T - Run

[2, 4, 2, 1, 4, 3], [3, 6, 5, 6, 6, 5]

$\tilde{\pi}$ = [2, 3, 4, 4, 5, 6]
$\boldsymbol{\delta}$ = [1, 2, 2, 2, 2, 3]

POSSIBLE RANKS

1 x 24
2 x 12
3 x 8
4 x 6

BASE DETERMINANT 55/256, .2148437500

NullSpace of Δ

{2, 3, 5}, {1, 4, 6}

Nullspace of A

[{6},{1, 4}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[6] v[5]

Degree 3
v[4] v[6] v[5] + v[3] v[6] v[5] + v[2] v[3] v[6] + v[1] v[2] v[4]

Degree 4
v[3] v[4] v[6] v[5] + v[2] v[3] v[4] v[6] + v[1] v[3] v[4] v[5] + v[1] v[2] v[3] v[4]

Degree 5
2 v[1] v[2] v[4] v[6] v[5]

Degree 6
4 v[1] v[2] v[3] v[4] v[6] v[5]

==

20, [1, -1, 1, -1, -1, 1]

==

{2, 5, 6}

R: [2, 6, 2, 1, 6, 5]
B: [3, 4, 5, 6, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-3}{1024}(-1+s)(220+17s+46s^2+36s^3-2s^4+3s^5)$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[6] v[5]
"B CYCLES", 1 + v[3] v[4] v[6] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, −1, 0, 0, 0], [0, 1, 0, 0, −1, 0]}

NullSpace of B^*

{[0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 6 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 0 & 3 & 0 & 0 & 15 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}\,(2v[1] + 3v[2] + 4v[3] + 4v[4] + 5v[5] + 6v[6])$

degree 2: $\frac{1}{3}\,(2v[1]v[2] + v[2]v[6] + 4v[3]v[4] + 5v[6]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$\pi 2 = [2, 0, 0, 0, 0, 0, 0, 0, 1, 4, 0, 0, 0, 0, 5]$$

supp $\pi 2$ = {1, 9, 10, 15}

$$u2 = [3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

Action of R on ranges, [[1], [1], [4], [2]]
Action of B on ranges, [[3], [3], [1], [3]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{5}{12} & \frac{1}{12} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 3, 2, 3]

B–BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N – check: true

$b_1 = \{2, 3, 5\}$
$b_2 = \{1, 3, 6\}$
$b_3 = \{2, 4, 5\}$
$b_4 = \{1, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 ⊕ 11/9

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6}}, true

Ω_B in Vec(K)? , {{3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{1}{24} & \frac{5}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{-7}{48} & \frac{1}{8} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{-1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{-1}{4} & \frac{-1}{2} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ \frac{1}{24} & \frac{-3}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{17}{48} & \frac{1}{8} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [5, 6], [[6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 6, 5]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 2, 1]]

2, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [5, 6], [[6, 5, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$

{1, 9, 10, 15}

$u2 = \begin{pmatrix} 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{8} & \frac{1}{6} & \frac{1}{6} & \frac{5}{24} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_3 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ idem−checks N0−checks

$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{9} & \frac{1}{12} & \frac{1}{3} & 0 & \frac{5}{36} & \frac{1}{3} \\ \frac{1}{18} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{5}{18} & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ 1 & \frac{3}{4} & 3 & 0 & \frac{5}{4} & 3 \\ \frac{1}{2} & \frac{3}{2} & 0 & 3 & \frac{5}{2} & \frac{3}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [2, 3, -4, -4, 1, 2]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & -t & -t & t & -s+t \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 2 & 1 & -4 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & -t \\ -s & t \\ s+t & 0 \\ -s-t & 0 \\ -s & t \\ s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & 0 & s \\ -t & s+t & t \\ 0 & 0 & s+t \\ 0 & s+t & 0 \\ -t & s+t & t \\ t & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 \end{pmatrix}$

