {2, 3}

Nullspace of A

[[5, 6], {1, 4}] ` , ` [[3], {2}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
 $v[6] v[5] + v[3] v[4] + v[2] v[4]$

Degree 3
 $v[2] v[4] v[6] + v[1] v[3] v[5]$

Degree 4
 $2 v[3] v[4] v[6] v[5] + v[2] v[4] v[6] v[5] + v[1] v[2] v[6] v[5]$

Degree 5
 $2 v[1] v[2] v[3] v[4] v[5]$

Degree 6
 $4 v[1] v[2] v[3] v[4] v[6] v[5]$

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R: [2, 4, 4, 2, 6, 5]
B: [3, 6, 5, 3, 1, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{1}{2048} (-7 + s) (-1 + s) (11 - 4s + s^2) (3 + s)$$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", $(1 + v[2] v[4]) (1 + v[6] v[5])$

"B CYCLES", $1 + v[1] v[3] v[5]$

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, -1, 0, 0], [0, 1, -1, 0, 0, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 0 & 4 & 2 & 0 & 3 & 3 \\ 1 & 2 & 2 & 3 & 0 & 0 \\ 1 & 2 & 2 & 3 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{12} (2v[1]v[3] + v[1]v[5] + v[1]v[6] + 4v[2]v[4] + 2v[2]v[5] + 2v[2]v[6] + 2v[3]v[4] + 2v[3]v[5] + 2v[3]v[6] + 3v[4]v[5] + 3v[4]v[6])$

degree 3 : $\frac{1}{8} (v[1]v[3] + 2v[2]v[4] + v[3]v[4]) (v[5] + v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

$$\pi_3 = [0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 2, 2, 0, 1, 1, 0, 0]$$

supp $\pi_3 = \{6, 7, 14, 15, 17, 18\}$

$$u_3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $u_3 = \{3, 4, 6, 7, 14, 15, 17, 18\}$

Action of R on ranges, [[4], [3], [4], [3], [4], [3]]

Action of B on ranges, [[2], [6], [1], [5], [2], [6]]

$$\beta = \left(\frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{8} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1, 3]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1 = \{1, 4\}$

$b_2 = \{2, 3\}$

$b_3 = \{5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 \oplus 14/12

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4}, {5, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{6} \ \frac{1}{6}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0\right) \text{ vs } \left(\frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 3, 6], [[6, 3, 3, 6, 1, 1], [6, 1, 1, 6, 3, 3], [3, 6, 6, 3, 1, 1], [3, 1, 1, 3, 6, 6], [1, 6, 6, 1, 3, 3], [1, 3, 3, 1, 6, 6]]

6, "range", [1, 3, 5], [[5, 3, 3, 5, 1, 1], [5, 1, 1, 5, 3, 3], [3, 5, 5, 3, 1, 1], [3, 1, 1, 3, 5, 5], [1, 5, 5, 1, 3, 3], [1, 3, 3, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$$g_1 = [[1, 2, 3]]$$

$$g_2 = [[2, 3]]$$

$$g_3 = [[1, 3]]$$

$$g_4 = []$$

$$g_5 = [[1, 3, 2]]$$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$$(h[2] \ 0 \ 0 \ 2h[1] \ h[2])$$

"Basis for Z(G)"

1, "coeff", 2

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 2 \ 0 \ 1 \ 1 \ 0 \ 0)$$

{6, 7, 14, 15, 17, 18}

$$u3 = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0)$$

{3, 4, 6, 7, 14, 15, 17, 18}

$$picheck \ (2 \ 4 \ 4 \ 6 \ 4 \ 4)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi2 = (0 \ 2 \ 0 \ 1 \ 1 \ 0 \ 4 \ 2 \ 2 \ 2 \ 2 \ 2 \ 3 \ 3 \ 0)$$

$$u2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \right)$$

$$picheck \ (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

$$\pi1 = (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

$$u1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \right)$$

$$picheck \ (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 & 4 \\ 1 & 2 & 2 & 3 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, 2, -2, 1, 0, 0]$$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & t & s & -s \end{pmatrix} \text{ RB checks}$$

$$\pi\Delta \text{ via ker NC } (-2 \ 1 \ 0)$$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -s-t & -s-t \\ s & t \\ s & t \\ -s-t & -s-t \\ t & s \\ t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ t & s & 0 \\ t & s & 0 \\ 0 & 0 & s+t \\ s & t & 0 \\ s & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (2 \ 2 \ 2)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 5, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -2 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 3 & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & 1 & 1 & \frac{3}{2} & 2 & 0 \\ \frac{1}{2} & 1 & 1 & \frac{3}{2} & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad \frac{3}{4} \quad 0 \quad 0 \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\text{NM} (4 \quad 2 \quad 1 \quad 2 \quad 8 \quad 4 \quad 4 \quad 1 \quad 2 \quad 2 \quad 6 \quad 2 \quad 2 \quad 2)$$

"IS MN in Vec(K)?", false

$$\text{MN} (4 \quad 2 \quad 2 \quad 2 \quad 8 \quad 4 \quad 4 \quad 2 \quad 2 \quad 2 \quad 4 \quad 2 \quad 2 \quad 4)$$

$$\tau = 12/1, \text{rank} = 3, \text{ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 13/72, \text{min } \tau = 13/2, \tau\text{-check is positive? } 11/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega?, true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 36

dim span idems 4 vs no. of idems 6

$$\text{"PT1"} = \{\{1, 4\}, \{2, 3\}, \{5, 6\}\}$$

$$\text{"RG1"} = \{3, 4, 6\}$$

$$\text{"RG2"} = \{3, 4, 5\}$$

$$\text{"RG3"} = \{2, 4, 6\}$$

$$\text{"RG4"} = \{2, 4, 5\}$$

$$\text{"RG5"} = \{1, 3, 6\}$$

$$\text{"RG6"} = \{1, 3, 5\}$$

$$M_c = \begin{pmatrix} \frac{3}{4} & \frac{-1}{2} & \frac{1}{2} & \frac{-3}{4} & 0 & 0 \\ \frac{-1}{2} & 1 & -1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -1 & 1 & \frac{-1}{2} & 0 & 0 \\ \frac{-3}{4} & \frac{1}{2} & \frac{-1}{2} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{3} & \frac{2}{3} & -1 & 0 & 0 \\ \frac{-1}{2} & 1 & -1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -1 & 1 & \frac{-1}{2} & 0 & 0 \\ -1 & \frac{2}{3} & \frac{-2}{3} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2., 2.780776406, 0.719223594]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues $M_C\text{-scaled}$

[0., 0., 0., 2., 3.154700539, 0.845299461]

Eigenvalues $N_C\text{-scaled}$

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, 0, 0, 0, 1, 1], [0, 1, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0]}

NullSpace N_C

{[0, 1, -1, 0, 0, 0], [0, 0, 0, 0, 1, -1], [-1, 0, 0, 1, 0, 0]}

Eigenvalues M_0

[0., 0., 2., 6.672823700, 0.6791154995, 2.648060798]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, -1, -1, 1, 0, 0], [0, -1, -1, 0, 1, 1]}

NullSpace N_0

{[0, 0, 0, 0, 1, -1], [0, -1, 1, 0, 0, 0], [1, 0, 0, -1, 0, 0]}

Eigenvalues M

[0., -2., 0.9044505452, 4.333539961, -0.8573820427, -2.380608463]

Eigenvalues N

[4., -2., -2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, 0, -1, 1]}

NullSpace N

{[0, 0, 0, 0, -1, 1], [0, -1, 1, 0, 0, 0], [-1, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

=====

{3}

R: [2, 4, 5, 2, 6, 5]
B: [3, 6, 4, 3, 1, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{-1}{2048} (11 + 9s + 3s^2 + s^3) (-1 + s) (-7 + s^2) (-3 + s)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", $(1 + v[6] v[5]) (1 + v[2] v[4])$

"B CYCLES", $1 + v[3] v[4]$

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [0., 0., 0., 0., 1., -1.]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[1, 0, 0, -1, 0, 0], [0, 0, 1, 0, 0, -1]}

NullSpace of B^*

{[1, 0, 0, -1, 0, 0], [0, 0, -1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 3 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 3 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12} (v[1] + 2 v[2] + 2 v[3] + 3 v[4] + 2 v[5] + 2 v[6])$

degree 2: $\frac{1}{6} (v[1]v[6] + 2 v[2]v[5] + 2 v[3]v[4] + v[4]v[6])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

$$\text{"PT1"} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$$

$$\text{"RG1"} = \{4, 6\}$$

$$\text{"RG2"} = \{3, 4\}$$

$$\text{"RG3"} = \{2, 5\}$$

$$\text{"RG4"} = \{1, 6\}$$

$$\pi_2 = [0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]$$

$$\text{supp } \pi_2 = \{5, 8, 10, 14\}$$

$$u_2 = [0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0]$$

$$\text{supp } u_2 = \{2, 4, 5, 6, 8, 9, 10, 13, 14\}$$

Action of R on ranges, [[3], [3], [1], [3]]

Action of B on ranges, [[2], [2], [4], [2]]

$$\beta = \left(\frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{6} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 2, 4\}$$

$$b_2 = \{3, 5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 9, Shape: $0 \oplus 9/7$

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4}, {5, 6}}, true

Ω_B in Vec(K)? , {{3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \left(0 \ \frac{1}{4} \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 2, 4}, {3, 5, 6}}

