New Graph

[4, 4, 4, 7, 7, 7, 1, 1, 1], [2, 9, 5, 8, 3, 8, 5, 6, 2]

π = [3, 2, 1, 3, 2, 1, 3, 2, 1]

POSSIBLE RANKS

1 x 18
2 x 9
3 x 6

BASE DETERMINANT 2151937075/68719476736, .3131480589e-1

NullSpace of Δ

{3, 7, 8}, {1, 2, 4, 5, 6, 9}

Range of Δ: $[-\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 - \lambda_7, \lambda_1, -\lambda_5 - \lambda_6, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7]$

1 . Coloring, {}

R: [4, 4, 4, 7, 7, 7, 1, 1, 1] **B**: [2, 9, 5, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:
‘[‘3‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , -3‘ (‘ - 1 + τ ‘)‘ ^{2} , 3‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , -3‘ (‘ - 1 + τ ‘)‘ ^{2} , 3‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , -3‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-15, -4, -1, -15, -4, -1, -15, -4, -1] . FixedPtCheck, [15, 4, 1, 15, 4, 1, 15, 4, 1]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[6, 0, 0, 6, 0, 0, 6, 0, 0] , [3, 1, 2, 3, 1, 2, 3, 1, 2] , [6, 3, 3, 6, 3, 3, 6, 3, 3] , [12, 7, 5, 12, 7, 5, 12, 7, 5] , [24, 15, 9, 24, 15, 9, 24, 15, 9] , [48, 31, 17, 48, 31, 17, 48, 31, 17] , [96, 63, 33, 96, 63, 33, 96, 63, 33]] $
$ [[0, 4, 2, 0, 4, 2, 0, 4, 2] , [3, 3, 0, 3, 3, 0, 3, 3, 0] , [6, 5, 1, 6, 5, 1, 6, 5, 1] , [12, 9, 3, 12, 9, 3, 12, 9, 3] , [24, 17, 7, 24, 17, 7, 24, 17, 7] , [48, 33, 15, 48, 33, 15, 48, 33, 15] , [96, 65, 31, 96, 65, 31, 96, 65, 31]] $
$ [[3, -2, -1, 3, -2, -1, 3, -2, -1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, -1, 1, 0, -1, 1, 0, -1, 1]] $

$$[-y_2 - y_1, y_1, y_2, -y_2 - y_1, y_1, y_2, -y_2 - y_1, y_1, y_2]$$

$p' = s^4 - 4s^6$ $p' = s^5 - 2s^6$ $p = s^2 - 32s^7$

S+ \; S- \; NM

$ [[10, 10, 4, 11, 7, 3, 12, 5, 4] , [13, 5, 1, 11, 10, 3, 9, 7, 7] , [13, 4, 3, 12, 14, 2, 8, 4, 6] , [10, 7, 4, 10, 4, 4, 13, 11, 3] , [9, 10, 8, 8, 5, 2, 16, 7, 1] , [10, 7, 4, 10, 4, 4, 13, 11, 3] , [13, 5, 3, 12, 11, 4, 8, 6, 4] , [11, 7, 2, 14, 7, 6, 8, 8, 3] , [10, 11, 4, 11, 4, 5, 12, 7, 2]] $ $ [[10, 7, 3, 14, 6, 4, 9, 9, 4] , [11, 7, 2, 14, 8, 5, 8, 7, 4] , [11, 5, 1, 15, 10, 6, 7, 7, 4] , [9, 9, 6, 9, 6, 2, 15, 7, 3] , [8, 8, 6, 10, 4, 3, 15, 10, 2] , [9, 9, 6, 9, 6, 2, 15, 7, 3] , [14, 6, 2, 10, 10, 5, 9, 6, 4] , [14, 7, 3, 9, 10, 3, 10, 5, 5] , [13, 8, 4, 9, 6, 3, 11, 8, 4]] $ $ [[36, 18, 9, 18, 16, 6, 18, 14, 9] , [27, 24, 10, 24, 12, 8, 21, 12, 6] , [27, 20, 12, 18, 12, 6, 27, 16, 6] , [18, 16, 6, 36, 20, 12, 18, 12, 6] , [24, 12, 6, 30, 24, 10, 18, 12, 8] , [18, 16, 6, 36, 20, 12, 18, 12, 6] , [18, 14, 9, 18, 12, 6, 36, 22, 9] , [21, 12, 8, 18, 12, 6, 33, 24, 10] , [27, 12, 6, 18, 16, 6, 27, 20, 12]] $
CmmCk *true, true, true*
$p' = s^2 - 16s^6$ $p' = s^3 - 8s^6$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 7	2 *vs* 7	2 *vs* 7	1 *vs* 3	2 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 1 . order: 3

$ [[6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1, 0, 0, y_1, 0, 0, y_1, 0, 0]$$

$p = -s + s^2$ $p = -s + s^3$

Omega Rank for B : cycles: {{2, 9}, {3, 5}, {6, 8}}, net cycles: 3 . order: 2

$ [[0, 4, 2, 0, 4, 2, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 2, 4] , [0, 4, 2, 0, 4, 2, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 2, 4] , [0, 4, 2, 0, 4, 2, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 2, 4]] $

$$[0, y_1, y_2, 0, y_1, y_2, 0, y_1, y_2]$$

$p' = s - s^3$ $p' = -s^3 + s^5$ $p = s - s^5$ $p' = s^2 - s^4$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[0, x_1, x_2, x_3, x_4, x_5, -32\,x_1 - 16\,x_2 - 8\,x_3 - 4\,x_4 - 2\,x_5]$
For A+2Δ : $[y_4, y_5, y_6, y_7, -3\,y_4 - y_5 - 3\,y_7 - 3\,y_1 - y_2, -3\,y_4 - y_6 - 3\,y_7 - 3\,y_1 - y_3, y_1, y_2, y_3]$
For A-2Δ : $[-3\,y_1 - 3\,y_7 - 3\,y_6 - y_2 - y_4, y_1 - y_3 + y_7 - y_5 + y_6, y_1, y_2, y_3, y_7, y_4, y_5, y_6]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2]$

rank of M is 9 , rank of N is 6

M \; N

$ [[0, 0, 0, 3, 0, 0, 3, 0, 0] , [0, 0, 0, 0, 2, 0, 0, 2, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 1] , [3, 0, 0, 0, 0, 0, 3, 0, 0] , [0, 2, 0, 0, 0, 0, 0, 2, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 1] , [3, 0, 0, 3, 0, 0, 0, 0, 0] , [0, 2, 0, 0, 2, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 1, 0, 0, 0]] $ $ [[0, 3, 3, 6, 4, 6, 6, 5, 3] , [3, 0, 2, 4, 6, 4, 5, 6, 6] , [3, 2, 0, 6, 6, 6, 3, 4, 6] , [6, 4, 6, 0, 2, 0, 6, 6, 6] , [4, 6, 6, 2, 0, 2, 6, 6, 4] , [6, 4, 6, 0, 2, 0, 6, 6, 6] , [6, 5, 3, 6, 6, 6, 0, 1, 3] , [5, 6, 4, 6, 6, 6, 1, 0, 2] , [3, 6, 6, 6, 4, 6, 3, 2, 0]] $

Check is ΩΔN zero? *true*, $\pi\Delta$= [3, -2, -1, 3, -2, -1, 3, -2, -1]

ker M, [0, 0, 0, 0, 0, 0, 0, 0, 0]
Range M, $[x_7, x_4, x_5, x_6, x_1, x_2, x_3, x_8, x_9]$

τ= 27 , r'= 2/3

Ranges

Action of R on ranges, [[1], [1], [1]]
Action of B on ranges, [[2], [3], [2]]
β({1, 4, 7}) = 1/2
β({2, 5, 8}) = 1/3
β({3, 6, 9}) = 1/6

ker N, $[\mu_2, \mu_3, -\mu_2 - \mu_3, -\mu_3 - \mu_1, \mu_3, \mu_1, \mu_2, \mu_3, -\mu_2 - \mu_3]$
Range of N
$[y_3 - y_5 + y_6, y_3 - y_1 + y_2 - y_4 + y_6, y_3, y_2, y_1, y_2, y_5, y_4, y_6]$

Partitions

Action of R on partitions, [[2], [2], [2]]
Action of B on partitions, [[3], [3], [1]]

α([{1, 8, 9}, {2, 3, 7}, {4, 5, 6}]) = 1/6
α([{1, 2, 3}, {4, 5, 6}, {7, 8, 9}]) = 1/2
α([{1, 5, 9}, {2, 4, 6}, {3, 7, 8}]) = 1/3
b1 = {1, 2, 3} ‘ , ‘ b2 = {1, 5, 9} ‘ , ‘ b3 = {1, 8, 9} ‘ , ‘ b4 = {2, 3, 7} ‘ , ‘ b5 = {2, 4, 6} ‘ , ‘ b6 = {3, 7, 8} ‘ , ‘ b7 = {4, 5, 6} ‘ , ‘ b8 = {7, 8, 9}

Action of R and B on the blocks of the partitions: $ [[0, 1, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 1] , [0, 1, 0, 0, 0, 0, 1, 0] , [1, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 2, 0] , [1, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 1, 0]] $ = $ [[0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0]] $ + $ [[0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0]] $
[‘8‘, ‘8‘, ‘8‘, ‘7‘, ‘1‘, ‘7‘, ‘1‘, ‘7‘], [‘2‘, ‘4‘, ‘5‘, ‘2‘, ‘3‘, ‘7‘, ‘6‘, ‘5‘] with invariant measure [3, 2, 1, 1, 2, 2, 4, 3]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Sandwich	
Coloring	{}
Rank	3
R,B	[4, 4, 4, 7, 7, 7, 1, 1, 1], [2, 9, 5, 8, 3, 8, 5, 6, 2]
π_2	[0, 0, 3, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0]
$\mathbf{u}_2$	[3, 3, 6, 4, 6, 6, 5, 3, 2, 4, 6, 4, 5, 6, 6, 6, 6, 6, 3, 4, 6, 2, 0, 6, 6, 6, 2, 6, 6, 4, 6, 6, 6, 1, 3, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3, 3, 3, 3]
π_3	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$\mathbf{u}_3$	[2, 1, 1, 1, 2, 2, 0, 3, 1, 3, 0, 0, 0, 0, 0, 6, 5, 3, 0, 4, 3, 3, 6, 5, 3, 0, 0, 0, 0, 2, 0, 0, 0, 2, 0, 0, 3, 4, 4, 0, 5, 6, 4, 3, 4, 4, 0, 2, 2, 2, 0, 3, 4, 6, 2, 3, 4, 4, 3, 4, 6, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 1, 3, 2, 2, 2, 0, 1, 1, 0, 1, 3, 2, 0]

2 . Coloring, {2}

R: [4, 9, 4, 7, 7, 7, 1, 1, 1]   **B**: [2, 4, 5, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘]‘

For τ=1/2, [-741, -228, -43, -645, -172, -43, -645, -172, -171] . FixedPtCheck, [741, 228, 43, 645, 172, 43, 645, 172, 171]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 0, 4, 0, 0, 6, 0, 2] , [4, 0, 2, 5, 1, 2, 2, 2, 0] , [4, 4, 3, 10, 4, 2, 8, 1, 0] , [9, 12, 4, 11, 5, 7, 16, 4, 4] , [24, 19, 11, 17, 12, 12, 23, 14, 12] , [49, 28, 20, 48, 30, 18, 41, 35, 19] , [95, 60, 34, 105, 67, 29, 96, 62, 28]] $ $ [[0, 4, 2, 2, 4, 2, 0, 4, 0] , [2, 4, 0, 1, 3, 0, 4, 2, 2] , [8, 4, 1, 2, 4, 2, 4, 7, 4] , [15, 4, 4, 13, 11, 1, 8, 12, 4] , [24, 13, 5, 31, 20, 4, 25, 18, 4] , [47, 36, 12, 48, 34, 14, 55, 29, 13] , [97, 68, 30, 87, 61, 35, 96, 66, 36]] $ $ [[3, -2, -1, 1, -2, -1, 3, -2, 1] , [1, -2, 1, 2, -1, 1, -1, 0, -1] , [-2, 0, 1, 4, 0, 0, 2, -3, -2] , [-3, 4, 0, -1, -3, 3, 4, -4, 0] , [0, 3, 3, -7, -4, 4, -1, -2, 4] , [1, -4, 4, 0, -2, 2, -7, 3, 3] , [-1, -4, 2, 9, 3, -3, 0, -2, -4]] $

$[y_6, y_5, y_4, y_3, y_2, y_1, y_6 + 2\,y_5 + y_3 - y_2 - y_1, -y_4 - y_6 - 2\,y_5 - y_3 + y_2 + y_1, -y_5 - y_2 - y_1 - y_6 - y_3]$

$p = s^3 - s^4 - 8s^7$

S+ \ ; S- \ ; NM

$ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 4	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$$[y_3, 0, 0, y_1, 0, 0, y_2, 0, y_4]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 1/54, 5/27, -4/27] , [1/2, -4/27, 1/54, -17/54] , [0, 1/54, 5/27, -4/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54]] $ x $ [[6, 0, 0, 4, 0, 0, 6, 0, 2] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] $

Omega Rank for B : cycles: {{3, 5}, {6, 8}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 2, 4, 2, 0, 4, 0] , [0, 0, 4, 4, 2, 4, 0, 4, 0] , [0, 0, 2, 0, 4, 4, 0, 8, 0] , [0, 0, 4, 0, 2, 8, 0, 4, 0] , [0, 0, 2, 0, 4, 4, 0, 8, 0] , [0, 0, 4, 0, 2, 8, 0, 4, 0]] $

$$[0, 2\,y_2 - y_3, y_1, 2\,y_1 - y_4, y_2, y_4, 0, y_3, 0]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$

Â» SYNC'D 15/512 , 0.02929687500

3 . Coloring, {3}

R: [4, 4, 5, 7, 7, 7, 1, 1, 1] **B**: [2, 9, 4, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-1470, -392, -114, -1425, -456, -95, -1482, -380, -98] . FixedPtCheck, [1470, 392, 114, 1425, 456, 95, 1482, 380, 98]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	2 *vs* 4	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$ [[6, 0, 0, 5, 1, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_2, 0, 0, y_2 - y_1, y_1, 0, y_2, 0, 0]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$

Omega Rank for B : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 1, 3, 2, 0, 4, 2] , [0, 2, 3, 2, 0, 4, 0, 3, 4] , [0, 4, 0, 3, 0, 3, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 6, 4] , [0, 4, 0, 0, 0, 6, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 6, 4] , [0, 4, 0, 0, 0, 6, 0, 6, 2]] $

$$[0, y_1 + y_2 + y_3 - y_5, y_1 + y_2 + y_3 - y_4, y_1, y_2, y_3, 0, y_4, y_5]$$

$p = -s^4 + s^6$ $p' = -s^4 + s^6$

Â» SYNC'D 15/4096 , 0.003662109375

4 . Coloring, {4}

R: [4, 4, 4, 8, 7, 7, 1, 1, 1] **B**: [2, 9, 5, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 4τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 4τ + τ 2 ‘)‘ , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 4τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-585, -156, -19, -570, -76, -114, -285, -456, -39] . FixedPtCheck, [585, 156, 19, 570, 76, 114, 285, 456, 39]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	7 *vs* 8	8 *vs* 8	2 *vs* 4	3 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$ [[6, 0, 0, 6, 0, 0, 3, 3, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[y_1, 0, 0, y_1, 0, 0, y_1 - y_2, y_2, 0]$$

p = s ^ 2 - s ^ 4 p' = s ^ 2 - s ^ 3

Omega Rank for B : cycles: {{6, 8}, {3, 5}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 4, 2, 0, 4, 2, 3, 1, 2] , [0, 2, 4, 0, 5, 1, 0, 2, 4] , [0, 4, 5, 0, 4, 2, 0, 1, 2] , [0, 2, 4, 0, 5, 1, 0, 2, 4] , [0, 4, 5, 0, 4, 2, 0, 1, 2] , [0, 2, 4, 0, 5, 1, 0, 2, 4] , [0, 4, 5, 0, 4, 2, 0, 1, 2]] $

$$[0, 2 y_1, 2 y_1 + y_3 - y_2, 0, y_1 + 2 y_3, y_1, y_2, y_3, 2 y_3]$$

p' = - s ^ 2 + s ^ 6 p = - s ^ 2 + s ^ 4 p' = - s ^ 2 + s ^ 4 p = - s ^ 2 + s ^ 6