$$\mathrm{R} N_0 \mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \ \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B} N_0 \mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \ \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{2} & \frac{9}{4} & 3 \\ \frac{-3}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{-3}{4} & 0 & \frac{3}{4} \\ -3 & \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{4} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{36} & \frac{1}{18} & \frac{1}{6} & 0 \\ \frac{-1}{9} & \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & \frac{-1}{6} & \frac{-1}{36} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{1}{6} \\ \frac{-1}{24} & 0 & \frac{1}{24} & \frac{1}{24} & \frac{1}{12} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{24} & \frac{-1}{24} & 0 & \frac{1}{24} \\ \frac{-1}{6} & \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{24} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & \frac{9}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{15}{4} & \frac{15}{4} \\ 0 & \frac{3}{4} & 0 & 0 & \frac{15}{4} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{5}{24} \ \frac{1}{6} \ \frac{1}{8} \ \frac{1}{12} \ \frac{1}{8} \ \frac{1}{12} \ \frac{1}{4} \ \frac{5}{24} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{8} \ \frac{1}{12} \right)$$

$$T \left(\frac{5}{36} \ \frac{1}{3} \ \frac{1}{12} \ \frac{1}{9} \ \frac{1}{4} \ 0 \ \frac{1}{2} \ 0 \ \frac{1}{9} \ \frac{2}{9} \ 0 \ \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{5}{4} \ 3 \ \frac{3}{4} \ 1 \ \frac{9}{4} \ 0 \ \frac{9}{2} \ 0 \ 1 \ 2 \ 0 \ \frac{3}{2} \right)$$

"IS MN in Vec(K)?", false

MN $\begin{pmatrix} 1 & 3 & 1 & 2 & 3 & 0 & 3 & 0 & 1 & 2 & 0 & 3 \end{pmatrix}$

$$\tau = 18/1 \text{ , rank } = 2 \text{ , ratio } = 9/1 \text{ , } n^2 / r = 18/1$$

$$\tau' = 18/1 \text{ , } r' = 1/2 \text{ , } \tau / n^2 = 1/2$$

$$p^2 = 53/288 \text{ , min } \tau = 53/8 \text{ , } \tau\text{-check is positive? } 91/8$$

$$\text{max r} = 288/53 \text{ , } r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{5}{4} & \frac{9}{8} & \frac{-1}{2} & \frac{-1}{2} & \frac{-5}{8} & \frac{-3}{4} \\ \frac{9}{8} & \frac{27}{16} & \frac{-3}{4} & \frac{-3}{4} & \frac{-15}{16} & \frac{-3}{8} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-5}{8} & \frac{-15}{16} & \frac{-5}{4} & \frac{-5}{4} & \frac{35}{16} & \frac{15}{8} \\ \frac{-3}{4} & \frac{-3}{8} & \frac{-3}{2} & \frac{-3}{2} & \frac{15}{8} & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{139}{288} & \frac{43}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{9}{10} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{2} & \frac{-3}{5} \\ \frac{2}{3} & 1 & \frac{-4}{9} & \frac{-4}{9} & \frac{-5}{9} & \frac{-2}{9} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-2}{7} & \frac{-3}{7} & \frac{-4}{7} & \frac{-4}{7} & 1 & \frac{6}{7} \\ \frac{-1}{3} & \frac{-1}{6} & \frac{-2}{3} & \frac{-2}{3} & \frac{5}{6} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ \frac{139}{235} & \frac{43}{235} & 1 & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} \\ \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} & 1 & \frac{139}{235} & \frac{43}{235} \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.811196768, 0.629770625, 3.934032609]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.236783071, 0.365404790, 2.397812138]

Eigenvalues N_C–scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[0, 1, 0, 1, 1, 0], [1, 0, 0, 1, 0, 1], [0, 0, 1, −1, 0, 0]}

