New Graph

[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]

π = [1, 1, 1, 1, 1, 1]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 91/512, .1777343750

NullSpace of Δ

{1, 2, 3, 4}, {5, 6}

Nullspace of A

[{6},{5}] , [{2, 4},{1, 3}]

1 . Coloring, {}

Ωp(Δ)=0: $p' = s^3$ $p = s^2$ $p' = s^2$

R: [3, 3, 5, 5, 1, 1]
B: [2, 4, 6, 6, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 4	1 *vs* 4	1 *vs* 4	1 *vs* 3	1 *vs* 3

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$[y_1, 0, y_1, 0, y_1, 0]$

$p = - s + s^3$ $p = - s + s^2$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$[0, y_1, 0, y_1, 0, y_1]$

$p = s - s^3 \quad p' = s - s^2$

See 3-level graph

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 2 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 \end{bmatrix}$$

$\tau = 12$, $r' = 2/3$

R: [3, 3, 5, 5, 1, 1]
B: [2, 4, 6, 6, 4, 2]

Ranges

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

Cycles: R , {{1, 3, 5}}, B , {{2, 4, 6}}

$\beta(\{1, 3, 5\}) = 1/2$
$\beta(\{2, 4, 6\}) = 1/2$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [2]]

$\alpha([\{1, 2\}, \{5, 6\}, \{3, 4\}]) = 1/2$
$\alpha([\{3, 4\}, \{2, 5\}, \{1, 6\}]) = 1/2$

b1 = {1, 2} , b2 = {5, 6} , b3 = {3, 4} , b4 = {2, 5} , b5 = {1, 6}

Action of R and B on the blocks of the partitions: = [2, 3, 1, 3, 2] [5, 3, 4, 5, 3]
with invariant measure [1, 1, 2, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Sandwich	
Coloring	{}
Rank	3
R,B	[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]

π_2	[0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
u_3	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

2 . Coloring, {2}

$\Omega p(\Delta)=0$: $p = s^3$ $p' = s^3$

R: [3, 4, 5, 5, 1, 1]
B: [2, 3, 6, 6, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	2 *vs* 5	2 *vs* 5	2 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$[y_1, 0, y_1 - y_2, y_2, y_1, 0]$

$p = - s^2 + s^4$ $p = - s^2 + s^3$

Omega Rank for B : cycles: {{2, 3, 6}} order: 3

See Matrix

$[0, y_2, -y_1 + y_2, y_1, 0, y_2]$

$p = - s^2 + s^3$ $p = - s^2 + s^4$

See 3-level graph

M N

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 2 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 \end{bmatrix}$$

τ= 12 , r'= 2/3

R: [3, 4, 5, 5, 1, 1]
B: [2, 3, 6, 6, 4, 2]

Ranges

Action of R on ranges, [[1], [1], [2], [2]]
Action of B on ranges, [[4], [4], [3], [3]]

Cycles: R , {{1, 3, 5}}, B , {{2, 3, 6}}

β({1, 3, 5}) = 1/4
β({1, 4, 5}) = 1/4
β({2, 3, 6}) = 1/4
β({2, 4, 6}) = 1/4

Partitions

Action of R on partitions, [[2], [2]]
Action of B on partitions, [[1], [1]]

α([{3, 4}, {2, 5}, {1, 6}]) = 1/2
α([{1, 2}, {5, 6}, {3, 4}]) = 1/2

b1 = {1, 2} , b2 = {5, 6} , b3 = {3, 4} , b4 = {2, 5} , b5 = {1, 6}

Action of R and B on the blocks of the partitions: = [2, 3, 1, 3, 2] [5, 3, 4, 5, 3]
with invariant measure [1, 1, 2, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Sandwich	
Coloring	{2}
Rank	3
R,B	[3, 4, 5, 5, 1, 1], [2, 3, 6, 6, 4, 2]
π_2	[0, 1, 1, 2, 0, 1, 1, 0, 2, 0, 1, 1, 1, 1, 0]
u_2	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]

n_3	[0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]
u_3	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

3 . Coloring, {3}

R: [3, 3, 6, 5, 1, 1]
B: [2, 4, 5, 6, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 5	5 *vs* 5	2 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$$[y_1, 0, y_1, 0, y_1 - y_2, y_2]$$