Â» SYNC'D 243/131072 , 0.001853942871

5 . Coloring, {5}

R: [4, 4, 4, 7, 3, 7, 1, 1, 1] **B**: [2, 9, 5, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [255, 68, 57, 285, 76, 19, 247, 76, 17] . FixedPtCheck, [255, 68, 57, 285, 76, 19, 247, 76, 17]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 2, 6, 0, 0, 4, 0, 0] , [2, 1, 0, 4, 1, 2, 5, 1, 2] , [8, 4, 1, 3, 3, 3, 9, 2, 3] , [14, 5, 3, 13, 6, 6, 11, 10, 4] , [25, 14, 6, 22, 18, 6, 29, 13, 11] , [53, 28, 18, 45, 29, 19, 42, 36, 18] , [96, 57, 29, 99, 68, 28, 99, 64, 36]] $ $ [[0, 4, 0, 0, 4, 2, 2, 4, 2] , [4, 3, 2, 2, 3, 0, 1, 3, 0] , [4, 4, 3, 9, 5, 1, 3, 6, 1] , [10, 11, 5, 11, 10, 2, 13,

6, 4] , [23, 18, 10, 26, 14, 10, 19, 19, 5] , [43, 36, 14, 51, 35, 13, 54, 28, 14] , [96, 71, 35, 93, 60, 36, 93, 64, 28]] $ $ [[3, -2, 1, 3, -2, -1, 1, -2, -1] , [-1, -1, -1, 1, -1, 1, 2, -1, 1] , [2, 0, -1, -3, -1, 1, 3, -2, 1] , [2, -3, -1, 1, -2, 2, -1, 2, 0] , [1, -2, -2, -2, 2, -2, 5, -3, 3] , [5, -4, 2, -3, -3, 3, -6, 4, 2] , [0, -7, -3, 3, 4, -4, 3, 0, 4]] $

$$[-y_4 - 3\,y_2 - 3\,y_3 + y_6 - y_1, y_4 + 2\,y_2 + 2\,y_3 - y_6 - y_5, y_4, y_1, y_2, y_3, -y_4 - y_6, y_6, y_5]$$

$p = s^3 - s^4 - 4s^5 + 8s^7$

S+ \; S- \; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 4	2 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$$[y_4, 0, y_2, y_3, 0, 0, y_1, 0, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 1/54, 5/27, -4/27] , [1/2, -4/27, 1/54, -17/54] , [0, 1/54, 5/27, -4/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27]] $ x $ [[6, 0, 2, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0]] $

Omega Rank for B : cycles: {{6, 8}, {2, 9}, {5, 7}}, net cycles: 3 . order: 2

$ [[0, 4, 0, 0, 4, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 2, 4]] $

$$[0, y_1, 0, 0, y_1, y_2, y_2, y_1, y_2]$$

$p = -s + s^3$ $p' = -s + s^3$ $p = -s + s^5$ $p' = -s + s^5$

Â» SYNC’D 81/16384 , 0.004943847656

6 . Coloring, {6}

R: [4, 4, 4, 7, 7, 8, 1, 1, 1] **B**: [2, 9, 5, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [1485, 396, 95, 1482, 380, 114, 1425, 456, 99] . FixedPtCheck, [1485, 396, 95, 1482, 380, 114, 1425, 456, 99]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 8	8 *vs* 8	2 *vs* 4	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$ [[6, 0, 0, 6, 0, 0, 5, 1, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1 + y_2, 0, 0, y_1 + y_2, 0, 0, y_1, y_2, 0]$$

p = - s ^ 2 + s ^ 3 p = - s ^ 2 + s ^ 4

Omega Rank for B : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 0, 4, 2, 1, 3, 2] , [0, 2, 4, 0, 3, 3, 2, 0, 4] , [0, 4, 3, 0, 6, 0, 3, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2]] $

$$[0, y_2, y_2 - y_3 - y_4 + y_5, 0, y_2 - y_1 + y_5, y_1, y_3, y_4, y_5]$$

p = s ^ 4 - s ^ 6 p’ = - s ^ 4 + s ^ 6

Â» SYNC’D 1371/524288 , 0.002614974976

7 . Coloring, {7}

R: [4, 4, 4, 7, 7, 7, 5, 1, 1] **B**: [2, 9, 5, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘ - 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -27‘ (‘ - 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} ‘]‘

For τ=1/2, [-210, -56, -114, -285, -456, -19, -570, -76, -14] . FixedPtCheck, [210, 56, 114, 285, 456, 19, 570, 76, 14]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 0, 0, 6, 3, 0, 6, 0, 0] , [0, 5, 1, 3, 8, 4, 9, 2, 4] , [9, 12, 0, 6, 12, 6, 15, 9, 3] , [21, 20, 4, 21, 23, 7, 24, 20, 4] , [48, 39, 9, 45, 36, 12, 51, 36, 12] , [93, 68, 28, 96, 74, 28, 93, 71, 25] , [195, 138, 54, 189, 129, 57, 198, 132, 60]] $ $ [[3, 4, 2, 0, 1, 2, 0, 4, 2] , [12, 3, 3, 9, 0, 0, 3, 6, 0] , [15, 4, 8, 18, 4, 2, 9, 7, 5] , [27, 12, 12, 27, 9, 9, 24, 12, 12] , [48, 25, 23, 51, 28, 20, 45, 28, 20] , [99, 60, 36, 96, 54, 36, 99, 57, 39] , [189, 118, 74, 195, 127, 71, 186, 124, 68]] $ $ [[0, -2, -1, 3, 1, -1, 3, -2, -1] , [-6, 1, -1, -3, 4, 2, 3, -2, 2] , [-3, 4, -4, -6, 4, 2, 3, 1, -1] , [-3, 4, -4, -3, 7, -1, 0, 4, -4] , [0, 7, -7, -3, 4, -4, 3, 4, -4] , [-3, 4, -4, 0, 10, -4, -3, 7, -7] , [3, 10, -10, -3, 1, -7, 6, 4, -4]] $

$[-y_1 - y_5, y_3 + 2\,y_4 + y_5, -y_3 - y_4, -y_3 - 2\,y_4 - y_2 - y_5, y_1, y_2, y_3, y_4, y_5]$

p = s ^{3} - 16s ^{5} + 8s ^{6} + 32s ^{7}

S+ \ ; S- \ ; NM

$ [[37, 25, 12, 40, 24, 12, 36, 27, 15] , [42, 24, 5, 41, 32, 16, 29, 23, 16] , [42, 15, 8, 43, 46, 14, 26, 20, 14] , [34, 32, 16, 33, 17, 11, 48, 29, 8] , [29, 28, 27, 32, 17, 9, 52, 28, 6] , [34, 32, 16, 33, 17, 11, 48, 29, 8] , [43, 20, 9, 41, 36, 14, 30, 21, 14] , [43, 22, 8, 41, 25, 15, 33, 23, 18] , [38, 30, 13, 38, 14, 12, 40, 28, 15]] $ $ [[36, 29, 9, 40, 22, 12, 39, 27, 14] , [47, 18, 6, 41, 28, 16, 28, 23, 21] , [46, 13, 6, 43, 40, 14, 28, 20, 18] , [35, 28, 19, 33, 19, 11, 45, 29, 9] , [28, 32, 24, 32, 15, 9, 55, 28, 5] , [35, 28, 19, 33, 19, 11, 45, 29, 9] , [43, 20, 9, 41, 36, 14, 30, 21, 14] , [39, 24, 10, 41, 31, 15, 31, 23, 14] , [33, 36, 12, 38, 18, 12, 41, 28, 10]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

p’ = s ^{3} - 4s ^{4} + 8s ^{6}

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 4	3 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 0, 6, 3, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 + y_2 - y_3, 0, 0, y_1, y_2, 0, y_3, 0, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{3, 5}, {2, 9}, {6, 8}}, net cycles: 2 . order: 2

$ [[3, 4, 2, 0, 1, 2, 0, 4, 2] , [0, 5, 1, 0, 2, 4, 0, 2, 4] , [0, 4, 2, 0, 1, 2, 0, 4, 5] , [0, 5, 1, 0, 2, 4, 0, 2, 4] , [0, 4, 2, 0, 1, 2, 0, 4, 5] , [0, 5, 1, 0, 2, 4, 0, 2, 4] , [0, 4, 2, 0, 1, 2, 0, 4, 5]] $

$$[2 y_1 + y_2 - y_3, y_1 + 2 y_2, y_1, 0, y_2, 2 y_2, 0, 2 y_1, y_3]$$

$p = - s^2 + s^4$ p’ = $- s^2 + s^4$ $p = - s^2 + s^6$ p’ = $- s^2 + s^6$

Â» SYNC’D 675/262144 , 0.002574920654

8 . Coloring, {8}

R: [4, 4, 4, 7, 7, 7, 1, 6, 1] **B**: [2, 9, 5, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [735, 196, 57, 741, 228, 171, 855, 228, 49] . FixedPtCheck, [735, 196, 57, 741, 228, 171, 855, 228, 49]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[4, 0, 0, 6, 0, 2, 6, 0, 0] , [5, 2, 2, 2, 1, 0, 4, 0, 2] , [10, 1, 3, 9, 2, 0, 3, 6, 2] , [7, 4, 6, 14, 10, 6, 11, 7, 7] , [27, 18, 6, 17, 15, 7, 30, 12, 12] , [62, 25, 17, 51, 28, 12, 39, 40, 14] , [77, 52, 36, 104, 72, 40, 91, 65, 39]] $ $ [[2, 4, 2, 0, 4, 0, 0, 4, 2] , [1, 2, 0, 4, 3, 2, 2, 4, 0] , [2, 7, 1, 3, 6, 4, 9, 2, 2] , [17, 12, 2, 10, 6, 2, 13, 9, 1] , [21, 14, 10, 31, 17, 9, 18, 20, 4] , [34, 39, 15, 45, 36, 20, 57, 24, 18] , [115, 76, 28, 88, 56, 24, 101, 63, 25]] $ $ [[1, -2, -1, 3, -2, 1, 3, -2, -1] , [2, 0, 1, -1, -1, -1, 1, -2, 1] , [4, -3, 1, 3, -2, -2, -3, 2, 0] , [-5, -4, 2, 2, 2, 2, -1, -1, 3] , [3, 2, -2, -7, -1, -1, 6, -4, 4] , [14, -7, 1, 3, -4, -4, -9, 8, -2] , [-19, -12, 4, 8, 8, 8, -5, 1, 7]] $

$$[-y_4 - y_1 - 3\,y_2 + y_3, y_4 + 2\,y_2 - 2\,y_3 - y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$$

$p = s^2 - 6s^4 + 16s^7$

S+ \; S- \; NM

$ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 4	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$$[y_1, 0, 0, y_2, 0, y_3, y_4, 0, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 1/54, 5/27, -4/27] , [1/2, -4/27, 1/54, -17/54] , [0, 1/54, 5/27, -4/27]] $ x $ [[4, 0, 0, 6, 0, 2, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0]] $

Omega Rank for B : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 2, 0, 4, 0, 0, 4, 2] , [4, 4, 4, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 2, 0, 0, 0, 8]] $

$$[2\,y_1 - y_4, 2\,y_2 - y_3, y_1, 0, y_2, 0, 0, y_3, y_4]$$

$p = -s^3 + s^5 \quad p' = -s^3 + s^5$

Â» SYNC'D 9/256 , 0.03515625000

9 . Coloring, {9}

R: [4, 4, 4, 7, 7, 7, 1, 1, 2] **B**: [2, 9, 5, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ ² ‘)‘‘ (‘ 3 + τ ² ‘)‘ , -18‘ (‘ - 5 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² ‘]‘

For τ=1/2, [-247, -76, -17, -255, -68, -17, -255, -68, -19] . FixedPtCheck, [247, 76, 17, 255, 68, 17, 255, 68, 19]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	2 *vs* 4	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: 0 . order: 3

$ [[5, 1, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_2, y_1, 0, y_2 + y_1, 0, 0, y_2 + y_1, 0, 0]$$

$p = -s^2 + s^4 \quad p = -s^2 + s^3$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}, {6, 8}}, net cycles: 3 . order: 6

$ [[1, 3, 2, 0, 4, 2, 0, 4, 2] , [2, 1, 4, 0, 2, 4, 0, 2, 3] , [3, 2, 2, 0, 4, 2, 0, 4, 1] , [1, 3, 4, 0, 2, 4, 0, 2, 2] , [2, 1, 2, 0, 4, 2, 0, 4, 3] , [3, 2, 4, 0, 2, 4, 0, 2, 1] , [1, 3, 2, 0, 4, 2, 0, 4, 2]] $

$$[y_3, y_4, y_3 + y_4 - y_1 + y_2, 0, y_1, y_3 + y_4 - y_1 + y_2, 0, y_1, y_2]$$

$p' = s - s^3 - s^4 + s^6$ $p' = s^2 + s^3 - s^5 - s^6$ $p = s - s^7$

Â» SYNC'D 3885/1048576 , 0.003705024719

10 . Coloring, {2, 3}

R: [4, 9, 5, 7, 7, 7, 1, 1, 1] **B**: [2, 4, 4, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [3822, 1176, 258, 3225, 1032, 215, 3354, 860, 882] . FixedPtCheck, [3822, 1176, 258, 3225, 1032, 215, 3354, 860, 882]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 3, 1, 0, 6, 0, 2] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0]] $

$$[y_2, 0, 0, y_3, y_4, 0, y_1, 0, 2\, y_4]$$

$p = - s^2 + s^5$

Omega Rank for B : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[0, 4, 2, 3, 3, 2, 0, 4, 0] , [0, 0, 3, 6, 0, 4, 0, 5, 0] , [0, 0, 0, 3, 0, 5, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

$$[0, 4\, y_3, 3\, y_1, 3\, y_2, 3\, y_3, 3\, y_4, 0, 3\, y_5, 0]$$

$p = -s^4 + s^6$

Â» SYNC'D 15/256 , 0.05859375000

11 . Coloring, {2, 4}

R: [4, 9, 4, 8, 7, 7, 1, 1, 1] **B**: [2, 4, 5, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘ ² ‘ (‘ 3 + τ ² ‘)‘ , 18‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ² , 9‘ (‘ - 1 + τ ‘)‘ ³ ‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ² ‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘ , 9‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ³ ‘]‘