NullSpace N_C

{[−1, 0, 0, 0, 0, 1], [1, 1, −1, −1, 0, 0], [1, 0, −1, −1, 1, 0]}

Eigenvalues M_0

[0., 0., 6., 7.951833037, 0.618821322, 3.429345642]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[1, −1, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[1, 1, −1, −1, 0, 0], [1, 0, −1, −1, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[−3., 3., −3.837504551, 3.837504551, −1.465796307, 1.465796307]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[1, 0, −1, −1, 1, 0], [0, 1, 0, 0, −1, 0], [0, 0, −1, −1, 1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 3 & 0 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 5, 6}

R: [2, 6, 5, 1, 6, 5]
B: [3, 4, 2, 6, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{3}{1024}(-1+s)(-220-55s-58s^2+12s^3-2s^4+3s^5)$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[6] v[5]
"B CYCLES", 1 + v[2] v[3] v[4] v[6]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, 0, 1, 0, 0, −1], [0, 1, 0, 0, −1, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 9 \\ 0 & 9 & 0 & 0 & 9 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}\left(2v[1] + 3v[2] + 4v[3] + 4v[4] + 5v[5] + 6v[6]\right)$

degree 2: $\frac{1}{3}$ ($2 v[1] v[5]+3 v[2] v[6]+4 v[3] v[4]+3 v[6] v[5]$)

Group spectrum $1+t+t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 5}

$$\pi 2 = [0, 0, 0, 2, 0, 0, 0, 0, 3, 4, 0, 0, 0, 0, 3]$$

supp $\pi 2$ = {4, 9, 10, 15}

$$u2 = [3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

Action of R on ranges, [[1], [1], [4], [2]]
Action of B on ranges, [[3], [3], [2], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 3, 2, 3]

B–BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N – check: true

b_1 = {2, 3, 5}
b_2 = {1, 3, 6}
b_3 = {2, 4, 5}
b_4 = {1, 4, 6}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 ⊕ 11/9

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6}}, true

Ω_B in Vec(K)? , {{2, 3, 4, 6}}, true

$$V = \begin{pmatrix} \frac{1}{24} & \frac{5}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{-7}{48} & \frac{1}{8} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ 0 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{-1}{4} & \frac{-1}{2} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ \frac{1}{24} & \frac{-3}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{17}{48} & \frac{1}{8} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [5, 6], [[6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 6, 5]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 4, 3]]

4, "range", [1, 5], [[5, 1, 5, 1, 1, 5], [1, 5, 1, 5, 5, 1]]

2, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [5, 6], [[6, 5, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5], [1, 5, 5, 1, 5, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$

{4, 9, 10, 15}

$u2 = \begin{pmatrix} 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{8} & \frac{1}{6} & \frac{1}{6} & \frac{5}{24} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{9} & \frac{1}{12} & \frac{1}{3} & 0 & \frac{5}{36} & \frac{1}{3} \\ \frac{1}{18} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{5}{18} & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ 1 & \frac{3}{4} & 3 & 0 & \frac{5}{4} & 3 \\ \frac{1}{2} & \frac{3}{2} & 0 & 3 & \frac{5}{2} & \frac{3}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [2, -1, -4, -4, 5, 2]$