$p = - s^2 + s^4$ $p = - s^2 + s^3$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_4, 0, y_3, y_2, y_1]$$

4 . Coloring, {4}

R: [3, 3, 5, 6, 1, 1]
B: [2, 4, 6, 5, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_2, 0, y_2, 0, y_2 - y_1, y_1]$$

$p = - s^2 + s^3 \quad p = - s^2 + s^4$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$[0, y_1 - y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = - s^3 + s^4$

5 . Coloring, {5}

R: [3, 3, 5, 5, 4, 1]
B: [2, 4, 6, 6, 1, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$[y_1, 0, y_1 - y_3 + y_2, y_3, y_2, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[y_4, y_3, 0, y_1, 0, y_2]$$

6 . Coloring, {6}

R: [3, 3, 5, 5, 1, 2]
B: [2, 4, 6, 6, 4, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[-y_1 + y_2, y_1, y_2, 0, y_2, 0]$$

$p = - s^2 + s^4$ $p = - s^2 + s^3$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$$[y_2, y_3, 0, -y_2 + y_3 + y_1, 0, y_1]$$

$p = - s + s^2 - s^3 + s^4$

7 . Coloring, {2, 3}

R: [3, 4, 6, 5, 1, 1]
B: [2, 3, 5, 6, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$$[y_2 + y_3, 0, y_2 + y_3 - y_1, y_1, y_2, y_3]$$

$p = s^3 - s^4$ $p' = - s^3 + s^4$

Omega Rank for B : cycles: {{2, 3, 4, 5, 6}} order: 5

See Matrix

$$[0, y_5, y_4, y_2, y_3, y_1]$$

8 . Coloring, {2, 4}

R: [3, 4, 5, 6, 1, 1]
B: [2, 3, 6, 5, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_3 + y_2, 0, y_3 + y_2 - y_1, y_1, y_3, y_2]$$

$p = - s^3 + s^5$ $p = - s^3 + s^4$

Omega Rank for B : cycles: {{4, 5}, {2, 3, 6}} order: 6

See Matrix

$$[0, -y_2 + 4 y_1 - y_3, y_2, y_1, y_1, y_3]$$

$p' = - s + s^4$ $p = s - s^4$

9 . Coloring, {2, 5}

R: [3, 4, 5, 5, 4, 1]
B: [2, 3, 6, 6, 1, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	3 *vs* 4	4 *vs* 4	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$[y_3, 0, y_3 - y_1 + y_2, y_1, y_2, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{2, 3, 6}} order: 3

See Matrix

$$[y_2, y_1, y_3, 0, 0, y_4]$$

10 . Coloring, {2, 6}

R: [3, 4, 5, 5, 1, 2]
B: [2, 3, 6, 6, 4, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[-y_1 + y_3, y_1, -y_2 + y_3, y_2, y_3, 0]$$

$p = - s^3 + s^5$ $p = - s^3 + s^4$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$[y_1, y_1 + y_3 + y_2 - y_4, y_3, y_2, 0, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

11 . Coloring, {3, 4}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$

R: [3, 3, 6, 6, 1, 1]
B: [2, 4, 5, 5, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	4 *vs* 4	4 *vs* 4	1 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$$[y_1, 0, y_1, 0, 0, y_1]$$

$p = - s + s^2$ $p = - s + s^3$

Omega Rank for B : cycles: {{4, 5}} order: 2

See Matrix

$$[0, y_1, 0, y_3, y_2, 0]$$

12 . Coloring, {3, 5}

R: [3, 3, 6, 5, 4, 1]
B: [2, 4, 5, 6, 1, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{4, 5}, {1, 3, 6}} order: 6

See Matrix

$$[-y_1 + 4 y_2 - y_3, 0, y_1, y_2, y_2, y_3]$$

$p' = s - s^4$ $p = s - s^4$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[y_1, y_1 + y_3, 0, y_1 + y_3 - y_2, y_2, y_3]$$

$p' = s^3 - s^4$ $p = s^3 - s^5$

13 . Coloring, {3, 6}

R: [3, 3, 6, 5, 1, 2]
B: [2, 4, 5, 6, 4, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 3, 6}} order: 3

See Matrix

$$[-y_1 + y_2 + y_3, y_1, y_2 + y_3, 0, y_2, y_3]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$$[y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