For τ=1/2, [-1521, -468, -43, -1290, -172, -258, -645, -1032, -351] . FixedPtCheck, [1521, 468, 43, 1290, 172, 258, 645, 1032, 351]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 4, 0, 0, 3, 3, 2] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0]] $

$$[2\,y_1, 0, 0, 2\,y_2, 0, 0, 3\,y_4, 2\,y_3, 2\,y_4]$$

$p = -s^2 + s^5$

Omega Rank for B : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 2, 4, 2, 3, 1, 0] , [0, 0, 4, 4, 5, 1, 2, 2, 0] , [0, 0, 5, 0, 6, 2, 4, 1, 0] , [0, 0, 6, 0, 9, 1, 0, 2, 0] , [0, 0, 9, 0, 6, 2, 0, 1, 0] , [0, 0, 6, 0, 9, 1, 0, 2, 0] , [0, 0, 9, 0, 6, 2, 0, 1, 0]] $

$$[0, y_1, -y_1 + 4\,y_4 - y_3 + y_2, -y_5 + y_4 + 4\,y_2, y_5, y_4, y_3, y_2, 0]$$

p = s 4 - s 6 p’ = - s 4 + s 6

Â» SYNC’D 175/8192 , 0.02136230469

12 . Coloring, {2, 5}

R: [4, 9, 4, 7, 3, 7, 1, 1, 1] **B**: [2, 4, 5, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-663, -204, -129, -645, -172, -43, -559, -172, -153] . FixedPtCheck, [663, 204, 129, 645, 172, 43, 559, 172, 153]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 2, 4, 0, 0, 4, 0, 2] , [3, 0, 0, 6, 1, 2, 4, 2, 0] , [6, 5, 1, 7, 4, 2, 11, 0, 0] , [11, 10, 4, 10, 4, 8, 13, 7, 5] , [25, 16, 4, 21, 15, 9, 30, 14, 10] , [54, 29, 15, 45, 30, 18, 47, 34, 16] , [97, 58, 30, 104, 66, 30, 97, 65, 29]] $ $ [[0, 4, 0, 2, 4, 2, 2, 4, 0] , [3, 4, 2, 0, 3, 0, 2, 2, 2] , [6, 3, 3, 5, 4, 2, 1, 8, 4] , [13, 6, 4, 14, 12, 0, 11, 9, 3] , [23, 16, 12, 27, 17, 7, 18, 18, 6] , [42, 35, 17, 51, 34, 14, 49, 30, 16] , [95, 70, 34, 88, 62, 34, 95, 63, 35]] $ $ [[3, -2, 1, 1, -2, -1, 1, -2, 1] , [0, -2, -1, 3, -1, 1, 1, 0, -1] , [0, 1, -1, 1, 0, 0, 5, -4, -2] , [-1, 2, 0, -2, -4, 4, 1, -1, 1] , [1, 0, -4, -3, -1, 1, 6, -2, 2] , [6, -3, -1, -3, -2, 2, -1, 2, 0] , [1, -6, -2, 8, 2, -2, 1, 1, -3]] $

$[y_4 + 2 y_5 - y_1 - 3 y_2 - 3 y_3 - 2 y_6, -y_4 - 2 y_5 + 2 y_2 + 2 y_3 + y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$

p = s 2 - 2s 4 - 16s 7
S+ \ ; S- \ ; NM
$ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12,

8, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[6, 0, 2, 4, 0, 0, 4, 0, 2] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0]] $

$$[y_1, 0, y_4, y_2, 0, 0, y_3, 0, y_4]$$

$p = - s^2 + s^5$

Omega Rank for B : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 2, 4, 2, 2, 4, 0] , [0, 0, 0, 4, 2, 4, 4, 4, 0] , [0, 0, 0, 0, 4, 4, 2, 8, 0] , [0, 0, 0, 0, 2, 8, 4, 4, 0] , [0, 0, 0, 0, 4, 4, 2, 8, 0] , [0, 0, 0, 0, 2, 8, 4, 4, 0]] $

$$[0, 2\,y_1 - y_4, 0, -y_2 + 2\,y_3, y_1, y_2, y_3, y_4, 0]$$

$p' = - s^3 + s^5$ $p = - s^3 + s^5$

Â» SYNC’D 9/256 , 0.03515625000

13 . Coloring, {2, 6}

R: [4, 9, 4, 7, 7, 8, 1, 1, 1] **B**: [2, 4, 5, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [3861, 1188, 215, 3354, 860, 258, 3225, 1032, 891] . FixedPtCheck, [3861, 1188, 215, 3354, 860, 258, 3225, 1032, 891]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 4, 0, 0, 5, 1, 2] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0]] $

$$[y_1, 0, 0, y_2, 0, 0, y_3, y_4, 2\, y_4]$$

$p = -s^2 + s^5$

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[1/4, -1/8, -1/8, 1/32, 5/64, -37/288, 43/576] , [0, 1/4, -1/8, -1/8, 1/32, 11/72, -37/288] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 1/4, -1/8, -1/8, -7/72, 11/72] , [0, 0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 0, 1/4, -2/9, 1/36] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 1/4, -1/8, 1/36, -7/72] , [1/4, -1/8, -1/8, 1/32, 5/64, -37/288, 43/576]] $ x $ [[0, 4, 2, 2, 4, 2, 1, 3, 0] , [0, 0, 4, 4, 3, 3, 2, 2, 0] , [0, 0, 3, 0, 6, 2, 3, 4, 0] , [0, 0, 6, 0, 6, 4, 2, 0, 0] , [0, 0, 6, 0, 8, 0, 4, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0]] $

Â» SYNC’D 495/8192 , 0.06042480469

14 . Coloring, {2, 7}

R: [4, 9, 4, 7, 7, 7, 5, 1, 1] **B**: [2, 4, 5, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-546, -168, -258, -645, -1032, -43, -1290, -172, -126] . FixedPtCheck, [546, 168, 258, 645, 1032, 43, 1290, 172, 126]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 0, 4, 3, 0, 6, 0, 2] , [2, 0, 0, 3, 6, 0, 7, 0, 0] , [0, 0, 0, 2, 7, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_2, 0, 0, y_3, y_1, 0, -y_2 + y_3 + y_1 + y_4, 0, y_4]$$

p = - s ^{4} + s ^{5}

Omega Rank for B : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[3, 4, 2, 2, 1, 2, 0, 4, 0] , [0, 3, 1, 4, 2, 4, 0, 4, 0] , [0, 0, 2, 3, 1, 4, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0]] $

$$[2\,y_1 - y_2 + 3\,y_3 - y_5, 3\,y_1 + 2\,y_3 - y_4, y_1, y_2, y_3, y_5, 0, y_4, 0]$$

p = s ^{4} - s ^{6} p’ = - s ^{4} + s ^{6}

Â» SYNC’D 1571/65536 , 0.02397155762

15 . Coloring, {2, 8}

R: [4, 9, 4, 7, 7, 7, 1, 6, 1] **B**: [2, 4, 5, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-637, -196, -43, -559, -172, -129, -645, -172, -147] . FixedPtCheck, [637, 196, 43, 559, 172, 129, 645, 172, 147]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

\+ \ ; - \ ; Δ

$ [[4, 0, 0, 4, 0, 2, 6, 0, 2] , [6, 1, 2, 4, 1, 0, 3, 1, 0] , [6, 2, 3, 11, 3, 1, 5, 4, 1] , [10, 9, 5, 15, 8, 4, 15, 4, 2] , [29, 20, 8, 22, 12, 4, 27, 13, 9] , [55, 26, 20, 49, 29, 13, 38, 38, 20] , [84, 53, 35, 113, 70, 38, 91, 66, 26]] $ $ [[2, 4, 2, 2, 4, 0, 0, 4, 0] , [0, 3, 0, 2, 3, 2, 3, 3, 2] , [6, 6, 1, 1, 5, 3, 7, 4, 3] , [14, 7, 3, 9, 8, 4, 9, 12, 6] , [19, 12, 8, 26, 20, 12, 21, 19, 7] , [41, 38, 12, 47, 35, 19, 58, 26, 12] , [108, 75, 29, 79, 58, 26, 101, 62, 38]] $ $ [[1, -2, -1, 1, -2, 1, 3, -2, 1] , [3, -1, 1, 1, -1, -1, 0, -1, -1] , [0, -2, 1, 5, -1, -1, -1, 0, -1] , [-2, 1, 1, 3, 0, 0, 3, -4, -2] , [5, 4, 0, -2, -4, -4, 3, -3, 1] , [7, -6, 4, 1, -3, -3, -10, 6, 4] , [-12, -11, 3, 17, 6, 6, -5, 2, -6]] $

$[-3 y_2 + y_6 - 2 y_5 - y_3 - y_1, 2 y_2 - 2 y_6 + y_5 + y_3, -y_3 - y_4, y_1, y_2, y_6, y_3, y_4, y_5]$

p = /1\ /1\ - || |-| || s || ^ || 3 || || - || |-| || s || ^ || 4 || \4/ \4/ || + || s || ^ || 5 || || + || 2 || s || ^ || 7 ||
S+ \ ; S- \ ; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	2 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[4, 0, 0, 4, 0, 2, 6, 0, 2] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0]] $

$$[y_1, 0, 0, y_2, 0, y_3, y_4, 0, y_3]$$

$p = - s^2 + s^5$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 2 . order: 4

$ [[2, 4, 2, 2, 4, 0, 0, 4, 0] , [4, 2, 4, 4, 2, 0, 0, 2, 0] , [2, 4, 2, 2, 4, 0, 0, 4, 0] , [4, 2, 4, 4, 2, 0, 0, 2, 0] , [2, 4, 2, 2, 4, 0, 0, 4, 0] , [4, 2, 4, 4, 2, 0, 0, 2, 0]] $

$$[y_1, y_2, y_1, y_1, y_2, 0, 0, y_2, 0]$$

$p' = - s + s^3$ $p' = - s^2 + s^4$ $p' = - s + s^5$ $p = s - s^3$

Â» SYNC'D 675/16384 , 0.04119873047

16 . Coloring, {2, 9}

R: [4, 9, 4, 7, 7, 7, 1, 1, 2] **B**: [2, 4, 5, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-399, -228, -25, -375, -100, -25, -375, -100, -171] . FixedPtCheck, [399, 228, 25, 375, 100, 25, 375, 100, 171]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[5, 1, 0, 4, 0, 0, 6, 0, 2] , [6, 2, 0, 5, 0, 0, 4, 0, 1] , [4, 1, 0, 6, 0, 0, 5, 0, 2] , [5, 2, 0, 4, 0, 0, 6, 0, 1] , [6, 1, 0, 5, 0, 0, 4, 0, 2]] $

$$[5\,y_1 - y_2 - y_4 + 5\,y_3, y_1, 0, y_2, 0, 0, y_4, 0, y_3]$$

$p = -s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{3, 5}, {6, 8}}, net cycles: 1 . order: 4

$$[[1, 3, 2, 2, 4, 2, 0, 4, 0] , [0, 1, 4, 3, 2, 4, 0, 4, 0] , [0, 0, 2, 1, 4, 4, 0, 7, 0] , [0, 0, 4, 0, 2, 7, 0, 5, 0] , [0, 0, 2, 0, 4, 5, 0, 7, 0] , [0, 0, 4, 0, 2, 7, 0, 5, 0] , [0, 0, 2, 0, 4, 5, 0, 7, 0]]$$

$$[3\,y_2 - y_3 - 4\,y_4 - y_5 + 3\,y_1, y_2, 2\,y_2 - 3\,y_4 + 2\,y_1, y_3, y_4, y_5, 0, y_1, 0]$$

$p' = -s^4 + s^6 \quad p = -s^4 + s^6$

Â» SYNC'D 1145/131072 , 0.008735656738

17 . Coloring, {3, 4}

R: [4, 4, 5, 8, 7, 7, 1, 1, 1] **B**: [2, 9, 4, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [495, 132, 19, 475, 76, 95, 247, 380, 33] . FixedPtCheck, [495, 132, 19, 475, 76, 95, 247, 380, 33]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 4 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	3 *vs* 5	4 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$$[[6, 0, 0, 5, 1, 0, 3, 3, 0] , [6, 0, 0, 6, 0, 0, 1, 5, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]]$$

$$[y_2 + y_3, 0, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$p = s^3 - s^5 \quad p' = s^3 - s^4$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {2, 9}, {6, 8}}, net cycles: 3 . order: 4

$ [[0, 4, 2, 1, 3, 2, 3, 1, 2] , [0, 2, 3, 2, 3, 1, 1, 2, 4] , [0, 4, 3, 3, 1, 2, 2, 1, 2] , [0, 2, 1, 3, 2, 1, 3, 2, 4] , [0, 4, 2, 1, 3, 2, 3, 1, 2] , [0, 2, 3, 2, 3, 1, 1, 2, 4] , [0, 4, 3, 3, 1, 2, 2, 1, 2] , [0, 2, 1, 3, 2, 1, 3, 2, 4]] $

$$[0, 2\,y_2, 2\,y_2 - y_3 + y_4, y_2 + 2\,y_4 - y_1, y_1, y_2, y_3, y_4, 2\,y_4]$$

$p' = -s^2 + s^6 \quad p' = -s^3 + s^7 \quad p = -s + s^5 \quad p' = -s + s^5$

Â» SYNC'D 155043/33554432 , 0.004620641470

18 . Coloring, {3, 5}

R: [4, 4, 5, 7, 3, 7, 1, 1, 1] **B**: [2, 9, 4, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ^2 ‘)‘ , 9‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ^2 ‘)‘ , -18‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-270, -72, -114, -285, -152, -19, -266, -76, -18] . FixedPtCheck, [270, 72, 114, 285, 152, 19, 266, 76, 18]

$\det(A + \tau\,\Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	3 *vs* 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[6, 0, 2, 5, 1, 0, 4, 0, 0] , [4, 0, 1, 6, 2, 0, 5, 0, 0] , [5, 0, 2, 4, 1, 0, 6, 0, 0] , [6, 0, 1, 5, 2, 0, 4, 0, 0] , [4, 0, 2, 6, 1, 0, 5, 0, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4, 0, y_1, y_2, y_3, 0, y_4, 0, 0]$$

$p = -s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{5, 7}, {2, 9}, {6, 8}}, net cycles: 2 . order: 2

$ [[0, 4, 0, 1, 3, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 3, 4] , [0, 4, 0, 0, 3, 3, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 3, 4] , [0, 4, 0, 0, 3, 3, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 3, 4] , [0, 4, 0, 0, 3, 3, 2, 4, 2]] $

$[0, 8\,y_1 + 8\,y_2 - 10\,y_3, 0, y_1, 5\,y_1 + 5\,y_2 - 6\,y_3, y_2, y_3, 6\,y_1 + 6\,y_2 - 7\,y_3, -2\,y_1 - 2\,y_2 + 4\,y_3]$

$p' = -s^2 + s^6 \quad p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6$

Â» SYNC'D 645/524288 , 0.001230239868

19 . Coloring, {3, 6}

R: [4, 4, 5, 7, 7, 8, 1, 1, 1] **B**: [2, 9, 4, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-255, -68, -19, -247, -76, -19, -247, -76, -17] . FixedPtCheck, [255, 68, 19, 247, 76, 19, 247, 76, 17]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 4

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	2 *vs* 5	4 *vs* 8

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 5, 1, 0, 5, 1, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$[y_1 + y_2, 0, 0, y_1, y_2, 0, y_1, y_2, 0]$

$p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^3$

Omega Rank for B : cycles: {{3, 4, 5, 6, 7, 8}, {2, 9}}, net cycles: 2 . order: 6

$ [[0, 4, 2, 1, 3, 2, 1, 3, 2] , [0, 2, 3, 2, 1, 3, 2, 1, 4] , [0, 4, 1, 3, 2, 1, 3, 2, 2] , [0, 2, 2, 1, 3, 2, 1, 3, 4] , [0, 4, 3, 2, 1, 3, 2, 1, 2] , [0, 2, 1, 3, 2, 1, 3, 2, 4] , [0, 4, 2, 1, 3, 2, 1, 3, 2] , [0, 2, 3, 2, 1, 3, 2, 1, 4]] $

$$[0, y_1 + y_2 + y_3 - y_4, y_1, y_2, y_3, y_1, y_2, y_3, y_4]$$

p’ = $s^3 + s^4 - s^6 - s^7$ p’ = $s^2 - s^4 - s^5 + s^7$ p’ = $s - s^7$ p = $s - s^7$

Â» SYNC’D 209385/33554432 , 0.006240159273

20 . Coloring, {3, 7}

R: [4, 4, 5, 7, 7, 7, 5, 1, 1] **B**: [2, 9, 4, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ^2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ^2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ^2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ - 5 + τ^2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ 3 + τ^2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ^2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ^2 ‘)‘‘]‘