- 1, "range", [4, 6], [[6, 6, 4, 6, 4, 4], [4, 4, 6, 4, 6, 6]]
- 2, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]
- 3, "range", [2, 5], [[5, 5, 2, 5, 2, 2], [2, 2, 5, 2, 5, 5]]
- 4, "range", [1, 6], [[6, 6, 1, 6, 1, 1], [1, 1, 6, 1, 6, 6]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$g_1 = [[1, 2]]$$

$$g_2 = []$$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
(h[2] h[1])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 2 \ 0 \ 2 \ 0 \ 0 \ 0 \ 1 \ 0)$$

{5, 8, 10, 14}

$$u_2 = (0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0)$$

{2, 4, 5, 6, 8, 9, 10, 13, 14}

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_1 = (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$u_1 = \left(\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \right)$$

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 3 & 0 & \frac{9}{2} & 0 & 0 \\ \frac{3}{2} & 3 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 3 \\ \frac{3}{2} & 3 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 3 \\ 0 & 0 & 3 & 0 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, 2, -2, -1, 2, 0]$$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} -t & 0 & 0 & t & s & -s \\ 0 & s & t & -s & 0 & -t \\ 0 & s & t & -s & 0 & -t \\ -t & 0 & 0 & t & s & -s \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker NC \begin{pmatrix} -2 & -1 & -1 & 0 \end{pmatrix}$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t & -s \\ -s+t & 0 \\ -t & s \\ t & -s \\ -t+s & 0 \\ -t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s & s+t & -s \\ s & t & 0 \\ t & 0 & s \\ s & s+t & -s \\ t & s & 0 \\ t & 0 & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (3 \ 3 \ 0)$$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & 3 & \frac{3}{2} & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -3 & \frac{-3}{2} & \frac{-3}{2} & 0 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{-1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & \frac{9}{2} & 0 & \frac{3}{2} \\ 0 & 3 & 0 & 0 & 3 & 0 \\ \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{3} \quad 0 \quad 0 \quad 0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{3} \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(3 \quad 0 \quad 0 \quad 0 \quad \frac{9}{2} \quad 0 \quad 3 \quad \frac{3}{2} \right)$$

"IS MN in Vec(K)?", false

$$MN \left(3 \quad 0 \quad 0 \quad 0 \quad 3 \quad 0 \quad 3 \quad 3 \right)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2 / r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau / n^2 = 1/2$$

$$p^2 = 13/72, \text{ min } \tau = 13/2, \tau\text{-check is positive? } 23/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
 out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {4, 6}

"RG2" = {3, 4}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$M_C = \begin{pmatrix} \frac{5}{4} & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{4} & -\frac{1}{2} & 1 \\ -\frac{1}{2} & 2 & -1 & -\frac{3}{2} & 2 & -1 \\ -\frac{1}{2} & -1 & 2 & \frac{3}{2} & -1 & -1 \\ -\frac{3}{4} & -\frac{3}{2} & \frac{3}{2} & \frac{9}{4} & -\frac{3}{2} & 0 \\ -\frac{1}{2} & 2 & -1 & -\frac{3}{2} & 2 & -1 \\ 1 & -1 & -1 & 0 & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} \\ \frac{59}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} \\ -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{59}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} \\ -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & \frac{59}{72} \\ -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -\frac{2}{5} & -\frac{2}{5} & -\frac{3}{5} & -\frac{2}{5} & \frac{4}{5} \\ -\frac{1}{4} & 1 & -\frac{1}{2} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{2} & 1 & \frac{3}{4} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} & 1 & -\frac{2}{3} & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{2} & -\frac{3}{4} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} \\ 1 & 1 & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} \\ -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & 1 & 1 \\ 1 & 1 & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} \\ -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & 1 & 1 \\ -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 0., 3., 1.916666667]

Eigenvalues M_C -scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C -scaled

[0., 0., 0., 0., 3.661016949, 2.338983051]

NullSpace M_C

{[0, 0, 1, 0, 1, 1], [0, 1, 0, 0, -1, 0], [1, 0, 0, 1, 1, 0]}

NullSpace N_C

{[1, 0, 0, -1, 0, 0], [0, 0, 0, 0, 1, -1], [0, 0, 1, 0, 0, -1], [0, 1, 0, -1, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[0, -1, 0, 0, 1, 0], [-1, 0, 1, -1, 0, 1]}

NullSpace N_0

{[0, 0, -1, 0, 0, 1], [1, 0, 0, -1, 0, 0], [0, 1, 0, -1, 0, 0], [0, 0, -1, 0, 1, 0]}

Eigenvalues M

[-3., 3., -3.432368417, 3.432368417, -1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 0., 3., -3.]

NullSpace M

{}

NullSpace N

{[0, 0, -1, 0, 1, 0], [0, 0, -1, 0, 0, 1], [1, 0, 0, -1, 0, 0], [0, 1, 0, -1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

=====

{4}

R: [2, 4, 4, 3, 6, 5]
 B: [3, 6, 5, 2, 1, 4]

TRACE TWO = 1

$$\det AT = \frac{1}{4} (t)^2 (-1 + t)^2$$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

$$\text{Level 2 det} = \frac{1}{8192} (-1848 + 1798s + 88s^2 - 187s^3 - 93s^4 - 5s^5 + 5s^6 + 2s^7) (-1 + s)$$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[6] v[5]) (1 + v[3] v[4])

"B CYCLES", (1 + v[1] v[3] v[5]) (1 + v[2] v[4] v[6])

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R*

{0, 1, -1, 0, 0, 0}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 1 & 0 & 2 & 3 \\ 4 & 0 & 0 & 6 & 6 & 4 \\ 1 & 0 & 0 & 9 & 4 & 6 \\ 0 & 6 & 9 & 0 & 8 & 7 \\ 2 & 6 & 4 & 8 & 0 & 0 \\ 3 & 4 & 6 & 7 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{15} (4v[1]v[2] + v[1]v[3] + 2v[1]v[5] + 3v[1]v[6] + 6v[2]v[4] + 6v[2]v[5] + 4v[2]v[6] + 9v[3]v[4] + 4v[3]v[5] + 6v[3]v[6] + 8v[4]v[5] + 7v[4]v[6])$

degree 3 : $\frac{1}{10} (2v[1]v[2]v[5] + 2v[1]v[2]v[6] + v[1]v[3]v[6] + 4v[2]v[4]v[5] + 2v[2]v[4]v[6] + 4v[3]v[4]v[5] + 5v[3]v[4]v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 2, 6}

"RG7" = {1, 2, 5}

$$\pi_3 = [0, 0, 2, 2, 0, 0, 1, 0, 0, 0, 0, 0, 4, 2, 0, 4, 5, 0, 0]$$

supp $\pi_3 = \{3, 4, 7, 14, 15, 17, 18\}$

$$u_3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $u_3 = \{3, 4, 6, 7, 14, 15, 17, 18\}$

Action of R on ranges, [[2], [1], [2], [1], [4], [4], [3]]

Action of B on ranges, [[4], [7], [3], [6], [2], [1], [5]]

$$\beta = \left(\frac{1}{4} \frac{1}{5} \frac{1}{10} \frac{1}{5} \frac{1}{20} \frac{1}{10} \frac{1}{10} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1, 3]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 4\}$$

$$b_2 = \{2, 3\}$$

$$b_3 = \{5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 11 \oplus 11/9

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6}, {3, 4}}, true

Ω_B in Vec(K)? , {{1, 3, 5}, {2, 4, 6}}, false

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-5}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$$\pi_R = \left(0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{6} \ \frac{1}{6}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 3, 6], [[6, 3, 3, 6, 1, 1], [6, 1, 1, 6, 3, 3], [3, 6, 6, 3, 1, 1], [3, 1, 1, 3, 6, 6], [1, 6, 6, 1, 3, 3], [1, 3, 3, 1, 6, 6]]

6, "range", [1, 2, 6], [[6, 2, 2, 6, 1, 1], [6, 1, 1, 6, 2, 2], [2, 6, 6, 2, 1, 1], [2, 1, 1, 2, 6, 6], [1, 6, 6, 1, 2, 2], [1, 2, 2, 1, 6, 6]]

7, "range", [1, 2, 5], [[5, 2, 2, 5, 1, 1], [5, 1, 1, 5, 2, 2], [2, 5, 5, 2, 1, 1], [2, 1, 1, 2, 5, 5], [1, 5, 5, 1, 2, 2], [1, 2, 2, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 = [[1, 2, 3]]$

$g_2 = [[2, 3]]$

$g_3 = [[1, 3]]$

$g_4 = []$

$g_5 = [[1, 3, 2]]$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
 $(h[2] \ 0 \ 0 \ 2h[1] \ h[2])$

"Basis for Z(G)"

1, "coeff", 2

$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 2 \ 2 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 2 \ 0 \ 4 \ 5 \ 0 \ 0)$$

{3, 4, 7, 14, 15, 17, 18}

$$\mu_3 = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0)$$

{3, 4, 6, 7, 14, 15, 17, 18}

picheck (5 10 10 15 10 10)

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_2 = (4 \ 1 \ 0 \ 2 \ 3 \ 0 \ 6 \ 6 \ 4 \ 9 \ 4 \ 6 \ 8 \ 7 \ 0)$$

$$\mu_2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \right)$$

picheck (10 20 20 30 20 20)

$$\pi_1 = (10 \ 20 \ 20 \ 30 \ 20 \ 20)$$

$$\mu_1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \right)$$

picheck (10 20 20 30 20 20)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 & 4 \\ 1 & 2 & 2 & 3 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, -1, 1, 1, 0, 0]$$