14 . Coloring, {4, 5}

R: [3, 3, 5, 6, 4, 1]
B: [2, 4, 6, 5, 1, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

$$[y_5, 0, y_4, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$[y_1 - y_2 + y_3 - y_4, y_1, 0, y_2, y_3, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

15 . Coloring, {4, 6}

R: [3, 3, 5, 6, 1, 2]
B: [2, 4, 6, 5, 4, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_1, -y_1 + y_3 + y_2, y_3 + y_2, 0, y_3, y_2]$$

$p' = s^3 - s^4$ $p = s^3 - s^5$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$[y_1 - y_4 + y_2 + y_3, y_1, 0, y_4, y_2, y_3]$$

$p = - s^4 + s^5$

16 . Coloring, {5, 6}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$

R: [3, 3, 5, 5, 4, 2]
B: [2, 4, 6, 6, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$[0, y_1, y_2, y_3, -y_1 + y_2 + y_3, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$$[y_3, y_2, 0, -y_3 + y_2 + y_1, 0, y_1]$$

$p = - s + s^2 - s^3 + s^4$

17 . Coloring, {2, 3, 4}

R: [3, 4, 6, 6, 1, 1]
B: [2, 3, 5, 5, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$$[y_2, 0, -y_1 + y_2, y_1, 0, y_2]$$

$p = s^2 - s^4$ $p' = s^2 - s^3$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$[0, y_4, y_3, y_2, y_1, 0]$$

18 . Coloring, {2, 3, 5}

$\Omega p(\Delta)=0$: $p = s + 3s^2 + 4s^3 + 4s^4$

R: [3, 4, 6, 5, 4, 1]
B: [2, 3, 5, 6, 1, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 5	4 *vs* 5	2 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{4, 5}, {1, 3, 6}} order: 6

See Matrix

$$[y_2, 0, y_2, 3\,y_2 - y_1, y_1, y_2]$$

$p = s - s^5$ $p' = s^2 - s^4$ $p' = -s + s^3$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$$[y_4 - y_3 + y_2 - y_1, y_4, y_3, 0, y_2, y_1]$$

$p = s^2 - s^3 + s^4 - s^5$

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

τ= 18 , r'= 1/2

R: [3, 4, 6, 5, 4, 1]
B: [2, 3, 5, 6, 1, 2]

Ranges

Action of R on ranges, [[6], [7], [6], [8], [2], [9], [8], [3], [2]]
Action of B on ranges, [[4], [5], [1], [7], [4], [9], [3], [5], [1]]

Cycles: R , {{4, 5}, {1, 3, 6}}, B , {{1, 2, 3, 5}}

β({1, 2}) = 1/9
β({1, 4}) = 1/9
β({1, 5}) = 1/9
β({2, 3}) = 1/9
β({2, 6}) = 1/9
β({3, 4}) = 1/9
β({3, 5}) = 1/9
β({4, 6}) = 1/9
β({5, 6}) = 1/9

Partitions
α([{1, 3, 6}, {2, 4, 5}]) = 1/1

b1 = {1, 3, 6} , b2 = {2, 4, 5}

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Right Group	
Coloring	{2, 3, 5}
Rank	2
R,B	[3, 4, 6, 5, 4, 1], [2, 3, 5, 6, 1, 2]
π_2	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]
u_2	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

19 . Coloring, {2, 3, 6}

$\Omega p(\Delta)=0$: $p = s$ $p' = s$ $p' = s^2$ $p' = s^3$

R: [3, 4, 6, 5, 1, 2]
B: [2, 3, 5, 6, 4, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
0 *vs* 4	1 *vs* 6	1 *vs* 6	1 *vs* 6	1 *vs* 6

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$[y_1, y_1, y_1, y_1, y_1, y_1]$

$p' = -1 + s^4$ $p' = -1 + s$ $p' = -1 + s^3$ $p' = -1 + s^5$ $p' = -1 + s^2$

Omega Rank for B : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$[y_1, y_1, y_1, y_1, y_1, y_1]$

$p' = 1 - s$ $p' = -s + s^5$ $p' = -s + s^4$ $p' = -s + s^3$ $p' = -s + s^2$

See 6-level graph

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

τ= 6 , r'= 5/6

R: [3, 4, 6, 5, 1, 2]
B: [2, 3, 5, 6, 4, 1]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R , {{1, 2, 3, 4, 5, 6}}, B , {{1, 2, 3, 4, 5, 6}}

β({1, 2, 3, 4, 5, 6}) = 1/1

Partitions
α([{2}, {1}, {5}, {6}, {3}, {4}]) = 1/1

b1 = {2} , b2 = {1} , b3 = {5} , b4 = {6} , b5 = {3} , b6 = {4}

Action of R and B on the blocks of the partitions: = [4, 3, 6, 5, 2, 1] [2, 4, 5, 6, 1, 3]
with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-6 partition graph.