For τ=1/2, [-255, -68, -171, -285, -684, -19, -741, -76, -17] . FixedPtCheck, [255, 68, 171, 285, 684, 19, 741, 76, 17]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 0, 0, 5, 4, 0, 6, 0, 0] , [0, 5, 0, 5, 6, 4, 9, 3, 4] , [10, 12, 2, 9, 9, 5, 15, 7, 3] , [19, 19, 7, 28, 17, 9, 23, 18, 4] , [47, 41, 15, 47, 30, 14, 54, 27, 13] , [82, 68, 34, 105, 69, 37, 91, 67, 23] , [191, 151, 59, 180, 125, 61, 211, 114, 60]] $ $ [[3, 4, 2, 1, 0, 2, 0, 4, 2] , [12, 3, 4, 7, 2, 0, 3, 5, 0] , [14, 4, 6, 15, 7, 3, 9, 9, 5] , [29, 13, 9, 20, 15, 7, 25, 14, 12] , [49, 23, 17, 49, 34, 18, 42, 37, 19] , [110, 60, 30, 87, 59, 27, 101, 61, 41] , [193, 105, 69, 204, 131, 67, 173, 142, 68]] $ $ [[0, -2, -1, 2, 2, -1, 3, -2, -1] , [-6, 1, -2, -1, 2, 2, 3, -1, 2] , [-2, 4, -2, -3, 1, 1, 3, -1, -1] , [-5, 3, -1, 4, 1, 1, -1, 2, -4] , [-1, 9, -1, -1, -2, -2, 6, -5, -3] , [-14, 4, 2, 9, 5, 5, -5, 3, -9] , [-1, 23, -5, -12, -3, -3, 19, -14, -4]] $

$$[-y_1 + y_2 - 3\,y_3 - y_4, -2\,y_2 + 2\,y_3 + y_4 - y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$$

$p = s^{3} - s^{4} + 4s^{5} - 8s^{7}$

S+ \; S- \; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	3 *vs* 4	4 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 0, 5, 4, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 + y_2 - y_3, 0, 0, y_1, y_2, 0, y_3, 0, 0]$$

$p = - s^{3} + s^{4}$

Omega Rank for B : cycles: {{6, 8}, {2, 9}}, net cycles: 0 . order: 4

$ [[3, 4, 2, 1, 0, 2, 0, 4, 2] , [0, 5, 0, 2, 0, 4, 0, 3, 4] , [0, 4, 0, 0, 0, 3, 0, 6, 5] , [0, 5, 0, 0, 0, 6, 0, 3, 4] , [0, 4, 0, 0, 0, 3, 0, 6, 5] , [0, 5, 0, 0, 0, 6, 0, 3, 4] , [0, 4, 0, 0, 0, 3, 0, 6, 5]] $

$$[9 y_1 - 6 y_2 - 6 y_3 - 3 y_4, 2 y_1, 6 y_1 - 4 y_2 - 4 y_3 - 2 y_4, 2 y_2, 0, 2 y_3, 0, 2 y_4, -5 y_1 + 4 y_2 + 4 y_3 + 3 y_4]$$

$p = - s^{3} + s^{5}$ $p' = - s^{3} + s^{5}$ $p = - s^{3} + s^{7}$

Â» SYNC’D 69/4096 , 0.01684570312

21 . Coloring, {3, 8}

R: [4, 4, 5, 7, 7, 7, 1, 6, 1] **B**: [2, 9, 4, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -27‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-3810, -1016, -342, -3705, -1368, -855, -4446, -1140, -254] . FixedPtCheck, [3810, 1016, 342, 3705, 1368, 855, 4446, 1140, 254]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[4, 0, 0, 5, 1, 2, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] $

$$[y_3, 0, 0, y_4, y_2, 2\,y_2, y_1, 0, 0]$$

p = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 6

$$[y_7, y_5, y_6, y_4, y_3, 0, 0, y_1, y_2]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 1/3, -2/9, 1/27, 1/2, -16/27] , [0, 0, 0, 1/3, -2/9, -5/9, 1/2] , [0, 1/3, -2/9, 1/27, -32/81, -16/27, 145/162] , [0, 0, 0, 1/3, -2/9, -5/9, 1/2] , [1/3, -2/9, 1/27, -32/81, 79/243, 145/162, -223/243] , [0, 0, 0, 0, 1/3, 5/18, -5/9] , [0, 0, 0, 0, 0, -2/9, 5/18]] $ x $ [[2, 4, 2, 1, 3, 0, 0, 4, 2] , [4, 4, 3, 2, 0, 0, 0, 1, 4] , [1, 8, 0, 3, 0, 0, 0, 2, 4] , [2, 5, 0, 0, 0, 0, 0, 3, 8] , [3, 10, 0, 0, 0, 0, 0, 0, 5] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC’D 15005/262144 , 0.05723953247

22 . Coloring, {3, 9}

R: [4, 4, 5, 7, 7, 7, 1, 1, 2] **B**: [2, 9, 4, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-1274, -392, -102, -1275, -408, -85, -1326, -340, -98] . FixedPtCheck, [1274, 392, 102, 1275, 408, 85, 1326, 340, 98]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	2 *vs* 5	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 5, 1, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_2, y_1, 0, y_2, y_1, 0, y_2 + y_1, 0, 0]$$

p’ = - s ^ 3 + s ^ 4 p’ = s ^ 2 - s ^ 3 p = s ^ 2 - s ^ 4

Omega Rank for B : cycles: {{1, 2, 9}, {6, 8}}, net cycles: 1 . order: 6

$ [[1, 3, 2, 1, 3, 2, 0, 4, 2] , [2, 1, 3, 2, 0, 4, 0, 3, 3] , [3, 2, 0, 3, 0, 3, 0, 6, 1] , [1, 3, 0, 0, 0, 6, 0, 6, 2] , [2, 1, 0, 0, 0, 6, 0, 6, 3] , [3, 2, 0, 0, 0, 6, 0, 6, 1] , [1, 3, 0, 0, 0, 6, 0, 6, 2] , [2, 1, 0, 0, 0, 6, 0, 6, 3]] $

$$[y_5, y_6, y_4, y_2, y_3, y_1, 0, y_2 + y_3 + y_1 - y_4, -y_5 - y_6 + y_2 + y_3 + y_1]$$

p = - s ^ 4 + s ^ 7 p’ = - s ^ 4 + s ^ 7

Â» SYNC’D 675/262144 , 0.002574920654

23 . Coloring, {4, 5}

R: [4, 4, 4, 8, 3, 7, 1, 1, 1] **B**: [2, 9, 5, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [555, 148, 57, 570, 76, 114, 247, 456, 37] . FixedPtCheck, [555, 148, 57, 570, 76, 114, 247, 456, 37]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	2 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 2, 6, 0, 0, 1, 3, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0]] $

$$[y_1, 0, 2\,y_3, y_2, 0, 0, y_3, y_4, 0]$$

p = - s ^{2} + s ^{5}

Omega Rank for B : cycles: {{2, 9}, {5, 7}, {6, 8}}, net cycles: 3 . order: 2

$ [[0, 4, 0, 0, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4]] $

$$[0, 2\,y_1, 0, 0, y_1 + 2\,y_2, y_1, 2\,y_1 + y_2, y_2, 2\,y_2]$$

p = - s + s ^{3} p’ = - s + s ^{3} p = - s + s ^{5} p’ = - s + s ^{5}

Â» SYNC’D 171/32768 , 0.005218505859

24 . Coloring, {4, 6}

R: [4, 4, 4, 8, 7, 8, 1, 1, 1] **B**: [2, 9, 5, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-765, -204, -19, -741, -76, -171, -285, -684, -51] . FixedPtCheck, [765, 204, 19, 741, 76, 171, 285, 684, 51]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 0, 6, 0, 0, 2, 4, 0] , [3, 1, 2, 3, 3, 0, 1, 3, 2] , [6, 3, 1, 6, 5, 1, 8, 3, 3] , [14, 7, 3, 10, 7, 5, 14, 7, 5] , [26, 13, 9, 24, 15, 9, 24, 15, 9] , [48, 29, 17, 48, 31, 17, 46, 33, 19] , [98, 61, 33, 94, 65, 31, 94, 65, 35]] $
$ [[0, 4, 2, 0, 4, 2, 4, 0, 2] , [3, 3, 0, 3, 1, 2, 5, 1, 0] , [6, 5, 3, 6, 3, 3, 4, 5, 1] , [10, 9, 5, 14, 9, 3, 10, 9, 3] , [22, 19, 7, 24, 17, 7, 24, 17, 7] , [48, 35, 15, 48, 33, 15, 50, 31, 13] , [94, 67, 31, 98, 63, 33, 98, 63, 29]] $
$ [[3, -2, -1, 3, -2, -1, -1, 2, -1] , [0, -1, 1, 0, 1, -1, -2, 1, 1] , [0, -1, -1, 0, 1, -1, 2, -1, 1] , [2, -1, -1, -2, -1, 1, 2, -1, 1] , [2, -3, 1, 0, -1, 1, 0, -1, 1] , [0, -3, 1, 0, -1, 1, -2, 1, 3] , [2, -3, 1, -2, 1, -1, -2, 1, 3]] $

$[y_2, y_3, y_4, y_5, y_6, -y_2 - y_3 - y_5 - y_6 - y_1, -3\,y_3 - 2\,y_4 - 2\,y_2 - 2\,y_5 - 3\,y_1, 3\,y_3 + y_4 + 2\,y_2 + 2\,y_5 + 3\,y_1, y_1]$

p = s 2 + 2s 4 - 16s 7

S+ \ ; S- \ ; NM

$ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	6 *vs* 7	7 *vs* 7	2 *vs* 4	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$ [[6, 0, 0, 6, 0, 0, 2, 4, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[y_1, 0, 0, y_1, 0, 0, y_1 - y_2, y_2, 0]$$

p = s ^ 2 - s ^ 4 p' = s ^ 2 - s ^ 3

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 0, 4, 2, 4, 0, 2] , [0, 2, 4, 0, 6, 0, 2, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4]] $

$$[0, y_1 + y_2 - y_4, -y_3 + y_1 + y_2, 0, y_1, y_2, y_3, 0, y_4]$$

p = s ^ 3 - s ^ 5 p' = s ^ 3 - s ^ 5

Â» SYNC'D 69/8192 , 0.008422851562

25 . Coloring, {4, 7}

R: [4, 4, 4, 8, 7, 7, 5, 1, 1] **B**: [2, 9, 5, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [255, 68, 57, 285, 228, 57, 285, 228, 17] . FixedPtCheck, [255, 68, 57, 285, 228, 57, 285, 228, 17]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 4

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

$$[3, 2, 1, 3, 2, 1, 3, 2, 1]$$

+ \ ; - \ ; Δ

$ [[3, 0, 0, 6, 3, 0, 3, 3, 0] , [6, 5, 1, 3, 5, 1, 3, 8, 4] , [21, 6, 3, 12, 6, 0, 15, 6, 3] , [9, 4, 5, 15, 10, 5, 9, 10, 5] , [15, 9, 3, 9, 6, 3, 12, 9, 6] , [27, 11, 10, 27, 17, 7, 24, 14, 7] , [45, 30, 15, 48, 30, 18, 45, 36, 21]] $ $ [[3, 4, 2, 0, 1, 2, 3, 1, 2] , [6, 3, 3, 9, 3, 3, 9, 0, 0] , [3, 10, 5, 12, 10, 8, 9, 10, 5] , [15, 12, 3, 9, 6, 3, 15, 6, 3] , [9, 7, 5, 15, 10, 5, 12, 7, 2] , [21, 21, 6, 21, 15, 9, 24, 18, 9] , [51, 34, 17, 48, 34, 14, 51, 28, 11]] $ $ [[0, -2, -1, 3, 1, -1, 0, 1, -1] , [0, 1, -1, -3, 1, -1, -3, 4, 2] , [9, -2, -1, 0, -2, -4, 3, -2, -1] , [-3, -4, 1, 3, 2, 1, -3, 2, 1] , [3, 1, -1, -3, -2, -1, 0, 1, 2] , [3, -5, 2, 3, 1, -1, 0, -2, -1] , [-3, -2, -1, 0, -2, 2, -3, 4, 5]] $

$$[-y_1 - y_2 - y_3 + y_4, y_1, -y_4 - y_5, y_2, y_1 + y_2 - y_3 - 2\,y_4 - y_5, y_3, y_4, y_5, -y_1 + y_4 + y_5 - y_2 + y_3]$$

$p = s - 4s^{3} - 4s^{4} + 4s^{5} + 8s^{6}$

S+ \; S- \; NM

$ [[17, 12, 3, 13, 8, 6, 12, 9, 4] , [15, 10, 4, 17, 5, 5, 11, 9, 8] , [16, 6, 8, 14, 12, 4, 13, 8, 3] , [13, 8, 5, 16, 8, 4, 14, 11, 5] , [10, 8, 8, 13, 14, 4, 18, 7, 2] , [10, 11, 4, 15, 10, 4, 16, 10, 4] , [12, 9, 5, 13, 13, 3, 16, 9, 4] , [17, 8, 4, 12, 7, 7, 13, 10, 6] , [16, 12, 1, 13, 7, 5, 13, 11, 6]] $ $ [[17, 12, 3, 13, 8, 6, 12, 9, 4] , [12, 13, 3, 16, 7, 5, 13, 8, 7] , [12, 10, 6, 13, 15, 5, 16, 7, 0] , [10, 11, 4, 15, 10, 4, 16, 10, 4] , [14, 4, 10, 14, 11, 3, 15, 8, 5] , [13, 8, 5, 16, 8, 4, 14, 11, 5] , [15, 6, 6, 14, 11, 3, 14, 10, 5] , [16, 9, 3, 12, 8, 8, 14, 10, 4] , [17, 11, 2, 13, 6, 4, 12, 11, 8]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

$p' = s - 4s^{3} - 4s^{4} + 4s^{5} + 8s^{6}$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
5 *vs* 7	9 *vs* 9	9 *vs* 9	3 *vs* 5	4 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 2 . order: 6

$ [[3, 0, 0, 6, 3, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 3, 6, 0] , [6, 0, 0, 3, 3, 0, 3, 3, 0] , [3, 0, 0, 6, 3, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 3, 6, 0]] $

$$[-y_1 + 4\,y_2 - y_3, 0, 0, y_1, y_2, 0, y_2, y_3, 0]$$

$p = s - s^{4}$ $p' = -s + s^{4}$

Omega Rank for B : cycles: {{6, 8}, {2, 9}, {3, 5}}, net cycles: 2 . order: 4

$ [[3, 4, 2, 0, 1, 2, 3, 1, 2] , [3, 5, 1, 0, 2, 1, 0, 2, 4] , [0, 7, 2, 0, 1, 2, 0, 1, 5] , [0, 5, 1, 0, 2, 1, 0, 2, 7] , [0, 7, 2, 0, 1, 2, 0, 1, 5] , [0, 5, 1, 0, 2, 1, 0, 2, 7] , [0, 7, 2, 0, 1, 2, 0, 1, 5] , [0, 5, 1, 0, 2, 1, 0, 2, 7]] $

$$[y_1 + 3\,y_3 - y_4, 3\,y_1 + y_3 - y_2, y_1, 0, y_3, y_1, y_2, y_3, y_4]$$

$p = -s^{3} + s^{5}$ $p' = -s^{3} + s^{5}$ $p = -s^{3} + s^{7}$ $p' = -s^{3} + s^{7}$