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & s-t & t-s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -t & t \\ t & 0 & 0 & -t & -s & s \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \quad (-1 \quad -1 \quad 0)$$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -s-t & -s-t \\ s & t \\ s & t \\ -s-t & -s-t \\ t & s \\ t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ s & t & 0 \\ s & t & 0 \\ 0 & 0 & s+t \\ t & s & 0 \\ t & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (2 \ 2 \ 2)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 6, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -2 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \text{ Skew T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & 0 & \frac{-1}{12} & \frac{-1}{12} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{4}{5} & \frac{1}{5} & 0 & \frac{2}{5} & \frac{3}{5} \\ \frac{4}{5} & 2 & 0 & \frac{6}{5} & \frac{6}{5} & \frac{4}{5} \\ \frac{1}{5} & 0 & 2 & \frac{9}{5} & \frac{4}{5} & \frac{6}{5} \\ 0 & \frac{6}{5} & \frac{9}{5} & 3 & \frac{8}{5} & \frac{7}{5} \\ \frac{2}{5} & \frac{6}{5} & \frac{4}{5} & \frac{8}{5} & 2 & 0 \\ \frac{3}{5} & \frac{4}{5} & \frac{6}{5} & \frac{7}{5} & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad \frac{3}{4} \quad 0 \quad 0 \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM \ (4 \ 2 \ 1 \ 2 \ 8 \ 4 \ 4 \ 1 \ 2 \ 2 \ 6 \ 2 \ 2 \ 2)$$

"IS MN in Vec(K)?", false

$$MN \ (4 \ 2 \ 2 \ 2 \ 8 \ 4 \ 4 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 4)$$

$$\tau = 12/1, \text{ rank} = 3, \text{ ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 13/72, \text{ min } \tau = 13/2, \tau\text{-check is positive? } 11/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 7, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 42

dim span idems 4 vs no. of idems 7

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 2, 6}

"RG7" = {1, 2, 5}

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{3}{10} & \frac{-3}{10} & \frac{-3}{4} & \frac{-1}{10} & \frac{1}{10} \\ \frac{3}{10} & 1 & -1 & \frac{-3}{10} & \frac{1}{5} & \frac{-1}{5} \\ \frac{-3}{10} & -1 & 1 & \frac{3}{10} & \frac{-1}{5} & \frac{1}{5} \\ \frac{-3}{4} & \frac{-3}{10} & \frac{3}{10} & \frac{3}{4} & \frac{1}{10} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{1}{5} & \frac{-1}{5} & \frac{1}{10} & 1 & -1 \\ \frac{1}{10} & \frac{-1}{5} & \frac{1}{5} & \frac{-1}{10} & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & -1 & \frac{-2}{15} & \frac{2}{15} \\ \frac{3}{10} & 1 & -1 & \frac{-3}{10} & \frac{1}{5} & \frac{-1}{5} \\ \frac{-3}{10} & -1 & 1 & \frac{3}{10} & \frac{-1}{5} & \frac{1}{5} \\ -1 & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{2}{15} & \frac{-2}{15} \\ \frac{-1}{10} & \frac{1}{5} & \frac{-1}{5} & \frac{1}{10} & 1 & -1 \\ \frac{1}{10} & \frac{-1}{5} & \frac{1}{5} & \frac{-1}{10} & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2.519209951, 0.9514021828, 2.029387863]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C -scaled

[0., 0., 0., 2.717609456, 1.087019456, 2.195371088]

Eigenvalues N_C -scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, 1, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 1]}

NullSpace N_C

{[1, 0, 0, -1, 0, 0], [0, -1, 1, 0, 0, 0], [0, 0, 0, 0, -1, 1]}

Eigenvalues M_0

[0., 0., 0.9007139947, 1.961107586, 2.482271186, 6.655907233]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[-1, 1, 1, -1, 0, 0], [-1, 0, 0, -1, 1, 1]}

NullSpace N_0

{[-1, 0, 0, 1, 0, 0], [0, 1, -1, 0, 0, 0], [0, 0, 0, 0, -1, 1]}

Eigenvalues M

[-2., 0.1707147561, 0.5956019496, 4.322894958, -0.7455189694, -2.343692694]

Eigenvalues N

[4., -2., -2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, -1, 1, 0, 0, 0], [-1, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, -1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

=====

{5}

R: [2, 4, 4, 2, 1, 5]
 B: [3, 6, 5, 3, 6, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{1}{2048} (7 + 2s + s^2) (-33 - 36s - 12s^2 + s^4) (-1 + s)$$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[4]

"B CYCLES", 1 + v[3] v[4] v[6] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R*

{[0, 1, -1, 0, 0, 0], [1, 0, 0, -1, 0, 0]}

NullSpace of B*

{[1, 0, 0, -1, 0, 0], [0, 1, 0, 0, -1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 3 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 1 \\ 0 & 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{6} (v[1]v[2] + v[2]v[4] + 2v[3]v[6] + 2v[4]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {3, 6}

"RG2" = {4, 5}

"RG3" = {2, 4}

"RG4" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]$$

supp $\pi_2 = \{1, 7, 12, 13\}$

$$u_2 = [3, 2, 0, 3, 1, 1, 3, 0, 2, 2, 1, 3, 3, 1, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15\}$

Action of R on ranges, [[2], [4], [3], [3]]

Action of B on ranges, [[2], [1], [1], [1]]

$$\beta = \left(\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \right)$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \left(\frac{2}{3} \quad \frac{1}{3} \right)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 1, 3]

B-BLOCKS,

[2, 3, 4, 1]

with invariant measure, [2, 1, 2, 1]

N by blocks, N - check: true

$$b_1 = \{2, 3, 5\}$$

$$b_2 = \{1, 3, 4\}$$

$$b_3 = \{1, 4, 6\}$$

$$b_4 = \{2, 5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 \oplus 11/9

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4}}, true

Ω_B in Vec(K)? , {{3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } \left(0 \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

- 1, "partition", {{2, 3, 5}, {1, 4, 6}}
- 1, "range", [3, 6], [[6, 3, 3, 6, 3, 6], [3, 6, 6, 3, 6, 3]]
- 2, "range", [4, 5], [[5, 4, 4, 5, 4, 5], [4, 5, 5, 4, 5, 4]]
- 3, "range", [2, 4], [[4, 2, 2, 4, 2, 4], [2, 4, 4, 2, 4, 2]]
- 4, "range", [1, 2], [[2, 1, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1]]
- 2, "partition", {{1, 3, 4}, {2, 5, 6}}
- 1, "range", [3, 6], [[6, 3, 6, 6, 3, 3], [3, 6, 3, 3, 6, 6]]
- 2, "range", [4, 5], [[5, 4, 5, 5, 4, 4], [4, 5, 4, 4, 5, 5]]
- 3, "range", [2, 4], [[4, 2, 4, 4, 2, 2], [2, 4, 2, 2, 4, 4]]

4, "range", [1, 2], [[2, 1, 2, 2, 1, 1], [1, 2, 1, 1, 2, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 2 \ 2 \ 0 \ 0)$$

{1, 7, 12, 13}

$$u_2 = (3 \ 2 \ 0 \ 3 \ 1 \ 1 \ 3 \ 0 \ 2 \ 2 \ 1 \ 3 \ 3 \ 1 \ 2)$$

{1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15}

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_1 = (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$u_1 = \left(\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \right)$$

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{1}{2} & 0 & \frac{2}{9} \\ 0 & \frac{1}{3} & \frac{2}{9} & 0 & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{18} & \frac{2}{9} & \frac{1}{3} & \frac{1}{6} & \frac{2}{9} & 0 \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{1}{2} & 0 & \frac{2}{9} \\ 0 & \frac{1}{3} & \frac{2}{9} & 0 & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{3} & \frac{1}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 1 & \frac{9}{2} & 0 & 2 \\ 0 & 3 & 2 & 0 & 3 & 1 \\ \frac{1}{2} & 2 & 3 & \frac{3}{2} & 2 & 0 \\ \frac{3}{2} & 0 & 1 & \frac{9}{2} & 0 & 2 \\ 0 & 3 & 2 & 0 & 3 & 1 \\ 1 & 1 & 0 & 3 & 1 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [1, 2, -2, 1, 0, -2]$$

$$\ker N_C = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & 0 & 0 & s & 0 & 0 \\ -s & -s & -t & s+t & s+t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\Delta$ via $\ker NC$ (2 -2 1)

$$\ker M_0 = \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -t & s \\ t & -s \\ 0 & -s+t \\ -t & s \\ t & -s \\ 0 & -t+s \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & s+t & -t \\ s & 0 & t \\ s & t & 0 \\ t & s+t & -t \\ s & 0 & t \\ t & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (3 \ 3 \ 0)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ 1 & 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & 3 & \frac{3}{2} & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -3 & \frac{-3}{2} & \frac{-3}{2} & 0 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{3} & 0 & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{18} & 0 & 0 & \frac{1}{18} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & 0 & \frac{-1}{12} & \frac{-1}{12} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & 3 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & \frac{3}{2} & 0 & \frac{9}{2} & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{6} \quad \frac{1}{3} \quad \frac{2}{9} \quad \frac{1}{18} \quad \frac{1}{3} \quad 0 \quad \frac{2}{9} \quad 0 \quad \frac{1}{2} \quad \frac{1}{9} \quad 0 \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{3}{2} \quad 3 \quad 2 \quad \frac{1}{2} \quad 3 \quad 0 \quad 2 \quad 0 \quad \frac{9}{2} \quad 1 \quad 0 \quad \frac{3}{2} \right)$$