Right Group	
Coloring	{2, 3, 6}
Rank	6
R,B	[3, 4, 6, 5, 1, 2], [2, 3, 5, 6, 4, 1]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 1)
wpp	[1, 1, 1, 1, 1, 1]

π_6	[1]
u_6	[1]

20 . Coloring, {2, 4, 5}

R: [3, 4, 5, 6, 4, 1]
B: [2, 3, 6, 5, 1, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

$$[y_3, 0, y_1, y_2, y_4, y_5]$$

Omega Rank for B : cycles: {{2, 3, 6}} order: 3

See Matrix

$$[y_2 + y_3 - y_1, y_2 + y_3, y_2, 0, y_3, y_1]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$

21 . Coloring, {2, 4, 6}

$\Omega p(\Delta)=0$: $p' = s^2$ $p' = s^3$ $p' = s$ $p = s$

R: [3, 4, 5, 6, 1, 2]
B: [2, 3, 6, 5, 4, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
0 *vs* 4	1 *vs* 6	1 *vs* 6	1 *vs* 6	1 *vs* 6

Omega Rank for R : cycles: {{1, 3, 5}, {2, 4, 6}} order: 3

See Matrix

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$p' = -s^4 + s^5$ $p' = 1 - s^4$ $p' = s - s^4$ $p' = s^2 - s^4$ $p' = s^3 - s^4$

Omega Rank for B : cycles: {{1, 2, 3, 6}, {4, 5}} order: 4

See Matrix

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$p' = -1 + s$ $p' = -1 + s^2$ $p' = -1 + s^3$ $p' = -1 + s^4$ $p' = -1 + s^5$

See 6-level graph

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$\tau = 6$, $r' = 5/6$

R: [3, 4, 5, 6, 1, 2]
B: [2, 3, 6, 5, 4, 1]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R , {{1, 3, 5}, {2, 4, 6}}, B , {{1, 2, 3, 6}, {4, 5}}

β({1, 2, 3, 4, 5, 6}) = 1/1

Partitions
α([{2}, {1}, {5}, {6}, {3}, {4}]) = 1/1

b1 = {2} , b2 = {1} , b3 = {5} , b4 = {6} , b5 = {3} , b6 = {4}

Action of R and B on the blocks of the partitions: = [4, 3, 5, 6, 2, 1] [2, 4, 6, 5, 1, 3]
with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-6 partition graph.

Right Group	
Coloring	{2, 4, 6}
Rank	6
R,B	[3, 4, 5, 6, 1, 2], [2, 3, 6, 5, 4, 1]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 2)
wpp	[1, 1, 1, 1, 1, 1]
π_6	[1]
u_6	[1]

22 . Coloring, {2, 5, 6}

R: [3, 4, 5, 5, 4, 2]
B: [2, 3, 6, 6, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{4, 5}} order: 2

See Matrix

$$[0, y_1, y_1, y_2, y_2, 0]$$

$p' = s^2 - s^3 \quad p = s^2 - s^4$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$[y_1 - y_3 + y_2, y_1, y_3, 0, 0, y_2]$$

$p = - s + s^2 - s^3 + s^4$

23 . Coloring, {3, 4, 5}

R: [3, 3, 6, 6, 4, 1]
B: [2, 4, 5, 5, 1, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$$[y_4, 0, y_3, y_2, 0, y_1]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$[y_1, y_2, 0, y_1, y_2, 0]$$

$p' = s - s^3$ $p = s - s^3$

24 . Coloring, {3, 4, 6}

R: [3, 3, 6, 6, 1, 2]
B: [2, 4, 5, 5, 4, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 3, 6}} order: 3