Â» SYNC’D 4725/2097152 , 0.002253055573

26 . Coloring, {4, 8}

R: [4, 4, 4, 8, 7, 7, 1, 6, 1] **B**: [2, 9, 5, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [1485, 396, 95, 1482, 380, 1026, 1425, 1368, 99] . FixedPtCheck, [1485, 396, 95, 1482, 380, 1026, 1425, 1368, 99]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 5	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: 1 . order: 5

$$[y_5, 0, 0, y_4, 0, y_3, y_2, y_1, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[385/1278, -155/1278, 79/1278, -65/1278, -173/1278] , [385/1278, -155/1278, 79/1278, -65/1278, -173/1278] , [385/1278, -155/1278, 79/1278, -65/1278, -173/1278] , [-173/1278, 385/1278, -155/1278, 79/1278, -65/1278] , [79/1278, -65/1278, -173/1278, 385/1278, -155/1278] , [79/1278, -65/1278, -173/1278, 385/1278, -155/1278] , [-155/1278, 79/1278, -65/1278, -173/1278, 385/1278] , [-65/1278, -173/1278, 385/1278, -155/1278, 79/1278] , [-155/1278, 79/1278, -65/1278, -173/1278, 385/1278]] $ x $ [[4, 0, 0, 6, 0, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 2, 6, 0] , [2, 0, 0, 3, 0, 6, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 6, 3, 0] , [6, 0, 0, 3, 0, 3, 4, 2, 0]] $

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 0 . order: 4

$ [[2, 4, 2, 0, 4, 0, 3, 1, 2] , [1, 4, 4, 0, 5, 0, 0, 0, 4] , [0, 5, 5, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 5, 0, 0, 0, 5] , [0, 5, 5, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 5, 0, 0, 0, 5] , [0, 5, 5, 0, 4, 0, 0, 0, 4]] $

$$[y_2 - y_4, y_1 + 2\,y_3, y_1, 0, y_2, 0, 3\,y_3, y_3, y_4]$$

$p = -s^3 + s^7 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^5$

Â» SYNC'D 725/65536 , 0.01106262207

27 . Coloring, {4, 9}

R: [4, 4, 4, 8, 7, 7, 1, 1, 2] **B**: [2, 9, 5, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ 3 + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ³ , -9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , -9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - 4τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ² ‘]‘

For τ=1/2, [507, 156, 17, 510, 68, 102, 255, 408, 39] . FixedPtCheck, [507, 156, 17, 510, 68, 102, 255, 408, 39]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ² ‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ⁴

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	2 *vs* 5	5 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 6, 0, 0, 3, 3, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[2\,y_1 + y_2, y_1, 0, 3\,y_1 + y_2, 0, 0, 3\,y_1, y_2, 0]$$

$p = s^2 - s^5 \quad p' = s^2 - s^4 \quad p' = s^3 - s^4$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}, {6, 8}}, net cycles: 2 . order: 6

$ [[1, 3, 2, 0, 4, 2, 3, 1, 2] , [2, 1, 4, 0, 5, 1, 0, 2, 3] , [3, 2, 5, 0, 4, 2, 0, 1, 1] , [1, 3, 4, 0, 5, 1, 0, 2, 2] , [2, 1, 5, 0, 4, 2, 0, 1, 3] , [3, 2, 4, 0, 5, 1, 0, 2, 1] , [1, 3, 5, 0, 4, 2, 0, 1, 2] , [2, 1, 4, 0, 5, 1, 0, 2, 3]] $

$$[-y_1 + 2\,y_3 - 2\,y_4 + 2\,y_5 - y_2, y_1, y_3, 0, 2\,y_3 - 3\,y_4 + 2\,y_5, y_4, y_5, y_3 - 2\,y_4 + y_5, y_2]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$

Â» SYNC'D 53265/8388608 , 0.006349682808

28 . Coloring, {5, 6}

R: [4, 4, 4, 7, 3, 8, 1, 1, 1] **B**: [2, 9, 5, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [-1335, -356, -285, -1482, -380, -114, -1235, -456, -89] . FixedPtCheck, [1335, 356, 285, 1482, 380, 114, 1235, 456, 89]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[6, 0, 2, 6, 0, 0, 3, 1, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0]] $

$$[y_1, 0, 2\,y_3, y_2, 0, 0, y_4, y_3, 0]$$

$p = s^2 - s^5$

Omega Rank for B : cycles: {{2, 9}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 0, 4, 2, 3, 3, 2] , [0, 2, 0, 0, 3, 3, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4]] $

$$[0, y_2 + y_3 - y_4, 0, 0, y_2 + y_3 - y_1, y_1, y_2, y_3, y_4]$$

$p = -s^3 + s^5 \quad p' = -s^3 + s^5$

Â» SYNC'D 99/4096 , 0.02416992188

29 . Coloring, {5, 7}

R: [4, 4, 4, 7, 3, 7, 5, 1, 1] **B**: [2, 9, 5, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -54‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-690, -184, -1026, -1425, -1368, -95, -1482, -380, -46] . FixedPtCheck, [690, 184, 1026, 1425, 1368, 95, 1482, 380, 46]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 2, 6, 3, 0, 4, 0, 0] , [0, 0, 3, 5, 4, 0, 6, 0, 0] , [0, 0, 4, 3, 6, 0, 5, 0, 0] , [0, 0, 6, 4, 5, 0, 3, 0, 0] , [0, 0, 5, 6, 3, 0, 4, 0, 0]] $

$$[-y_1 + y_2 + y_3 - y_4, 0, y_1, y_2, y_3, 0, y_4, 0, 0]$$

$p = -s^2 + s^3 - s^4 + s^5$

Omega Rank for B : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[3, 4, 0, 0, 1, 2, 2, 4, 2] , [2, 5, 0, 0, 0, 4, 1, 2, 4] , [1, 6, 0, 0, 0, 2, 0, 4, 5] , [0, 6, 0, 0, 0, 4, 0, 2, 6] , [0, 6, 0, 0, 0, 2, 0, 4, 6] , [0, 6, 0, 0, 0, 4, 0, 2, 6] , [0, 6, 0, 0, 0, 2, 0, 4, 6]] $

$$[y_2, y_3, 0, 0, -y_2 + y_3 + y_1 - y_4, y_3 + y_1 - y_5, y_1, y_5, y_4]$$

p' = - s ^{4} + s ^{6} p = - s ^{4} + s ^{6}

Â» SYNC'D 4815/524288 , 0.009183883667

30 . Coloring, {5, 8}

R: [4, 4, 4, 7, 3, 7, 1, 6, 1] **B**: [2, 9, 5, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [645, 172, 171, 741, 228, 171, 741, 228, 43] . FixedPtCheck, [645, 172, 171, 741, 228, 171, 741, 228, 43]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[4, 0, 2, 6, 0, 2, 4, 0, 0] , [4, 2, 0, 3, 1, 0, 6, 0, 2] , [12, 2, 1, 6, 2, 0, 6, 5, 2] , [11, 2, 2, 15, 9, 5, 12, 10, 6] , [24, 15, 9, 15, 18, 10, 27, 12, 14] , [61, 26, 18, 48, 28, 12, 39, 39, 17] , [81, 50, 28, 105, 71, 39, 96, 68, 38]] $ $ [[2, 4, 0, 0, 4, 0, 2, 4, 2] , [2, 2, 2, 3, 3, 2, 0, 4, 0] , [0, 6, 3, 6, 6, 4, 6, 3, 2] , [13, 14, 6, 9, 7, 3, 12, 6, 2] , [24, 17, 7, 33, 14, 6, 21, 20, 2] , [35, 38, 14, 48, 36, 20, 57, 25, 15] , [111, 78, 36, 87, 57, 25, 96, 60, 26]] $ $ [[1, -2, 1, 3, -2, 1, 1, -2, -1] , [1, 0, -1, 0, -1, -1, 3, -2, 1] , [6, -2, -1, 0, -2, -2, 0, 1, 0] , [-1, -6, -2, 3, 1, 1, 0, 2, 2] , [0, -1, 1, -9, 2, 2, 3, -4, 6] , [13, -6, 2, 0, -4, -4, -9, 7, 1] , [-15, -14, -4, 9, 7, 7, 0, 4, 6]] $

$$[y_4 + 2\,y_5 - y_1 - 3\,y_2 + y_3, -y_4 - 2\,y_5 + 2\,y_2 - 2\,y_3 - y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$$

p = s ^{3} - 3s ^{4} + 8s ^{7}
S+ \ ; S- \ ; NM
$ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15,

10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[4, 0, 2, 6, 0, 2, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0]] $

$$[y_1, 0, y_2, y_4, 0, y_2, y_3, 0, 0]$$

$p = -s^2 + s^5$

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 0, 0, 4, 0, 2, 4, 2] , [4, 4, 0, 0, 2, 0, 4, 0, 4] , [0, 8, 0, 0, 4, 0, 2, 0, 4] , [0, 4, 0, 0, 2, 0, 4, 0, 8] , [0, 8, 0, 0, 4, 0, 2, 0, 4] , [0, 4, 0, 0, 2, 0, 4, 0, 8]] $

$$[2 y_1 - y_4, 2 y_3 - y_2, 0, 0, y_3, 0, y_1, y_2, y_4]$$

$p = s^3 - s^5$ $p' = s^3 - s^5$

Â» SYNC'D 15/512 , 0.02929687500

31 . Coloring, {5, 9}

R: [4, 4, 4, 7, 3, 7, 1, 1, 2] **B**: [2, 9, 5, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [13, 4, 3, 15, 4, 1, 13, 4, 1] . FixedPtCheck, [13, 4, 3, 15, 4, 1, 13, 4, 1]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[5, 1, 2, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0]] $

$$[y_1, y_2, 2\,y_2, y_3, 0, 0, y_4, 0, 0]$$

p = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}, {6, 8}}, net cycles: 3 . order: 6

$ [[1, 3, 0, 0, 4, 2, 2, 4, 2] , [2, 1, 0, 0, 2, 4, 4, 2, 3] , [3, 2, 0, 0, 4, 2, 2, 4, 1] , [1, 3, 0, 0, 2, 4, 4, 2, 2] , [2, 1, 0, 0, 4, 2, 2, 4, 3] , [3, 2, 0, 0, 2, 4, 4, 2, 1] , [1, 3, 0, 0, 4, 2, 2, 4, 2]] $

$$[-y_1 + y_3 + y_2 - y_4, y_1, 0, 0, y_3, y_2, y_2, y_3, y_4]$$

p’ = s ^ 2 + s ^ 3 - s ^ 5 - s ^ 6 p’ = - s - s ^ 2 + s ^ 4 + s ^ 5 p = s - s ^ 7

Â» SYNC’D 30495/2097152 , 0.01454114914

32 . Coloring, {6, 7}

R: [4, 4, 4, 7, 7, 8, 5, 1, 1] **B**: [2, 9, 5, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [555, 148, 285, 741, 1140, 57, 1425, 228, 37] . FixedPtCheck, [555, 148, 285, 741, 1140, 57, 1425, 228, 37]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ ^{4} ‘ (‘ 1 + τ ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 5	6 *vs* 8

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$$[y_1, 0, 0, y_3, y_2, 0, y_4, y_5, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -2/9, -13/18] , [0, 0, 1, -2/9, -13/18] , [0, 0, 1, -2/9, -13/18] , [0, 0, 0, 5/18, -2/9] , [0, 0, 0, 5/18, -2/9] , [1, -3, 3, 25/9, -67/18] , [0, 0, 0, -2/9, 5/18] , [0, 1, -3, -13/18, 25/9] , [0, 1, -3, -13/18, 25/9]] $ x $ [[3, 0, 0, 6, 3, 0, 5, 1, 0] , [1, 0, 0, 3, 5, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 1 . order: 6

$ [[3, 4, 2, 0, 1, 2, 1, 3, 2] , [1, 5, 1, 0, 2, 3, 2, 0, 4] , [2, 5, 2, 0, 1, 0, 3, 0, 5] , [3, 7, 1, 0, 2, 0, 0, 0, 5] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8]] $

$$[2\,y_1 + 3\,y_2 - y_3 - y_6, 3\,y_1 + 2\,y_2 - y_5 - y_4, y_1, 0, y_2, y_3, y_5, y_4, y_6]$$

p = s ^{5} - s ^{7} p’ = s ^{5} - s ^{7}

Â» SYNC’D 128899/8388608 , 0.01536595821

33 . Coloring, {6, 8}

R: [4, 4, 4, 7, 7, 8, 1, 6, 1] **B**: [2, 9, 5, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [795, 212, 57, 798, 228, 342, 855, 456, 53] . FixedPtCheck, [795, 212, 57, 798, 228, 342, 855, 456, 53]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	4 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[4, 0, 0, 6, 0, 2, 5, 1, 0] , [5, 0, 0, 4, 0, 1, 6, 2, 0] , [6, 0, 0, 5, 0, 2, 4, 1, 0] , [4, 0, 0, 6, 0, 1, 5, 2, 0] , [5, 0, 0, 4, 0, 2, 6, 1, 0]] $

$$[-y_1 + 5 y_2 - y_3 + 5 y_4, 0, 0, y_1, 0, y_2, y_3, y_4, 0]$$

$p = - s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 0 . order: 4

$ [[2, 4, 2, 0, 4, 0, 1, 3, 2] , [3, 4, 4, 0, 3, 0, 0, 0, 4] , [0, 7, 3, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 3, 0, 0, 0, 7] , [0, 7, 3, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 3, 0, 0, 0, 7] , [0, 7, 3, 0, 4, 0, 0, 0, 4]] $

$$[-16 y_1 + 33 y_2 - 48 y_3 - 5 y_4, 5 y_1, -7 y_1 + 16 y_2 - 26 y_3, 0, 5 y_2, 0, 5 y_3, 15 y_3, 5 y_4]$$

$p = - s^3 + s^5$ p’ $= - s^3 + s^5$ $p = - s^3 + s^7$

Â» SYNC’D 957/65536 , 0.01460266113

34 . Coloring, {6, 9}

R: [4, 4, 4, 7, 7, 8, 1, 1, 2] **B**: [2, 9, 5, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-1287, -396, -85, -1326, -340, -102, -1275, -408, -99] . FixedPtCheck, [1287, 396, 85, 1326, 340, 102, 1275, 408, 99]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ 4 ‘ (‘ 1 + τ ‘)‘‘ (‘ τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	2 *vs* 5	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 6, 0, 0, 5, 1, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1, y_2, 0, y_1 + y_2, 0, 0, y_1, y_2, 0]$$

p = - s 2 + s 3 p = - s 2 + s 4 p = - s 2 + s 5

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 1 . order: 6

$ [[1, 3, 2, 0, 4, 2, 1, 3, 2] , [2, 1, 4, 0, 3, 3, 2, 0, 3] , [3, 2, 3, 0, 6, 0, 3, 0, 1] , [1, 3, 6, 0, 6, 0, 0, 0, 2] , [2, 1, 6, 0, 6, 0, 0, 0, 3] , [3, 2, 6, 0, 6, 0, 0, 0, 1] , [1, 3, 6, 0, 6, 0, 0, 0, 2] , [2, 1, 6, 0, 6, 0, 0, 0, 3]] $

$$[y_3 - y_2 - y_6 + y_4 + y_5, y_2, y_3, 0, y_3 + y_4 + y_5 - y_1, y_1, y_4, y_5, y_6]$$

p = - s 4 + s 7 p’ = - s 4 + s 7

Â» SYNC’D 213555/33554432 , 0.006364434958

35 . Coloring, {7, 8}

R: [4, 4, 4, 7, 7, 7, 5, 6, 1] **B**: [2, 9, 5, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [510, 136, 342, 741, 1368, 171, 1710, 228, 34] . FixedPtCheck, [510, 136, 342, 741, 1368, 171, 1710, 228, 34]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	4 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 0, 6, 3, 2, 6, 0, 0] , [0, 0, 0, 1, 6, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

$$[y_1, 0, 0, y_2, y_3, 2\,y_1, y_4, 0, 0]$$

$p = -s^3 + s^5$

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 1 . order: 4

$ [[5, 4, 2, 0, 1, 0, 0, 4, 2] , [4, 7, 1, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8]] $

$$[2\,y_1 + 3\,y_2 - y_4, 3\,y_1 + 2\,y_2 - y_3, y_1, 0, y_2, 0, 0, y_3, y_4]$$

$p' = -s^3 + s^5$ $p = -s^3 + s^5$

Â» SYNC’D 705/16384 , 0.04302978516

36 . Coloring, {7, 9}

R: [4, 4, 4, 7, 7, 7, 5, 1, 2] **B**: [2, 9, 5, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 + 3τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ - 5 + 3τ ‘)‘‘]‘