"IS MN in Vec(K)?", false

MN (1 3 2 1 3 0 2 0 3 1 0 3)

$$\tau = 18/1, \text{rank} = 2, \text{ratio} = 9/1, n^2 / r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau / n^2 = 1/2$$

$$p^2 = 13/72, \text{min } \tau = 13/2, \tau\text{-check is positive? } 23/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega?, true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {3, 6}

"RG2" = {4, 5}

"RG3" = {2, 4}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{5}{4} & 1 & \frac{-1}{2} & \frac{-3}{4} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & 2 & -1 & 0 & -1 & -1 \\ \frac{-1}{2} & -1 & 2 & \frac{-3}{2} & -1 & 2 \\ \frac{-3}{4} & 0 & \frac{-3}{2} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{2} \\ \frac{-1}{2} & -1 & -1 & \frac{3}{2} & 2 & -1 \\ \frac{-1}{2} & -1 & 2 & \frac{-3}{2} & -1 & 2 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{35}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{11}{72} \\ \frac{11}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} & \frac{35}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{35}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{11}{72} \\ \frac{35}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} & \frac{11}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{4}{5} & \frac{-2}{5} & \frac{-3}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{1}{2} & 1 & \frac{-1}{2} & 0 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-3}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{3} & 0 & \frac{-2}{3} & 1 & \frac{2}{3} & \frac{-2}{3} \\ \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-3}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{11}{59} & 1 & \frac{-13}{59} & \frac{35}{59} \\ \frac{-13}{59} & 1 & \frac{35}{59} & \frac{-13}{59} & 1 & \frac{11}{59} \\ \frac{11}{59} & \frac{35}{59} & 1 & \frac{11}{59} & \frac{35}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{11}{59} & 1 & \frac{-13}{59} & \frac{35}{59} \\ \frac{-13}{59} & 1 & \frac{35}{59} & \frac{-13}{59} & 1 & \frac{11}{59} \\ \frac{35}{59} & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} & \frac{11}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 1.916666667, 2.187184271, 0.8128157289]

Eigenvalues $M_C\text{-scaled}$

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues $N_C\text{-scaled}$

[0., 0., 0., 2.338983051, 2.669106229, 0.9919107206]

NullSpace M_C

{[0, 0, -1, 0, 0, 1], [1, 0, 1, 1, 0, 0], [0, 1, 1, 0, 1, 0]}

NullSpace N_C

{[0, 0, -1, 1, 1, -1], [1, 0, -1, 0, 1, -1], [0, 1, 0, 0, -1, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[1, -1, 0, 1, -1, 0], [0, 0, 1, 0, 0, -1]}

NullSpace N_0

{[-1, 0, 0, 1, 0, 0], [-1, 0, 1, 0, -1, 1], [0, 1, 0, 0, -1, 0]}

Eigenvalues M

[-3., 3., -3.432368417, 3.432368417, -1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[0, -1, 0, 0, 1, 0], [-1, -1, 1, 0, 0, 1], [-1, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 0 & 3 & 1 \\ 3 & 0 & 1 & 3 & 0 & 2 \\ 2 & 1 & 0 & 2 & 1 & 3 \\ 0 & 3 & 2 & 0 & 3 & 1 \\ 3 & 0 & 1 & 3 & 0 & 2 \\ 1 & 2 & 3 & 1 & 2 & 0 \end{pmatrix}$$

=====

{2, 6}

R: [2, 6, 4, 2, 6, 4]
B: [3, 4, 5, 3, 1, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{1}{2048} (33 + 15s - s^2 + s^3) (-1 + s) (1 + s)^2 (-7 + s)$$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", $1 + v[2] v[4] v[6]$

"B CYCLES", $1 + v[1] v[3] v[5]$

Eigenvalues

R: $[0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]$

B: $[0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]$

NullSpace of R

$\{[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]\}$

NullSpace of B

$\{[0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0]\}$

NullSpace of R^*

$\{[0, -1, 0, 0, 1, 0], [0, 0, 1, 0, 0, -1], [-1, 0, 0, 1, 0, 0]\}$

NullSpace of B^*

$\{[0, 0, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0]\}$

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12} (v[1] + 2 v[2] + 2 v[3] + 3 v[4] + 2 v[5] + 2 v[6])$

degree 2: $\frac{1}{12} (v[1]v[3] + v[1]v[5] + 2 v[2]v[4] + 2 v[2]v[6] + v[3]v[4] + 2 v[3]v[5] + v[4]v[5] + 2 v[4]v[6])$

degree 3 : $\frac{1}{4} (v[1]v[3]v[5] + 2 v[2]v[4]v[6] + v[3]v[4]v[5])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {3, 6}, {2, 5}}

"RG1" = {3, 4, 5}

"RG2" = {2, 4, 6}

"RG3" = {1, 3, 5}

$$\pi_3 = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0]$$

supp $\pi_3 = \{6, 15, 17\}$

$$u_3 = [1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1]$$

supp $u_3 = \{1, 4, 6, 10, 11, 15, 17, 20\}$

Action of R on ranges, [[2], [2], [2]]

Action of B on ranges, [[3], [1], [3]]

$$\beta = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 4\}$$

$$b_2 = \{3, 6\}$$

$$b_3 = \{2, 5\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 8, Shape: $0 \oplus 8/6$

$$CLB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} -\frac{1}{12} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{1}{12} & -\frac{1}{6} & -\frac{1}{6} & -\frac{5}{12} & \frac{1}{6} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & -\frac{1}{2} & -\frac{1}{6} \\ -\frac{1}{12} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ -\frac{5}{12} & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & -\frac{1}{2} & -\frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3}\right) \text{ vs } \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0\right) \text{ vs } \left(\frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {3, 6}, {2, 5}}

1, "range", [3, 4, 5], [[5, 3, 4, 5, 3, 4], [4, 5, 3, 4, 5, 3], [3, 4, 5, 3, 4, 5]]

2, "range", [2, 4, 6], [[6, 4, 2, 6, 4, 2], [4, 2, 6, 4, 2, 6], [2, 6, 4, 2, 6, 4]]

3, "range", [1, 3, 5], [[5, 3, 1, 5, 3, 1], [3, 1, 5, 3, 1, 5], [1, 5, 3, 1, 5, 3]]

"group has", 3, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 = [[1, 2, 3]]$

$g_2 = []$

$g_3 = [[1, 3, 2]]$

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true

(h[2] h[1] h[3])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

1, 3, true

2, 3, true

$$\text{EIGS} = \begin{pmatrix} 1. & & & 1. & & & & & & 1. \\ 1. & -0.5000000000 + 0.8660254040i & & -0.5000000000 - 0.8660254040i & & & & & & \\ 1. & -0.5000000000 + 0.8660254040i & & -0.5000000000 - 0.8660254040i & & & & & & \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 0 \ 1 \ 0 \ 0 \ 0)$$

{6, 15, 17}

$$\mu_3 = (1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1)$$

{1, 4, 6, 10, 11, 15, 17, 20}

picheck (1 2 2 3 2 2)

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_2 = (0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 2 \ 0 \ 2 \ 1 \ 2 \ 0 \ 1 \ 2 \ 0)$$

$$\mu_2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \right)$$

picheck (2 4 4 6 4 4)

$$\pi_1 = (2 \ 4 \ 4 \ 6 \ 4 \ 4)$$

$$\mu_1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \right)$$

picheck (2 4 4 6 4 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 2 & 3 & 4 & 2 \\ 1 & 2 & 4 & 3 & 2 & 4 \\ 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 2 & 3 & 4 & 2 \\ 1 & 2 & 4 & 3 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, 2, -2, 1, -2, 2]$$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & 0 & -t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\pi\Delta \text{ via ker NC } (2 \quad -1 \quad -2)$$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & -t & -s-t \\ s & 0 & t \\ 0 & t & s \\ -s & -t & -s-t \\ s & 0 & t \\ 0 & t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & s+t & 0 & -t \\ s & 0 & t & 0 \\ 0 & 0 & s & t \\ t & s+t & 0 & -t \\ s & 0 & t & 0 \\ 0 & 0 & s & t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (2 \quad 2 \quad 2 \quad 0)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -2 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew } \Omega =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 1 & 0 & 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 3 & 1 & 2 \\ 1 & 0 & 2 & 1 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{12} \frac{1}{6} \frac{1}{12} \frac{1}{6} \frac{1}{6} \frac{1}{4} \frac{1}{6} \frac{1}{6} \frac{1}{12} \right)$$

$$\tau \left(0 \frac{1}{2} 0 0 0 \frac{3}{4} 0 0 \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM (1 \ 4 \ 1 \ 2 \ 2 \ 6 \ 2 \ 2 \ 2)$$

"IS MN in Vec(K)?", false

$$MN (2 \ 4 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 4)$$

$$\tau = 12/1, \text{ rank} = 3, \text{ ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 13/72, \text{ min } \tau = 13/2, \tau\text{-check is positive? } 11/2$$

$$\max r = 72/13, r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 9

dim span idems 3 vs no. of idems 3

$$\text{"PT1"} = \{\{1, 4\}, \{3, 6\}, \{2, 5\}\}$$

$$\text{"RG1"} = \{3, 4, 5\}$$

$$\text{"RG2"} = \{2, 4, 6\}$$

$$\text{"RG3"} = \{1, 3, 5\}$$

$$M_C = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{2} & -\frac{3}{4} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -1 & \frac{1}{2} & -1 & 1 \\ \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 1 & -1 \\ -\frac{3}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 1 & -1 \\ -\frac{1}{2} & 1 & -1 & \frac{1}{2} & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} \\ -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} \\ -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} \\ \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} \\ -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} \\ -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} & -\frac{13}{72} & -\frac{13}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -\frac{2}{3} & \frac{2}{3} & -1 & \frac{2}{3} & -\frac{2}{3} \\ -\frac{1}{2} & 1 & -1 & \frac{1}{2} & -1 & 1 \\ \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 1 & -1 \\ -1 & \frac{2}{3} & -\frac{2}{3} & 1 & -\frac{2}{3} & \frac{2}{3} \\ \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 1 & -1 \\ -\frac{1}{2} & 1 & -1 & \frac{1}{2} & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} \\ -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} \\ -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} & 1 \\ 1 & -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} \\ -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} \\ -\frac{13}{59} & -\frac{13}{59} & 1 & -\frac{13}{59} & -\frac{13}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 4.637458609, 0.862541391]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues M_C -scaled