$$\ker N_C = \begin{pmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -s & s-t & t & t & -s & s-t \\ -s & s-t & t & t & -s & s-t \\ 0 & -s & 0 & 0 & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 5 & 2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} -t & s \\ t & -s \\ 0 & t+s \\ 0 & -s-t \\ t & -s \\ -t & s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & t+s & s \\ s & 0 & t \\ -s-t & t+s & t+s \\ t+s & 0 & 0 \\ s & 0 & t \\ -s & t+s & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{2} & \frac{9}{4} & 3 \\ \frac{-3}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{-3}{4} & 0 & \frac{3}{4} \\ -3 & \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{4} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{36} & \frac{1}{18} & \frac{1}{6} & 0 \\ \frac{-1}{9} & \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & \frac{-1}{6} & \frac{-1}{36} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{1}{6} \\ \frac{-1}{24} & 0 & \frac{1}{24} & \frac{1}{24} & \frac{1}{12} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{24} & \frac{-1}{24} & 0 & \frac{1}{24} \\ \frac{-1}{6} & \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{24} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & \frac{9}{4} & 0 & 0 & 0 & \frac{9}{4} \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & \frac{15}{4} & \frac{9}{4} \\ 0 & \frac{9}{4} & 0 & 0 & \frac{9}{4} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{5}{24} \ \ \frac{1}{6} \ \ \frac{1}{8} \ \ \frac{1}{12} \ \ \frac{1}{8} \ \ \frac{1}{12} \ \ \frac{1}{4} \ \ \frac{5}{24} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{8} \ \ \frac{1}{12} \right)$

$T \left(\frac{5}{36} \ \ \frac{1}{3} \ \ \frac{1}{12} \ \ \frac{1}{9} \ \ \frac{1}{4} \ \ 0 \ \ \frac{1}{2} \ \ 0 \ \ \frac{1}{9} \ \ \frac{2}{9} \ \ 0 \ \ \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\text{NM} \left(\frac{5}{4} \ \ 3 \ \ \frac{3}{4} \ \ 1 \ \ \frac{9}{4} \ \ 0 \ \ \frac{9}{2} \ \ 0 \ \ 1 \ \ 2 \ \ 0 \ \ \frac{3}{2} \right)$

"IS MN in Vec(K)?", false

$\text{MN} \left(1 \ \ 3 \ \ 1 \ \ 2 \ \ 3 \ \ 0 \ \ 3 \ \ 0 \ \ 1 \ \ 2 \ \ 0 \ \ 3 \right)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 53/288 \text{ , min } \boldsymbol{\tau} = 53/8 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 91/8$$

$$\text{max r} = 288/53 \text{ , } r\text{–check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{5}{4} & \frac{-3}{8} & \frac{-1}{2} & \frac{-1}{2} & \frac{7}{8} & \frac{-3}{4} \\ \frac{-3}{8} & \frac{27}{16} & \frac{-3}{4} & \frac{-3}{4} & \frac{-15}{16} & \frac{9}{8} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{7}{8} & \frac{-15}{16} & \frac{-5}{4} & \frac{-5}{4} & \frac{35}{16} & \frac{3}{8} \\ \frac{-3}{4} & \frac{9}{8} & \frac{-3}{2} & \frac{-3}{2} & \frac{3}{8} & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{139}{288} & \frac{43}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-3}{10} & \frac{-2}{5} & \frac{-2}{5} & \frac{7}{10} & \frac{-3}{5} \\ \frac{-2}{9} & 1 & \frac{-4}{9} & \frac{-4}{9} & \frac{-5}{9} & \frac{2}{3} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{2}{5} & \frac{-3}{7} & \frac{-4}{7} & \frac{-4}{7} & 1 & \frac{6}{35} \\ \frac{-1}{3} & \frac{1}{2} & \frac{-2}{3} & \frac{-2}{3} & \frac{1}{6} & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ \frac{139}{235} & \frac{43}{235} & 1 & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} \\ \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} & 1 & \frac{139}{235} & \frac{43}{235} \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.410841895, 1.289330471, 3.674827635]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.140508746, 0.7826632797, 2.076827974]

Eigenvalues N_C–scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[0, 0, −1, 1, 0, 0], [1, 0, 1, 0, 0, 1], [0, 1, 1, 0, 1, 0]}

NullSpace N_C

{[−1, −1, 1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 6., 7.188657443, 1.155932192, 3.655410366]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[1, −1, 0, 0, −1, 1], [0, 0, 1, −1, 0, 0]}