For τ=1/2, [182, 56, 102, 255, 408, 17, 510, 68, 14] . FixedPtCheck, [182, 56, 102, 255, 408, 17, 510, 68, 14]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	3 *vs* 5	4 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[2, 1, 0, 6, 3, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[2\, y_1, y_1, 0, 3\, y_1 - y_3 + y_2, y_3, 0, y_2, 0, 0]$$

$p = s^3 - s^4$ $p' = -s^3 + s^4$

Omega Rank for B : cycles: {{1, 2, 9}, {6, 8}, {3, 5}}, net cycles: 3 . order: 6

$ [[4, 3, 2, 0, 1, 2, 0, 4, 2] , [2, 4, 1, 0, 2, 4, 0, 2, 3] , [3, 2, 2, 0, 1, 2, 0, 4, 4] , [4, 3, 1, 0, 2, 4, 0, 2, 2] , [2, 4, 2, 0, 1, 2, 0, 4, 3] , [3, 2, 1, 0, 2, 4, 0, 2, 4] , [4, 3, 2, 0, 1, 2, 0, 4, 2]] $

$$[y_1, -y_1 + 3\, y_3 + 3\, y_2 - y_4, y_3, 0, y_2, 2\, y_2, 0, 2\, y_3, y_4]$$

$p = -s + s^7$ $p = -s - s^2 + s^4 + s^5$ $p = s - s^3 - s^4 + s^6$

Â» SYNC'D 6591/524288 , 0.01257133484

37 . Coloring, {8, 9}

R: [4, 4, 4, 7, 7, 7, 1, 6, 2] **B**: [2, 9, 5, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘[‘9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-637, -196, -51, -663, -204, -153, -765, -204, -49] . FixedPtCheck, [637, 196, 51, 663, 204, 153, 765, 204, 49]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -1 . order: 3

$ [[3, 1, 0, 6, 0, 2, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] $

$$[y_1, y_2, 0, y_3, 0, 2\, y_2, y_4, 0, 0]$$

$p = - s^2 + s^5$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 1 . order: 6

$ [[3, 3, 2, 0, 4, 0, 0, 4, 2] , [6, 3, 4, 0, 2, 0, 0, 0, 3] , [3, 6, 2, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 2, 0, 0, 0, 6] , [6, 3, 2, 0, 4, 0, 0, 0, 3] , [3, 6, 4, 0, 2, 0, 0, 0, 3]] $

$$[-y_1 + 2\, y_2 + 2\, y_3 - y_4 - y_5, y_1, y_2, 0, y_3, 0, 0, y_4, y_5]$$

$p = - s^2 - s^3 + s^5 + s^6$

Â» SYNC’D 2267/32768 , 0.06918334961

38 . Coloring, {2, 3, 4}

R: [4, 9, 5, 8, 7, 7, 1, 1, 1] **B**: [2, 4, 4, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-1287, -396, -43, -1075, -172, -215, -559, -860, -297] . FixedPtCheck, [1287, 396, 43, 1075, 172, 215, 559, 860, 297]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 3, 1, 0, 3, 3, 2] , [8, 0, 0, 6, 0, 0, 1, 3, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0]] $

$$[y_1, 0, 0, y_2, y_4, 0, y_3, y_5, 2\, y_4]$$

p = - s 3 + s 6

Omega Rank for B : cycles: {{6, 8}, {3, 4, 5, 7}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 3, 3, 2, 3, 1, 0] , [0, 0, 3, 6, 3, 1, 3, 2, 0] , [0, 0, 3, 3, 3, 2, 6, 1, 0] , [0, 0, 3, 3, 6, 1, 3, 2, 0] , [0, 0, 6, 3, 3, 2, 3, 1, 0] , [0, 0, 3, 6, 3, 1, 3, 2, 0] , [0, 0, 3, 3, 3, 2, 6, 1, 0]] $

$$[0, -y_1 + 4\, y_3 - y_4 + y_5, y_1, -y_2 + y_3 + 4\, y_5, y_2, y_3, y_4, y_5, 0]$$

p’ = s 2 - s 6 p = - s 2 + s 6

Â» SYNC’D 16875/524288 , 0.03218650818

39 . Coloring, {2, 3, 5}

R: [4, 9, 5, 7, 3, 7, 1, 1, 1] **B**: [2, 4, 4, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ ‘)‘‘]‘

For τ=1/2, [702, 216, 258, 645, 344, 43, 602, 172, 162] . FixedPtCheck, [702, 216, 258, 645, 344, 43, 602, 172, 162]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[6, 0, 2, 3, 1, 0, 4, 0, 2] , [6, 0, 1, 6, 2, 0, 3, 0, 0] , [3, 0, 2, 6, 1, 0, 6, 0, 0] , [6, 0, 1, 3, 2, 0, 6, 0, 0] , [6, 0, 2, 6, 1, 0, 3, 0, 0] , [3, 0, 1, 6, 2, 0, 6, 0, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - y_5, 0, y_1, y_2, y_3, 0, y_4, 0, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{5, 7}, {6, 8}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 3, 3, 2, 2, 4, 0] , [0, 0, 0, 4, 2, 4, 3, 5, 0] , [0, 0, 0, 0, 3, 5, 2, 8, 0] , [0, 0, 0, 0, 2, 8, 3, 5, 0] , [0, 0, 0, 0, 3, 5, 2, 8, 0] , [0, 0, 0, 0, 2, 8, 3, 5, 0]] $

$$[0, -14\,y_1 - 14\,y_4 + 39\,y_2 - y_3, 0, y_1, -5\,y_1 - 5\,y_4 + 14\,y_2, y_4, y_2, y_3, 0]$$

$p = -s^3 + s^5$ p’ $= -s^3 + s^5$

Â» SYNC’D 51/4096 , 0.01245117188

40 . Coloring, {2, 3, 6}

R: [4, 9, 5, 7, 7, 8, 1, 1, 1] **B**: [2, 4, 4, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [663, 204, 43, 559, 172, 43, 559, 172, 153] . FixedPtCheck, [663, 204, 43, 559, 172, 43, 559, 172, 153]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[6, 0, 0, 3, 1, 0, 5, 1, 2] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0]] $

$$[y_1, 0, 0, y_2, y_4, 0, y_3, y_4, 2\, y_4]$$

p = - s ^ 2 + s ^ 5 p' = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{3, 4, 5, 6, 7, 8}}, net cycles: 0 . order: 6

$$[0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[1/4, -47/756, -29/756, -19/378, 61/756, -83/756, -11/756] , [0, 89/378, -47/756, -29/756, -19/378, 61/756, -83/756] , [0, 89/378, -47/756, -29/756, -19/378, 61/756, -83/756] , [0, -83/756, 89/378, -47/756, -29/756, -19/378, 61/756] , [0, -47/756, -29/756, -19/378, 61/756, -83/756, 89/378] , [0, -19/378, 61/756, -83/756, 89/378, -47/756, -29/756] , [0, -29/756, -19/378, 61/756, -83/756, 89/378, -47/756] , [0, 61/756, -83/756, 89/378, -47/756, -29/756, -19/378] , [1/4, -47/756, -29/756, -19/378, 61/756, -83/756, -11/756]] $ x $ [[0, 4, 2, 3, 3, 2, 1, 3, 0] , [0, 0, 3, 6, 1, 3, 2, 3, 0] , [0, 0, 1, 3, 2, 3, 3, 6, 0] , [0, 0, 2, 1, 3, 6, 3, 3, 0] , [0, 0, 3, 2, 3, 3, 6, 1, 0] , [0, 0, 3, 3, 6, 1, 3, 2, 0] , [0, 0, 6, 3, 3, 2, 1, 3, 0]] $

Â» SYNC'D 82215/2097152 , 0.03920316696

41 . Coloring, {2, 3, 7}

R: [4, 9, 5, 7, 7, 7, 5, 1, 1] **B**: [2, 4, 4, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [663, 204, 387, 645, 1548, 43, 1677, 172, 153] . FixedPtCheck, [663, 204, 387, 645, 1548, 43, 1677, 172, 153]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[3, 0, 0, 3, 4, 0, 6, 0, 2] , [2, 3, 0, 9, 6, 4, 7, 5, 0] , [10, 14, 2, 11, 7, 3, 19, 3, 3] , [11, 19, 9, 18, 21, 13, 21, 18, 14] , [59, 39, 11, 31, 30, 14, 52, 33, 19] , [96, 50, 34, 105, 63, 31, 75, 83, 39] , [239, 121, 65, 204, 109, 45, 199, 120, 50]] $ $ [[3, 4, 2, 3, 0, 2, 0, 4, 0] , [10, 5, 4, 3, 2, 0, 5, 3, 4] , [14, 2, 6, 13, 9, 5, 5, 13, 5] , [37, 13, 7, 30, 11, 3, 27, 14, 2] , [37, 25, 21, 65, 34, 18, 44, 31, 13] , [96, 78, 30, 87, 65, 33, 117, 45, 25] , [145, 135, 63, 180, 147, 83, 185, 136, 78]] $ $ [[0, -2, -1, 0, 2, -1, 3, -2, 1] , [-4, -1, -2, 3, 2, 2, 1, 1, -2] , [-2, 6, -2, -1, -1, -1, 7, -5, -1] , [-13, 3, 1, -6, 5, 5, -3, 2, 6] , [11, 7, -5, -17, -2, -2, 4, 1, 3] , [0, -14, 2, 9, -1, -1, -21, 19, 7] , [47, -7, 1, 12, -19, -19, 7, -8, -14]] $

$[y_2, y_3, y_4, y_5, y_6, y_1, 2 y_3 + 3 y_6 - y_1 + y_2 + y_5, -2 y_3 - 3 y_6 + y_1 - y_4 - y_2 - y_5, -y_2 - y_5 - y_6 - y_1 - y_3]$

$p = s^2 + 6s^4 + 16s^7$

S+ \; S- \; NM

$ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 0, 3, 4, 0, 6, 0, 2] , [2, 0, 0, 3, 6, 0, 7, 0, 0] , [0, 0, 0, 2, 7, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$[y_4 + y_3 - y_2 + y_1, 0, 0, y_4, y_3, 0, y_2, 0, y_1]$

$p = s^4 - s^5$

Omega Rank for B : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[3, 4, 2, 3, 0, 2, 0, 4, 0] , [0, 3, 0, 6, 0, 4, 0, 5, 0] , [0, 0, 0, 3, 0, 5, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

$$[3\,y_2, 2\,y_1, 2\,y_2, 2\,y_3, 0, 2\,y_4, 0, 2\,y_5, 0]$$

$p = -s^4 + s^6$

Â» SYNC'D 551/8192 , 0.06726074219

42 . Coloring, {2, 3, 8}

R: [4, 9, 5, 7, 7, 7, 1, 6, 1] **B**: [2, 4, 4, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-27‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 54‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 27‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-3302, -1016, -258, -2795, -1032, -645, -3354, -860, -762] . FixedPtCheck, [3302, 1016, 258, 2795, 1032, 645, 3354, 860, 762]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[4, 0, 0, 3, 1, 2, 6, 0, 2] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0]] $

$$[y_1, 0, 0, y_4, y_2, 2\,y_2, y_3, 0, 2\,y_2]$$

$p = -s^2 + s^5 \quad p' = -s^2 + s^5$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_6, y_3, y_4, y_5, y_2, 0, 0, y_1, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 1/3, 1/72, -17/72, 19/72, -23/72] , [0, 0, 19/72, 1/72, 1/72, -17/72] , [1/3, -2/9, -17/72, 19/72, -23/72, 17/72] , [0, 0, -17/72, 19/72, 1/72, 1/72] , [0, 0, 1/72, -17/72, 19/72, 1/72]] $ x $ [[2, 4, 2, 3, 3, 0, 0, 4, 0] , [4, 2, 3, 6, 0, 0, 0, 3, 0] , [3, 4, 0, 5, 0, 0, 0, 6, 0] , [6, 3, 0, 4, 0, 0, 0, 5, 0] , [5, 6, 0, 3, 0, 0, 0, 4, 0] , [4, 5, 0, 6, 0, 0, 0, 3, 0]] $

Â» SYNC'D 3645/131072 , 0.02780914307

43 . Coloring, {2, 3, 9}

R: [4, 9, 5, 7, 7, 7, 1, 1, 2] **B**: [2, 4, 4, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-2058, -1176, -150, -1875, -600, -125, -1950, -500, -882] . FixedPtCheck, [2058, 1176, 150, 1875, 600, 125, 1950, 500, 882]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

$$[3, 2, 1, 3, 2, 1, 3, 2, 1]$$

\+ \ ; - \ ; Δ

$ [[5, 1, 0, 3, 1, 0, 6, 0, 2] , [6, 3, 3, 10, 0, 4, 4, 5, 1] , [12, 7, 8, 12, 11, 3, 14, 2, 3] , [21, 15, 5, 21, 18, 14, 26, 17, 7] , [52, 34, 14, 49, 27, 15, 53, 29, 15] , [99, 59, 37, 100, 57, 35, 91, 64, 34] , [185, 127, 71, 195, 138, 64, 192, 121, 59]] $ $ [[1, 3, 2, 3, 3, 2, 0, 4, 0] , [6, 5, 1, 2, 8, 0, 8, 3, 3] , [12, 9, 0, 12, 5, 5, 10, 14, 5] , [27, 17, 11, 27, 14, 2, 22, 15, 9] , [44, 30, 18, 47, 37, 17, 43, 35, 17] , [93, 69, 27, 92, 71, 29, 101, 64, 30] , [199, 129, 57, 189, 118, 64, 192, 135, 69]] $ $ [[2, -1, -1, 0, -1, -1, 3, -2, 1] , [0, -1, 1, 4, -4, 2, -2, 1, -1] , [0, -1, 4, 0, 3, -1, 2, -6, -1] , [-3, -1, -3, -3, 2, 6, 2, 1, -1] , [4, 2, -2, 1, -5, -1, 5, -3, -1] , [3, -5, 5, 4, -7, 3, -5, 0, 2] , [-7, -1, 7, 3, 10, 0, 0, -7, -5]] $

$$[2\,y_1 + y_2 + y_3 - y_5 + 3\,y_6, y_1, -y_4 - y_5, y_2, -3\,y_1 - 2\,y_2 - 2\,y_3 + y_5 - 4\,y_6, y_3, y_4, y_5, y_6]$$