[0., 0., 0., 0., 4.914854215, 1.085145785]

Eigenvalues N_C -scaled

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, -1, 0, 0, 0, 1], [0, 1, 0, 0, 1, 0], [1, 0, 0, 1, 0, 0], [0, 1, 1, 0, 0, 0]}

NullSpace N_C

{[-1, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0], [0, 0, -1, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 0., 6.746568247, 0.798527660, 4.454904092]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, -1, -1, 1, 0, 0], [0, 0, -1, 0, 1, 0], [0, -1, 0, 0, 0, 1]}

NullSpace N_0

{[-1, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0], [0, 0, -1, 0, 0, 1]}

Eigenvalues M

[-2.387054170, 4.387054170, -0.589925799, 2.589925799, -2., -2.]

Eigenvalues N

[4., -2., -2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[1, 0, 0, -1, 0, 0], [0, -1, 0, 0, 1, 0], [0, 0, -1, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

=====

{2, 3, 5}

R: [2, 6, 5, 2, 1, 5]
 B: [3, 4, 4, 3, 6, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{1}{2048} (7 + 2s + s^2) (-33 - 7s - s^2 + s^3) (-1 + s) (1 + s)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 3

B ranking is 2, "vs", 3

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[2] v[6] v[5]

"B CYCLES", 1 + v[3] v[4]

Eigenvalues

R: [0., 0., -1., 1., 1. I, -1. I]

B: [0., 0., 0., 0., 1., -1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R*

{[-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 0, 1]}

NullSpace of B*

{[0, -1, 0, 0, 0, 1], [-1, 0, 0, 1, 0, 0], [0, -1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 3 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 3 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} & 1 \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} \\ 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{6} (v[1]v[6] + 2v[2]v[5] + 2v[3]v[4] + v[4]v[6])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 4, 5}, {2, 3, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 5}

"RG2" = {4, 6}

"RG3" = {3, 4}

"RG4" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]$$

supp $\pi_2 = \{5, 8, 10, 14\}$

$$u_2 = [2, 3, 0, 1, 3, 1, 2, 3, 1, 3, 2, 0, 1, 3, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15\}$

Action of R on ranges, [[4], [1], [1], [1]]

Action of B on ranges, [[2], [3], [3], [3]]

$$\beta = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [2, 1]

BPARTS [1, 1]

$$\alpha = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 4, 1, 3]

B-BLOCKS,

[3, 3, 2, 2]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

$$b_1 = \{1, 2, 4\}$$

$$b_2 = \{1, 4, 5\}$$

$$b_3 = \{2, 3, 6\}$$

$$b_4 = \{3, 5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 \oplus 10/8

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 5, 6}}, true

Ω_B in Vec(K)? , {{3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{-5}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(\frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } \left(\frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ \frac{1}{4}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4, 5}, {2, 3, 6}}

1, "range", [2, 5], [[5, 2, 2, 5, 5, 2], [2, 5, 5, 2, 2, 5]]

2, "range", [4, 6], [[6, 4, 4, 6, 6, 4], [4, 6, 6, 4, 4, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3], [3, 4, 4, 3, 3, 4]]

4, "range", [1, 6], [[6, 1, 1, 6, 6, 1], [1, 6, 6, 1, 1, 6]]

2, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [2, 5], [[5, 5, 2, 5, 2, 2], [2, 2, 5, 2, 5, 5]]

2, "range", [4, 6], [[6, 6, 4, 6, 4, 4], [4, 4, 6, 4, 6, 6]]

3, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

4, "range", [1, 6], [[6, 6, 1, 6, 1, 1], [1, 1, 6, 1, 6, 6]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 2 \ 0 \ 2 \ 0 \ 0 \ 0 \ 1 \ 0)$$

{5, 8, 10, 14}

$$u_2 = (2 \ 3 \ 0 \ 1 \ 3 \ 1 \ 2 \ 3 \ 1 \ 3 \ 2 \ 0 \ 1 \ 3 \ 2)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15}

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_1 = (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$u_1 = \left(\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \right)$$

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & 0 & \frac{1}{2} & \frac{2}{9} & 0 \\ \frac{1}{18} & \frac{1}{3} & \frac{2}{9} & \frac{1}{6} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{9} & 0 & \frac{1}{2} & \frac{2}{9} & 0 \\ \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{3} & \frac{1}{3} & \frac{1}{9} \\ 0 & \frac{2}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 1 & 0 & \frac{9}{2} & 2 & 0 \\ \frac{1}{2} & 3 & 2 & \frac{3}{2} & 0 & 2 \\ 0 & 2 & 3 & 0 & 1 & 3 \\ \frac{3}{2} & 1 & 0 & \frac{9}{2} & 2 & 0 \\ 1 & 0 & 1 & 3 & 3 & 1 \\ 0 & 2 & 3 & 0 & 1 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [1, 2, -2, -3, 2, 0]$$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ s & -s & -t & 0 & -s & s+t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\Delta$ via $\ker NC$ (0 2 1)

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t & -s \\ -t+s & 0 \\ -t & s \\ t & -s \\ -s+t & 0 \\ -t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s+t & -s & s \\ s & 0 & t \\ 0 & s & t \\ s+t & -s & s \\ t & 0 & s \\ 0 & s & t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (3 \ 0 \ 3)$$

$$RN_0R^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} & 0 \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} & 1 \\ 1 & \frac{2}{3} & 0 & 1 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & \frac{1}{3} & 1 & 0 & \frac{2}{3} & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 3, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & 3 & \frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -3 & -\frac{3}{2} & -\frac{3}{2} & 0 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{1}{18} & 0 & 0 & \frac{1}{18} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{18} & 0 & 0 & -\frac{1}{9} & 0 \\ -\frac{1}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & \frac{9}{2} & 0 & \frac{3}{2} \\ 0 & 3 & 0 & 0 & 3 & 0 \\ \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 0 \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} \\ 0 & \frac{2}{3} & 1 & 0 & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{3} \quad 0 \quad \frac{1}{6} \quad \frac{2}{9} \quad \frac{1}{3} \quad \frac{1}{18} \quad 0 \quad \frac{2}{9} \quad \frac{1}{2} \quad 0 \quad \frac{1}{9} \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(3 \quad 0 \quad \frac{3}{2} \quad 2 \quad 3 \quad \frac{1}{2} \quad 0 \quad 2 \quad \frac{9}{2} \quad 0 \quad 1 \quad \frac{3}{2} \right)$$

"IS MN in Vec(K)?", false

MN(3 0 1 2 3 1 0 2 3 0 1 3)

$$\tau = 18/1, \text{rank} = 2, \text{ratio} = 9/1, n^2 / r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau / n^2 = 1/2$$

$$p^2 = 13/72, \text{min } \tau = 13/2, \tau\text{-check is positive? } 23/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega?, true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 4, 5}, {2, 3, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 5}

"RG2" = {4, 6}

"RG3" = {3, 4}

"RG4" = {1, 6}

$$M_c = \begin{pmatrix} \frac{5}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-3}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & 2 & -1 & \frac{-3}{2} & 2 & -1 \\ \frac{-1}{2} & -1 & 2 & \frac{3}{2} & -1 & -1 \\ \frac{-3}{4} & \frac{-3}{2} & \frac{3}{2} & \frac{9}{4} & \frac{-3}{2} & 0 \\ \frac{-1}{2} & 2 & -1 & \frac{-3}{2} & 2 & -1 \\ 1 & -1 & -1 & 0 & -1 & 2 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} \\ \frac{11}{72} & \frac{59}{72} & \frac{35}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{35}{72} \\ \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} \\ \frac{59}{72} & \frac{11}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{35}{72} & \frac{-13}{72} \\ \frac{35}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{35}{72} & \frac{59}{72} & \frac{11}{72} \\ \frac{-13}{72} & \frac{35}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{11}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-3}{5} & \frac{-2}{5} & \frac{4}{5} \\ \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-3}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{3}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{3} & \frac{-2}{3} & \frac{2}{3} & 1 & \frac{-2}{3} & 0 \\ \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-3}{4} & 1 & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & 0 & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{11}{59} & \frac{-13}{59} & 1 & \frac{35}{59} & \frac{-13}{59} \\ \frac{11}{59} & 1 & \frac{35}{59} & \frac{11}{59} & \frac{-13}{59} & \frac{35}{59} \\ \frac{-13}{59} & \frac{35}{59} & 1 & \frac{-13}{59} & \frac{11}{59} & 1 \\ 1 & \frac{11}{59} & \frac{-13}{59} & 1 & \frac{35}{59} & \frac{-13}{59} \\ \frac{35}{59} & \frac{-13}{59} & \frac{11}{59} & \frac{35}{59} & 1 & \frac{11}{59} \\ \frac{-13}{59} & \frac{35}{59} & 1 & \frac{-13}{59} & \frac{11}{59} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 1.916666667, 2.187184271, 0.8128157289]