NullSpace N_0

{[0, 1, −1, −1, 0, 1], [0, −1, 0, 0, 1, 0], [1, 1, −1, −1, 0, 0]}

Eigenvalues M

[−3., 3., −3.372461083, 3.372461083, −1.000752839, 1.000752839]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1], [0, 0, 1, 1, −1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 3 & 0 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 4, 5, 6}

R: [2, 6, 2, 6, 6, 5]
B: [3, 4, 5, 1, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-3}{1024}(-1+s)(220+121s+14s^2-28s^3-10s^4+3s^5)$

RANK of R is 3

R ranking is 2, "vs", 3

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[6] v[5]
"B CYCLES", 1 + v[1] v[3] v[4] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[1, 0, −1, 0, 0, 0], [0, 0, 0, 1, −1, 0], [0, 1, 0, 0, −1, 0]}

NullSpace of B^*

{[0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO–M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 9 \\ 0 & 9 & 0 & 0 & 9 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($2v[1] + 3v[2] + 4v[3] + 4v[4] + 5v[5] + 6v[6]$)

degree 2: $\frac{1}{3}$ ($2v[1]v[5] + 3v[2]v[6] + 4v[3]v[4] + 3v[6]v[5]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 5}

$$\pi 2 = [0, 0, 0, 2, 0, 0, 0, 0, 3, 4, 0, 0, 0, 0, 3]$$

supp $\pi 2$ = {4, 9, 10, 15}

$$u2 = [3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

Action of R on ranges, [[1], [1], [2], [2]]
Action of B on ranges, [[3], [3], [4], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 3, 2, 3]

B−BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N − check: true

$b_1 = \{2, 3, 5\}$
$b_2 = \{1, 3, 6\}$
$b_3 = \{2, 4, 5\}$
$b_4 = \{1, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: 3 ⊕ 9/7

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 4, 5}}, true

$$V = \begin{pmatrix} \frac{1}{24} & \frac{5}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{-7}{48} & \frac{1}{8} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{-1}{2} & 0 \\ \frac{-1}{2} & \frac{-1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ \frac{1}{24} & \frac{-3}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{17}{48} & \frac{1}{8} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [5, 6], [[6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 6, 5]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 4, 3]]

4, "range", [1, 5], [[5, 1, 5, 1, 1, 5], [1, 5, 1, 5, 5, 1]]

2, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [5, 6], [[6, 5, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5], [1, 5, 5, 1, 5, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$

{4, 9, 10, 15}

$u2 = \begin{pmatrix} 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{8} & \frac{1}{6} & \frac{1}{6} & \frac{5}{24} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{9} & \frac{1}{12} & \frac{1}{3} & 0 & \frac{5}{36} & \frac{1}{3} \\ \frac{1}{18} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{5}{18} & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ 1 & \frac{3}{4} & 3 & 0 & \frac{5}{4} & 3 \\ \frac{1}{2} & \frac{3}{2} & 0 & 3 & \frac{5}{2} & \frac{3}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-2, 3, -4, -4, 1, 6]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & -t & -t & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 6 & 3 & -4 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & -t \\ s & t \\ -s-t & 0 \\ t+s & 0 \\ s & t \\ -s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & t+s & s \\ s & 0 & t \\ -s-t & t+s & t+s \\ t+s & 0 & 0 \\ s & 0 & t \\ -s & t+s & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 \end{pmatrix}$

$$\mathrm{R} N_0 \mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B} N_0 \mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{2} & \frac{9}{4} & 3 \\ \frac{-3}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{-3}{4} & 0 & \frac{3}{4} \\ -3 & \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{4} & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{36} & \frac{1}{18} & \frac{1}{6} & 0 \\ \frac{-1}{9} & \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & \frac{-1}{6} & \frac{-1}{36} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{1}{6} \\ \frac{-1}{24} & 0 & \frac{1}{24} & \frac{1}{24} & \frac{1}{12} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{24} & \frac{-1}{24} & 0 & \frac{1}{24} \\ \frac{-1}{6} & \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{24} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & \frac{9}{4} & 0 & 0 & 0 & \frac{9}{4} \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & \frac{15}{4} & \frac{9}{4} \\ 0 & \frac{9}{4} & 0 & 0 & \frac{9}{4} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{5}{24} \;\; \frac{1}{6} \;\; \frac{1}{8} \;\; \frac{1}{12} \;\; \frac{1}{8} \;\; \frac{1}{12} \;\; \frac{1}{4} \;\; \frac{5}{24} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{8} \;\; \frac{1}{12} \right)$