See Matrix

$$[-y_1 + y_2, y_1, y_2, 0, 0, y_2]$$

$p' = s^2 - s^3$ $p = s^2 - s^4$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$[y_1 - y_2 + y_3, y_1, 0, y_2, y_3, 0]$

$p = - s^3 + s^4$

25 . Coloring, {3, 5, 6}

R: [3, 3, 6, 5, 4, 2]
B: [2, 4, 5, 6, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 3, 6}, {4, 5}} order: 6

See Matrix

$[0, -y_3 + 4 y_2 - y_1, y_3, y_2, y_2, y_1]$

$p = - s + s^4$ $p' = - s + s^4$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$[y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4]$

$p = - s^2 + s^3 - s^4 + s^5$

26 . Coloring, {4, 5, 6}

Ωp(Δ)=0: $p' = s^2 + 2s^3$ $p = s^2 - 4s^4$

R: [3, 3, 5, 6, 4, 2]
B: [2, 4, 6, 5, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 3, 4, 5, 6}} order: 5

See Matrix

$[0, y_5, y_4, y_1, y_2, y_3]$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$[y_1, y_1 + y_4 - y_3 - y_2, 0, y_4, y_3, y_2]$

$p = s^2 - s^3 + s^4 - s^5$

27 . Coloring, {2, 3, 4, 5}

R: [3, 4, 6, 6, 4, 1]
B: [2, 3, 5, 5, 1, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 4	4 *vs* 4	4 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3

See Matrix

$[y_1, 0, y_2, y_4, 0, y_3]$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$[y_2, y_1, y_2, 0, y_1, 0]$

$p = s - s^3$ $p' = s - s^3$

28 . Coloring, {2, 3, 4, 6}

R: [3, 4, 6, 6, 1, 2]
B: [2, 3, 5, 5, 4, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[-y_1 + y_3, y_1, -y_2 + y_3, y_2, 0, y_3]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$[y_1 - y_2 - y_3 + y_4, y_1, y_2, y_3, y_4, 0]$$

$p = s^4 - s^5$

29 . Coloring, {2, 3, 5, 6}

R: [3, 4, 6, 5, 4, 2]
B: [2, 3, 5, 6, 1, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$[0, y_4, -y_4 + y_1 - y_2 + y_3, y_1, y_2, y_3]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$$[y_1 - y_3 + y_2 + y_4, y_1, y_3, 0, y_2, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

30 . Coloring, {2, 4, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 - 2s^3$

R: [3, 4, 5, 6, 4, 2]
B: [2, 3, 6, 5, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_1 - y_2, y_1 - y_3, y_1, y_2, y_3]$$

$p' = - s^3 + s^4$ $p = s^3 - s^4$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$[y_2, y_3, y_1, 0, y_2 - y_3 + y_1 - y_4, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

31 . Coloring, {3, 4, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2 + 4s^4$

R: [3, 3, 6, 6, 4, 2]
B: [2, 4, 5, 5, 1, 1]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 3, 6}} order: 3

See Matrix

$$[0, y_1, y_2, y_3, 0, y_4]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$[y_2, y_2 + y_1 - y_3, 0, y_1, y_3, 0]$$

$p = - s + s^{2} - s^{3} + s^{4}$

32 . Coloring, {2, 3, 4, 5, 6}

R: [3, 4, 6, 6, 4, 2]
B: [2, 3, 5, 5, 1, 1]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_4, y_3, y_2, 0, y_1]$$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$$[y_3, y_1, y_2, 0, y_3 - y_1 + y_2, 0]$$

$p = - s + s^{2} - s^{3} + s^{4}$

SUMMARY	
Graph Type	CC
ν(A)	2
ν(Δ)	2
π	[1, 1, 1, 1, 1, 1]

Dbly Stoch	true

SANDWICH		Total 2
No .	**Coloring**	**Rank**
1	{}	3
2	{2}	3

RT GROUPS		Total 3	
No .	**Coloring**	**Rank**	**Solv**
1	{2, 3, 5}	2	Not Solvable
2	{2, 4, 6}	6	["group", Not Solvable]
3	{2, 3, 6}	6	["group", Not Solvable]

CC Colorings		Total 1
No .	**Coloring**	**Sandwich,Rank**
1	{}	true, 3

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	**2^{n-1}**
22	0	24 , 27	7 , 6	5	32	32