Eigenvalues $M_C\text{-scaled}$

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues $N_C\text{-scaled}$

[0., 0., 0., 2.338983051, 2.669106229, 0.9919107206]

NullSpace M_C

{[1, 0, 0, 1, 1, 0], [-1, 0, 1, -1, 0, 1], [1, 1, 0, 1, 0, 0]}

NullSpace N_C

{[0, 0, 1, 0, 0, -1], [1, 0, 0, -1, 0, 0], [0, 1, 0, -1, 1, -1]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[-1, 0, 1, -1, 0, 1], [0, -1, 0, 0, 1, 0]}

NullSpace N_0

{[1, 0, 0, -1, 0, 0], [0, 1, -1, -1, 1, 0], [0, 0, -1, 0, 0, 1]}

Eigenvalues M

[-3., 3., -3.432368417, 3.432368417, -1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[1, -1, 1, 0, -1, 0], [-1, 0, 0, 1, 0, 0], [1, -1, 0, 0, -1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 0 & 1 & 3 \\ 2 & 0 & 1 & 2 & 3 & 1 \\ 3 & 1 & 0 & 3 & 2 & 0 \\ 0 & 2 & 3 & 0 & 1 & 3 \\ 1 & 3 & 2 & 1 & 0 & 2 \\ 3 & 1 & 0 & 3 & 2 & 0 \end{pmatrix}$$

=====

20, [1, -1, 1, -1, -1, 1]

=====

{3, 5, 6}

R: [2, 4, 5, 2, 1, 4]

B: [3, 6, 4, 3, 6, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{-1}{2048} (-7 + s^2) (-1 + s) (-33 - 20s + 4s^2 + s^4)$$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", $1 + v[2] v[4]$

"B CYCLES", $(1 + v[6] v[5]) (1 + v[3] v[4])$

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [1., -1., 1., -1., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 1, 0, 0, 0, -1], [1, 0, 0, -1, 0, 0]}

NullSpace of B^*

{[0, -1, 0, 0, 1, 0], [-1, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 3 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{6} (v[1]v[2] + v[2]v[4] + 2v[3]v[6] + 2v[4]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {4, 5}

"RG2" = {2, 4}

"RG3" = {3, 6}

"RG4" = {1, 2}

$$\pi_2 = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]$$

supp $\pi_2 = \{1, 7, 12, 13\}$

$$u_2 = [1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0]$$

supp $u_2 = \{1, 4, 5, 6, 7, 11, 12, 13, 14\}$

Action of R on ranges, [[4], [2], [1], [2]]

Action of B on ranges, [[3], [3], [1], [3]]

$$\beta = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1]

B-BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 3, 4\}$$

$$b_2 = \{2, 5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $0 \oplus 10/8$

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4}}, true

Ω_B in Vec(K)? , {{5, 6}, {3, 4}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$$\pi_R = \left(0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 3, 4}, {2, 5, 6}}

1, "range", [4, 5], [[5, 4, 5, 5, 4, 4], [4, 5, 4, 4, 5, 5]]

2, "range", [2, 4], [[4, 2, 4, 4, 2, 2], [2, 4, 2, 2, 4, 4]]

3, "range", [3, 6], [[6, 3, 6, 6, 3, 3], [3, 6, 3, 3, 6, 6]]

4, "range", [1, 2], [[2, 1, 2, 2, 1, 1], [1, 2, 1, 1, 2, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 = [[1, 2]]$

$g_2 = []$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[2] h[1])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 2 \ 2 \ 0 \ 0)$$

{1, 7, 12, 13}

$$u_2 = (1 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0)$$

{1, 4, 5, 6, 7, 11, 12, 13, 14}

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_1 = (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

$$u_1 = \left(\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \right)$$

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 2 \ 2)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 3 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 3 \\ \frac{3}{2} & 0 & 3 & \frac{9}{2} & 0 & 0 \\ \frac{3}{2} & 0 & 3 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 3 \\ 0 & 3 & 0 & 0 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [1, 2, -2, 1, 0, -2]$$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & t & -t \\ 0 & -s & -t & t & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & -s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \quad (-2 \quad -2 \quad 1 \quad 0)$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s & -t \\ -s & t \\ -t+s & 0 \\ s & -t \\ -s & t \\ -s+t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & s+t & -t \\ s & 0 & t \\ t & s & 0 \\ t & s+t & -t \\ s & 0 & t \\ s & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (3 \ 3 \ 0)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{3}{2} & \frac{3}{2} & 3 & \frac{3}{2} & \frac{3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ -3 & \frac{-3}{2} & \frac{-3}{2} & 0 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \frac{-3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 \end{pmatrix} \text{ Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & 3 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & \frac{3}{2} & 0 & \frac{9}{2} & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{3} \quad 0 \quad 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{3} \quad 0 \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(3 \quad 0 \quad 0 \quad 0 \quad \frac{9}{2} \quad 3 \quad 0 \quad \frac{3}{2} \right)$$

"IS MN in Vec(K)?", false

$$MN (3 \quad 0 \quad 0 \quad 0 \quad 3 \quad 3 \quad 0 \quad 3)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2 / r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau / n^2 = 1/2$$

$$p^2 = 13/72, \text{ min } \tau = 13/2, \tau\text{-check is positive? } 23/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 23/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
 out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {4, 5}

"RG2" = {2, 4}

"RG3" = {3, 6}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{5}{4} & 1 & \frac{-1}{2} & \frac{-3}{4} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & 2 & -1 & 0 & -1 & -1 \\ \frac{-1}{2} & -1 & 2 & \frac{-3}{2} & -1 & 2 \\ \frac{-3}{4} & 0 & \frac{-3}{2} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{2} \\ \frac{-1}{2} & -1 & -1 & \frac{3}{2} & 2 & -1 \\ \frac{-1}{2} & -1 & 2 & \frac{-3}{2} & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{4}{5} & \frac{-2}{5} & \frac{-3}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{1}{2} & 1 & \frac{-1}{2} & 0 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-3}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{3} & 0 & \frac{-2}{3} & 1 & \frac{2}{3} & \frac{-2}{3} \\ \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-3}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 4., 6.454163456, 1.045836544]

Eigenvalues N_C

[0., 0., 0., 0., 3., 1.916666667]

Eigenvalues M_C -scaled

[0., 0., 0., 3.139758004, 0.6081028905, 2.252139106]

Eigenvalues N_C -scaled

[0., 0., 0., 0., 3.661016949, 2.338983051]

NullSpace M_C

{[0, 1, 0, 0, 1, 1], [1, 0, 0, 1, 0, 1], [0, 0, 1, 0, 0, -1]}

NullSpace N_C

{[0, 0, 1, -1, 0, 0], [0, 0, 0, 0, 1, -1], [0, 1, 0, 0, 0, -1], [1, 0, 0, -1, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.220409754, 0.985615375, 3.793974871]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[0, 0, 1, 0, 0, -1], [1, -1, 0, 1, -1, 0]}

NullSpace N_0

{[0, -1, 0, 0, 1, 0], [0, 0, -1, 1, 0, 0], [0, -1, 0, 0, 0, 1], [1, 0, -1, 0, 0, 0]}

Eigenvalues M

[-3., 3., -3.432368417, 3.432368417, -1.311048073, 1.311048073]

Eigenvalues N

[0., 0., 0., 0., 3., -3.]

NullSpace M

{}

NullSpace N

{[0, -1, 0, 0, 0, 1], [0, 0, 1, -1, 0, 0], [1, 0, 0, -1, 0, 0], [0, -1, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

=====

{2, 3, 5, 6}

R: [2, 6, 5, 2, 1, 4]
 B: [3, 4, 4, 3, 6, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{1}{2048} (-7 + s) (11 - 4s + s^2) (-1 + s) (3 + s)$$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", $1 + v[2] v[4] v[6]$

"B CYCLES", $(1 + v[6] v[5]) (1 + v[3] v[4])$

Eigenvalues

R: [0., 0., 0., 1., $-0.5000000000 + 0.8660254040 I$, $-0.5000000000 - 0.8660254040 I$]

B: [1., -1., 1., -1., 0., 0.]