$T \left(\frac{5}{36} \;\; \frac{1}{3} \;\; \frac{1}{12} \;\; \frac{1}{9} \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; 0 \;\; \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(\frac{5}{4} \;\; 3 \;\; \frac{3}{4} \;\; 1 \;\; \frac{9}{4} \;\; 0 \;\; \frac{9}{2} \;\; 0 \;\; 1 \;\; 2 \;\; 0 \;\; \frac{3}{2} \right)$

"IS MN in Vec(K)?", false

MN $\begin{pmatrix} 1 & 3 & 1 & 2 & 3 & 0 & 3 & 0 & 1 & 2 & 0 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank } = 2 \text{ , ratio } = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 53/288 \text{ , min } \boldsymbol{\tau} = 53/8 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 91/8$$

$$\text{max r} = 288/53 \text{ , } r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{5}{4} & \frac{-3}{8} & \frac{-1}{2} & \frac{-1}{2} & \frac{7}{8} & \frac{-3}{4} \\ \frac{-3}{8} & \frac{27}{16} & \frac{-3}{4} & \frac{-3}{4} & \frac{-15}{16} & \frac{9}{8} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{7}{8} & \frac{-15}{16} & \frac{-5}{4} & \frac{-5}{4} & \frac{35}{16} & \frac{3}{8} \\ \frac{-3}{4} & \frac{9}{8} & \frac{-3}{2} & \frac{-3}{2} & \frac{3}{8} & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{139}{288} & \frac{43}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-3}{10} & \frac{-2}{5} & \frac{-2}{5} & \frac{7}{10} & \frac{-3}{5} \\ \frac{-2}{9} & 1 & \frac{-4}{9} & \frac{-4}{9} & \frac{-5}{9} & \frac{2}{3} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{2}{5} & \frac{-3}{7} & \frac{-4}{7} & \frac{-4}{7} & 1 & \frac{6}{35} \\ \frac{-1}{3} & \frac{1}{2} & \frac{-2}{3} & \frac{-2}{3} & \frac{1}{6} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ \frac{139}{235} & \frac{43}{235} & 1 & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} \\ \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} & 1 & \frac{139}{235} & \frac{43}{235} \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.410841895, 1.289330471, 3.674827635]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.140508746, 0.7826632797, 2.076827974]

Eigenvalues N_C–scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[1, 0, 0, 1, 0, 1], [−1, 1, 0, 0, 1, −1], [1, 0, 1, 0, 0, 1]}

NullSpace N_C

{[0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.188657443, 1.155932192, 3.655410366]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, 0, 1, −1, 0, 0], [1, −1, 0, 0, −1, 1]}

NullSpace N_0

{[0, 0, 1, 1, −1, −1], [0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1]}

Eigenvalues M

[−3., 3., −3.372461083, 3.372461083, −1.000752839, 1.000752839]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 3 & 0 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 4, 5, 6}

R: [2, 6, 5, 6, 6, 5]
B: [3, 4, 2, 1, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{3}{1024}(1+s)(-220+61s+9s^2-13s^3+3s^4)(-1+s)$