$p = s^2 + 2s^3 - 4s^5 - 8s^6 - 16s^7$

S+ \; S- \; NM

$ [[22, 15, 5, 20, 13, 6, 19, 16, 8] , [23, 13, 5, 22, 13, 11, 19, 10, 8] , [23, 10, 6, 21, 21, 6, 17, 13, 7] , [19, 15, 9, 20, 11, 8, 24, 13, 5] , [17, 14, 12, 19, 13, 5, 26, 14, 4] , [19, 15, 9, 20, 11, 8, 24, 13, 5] , [21, 12, 6, 22, 18, 5, 18, 14, 8] , [22, 14, 4, 21, 15, 5, 17, 17, 9] , [20, 15, 7, 21, 8, 11, 24, 11, 7]] $ $ [[22, 14, 6, 20, 12, 9, 21, 13, 7] , [23, 15, 3, 22, 15, 5, 15, 16, 10] , [23, 9, 7, 21, 20, 9, 19, 10, 6] , [19, 16, 8, 20, 12, 5, 22, 16, 6] , [17, 14, 12, 19, 13, 5, 26, 14, 4] , [19, 16, 8, 20, 12, 5, 22, 16, 6] , [21, 11, 7, 22, 17, 8, 20, 11, 7] , [22, 12, 6, 21, 13, 11, 21, 11, 7] , [20, 18, 4, 21, 11, 2, 18, 20, 10]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 3, 1, 0, 6, 0, 2] , [6, 2, 0, 5, 0, 0, 4, 0, 1] , [4, 1, 0, 6, 0, 0, 5, 0, 2] , [5, 2, 0, 4, 0, 0, 6, 0, 1] , [6, 1, 0, 5, 0, 0, 4, 0, 2] , [4, 2, 0, 6, 0, 0, 5, 0, 1]] $

$$[5\,y_1 - y_2 - y_3 - y_4 + 5\,y_5, y_1, 0, y_2, y_3, 0, y_4, 0, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[1, 3, 2, 3, 3, 2, 0, 4, 0] , [0, 1, 3, 5, 0, 4, 0, 5, 0] , [0, 0, 0, 4, 0, 5, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_3, y_4, -7\,y_3 + 3\,y_4, -11\,y_3 + 4\,y_4 - y_1 + y_2, 3\,y_3, y_1, 0, y_2, 0]$$

$p = -s^4 + s^6$ $p = -s^4 + s^5$ $p = -s^4 + s^7$

Â» SYNC’D 6645/262144 , 0.02534866333

44 . Coloring, {2, 4, 5}

R: [4, 9, 4, 8, 3, 7, 1, 1, 1] **B**: [2, 4, 5, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1443, 444, 129, 1290, 172, 258, 559, 1032, 333] . FixedPtCheck, [1443, 444, 129, 1290, 172, 258, 559, 1032, 333]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -2 . order: 3

$ [[6, 0, 2, 4, 0, 0, 1, 3, 2] , [6, 0, 0, 8, 0, 0, 0, 4, 0] , [4, 0, 0, 6, 0, 0, 0, 8, 0] , [8, 0, 0, 4, 0, 0, 0, 6, 0] , [6, 0, 0, 8, 0, 0, 0, 4, 0] , [4, 0, 0, 6, 0, 0, 0, 8, 0]] $

$$[y_1, 0, 2\,y_3, y_2, 0, 0, y_3, y_4, 2\,y_3]$$

p’ = - s ^ 2 + s ^ 5 p = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 2, 4, 2, 5, 1, 0] , [0, 0, 0, 4, 5, 1, 6, 2, 0] , [0, 0, 0, 0, 6, 2, 9, 1, 0] , [0, 0, 0, 0, 9, 1, 6, 2, 0] , [0, 0, 0, 0, 6, 2, 9, 1, 0] , [0, 0, 0, 0, 9, 1, 6, 2, 0]] $

$$[0, 4\,y_4 - y_2 + y_3, 0, -y_1 + y_4 + 4\,y_3, y_1, y_4, y_2, y_3, 0]$$

p = s ^ 3 - s ^ 5 p’ = s ^ 3 - s ^ 5

Â» SYNC’D 123/4096 , 0.03002929688

45 . Coloring, {2, 4, 6}

R: [4, 9, 4, 8, 7, 8, 1, 1, 1] **B**: [2, 4, 5, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [1989, 612, 43, 1677, 172, 387, 645, 1548, 459] . FixedPtCheck, [1989, 612, 43, 1677, 172, 387, 645, 1548, 459]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 0, 4, 0, 0, 2, 4, 2] , [4, 0, 2, 5, 3, 0, 2, 2, 0] , [4, 4, 1, 10, 4, 2, 6, 5, 0] , [11, 12, 4, 9, 9, 3, 8, 12, 4] , [24, 17, 7, 19, 20, 4, 29, 12, 12] , [53, 28, 12, 46, 28, 20, 61, 23, 17] , [101, 58, 36, 101, 55, 41, 90, 66, 28]] $ $ [[0, 4, 2, 2, 4, 2, 4, 0, 0] , [2, 4, 0, 1, 1, 2, 4, 2, 2] , [8, 4, 3, 2, 4, 2, 6, 3, 4] , [13, 4, 4, 15, 7, 5, 16, 4, 4] , [24, 15, 9, 29, 12, 12, 19, 20, 4] , [43, 36, 20, 50, 36, 12, 35, 41, 15] , [91, 70, 28, 91, 73, 23, 102, 62, 36]] $ $ [[3, -2, -1, 1, -2, -1, -1, 2, 1] , [1, -2, 1, 2, 1, -1, -1, 0, -1] , [-2, 0, -1, 4, 0, 0, 0, 1, -2] , [-1, 4, 0, -3, 1, -1, -4, 4, 0] , [0, 1, -1, -5, 4, -4, 5, -4, 4] , [5, -4, -4, -2, -4, 4, 13, -9, 1] , [5, -6, 4, 5, -9, 9, -6, 2, -4]] $

$[-3\,y_2 - 3\,y_3 - 2\,y_6 - y_4 - 2\,y_5 - y_1, 2\,y_2 + 2\,y_3 + y_6 + y_4 + 2\,y_5, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$

$p = s^3 + s^4 + 4s^5 + 8s^7$

S+ \ ; S- \ ; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 4, 0, 0, 2, 4, 2] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0]] $

$$[y_2, 0, 0, y_3, 0, 0, y_4, y_1, y_4]$$

$p = - s^2 + s^5$

Omega Rank for B : cycles: {{3, 5}}, net cycles: -1 . order: 4

$ [[0, 4, 2, 2, 4, 2, 4, 0, 0] , [0, 0, 4, 4, 6, 0, 4, 0, 0] , [0, 0, 6, 0, 8, 0, 4, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0]] $

$$[0, 2\,y_4, y_1, y_2, y_3, y_4, y_5, 0, 0]$$

$p = - s^4 + s^6$

Â» SYNC'D 175/4096 , 0.04272460938

46 . Coloring, {2, 4, 7}

R: [4, 9, 4, 8, 7, 7, 5, 1, 1] **B**: [2, 4, 5, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-221, -68, -43, -215, -172, -43, -215, -172, -51] . FixedPtCheck, [221, 68, 43, 215, 172, 43, 215, 172, 51]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	4 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 1 . order: 6

$ [[3, 0, 0, 4, 3, 0, 3, 3, 2] , [5, 0, 0, 3, 3, 0, 3, 4, 0] , [4, 0, 0, 5, 3, 0, 3, 3, 0] , [3, 0, 0, 4, 3, 0, 3, 5, 0] , [5, 0, 0, 3, 3, 0, 3, 4, 0] , [4, 0, 0, 5, 3, 0, 3, 3, 0]] $

$$[y_2, 0, 0, y_3, y_4, 0, y_4, -y_2 - y_3 + 4\,y_4 - y_1, y_1]$$

p = - s ^ 2 + s ^ 5 p’ = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}, {3, 5}}, net cycles: 3 . order: 4

$ [[3, 4, 2, 2, 1, 2, 3, 1, 0] , [3, 3, 1, 4, 2, 1, 2, 2, 0] , [2, 3, 2, 3, 1, 2, 4, 1, 0] , [4, 2, 1, 3, 2, 1, 3, 2, 0] , [3, 4, 2, 2, 1, 2, 3, 1, 0] , [3, 3, 1, 4, 2, 1, 2, 2, 0] , [2, 3, 2, 3, 1, 2, 4, 1, 0] , [4, 2, 1, 3, 2, 1, 3, 2, 0]] $

$$[y_2 - y_3 + 3\,y_4, y_1, y_2, y_3, y_4, y_2, -y_1 + 3\,y_2 + y_4, y_4, 0]$$

p = - s + s ^ 5 p’ = - s + s ^ 5 p’ = - s ^ 2 + s ^ 6 p’ = - s ^ 3 + s ^ 7

Â» SYNC’D 24105/16777216 , 0.001436769962

47 . Coloring, {2, 4, 8}

R: [4, 9, 4, 8, 7, 7, 1, 6, 1] **B**: [2, 4, 5, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [3861, 1188, 215, 3354, 860, 2322, 3225, 3096, 891] . FixedPtCheck, [3861, 1188, 215, 3354, 860, 2322, 3225, 3096, 891]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 6	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: 0 . order: 5

$$[y_1, 0, 0, y_2, 0, y_3, y_4, y_5, y_6]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 19/738, 181/738, -179/738, -35/738, 55/738] , [1/2, -179/738, -35/738, 55/738, 19/738, -94/369] , [0, 19/738, 181/738, -179/738, -35/738, 55/738] , [0, 55/738, 19/738, 181/738, -179/738, -35/738] , [0, -179/738, -35/738, 55/738, 19/738, 181/738] , [0, -179/738, -35/738, 55/738, 19/738, 181/738] , [0, 181/738, -179/738, -35/738, 55/738, 19/738] , [0, -35/738, 55/738, 19/738, 181/738, -179/738] , [0, 181/738, -179/738, -35/738, 55/738, 19/738]] $ x $ [[4, 0, 0, 4, 0, 2, 3, 3, 2] , [5, 0, 0, 4, 0, 3, 2, 4, 0] , [2, 0, 0, 5, 0, 4, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 4, 5, 0] , [4, 0, 0, 3, 0, 5, 4, 2, 0] , [4, 0, 0, 4, 0, 2, 5, 3, 0]] $

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -2, 0, 16/9, -13/18] , [0, 0, 0, 1, -2, -13/18, 16/9] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 0, 0, -2/9, 5/18] , [1, -2, 0, 6, -11, -38/9, 185/18] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 1, -2, 0, 6, -13/18, -38/9] , [0, 0, 1, -2, 0, 16/9, -13/18]] $ x $ [[2, 4, 2, 2, 4, 0, 3, 1, 0] , [1, 2, 4, 4, 5, 0, 2, 0, 0] , [0, 1, 5, 2, 6, 0, 4, 0, 0] , [0, 0, 6, 1, 9, 0, 2, 0, 0] , [0, 0, 9, 0, 8, 0, 1, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0]] $

Â» SYNC’D 18335/1048576 , 0.01748561859

48 . Coloring, {2, 4, 9}

R: [4, 9, 4, 8, 7, 7, 1, 1, 2] **B**: [2, 4, 5, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 4τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 18‘ (‘ 5 - 4τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - 4τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [819, 468, 25, 750, 100, 150, 375, 600, 351] . FixedPtCheck, [819, 468, 25, 750, 100, 150, 375, 600, 351]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ ^ 2

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[5, 1, 0, 4, 0, 0, 3, 3, 2] , [6, 3, 4, 8, 5, 1, 2, 6, 1] , [11, 7, 3, 15, 10, 2, 10, 11, 3] , [26, 16, 6, 23, 19, 5, 21, 21, 7] , [51, 29, 13, 48, 37, 11, 49, 34, 16] , [99, 61, 27, 99, 66, 30, 96, 69, 29] , [200, 122, 62, 193, 133, 59, 189, 133, 61]] $ $ [[1, 3, 2, 2, 4, 2, 3, 1, 0] , [6, 5, 0, 4, 3, 3, 10, 2, 3] , [13, 9, 5, 9, 6, 6, 14, 5, 5] , [22, 16, 10, 25, 13, 11, 27, 11, 9] , [45, 35, 19, 48, 27, 21, 47, 30, 16] , [93, 67, 37, 93, 62, 34, 96, 59, 35] , [184, 134, 66, 191, 123, 69, 195, 123, 67]] $ $ [[2, -1, -1, 1, -2, -1, 0, 1, 1] , [0, -1, 2, 2, 1, -1, -4, 2, -1] , [-1, -1, -1, 3, 2, -2, -2, 3, -1] , [2, 0, -2, -1, 3, -3, -3, 5, -1] , [3, -3, -3, 0, 5, -5, 1, 2, 0] , [3, -3, -5, 3, 2, -2, 0, 5, -3] , [8, -6, -2, 1, 5, -5, -3, 5, -3]] $

$[-2 y_1 - y_2 + y_5 + y_3 + y_6, y_1, -y_5 - y_4, y_2, y_1 - y_5 - 2 y_3 - 2 y_6, y_3, y_4, y_5, y_6]$

p = s ^ 2 - 2s ^ 3 + 4s ^ 5 + 8s ^ 6 - 16s ^ 7

S+ \ ; S- \ ; NM

$ [[22, 15, 6, 20, 13, 7, 20, 14, 7] , [23, 15, 5, 22, 15, 7, 17, 12, 8] , [23, 9, 6, 21, 20, 8, 18, 12, 7] , [19, 16, 9, 20, 12, 6, 23, 14, 5] , [17, 14, 12, 19, 13, 5, 26, 14, 4] , [19, 15, 8, 20, 11, 7, 23, 15, 6] , [21, 11, 6, 22, 17, 7, 19, 13, 8] , [22, 12, 4, 21, 13, 9, 19, 15, 9] , [20, 16, 5, 21, 9, 7, 21, 16, 9]] $ $ [[22, 14, 5, 20, 12, 8, 20, 15, 8] , [23, 13, 3, 22, 13, 9, 17, 14, 10] , [23, 10, 7, 21, 21, 7, 18, 11, 6] , [19, 15, 8, 20, 11, 7, 23, 15, 6] , [17, 14, 12, 19, 13, 5, 26, 14, 4] , [19, 16, 9, 20, 12, 6, 23, 14, 5] , [21, 12, 7, 22, 18, 6, 19, 12, 7] , [22, 14, 6, 21, 15, 7, 19, 13, 7] , [20, 17, 6, 21, 10, 6, 21, 15, 8]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 4, 0, 0, 3, 3, 2] , [6, 2, 0, 5, 0, 0, 0, 4, 1] , [4, 1, 0, 6, 0, 0, 0, 5, 2] , [5, 2, 0, 4, 0, 0, 0, 6, 1] , [6, 1, 0, 5, 0, 0, 0, 4, 2] , [4, 2, 0, 6, 0, 0, 0, 5, 1]] $

$$[5\,y_5 - y_1 - y_2 - y_3 + 5\,y_4, y_5, 0, y_1, 0, 0, y_2, y_3, y_4]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{3, 5}, {6, 8}}, net cycles: 1 . order: 6

$ [[1, 3, 2, 2, 4, 2, 3, 1, 0] , [0, 1, 4, 3, 5, 1, 2, 2, 0] , [0, 0, 5, 1, 6, 2, 3, 1, 0] , [0, 0, 6, 0, 8, 1, 1, 2, 0] , [0, 0, 8, 0, 7, 2, 0, 1, 0] , [0, 0, 7, 0, 8, 1, 0, 2, 0] , [0, 0, 8, 0, 7, 2, 0, 1, 0] , [0, 0, 7, 0, 8, 1, 0, 2, 0]] $

$$[-y_2 - y_3 + 2\,y_4 + 3\,y_6, -y_1 + 3\,y_4 - y_5 + 2\,y_6, y_1, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = -s^5 + s^7$ p' = $-s^5 + s^7$

Â» SYNC'D 96495/16777216 , 0.005751550198

49 . Coloring, {2, 5, 6}

R: [4, 9, 4, 7, 3, 8, 1, 1, 1] **B**: [2, 4, 5, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-3471, -1068, -645, -3354, -860, -258, -2795, -1032, -801] . FixedPtCheck, [3471, 1068, 645, 3354, 860, 258, 2795, 1032, 801]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[6, 0, 2, 4, 0, 0, 3, 1, 2] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0]] $

$$[y_1, 0, 2\,y_4, y_2, 0, 0, y_3, y_4, 2\,y_4]$$

$p = s^2 - s^5$ $p' = s^2 - s^5$

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[0, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[1/4, -1/8, -1/8, 1/32, 11/72, -37/288] , [0, 1/4, -1/8, -1/8, -7/72, 11/72] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 1/4, -1/8, 1/36, -7/72] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 1/4, -2/9, 1/36] , [1/4, -1/8, -1/8, 1/32, 11/72, -37/288]] $ x $ [[0, 4, 0, 2, 4, 2, 3, 3, 0] , [0, 0, 0, 4, 3, 3, 6, 2, 0] , [0, 0, 0, 0, 6, 2, 6, 4, 0] , [0, 0, 0, 0, 6, 4, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Â» SYNC'D 141/2048 , 0.06884765625