NullSpace of R

{[0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, -1, 0, 0]}

NullSpace of B^*

{[0, -1, 1, 0, 0, 0], [-1, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 2 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 4 & 2 & 2 \\ 0 & 2 & 4 & 0 & 3 & 3 \\ 1 & 2 & 2 & 3 & 0 & 0 \\ 1 & 2 & 2 & 3 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{12} (2v[1]v[2] + v[1]v[5] + v[1]v[6] + 2v[2]v[4] + 2v[2]v[5] + 2v[2]v[6] + 4v[3]v[4] + 2v[3]v[5] + 2v[3]v[6] + 3v[4]v[5] + 3v[4]v[6])$

degree 3 : $\frac{1}{8} (v[1]v[2] + v[2]v[4] + 2v[3]v[4]) (v[5] + v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 2, 6}

"RG6" = {1, 2, 5}

$$\pi_3 = [0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 2, 2, 0, 0]$$

supp $\pi_3 = \{3, 4, 14, 15, 17, 18\}$

$$u_3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $u_3 = \{3, 4, 6, 7, 14, 15, 17, 18\}$

Action of R on ranges, [[4], [6], [3], [5], [3], [5]]

Action of B on ranges, [[2], [1], [2], [1], [2], [1]]

$$\beta = \left(\frac{1}{4} \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{8} \frac{1}{8} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 2]

B-BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 4\}$$

$$b_2 = \{2, 3\}$$

$$b_3 = \{5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: $3 \oplus 14/12$

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)?, {{2, 4, 6}}, true

Ω_B in Vec(K)? , {{5, 6}, {3, 4}}, true

$$V = \begin{pmatrix} -\frac{1}{12} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{1}{12} & -\frac{1}{6} & -\frac{1}{6} & -\frac{5}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{6} & -\frac{1}{6} & -\frac{7}{12} & \frac{1}{2} & \frac{1}{6} \\ -\frac{1}{12} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{12} & -\frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & -\frac{1}{2} & -\frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$$\pi_R = \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3}\right) \text{ vs } \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{6} \ \frac{1}{6}\right) \text{ vs } (0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 2, 6], [[6, 2, 2, 6, 1, 1], [6, 1, 1, 6, 2, 2], [2, 6, 6, 2, 1, 1], [2, 1, 1, 2, 6, 6], [1, 6, 6, 1, 2, 2], [1, 2, 2, 1, 6, 6]]

6, "range", [1, 2, 5], [[5, 2, 2, 5, 1, 1], [5, 1, 1, 5, 2, 2], [2, 5, 5, 2, 1, 1], [2, 1, 1, 2, 5, 5], [1, 5, 5, 1, 2, 2], [1, 2, 2, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 = [[1, 2, 3]]$

$g_2 = [[2, 3]]$

$g_3 = [[1, 3]]$

$$g_4 = []$$

$$g_5 = [[1, 3, 2]]$$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$$(h[2] \ 0 \ 0 \ 2h[1] \ h[2])$$

"Basis for Z(G)"

1, "coeff", 2

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 2 \ 2 \ 0 \ 0)$$

{3, 4, 14, 15, 17, 18}

$$u_3 = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0)$$

{3, 4, 6, 7, 14, 15, 17, 18}

$$\text{picheck } (2 \ 4 \ 4 \ 6 \ 4 \ 4)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_2 = (2 \ 0 \ 0 \ 1 \ 1 \ 0 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 3 \ 3 \ 0)$$

$$u_2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0 \right)$$

$$\text{picheck } (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

$$\pi_1 = (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

$$u_1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \right)$$

$$\text{picheck } (4 \ 8 \ 8 \ 12 \ 8 \ 8)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 & 4 \\ 1 & 2 & 2 & 3 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [1, 2, -2, -1, 0, 0]$$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & -s & -t & t \\ 0 & 0 & 0 & 0 & s & -s \end{pmatrix} \text{ RB checks}$$

$$\pi\Delta \text{ via ker NC } (-1 \ 0 \ -2)$$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -s-t & -s-t \\ t & s \\ t & s \\ -s-t & -s-t \\ s & t \\ s & t \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ t & s & 0 \\ t & s & 0 \\ 0 & 0 & s+t \\ s & t & 0 \\ s & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (2 \ 2 \ 2)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 4, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -2 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 1 & 1 \\ 0 & 1 & 2 & 3 & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & 1 & 1 & \frac{3}{2} & 2 & 0 \\ \frac{1}{2} & 1 & 1 & \frac{3}{2} & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad \frac{3}{4} \quad 0 \quad 0 \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\text{NM} (4 \quad 2 \quad 1 \quad 2 \quad 8 \quad 4 \quad 4 \quad 1 \quad 2 \quad 2 \quad 6 \quad 2 \quad 2 \quad 2)$$

"IS MN in Vec(K)?", false

$$\text{MN} (4 \quad 2 \quad 2 \quad 2 \quad 8 \quad 4 \quad 4 \quad 2 \quad 2 \quad 2 \quad 4 \quad 2 \quad 2 \quad 4)$$

$$\tau = 12/1, \text{rank} = 3, \text{ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 13/72, \text{min } \tau = 13/2, \tau\text{-check is positive? } 11/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega?, true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 36

dim span idems 4 vs no. of idems 6

$$\text{"PT1"} = \{\{1, 4\}, \{2, 3\}, \{5, 6\}\}$$

$$\text{"RG1"} = \{3, 4, 6\}$$

$$\text{"RG2"} = \{3, 4, 5\}$$

$$\text{"RG3"} = \{2, 4, 6\}$$

$$\text{"RG4"} = \{2, 4, 5\}$$

$$\text{"RG5"} = \{1, 2, 6\}$$

$$\text{"RG6"} = \{1, 2, 5\}$$

$$M_c = \begin{pmatrix} \frac{3}{4} & \frac{1}{2} & \frac{-1}{2} & \frac{-3}{4} & 0 & 0 \\ \frac{1}{2} & 1 & -1 & \frac{-1}{2} & 0 & 0 \\ \frac{-1}{2} & -1 & 1 & \frac{1}{2} & 0 & 0 \\ \frac{-3}{4} & \frac{-1}{2} & \frac{1}{2} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{-2}{3} & -1 & 0 & 0 \\ \frac{1}{2} & 1 & -1 & \frac{-1}{2} & 0 & 0 \\ \frac{-1}{2} & -1 & 1 & \frac{1}{2} & 0 & 0 \\ -1 & \frac{-2}{3} & \frac{2}{3} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2., 2.780776406, 0.719223594]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues $M_C\text{-scaled}$

[0., 0., 0., 2., 3.154700539, 0.845299461]

Eigenvalues $N_C\text{-scaled}$

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[0, 0, 0, 0, 1, 1], [0, 1, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0]}

NullSpace N_C

{[0, 0, 0, 0, -1, 1], [1, 0, 0, -1, 0, 0], [0, -1, 1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 2., 6.672823700, 0.6791154995, 2.648060798]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, 1, 1, 0, -1, -1], [1, 0, 0, 1, -1, -1]}

NullSpace N_0

{[0, 0, 0, 0, 1, -1], [-1, 0, 0, 1, 0, 0], [0, 1, -1, 0, 0, 0]}

Eigenvalues M

[0., -2., 0.9044505452, 4.333539961, -0.8573820427, -2.380608463]

Eigenvalues N

[4., -2., -2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, 0, -1, 1]}

NullSpace N

{[0, 1, -1, 0, 0, 0], [0, 0, 0, 0, 1, -1], [1, 0, 0, -1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

=====

{2, 3, 4, 5, 6}

R: [2, 6, 5, 3, 1, 4]
B: [3, 4, 4, 2, 6, 5]

TRACE TWO = 1

$$\det AT = \frac{-1}{4} (t)^2 (1+t)^2$$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 6

$$\text{Level 2 det} = \frac{-1}{8192} (1848 - 410s + 128s^2 - 143s^3 - 105s^4 - 17s^5 - 7s^6 + 2s^7) (-1 + s)$$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 4, "vs", 6

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 1, "vs", 4

"R CYCLES", $1 + v[1] v[2] v[3] v[4] v[6] v[5]$

"B CYCLES", $(1 + v[6] v[5]) (1 + v[2] v[4])$

Eigenvalues

R: $[-1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]$

B: $[1., -1., 1., -1., 0., 0.]$

NullSpace of R

{}

NullSpace of B

{ $[1, 0, 0, 0, 0, 0]$ }

NullSpace of R^*

{}

NullSpace of B^*

{ $[0, 1, -1, 0, 0, 0]$ }

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 2 & 7 & 0 & 4 & 5 \\ 2 & 0 & 0 & 16 & 10 & 8 \\ 7 & 0 & 0 & 11 & 8 & 10 \\ 0 & 16 & 11 & 0 & 14 & 13 \\ 4 & 10 & 8 & 14 & 0 & 0 \\ 5 & 8 & 10 & 13 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 3

degree 1: $\frac{1}{12} (v[1] + 2v[2] + 2v[3] + 3v[4] + 2v[5] + 2v[6])$

degree 2: $\frac{1}{54} (2v[1]v[2] + 7v[1]v[3] + 4v[1]v[5] + 5v[1]v[6] + 16v[2]v[4] + 10v[2]v[5] + 8v[2]v[6] + 11v[3]v[4] + 8v[3]v[5] + 10v[3]v[6] + 14v[4]v[5] + 13v[4]v[6])$

degree 3 : $\frac{1}{54} (4v[1]v[2]v[5] + 2v[1]v[2]v[6] + 8v[1]v[3]v[5] + 13v[1]v[3]v[6] + 26v[2]v[4]v[5] + 22v[2]v[4]v[6] + 16v[3]v[4]v[5] + 17v[3]v[4]v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 6}

"RG8" = {1, 2, 5}

$$\pi_3 = [0, 0, 4, 2, 0, 8, 13, 0, 0, 0, 0, 0, 26, 22, 0, 16, 17, 0, 0]$$

supp $\pi_3 = \{3, 4, 6, 7, 14, 15, 17, 18\}$

$$u_3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $u_3 = \{3, 4, 6, 7, 14, 15, 17, 18\}$

Action of R on ranges, [[2], [6], [1], [5], [4], [8], [3], [7]]

Action of B on ranges, [[4], [3], [4], [3], [2], [1], [2], [1]]

$$\beta = \left(\frac{17}{108} \frac{4}{27} \frac{11}{54} \frac{13}{54} \frac{13}{108} \frac{2}{27} \frac{1}{54} \frac{1}{27} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 2]

B-BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 4\}$$

$$b_2 = \{2, 3\}$$

$$b_3 = \{5, 6\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: $11 \oplus 11/9$

$$CLB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

Ω_B in Vec(K)? , {{2, 4}, {5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{6} & \frac{-1}{6} & \frac{-5}{12} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-7}{12} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{12} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{7}{12} & \frac{-1}{2} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{ vs } \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 3, 6], [[6, 3, 3, 6, 1, 1], [6, 1, 1, 6, 3, 3], [3, 6, 6, 3, 1, 1], [3, 1, 1, 3, 6, 6], [1, 6, 6, 1, 3, 3], [1, 3, 3, 1, 6, 6]]