RANK of R is 3

R ranking is 2, "vs", 3

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[6] v[5]
"B CYCLES", 1 + v[1] v[2] v[3] v[4]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, 0, 1, 0, 0, −1], [0, 1, 0, −1, 0, 0], [0, 0, 0, −1, 1, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 6 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 0 & 3 & 0 & 0 & 15 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}\left(2v[1] + 3v[2] + 4v[3] + 4v[4] + 5v[5] + 6v[6]\right)$

degree 2: $\frac{1}{3}$ ($2v[1]v[2] + v[2]v[6] + 4v[3]v[4] + 5v[6]v[5]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$\pi 2 = [2, 0, 0, 0, 0, 0, 0, 0, 1, 4, 0, 0, 0, 0, 5]$$

supp $\pi 2$ = {1, 9, 10, 15}

$$u2 = [3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

Action of R on ranges, [[1], [1], [1], [2]]
Action of B on ranges, [[3], [3], [4], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{5}{12} & \frac{1}{12} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 3, 2, 3]

B–BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N – check: true

b_1 = {2, 3, 5}
b_2 = {1, 3, 6}
b_3 = {2, 4, 5}
b_4 = {1, 4, 6}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 ⊕ 10/8

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4}}, true

$$V = \begin{pmatrix} \frac{1}{24} & \frac{5}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{-7}{48} & \frac{1}{8} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ 0 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{-1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{-5}{24} & \frac{-1}{16} & \frac{1}{12} & \frac{-5}{12} & \frac{11}{48} & \frac{3}{8} \\ \frac{1}{24} & \frac{-3}{16} & \frac{-5}{12} & \frac{1}{12} & \frac{17}{48} & \frac{1}{8} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [5, 6], [[6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 6, 5]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 2, 1]]

2, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [5, 6], [[6, 5, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$

{1, 9, 10, 15}

$u2 = \begin{pmatrix} 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{8} & \frac{1}{6} & \frac{1}{6} & \frac{5}{24} & \frac{1}{4} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 3 & 4 & 4 & 5 & 6 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{3} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{9} & \frac{1}{12} & \frac{1}{3} & 0 & \frac{5}{36} & \frac{1}{3} \\ \frac{1}{18} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{5}{18} & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{9} & \frac{2}{9} & \frac{5}{12} & 0 \\ \frac{1}{6} & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ 1 & \frac{3}{4} & 3 & 0 & \frac{5}{4} & 3 \\ \frac{1}{2} & \frac{3}{2} & 0 & 3 & \frac{5}{2} & \frac{3}{2} \\ 0 & \frac{9}{4} & 1 & 2 & \frac{15}{4} & 0 \\ \frac{3}{2} & 0 & 2 & 1 & 0 & \frac{9}{2} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-2, -1, -4, -4, 5, 6]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ t & -s+t & -t & -t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 6 & -1 & -4 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & -t \\ -s & t \\ t+s & 0 \\ -t-s & 0 \\ -s & t \\ s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & s & 0 \\ -t & t & t+s \\ 0 & t+s & 0 \\ 0 & 0 & t+s \\ -t & t & t+s \\ t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{2} & \frac{9}{4} & 3 \\ \frac{-3}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{2} & \frac{9}{4} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{2} \\ \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{-3}{4} & 0 & \frac{3}{4} \\ -3 & \frac{-9}{4} & \frac{-3}{2} & \frac{-3}{2} & \frac{-3}{4} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{36} & \frac{1}{18} & \frac{1}{6} & 0 \\ \frac{-1}{9} & \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & \frac{-1}{6} & \frac{-1}{36} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{1}{6} \\ \frac{-1}{24} & 0 & \frac{1}{24} & \frac{1}{24} & \frac{1}{12} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{24} & \frac{-1}{24} & 0 & \frac{1}{24} \\ \frac{-1}{6} & \frac{-1}{8} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{24} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\ \frac{3}{2} & \frac{9}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{15}{4} & \frac{15}{4} \\ 0 & \frac{3}{4} & 0 & 0 & \frac{15}{4} & \frac{9}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{5}{24} \ \ \frac{1}{6} \ \ \frac{1}{8} \ \ \frac{1}{12} \ \ \frac{1}{8} \ \ \frac{1}{12} \ \ \frac{1}{4} \ \ \frac{5}{24} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{8} \ \ \frac{1}{12} \right)$

$T \left(\frac{5}{36} \ \ \frac{1}{3} \ \ \frac{1}{12} \ \ \frac{1}{9} \ \ \frac{1}{4} \ \ 0 \ \ \frac{1}{2} \ \ 0 \ \ \frac{1}{9} \ \ \frac{2}{9} \ \ 0 \ \ \frac{1}{6} \right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(\frac{5}{4} \ \ 3 \ \ \frac{3}{4} \ \ 1 \ \ \frac{9}{4} \ \ 0 \ \ \frac{9}{2} \ \ 0 \ \ 1 \ \ 2 \ \ 0 \ \ \frac{3}{2} \right)$

"IS MN in Vec(K)?", false

$\mathrm{MN} \left(1 \ \ 3 \ \ 1 \ \ 2 \ \ 3 \ \ 0 \ \ 3 \ \ 0 \ \ 1 \ \ 2 \ \ 0 \ \ 3 \right)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 53/288 \text{ , min } \boldsymbol{\tau} = 53/8 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 91/8$$

$$\text{max r} = 288/53 \text{ , } r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"PT2" = {{2, 3, 5}, {1, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{5}{4} & \frac{9}{8} & \frac{-1}{2} & \frac{-1}{2} & \frac{-5}{8} & \frac{-3}{4} \\ \frac{9}{8} & \frac{27}{16} & \frac{-3}{4} & \frac{-3}{4} & \frac{-15}{16} & \frac{-3}{8} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{-3}{4} & 2 & 2 & \frac{-5}{4} & \frac{-3}{2} \\ \frac{-5}{8} & \frac{-15}{16} & \frac{-5}{4} & \frac{-5}{4} & \frac{35}{16} & \frac{15}{8} \\ \frac{-3}{4} & \frac{-3}{8} & \frac{-3}{2} & \frac{-3}{2} & \frac{15}{8} & \frac{9}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{139}{288} & \frac{43}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} \\ \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} \\ \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{9}{10} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{2} & \frac{-3}{5} \\ \frac{2}{3} & 1 & \frac{-4}{9} & \frac{-4}{9} & \frac{-5}{9} & \frac{-2}{9} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-3}{8} & 1 & 1 & \frac{-5}{8} & \frac{-3}{4} \\ \frac{-2}{7} & \frac{-3}{7} & \frac{-4}{7} & \frac{-4}{7} & 1 & \frac{6}{7} \\ \frac{-1}{3} & \frac{-1}{6} & \frac{-2}{3} & \frac{-2}{3} & \frac{5}{6} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ \frac{139}{235} & \frac{43}{235} & 1 & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} \\ \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} & 1 & \frac{139}{235} & \frac{43}{235} \\ \frac{-53}{235} & 1 & \frac{43}{235} & \frac{139}{235} & 1 & \frac{-53}{235} \\ 1 & \frac{-53}{235} & \frac{139}{235} & \frac{43}{235} & \frac{-53}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.811196768, 0.629770625, 3.934032609]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[0., 0., 0., 3.236783071, 0.365404790, 2.397812138]

Eigenvalues N_C–scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[0, 0, 1, −1, 0, 0], [0, 1, 0, 1, 1, 0], [1, 0, 0, 1, 0, 1]}

NullSpace N_c

{[0, −1, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 6., 7.951833037, 0.618821322, 3.429345642]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 1, −1]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0]}

Eigenvalues M

[−3., 3., −3.837504551, 3.837504551, −1.465796307, 1.465796307]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[1, 0, −1, −1, 1, 0], [−1, 0, 0, 0, 0, 1], [1, 1, −1, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 1 & 2 & 3 & 0 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$