6, "range", [1, 3, 5], [[5, 3, 3, 5, 1, 1], [5, 1, 1, 5, 3, 3], [3, 5, 5, 3, 1, 1], [3, 1, 1, 3, 5, 5], [1, 5, 5, 1, 3, 3], [1, 3, 3, 1, 5, 5]]

7, "range", [1, 2, 6], [[6, 2, 2, 6, 1, 1], [6, 1, 1, 6, 2, 2], [2, 6, 6, 2, 1, 1], [2, 1, 1, 2, 6, 6], [1, 6, 6, 1, 2, 2], [1, 2, 2, 1, 6, 6]]

8, "range", [1, 2, 5], [[5, 2, 2, 5, 1, 1], [5, 1, 1, 5, 2, 2], [2, 5, 5, 2, 1, 1], [2, 1, 1, 2, 5, 5], [1, 5, 5, 1, 2, 2], [1, 2, 2, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 =
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$g_1 = [[1, 2, 3]]$$

$$g_2 = [[2, 3]]$$

$$g_3 = [[1, 3]]$$

$$g_4 = []$$

$$g_5 = [[1, 3, 2]]$$

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
(h[2] 0 0 2h[1] h[2])

"Basis for Z(G)"

1, "coeff", 2

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {15, [2, 4, 6]}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 4 \ 2 \ 0 \ 8 \ 13 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 26 \ 22 \ 0 \ 16 \ 17 \ 0 \ 0)$$

{3, 4, 6, 7, 14, 15, 17, 18}

$$\mu_3 = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0)$$

{3, 4, 6, 7, 14, 15, 17, 18}

picheck (27 54 54 81 54 54)

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{6} \ \frac{1}{6} \right)$$

$$\pi_2 = (6 \ 21 \ 0 \ 12 \ 15 \ 0 \ 48 \ 30 \ 24 \ 33 \ 24 \ 30 \ 42 \ 39 \ 0)$$

$$u_2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ 0\right)$$

$$\text{picheck} (54 \ 108 \ 108 \ 162 \ 108 \ 108)$$

$$\pi_1 = (54 \ 108 \ 108 \ 162 \ 108 \ 108)$$

$$u_1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9}\right)$$

$$\text{picheck} (54 \ 108 \ 108 \ 162 \ 108 \ 108)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 1 & 4 & 4 & 3 & 2 & 2 \\ 2 & 2 & 2 & 6 & 2 & 2 \\ 1 & 2 & 2 & 3 & 4 & 4 \\ 1 & 2 & 2 & 3 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\Delta = [1, -1, 1, -1, 0, 0]$

$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & -s & -t & t \\ 0 & -s+t & -t+s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -s & s \end{pmatrix}$ RB checks

$\pi\Delta$ via $\ker NC (0 \ -1 \ -1)$

$\ker M_0 = \begin{pmatrix} -1 & -1 \\ 0 & 1 \\ 0 & 1 \\ -1 & -1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & s+t \\ -t+s & -t \\ -t+s & -t \\ 0 & s+t \\ -s+t & -s \\ -s+t & -s \end{pmatrix}$ RB checks

$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s+t & 0 \\ t & 0 & s \\ t & 0 & s \\ 0 & s+t & 0 \\ s & 0 & t \\ s & 0 & t \end{pmatrix}$ RB checks

$n\pi x^\dagger = (2 \ 2 \ 2)$

$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -2 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{12} & -\frac{1}{12} & 0 & -\frac{1}{12} & -\frac{1}{12} \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \\ -\frac{1}{12} & 0 & 0 & \frac{1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & \frac{2}{9} & \frac{7}{9} & 0 & \frac{4}{9} & \frac{5}{9} \\ \frac{2}{9} & 2 & 0 & \frac{16}{9} & \frac{10}{9} & \frac{8}{9} \\ \frac{7}{9} & 0 & 2 & \frac{11}{9} & \frac{8}{9} & \frac{10}{9} \\ 0 & \frac{16}{9} & \frac{11}{9} & 3 & \frac{14}{9} & \frac{13}{9} \\ \frac{4}{9} & \frac{10}{9} & \frac{8}{9} & \frac{14}{9} & 2 & 0 \\ \frac{5}{9} & \frac{8}{9} & \frac{10}{9} & \frac{13}{9} & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \left(\frac{1}{6} \frac{1}{6} \frac{1}{12} \frac{1}{6} \frac{1}{3} \frac{1}{6} \frac{1}{6} \frac{1}{12} \frac{1}{6} \frac{1}{6} \frac{1}{4} \frac{1}{6} \frac{1}{6} \frac{1}{12} \right)$$

$$T \left(\frac{1}{2} \ 0 \ 0 \ 0 \ 1 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0 \ 0 \ \frac{3}{4} \ 0 \ 0 \ \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM (4 \ 2 \ 1 \ 2 \ 8 \ 4 \ 4 \ 1 \ 2 \ 2 \ 6 \ 2 \ 2 \ 2)$$

"IS MN in Vec(K)?", false

$$MN (4 \ 2 \ 2 \ 2 \ 8 \ 4 \ 4 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 4)$$

$$\tau = 12/1, \text{ rank} = 3, \text{ ratio} = 4/1, n^2 / r = 12/1$$

$$\tau' = 24/1, r' = 2/3, \tau / n^2 = 1/3$$

$$p^2 = 13/72, \text{ min } \tau = 13/2, \tau\text{-check is positive? } 11/2$$

$$\text{max } r = 72/13, r\text{-check is positive? } 11/24$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

$$\text{"PT1"} = \{\{1, 4\}, \{2, 3\}, \{5, 6\}\}$$

$$\text{"RG1"} = \{3, 4, 6\}$$

$$\text{"RG2"} = \{3, 4, 5\}$$

$$\text{"RG3"} = \{2, 4, 6\}$$

$$\text{"RG4"} = \{2, 4, 5\}$$

$$\text{"RG5"} = \{1, 3, 6\}$$

$$\text{"RG6"} = \{1, 3, 5\}$$

$$\text{"RG7"} = \{1, 2, 6\}$$

$$\text{"RG8"} = \{1, 2, 5\}$$

$$M_C = \begin{pmatrix} \frac{3}{4} & \frac{-5}{18} & \frac{5}{18} & \frac{-3}{4} & \frac{-1}{18} & \frac{1}{18} \\ \frac{-5}{18} & 1 & -1 & \frac{5}{18} & \frac{1}{9} & \frac{-1}{9} \\ \frac{5}{18} & -1 & 1 & \frac{-5}{18} & \frac{-1}{9} & \frac{1}{9} \\ \frac{-3}{4} & \frac{5}{18} & \frac{-5}{18} & \frac{3}{4} & \frac{1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{9} & \frac{1}{18} & 1 & -1 \\ \frac{1}{18} & \frac{-1}{9} & \frac{1}{9} & \frac{-1}{18} & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{-13}{72} & \frac{-13}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \\ \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{-13}{72} & \frac{59}{72} & \frac{59}{72} \end{pmatrix}$$

$$M_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{-10}{27} & \frac{10}{27} & -1 & \frac{-2}{27} & \frac{2}{27} \\ \frac{-5}{18} & 1 & -1 & \frac{5}{18} & \frac{1}{9} & \frac{-1}{9} \\ \frac{5}{18} & -1 & 1 & \frac{-5}{18} & \frac{-1}{9} & \frac{1}{9} \\ -1 & \frac{10}{27} & \frac{-10}{27} & 1 & \frac{2}{27} & \frac{-2}{27} \\ \frac{-1}{18} & \frac{1}{9} & \frac{-1}{9} & \frac{1}{18} & 1 & -1 \\ \frac{1}{18} & \frac{-1}{9} & \frac{1}{9} & \frac{-1}{18} & -1 & 1 \end{pmatrix} \quad N_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & 1 & 1 & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} \\ 1 & \frac{-13}{59} & \frac{-13}{59} & 1 & \frac{-13}{59} & \frac{-13}{59} \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \\ \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & \frac{-13}{59} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 2.485072161, 1.139862126, 1.875065712]

Eigenvalues N_C

[0.9166666667, 2., 2., 0., 0., 0.]

Eigenvalues $M_{C\text{-scaled}}$

[0., 0., 0., 2.726443974, 1.351163554, 1.922392474]

Eigenvalues $N_{C\text{-scaled}}$

[1.118644068, 2.440677966, 2.440677966, 0., 0., 0.]

NullSpace M_C

{[1, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 1], [0, 1, 1, 0, 0, 0]}

NullSpace N_C

{[0, -1, 1, 0, 0, 0], [-1, 0, 0, 1, 0, 0], [0, 0, 0, 0, -1, 1]}

Eigenvalues M_0

[0., 0., 1.062640726, 1.860175760, 2.422740712, 6.654442803]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[-1, 0, 0, -1, 1, 1], [-1, 1, 1, -1, 0, 0]}

NullSpace N_0

{[-1, 0, 0, 1, 0, 0], [0, 0, 0, 0, -1, 1], [0, 1, -1, 0, 0, 0]}

Eigenvalues M

[-2., 0.5907398838, 4.322012694, -0.07291006400, -0.5049609398, -2.334881574]

Eigenvalues N

[4., -2., -2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 1, -1], [-1, 0, 0, 1, 0, 0], [0, -1, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$