50 . Coloring, {2, 5, 7}

R: [4, 9, 4, 7, 3, 7, 5, 1, 1] **B**: [2, 4, 5, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ² ‘)‘ , 54‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ² ‘ (‘ 5 + 3τ ² ‘)‘ , 9‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ‘)‘ ³ , -9‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ‘)‘ ² , 9‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , 9‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ² ‘)‘ , -18‘ (‘ 5 - τ + 3τ ² + τ ³ ‘)‘‘ (‘ 1 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , 27‘ (‘ 1 + τ ‘)‘ ² ‘ (‘ - 1 + τ ‘)‘ ² ‘ (‘ 5 + 3τ ² ‘)‘‘]‘

For τ=1/2, [1794, 552, 2322, 3225, 3096, 215, 3354, 860, 414] . FixedPtCheck, [1794, 552, 2322, 3225, 3096, 215, 3354, 860, 414]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	7 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$ [[3, 0, 2, 4, 3, 0, 4, 0, 2] , [2, 0, 3, 5, 4, 0, 4, 0, 0] , [0, 0, 4, 5, 4, 0, 5, 0, 0] , [0, 0, 4, 4, 5, 0, 5, 0, 0] , [0, 0, 5, 4, 5, 0, 4, 0, 0] , [0, 0, 5, 5, 4, 0, 4, 0, 0]] $

$$[-y_2 + y_3 + y_4 - y_1 + y_5, 0, y_2, y_3, y_4, 0, y_1, 0, y_5]$$

$p = - s^3 + s^4 - s^5 + s^6$

Omega Rank for B : cycles: {{6, 8}}, net cycles: 0 . order: 6

$$[y_3, y_4, 0, y_5, y_6, y_7, y_2, y_1, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 0, 1, -2, -11/9, 41/18] , [0, 0, 0, 0, 1, 5/18, -11/9] , [1, -2, 1, 0, 3, 41/18, -47/9] , [0, 0, 0, 0, 0, -2/9, 5/18] , [0, 1, -2, 1, 0, -20/9, 41/18] , [0, 0, 0, 0, 0, -2/9, 5/18] , [0, 0, 1, -2, 1, 41/18, -20/9] , [0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 1, -2, -11/9, 41/18]] $ x $ [[3, 4, 0, 2, 1, 2, 2, 4, 0] , [2, 3, 0, 4, 0, 4, 1, 4, 0] , [1, 2, 0, 3, 0, 4, 0, 8, 0] , [0, 1, 0, 2, 0, 8, 0, 7, 0] , [0, 0, 0, 1, 0, 7, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0]] $

Â» SYNC'D 35301/1048576 , 0.03366565704

51 . Coloring, {2, 5, 8}

R: [4, 9, 4, 7, 3, 7, 1, 6, 1] **B**: [2, 4, 5, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-3‘ (‘ 3 + τ^2 ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -3‘ (‘ 3 + τ^2 ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -3‘ (‘ 3 + τ^2 ‘)‘ , 6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-13, -4, -3, -13, -4, -3, -13, -4, -3] . FixedPtCheck, [13, 4, 3, 13, 4, 3, 13, 4, 3]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

$$[3, 2, 1, 3, 2, 1, 3, 2, 1]$$

+ \; - \; Δ

$ [[4, 0, 2, 4, 0, 2, 4, 0, 2] , [5, 1, 0, 5, 1, 0, 5, 1, 0] , [8, 3, 1, 8, 3, 1, 8, 3, 1] , [14, 7, 3, 14, 7, 3, 14, 7, 3] , [26, 15, 7, 26, 15, 7, 26, 15, 7] , [50, 31, 15, 50, 31, 15, 50, 31, 15] , [98, 63, 31, 98, 63, 31, 98, 63, 31]] $
$ [[2, 4, 0, 2, 4, 0, 2, 4, 0] , [1, 3, 2, 1, 3, 2, 1, 3, 2] , [4, 5, 3, 4, 5, 3, 4, 5, 3] , [10, 9, 5, 10, 9, 5, 10, 9, 5] , [22, 17, 9, 22, 17, 9, 22, 17, 9] , [46, 33, 17, 46, 33, 17, 46, 33, 17] , [94, 65, 33, 94, 65, 33, 94, 65, 33]] $
$ [[1, -2, 1, 1, -2, 1, 1, -2, 1] , [2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -1, -1, 2, -1, -1, 2, -1, -1]] $

$$[-y_2 - y_1, y_1, y_2, -y_2 - y_1, y_1, y_2, -y_2 - y_1, y_1, y_2]$$

p’ = $s^2 - 16s^6$ p’ = $s^4 - 4s^6$ p = $s^2 - 32s^7$
S+ \; S- \; NM
$ [[9, 6, 3, 14, 8, 5, 10, 8, 3] , [16, 7, 1, 10, 6, 1, 7, 9, 9] , [9, 6, 3, 14, 14, 6, 10, 2, 2] , [11, 11, 6, 8, 3, 1, 14, 8, 4] , [7, 6, 6, 11, 6, 4, 15, 10, 1] , [11, 11, 6, 8, 3, 1, 14, 8, 4] , [13, 5, 2, 11, 11, 5, 9, 6, 4] , [10, 9, 4, 12, 10, 6, 11, 3, 1] , [13, 5, 2, 11, 5, 4, 9, 12, 5]] $ $ [[11, 11, 4, 11, 5, 2, 11, 6, 5] , [8, 5, 2, 15, 12, 7, 10, 5, 2] , [15, 3, 1, 13, 10, 2, 5, 9, 8] , [8, 5, 4, 11, 7, 5, 14, 10, 2] , [10, 12, 8, 7, 3, 1, 16, 7, 2] , [8, 5, 4, 11, 7, 5, 14, 10, 2] , [14, 6, 3, 11, 10, 4, 8, 6, 4] , [15, 5, 1, 11, 7, 3, 7, 10, 7] , [10, 14, 6, 9, 5, 4, 14, 3, 1]] $ $ [[120, 48, 30, 60, 48, 20, 60, 64, 30] , [72, 80, 23, 84, 40, 28, 84, 40, 29] , [90, 46, 40, 60, 50, 20, 90, 64, 20] , [60, 56, 20, 120, 52, 40, 60, 52, 20] , [72, 40, 25, 78, 80, 26, 90, 40, 29] , [60, 56, 20, 120, 52, 40, 60, 52, 20] , [60, 56, 30, 60, 60, 20, 120, 44, 30] , [96, 40, 32, 78, 40, 26, 66, 80, 22] , [90, 58, 20, 60, 58, 20, 90, 44, 40]] $
CmmCk *true, true, true*
p’ = $s^3 - 8s^6$ p’ = $s^5 - 2s^6$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 7	2 *vs* 7	2 *vs* 7	2 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[4, 0, 2, 4, 0, 2, 4, 0, 2] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1, 0, y_2, y_1, 0, y_2, y_1, 0, y_2]$$

p = $-s^2 + s^4$ p = $-s^2 + s^5$ p = $-s^2 + s^3$ p = $-s^2 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[2, 4, 0, 2, 4, 0, 2, 4, 0] , [4, 2, 0, 4, 2, 0, 4, 2, 0] , [2, 4, 0, 2, 4, 0, 2, 4, 0] , [4, 2, 0, 4, 2, 0, 4, 2, 0] , [2, 4, 0, 2, 4, 0, 2, 4, 0] , [4, 2, 0, 4, 2, 0, 4, 2, 0]] $

$$[y_1, y_2, 0, y_1, y_2, 0, y_1, y_2, 0]$$

$p' = -s + s^5 \quad p = -s + s^3 \quad p' = -s + s^3 \quad p = -s + s^5$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[0, x_1, x_2, x_3, x_4, x_5, -32\,x_1 - 16\,x_2 - 8\,x_3 - 4\,x_4 - 2\,x_5]$
For A+2Δ : $[y_5, y_4, y_3, y_2, -y_4 + 9\,y_3 + 9\,y_1 + 9\,y_6 - y_7, y_1, -y_5 - 3\,y_3 - y_2 - 3\,y_1 - 3\,y_6, y_7, y_6]$
For A-2Δ : $[-3\,y_1 - y_2 - 3\,y_3 - y_4 - 3\,y_6, y_1, 9\,y_1 + 9\,y_3 + 9\,y_6 - y_5 - y_7, y_2, y_3, y_5, y_4, y_6, y_7]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_1 - \mu_2, \mu_1, \mu_2, -\mu_1 - \mu_2, \mu_1, \mu_2, -\mu_1 - \mu_2, \mu_1, \mu_2]$

rank of M is 9 , rank of N is 6

M \; N

$ [[0, 0, 0, 3, 0, 0, 3, 0, 0] , [0, 0, 0, 0, 2, 0, 0, 2, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 1] , [3, 0, 0, 0, 0, 0, 3, 0, 0] , [0, 2, 0, 0, 0, 0, 0, 2, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 1] , [3, 0, 0, 3, 0, 0, 0, 0, 0] , [0, 2, 0, 0, 2, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 1, 0, 0, 0]] $ $ [[0, 16, 10, 20, 16, 20, 20, 8, 10] , [16, 0, 17, 12, 20, 12, 12, 20, 11] , [10, 17, 0, 20, 15, 20, 10, 8, 20] , [20, 12, 20, 0, 14, 0, 20, 14, 20] , [16, 20, 15, 14, 0, 14, 10, 20, 11] , [20, 12, 20, 0, 14, 0, 20, 14, 20] , [20, 12, 10, 20, 10, 20, 0, 18, 10] , [8, 20, 8, 14, 20, 14, 18, 0, 18] , [10, 11, 20, 20, 11, 20, 10, 18, 0]] $

Check is ΩΔN zero? *true,* $\pi\Delta = [1, -2, 1, 1, -2, 1, 1, -2, 1]$

ker M, [0, 0, 0, 0, 0, 0, 0, 0, 0]
Range M, $[x_9, x_8, x_7, x_6, x_5, x_4, x_3, x_2, x_1]$

$\tau = 27$, r'= 2/3

Ranges

Action of R on ranges, [[1], [3], [1]]
Action of B on ranges, [[2], [1], [2]]
β({1, 4, 7}) = 1/2
β({2, 5, 8}) = 1/3
β({3, 6, 9}) = 1/6

ker N, $[-\mu_1 - \mu_3, \mu_3, \mu_1, -\mu_2 - \mu_3, \mu_3, \mu_2, -\mu_1 - \mu_3, \mu_3, \mu_1]$
Range of N
$[y_1 - y_4 + y_6, y_1 + y_6 + y_3 - y_2 - y_5, y_1, y_3, y_2, y_3, y_4, y_5, y_6]$

Partitions

Action of R on partitions, [[2], [6], [2], [2], [2], [6], [2], [2]]
Action of B on partitions, [[7], [5], [5], [8], [4], [1], [1], [3]]

α([{1, 2, 9}, {3, 5, 7}, {4, 6, 8}]) = 1/6
α([{1, 3, 8}, {2, 7, 9}, {4, 5, 6}]) = 1/4
α([{1, 8, 9}, {2, 3, 7}, {4, 5, 6}]) = 1/60
α([{1, 5, 9}, {2, 4, 6}, {3, 7, 8}]) = 1/15
α([{1, 5, 9}, {2, 3, 7}, {4, 6, 8}]) = 2/15
α([{1, 3, 8}, {2, 4, 6}, {5, 7, 9}]) = 1/4
α([{1, 8, 9}, {2, 4, 6}, {3, 5, 7}]) = 1/12
α([{1, 2, 9}, {3, 7, 8}, {4, 5, 6}]) = 1/30
b1 = {1, 2, 9} ‘ , ‘ b2 = {1, 3, 8} ‘ , ‘ b3 = {1, 5, 9} ‘ , ‘ b4 = {1, 8, 9} ‘ , ‘ b5 = {2, 3, 7} ‘ , ‘ b6 = {2, 4, 6} ‘ , ‘ b7 = {2, 7, 9} ‘ , ‘ b8 = {3, 5, 7} ‘ , ‘ b9 = {3, 7, 8} ‘ , ‘ b10 = {4, 5, 6} ‘ , ‘ b11 = {4, 6, 8} ‘ , ‘ b12 = {5, 7, 9}

Action of R and B on the blocks of the partitions: $ [[0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1] , [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0] , [0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0]] $ = $ [[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]] $ + $ [[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0]] $
[‘7‘, C, ‘7‘, ‘7‘, A, ‘2‘, ‘6‘, A, A, ‘2‘, ‘2‘, ‘6‘], [‘4‘, B, ‘9‘, B, ‘3‘, ‘1‘, ‘3‘, ‘8‘, A, ‘5‘, ‘6‘, ‘8‘] with invariant measure [4, 10, 4, 2, 3, 8, 5, 5, 2, 6, 6, 5]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Sandwich	
Coloring	{2, 5, 8}
Rank	3
R,B	[4, 9, 4, 7, 3, 7, 1, 6, 1], [2, 4, 5, 8, 7, 8, 5, 1, 2]
π_2	[0, 0, 3, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0]
$\mathbf{u}_2$	[16, 10, 20, 16, 20, 20, 8, 10, 17, 12, 20, 12, 12, 20, 11, 20, 15, 20, 10, 8, 20, 14, 0, 20, 14, 20, 14, 10, 20, 11, 20, 14, 20, 18, 10, 18] (dim 1)
wpp	[3, 3, 3, 3, 3, 3, 3, 3, 3]
π_3	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$\mathbf{u}_3$	[3, 8, 12, 8, 8, 4, 5, 10, 1, 10, 0, 6, 0, 10, 0, 20, 2, 10, 10, 6, 4, 5, 20, 2, 10, 6, 0, 0, 9, 12, 9, 5, 5, 8, 6, 0, 4, 6, 3, 6, 2, 20, 2, 4, 6, 3, 10, 3, 9, 9, 0, 10, 2, 20, 9, 5, 3, 6, 10, 2, 20, 0, 0, 6, 0, 4, 8, 5, 0, 0, 0, 12, 10, 12, 4, 8, 5, 8, 1, 9, 12, 10, 12, 6]

52 . Coloring, {2, 5, 9}

R: [4, 9, 4, 7, 3, 7, 1, 1, 2]   **B**: [2, 4, 5, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘]‘

For τ=1/2, [357, 204, 75, 375, 100, 25, 325, 100, 153] . FixedPtCheck, [357, 204, 75, 375, 100, 25, 325, 100, 153]

det(A + τ Δ) =   0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}, {2, 9}}, net cycles: 1 . order: 6

$ [[5, 1, 2, 4, 0, 0, 4, 0, 2] , [4, 2, 0, 7, 0, 0, 4, 0, 1] , [4, 1, 0, 4, 0, 0, 7, 0, 2] , [7, 2, 0, 4, 0, 0, 4, 0, 1] , [4, 1, 0, 7, 0, 0, 4, 0, 2] , [4, 2, 0, 4, 0, 0, 7, 0, 1]] $

$$[5\,y_4 - y_1 - y_2 - y_3 + 5\,y_5, y_4, y_1, y_2, 0, 0, y_3, 0, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 3, 0, 2, 4, 2, 2, 4, 0] , [0, 1, 0, 3, 2, 4, 4, 4, 0] , [0, 0, 0, 1, 4, 4, 2, 7, 0] , [0, 0, 0, 0, 2, 7, 4, 5, 0] , [0, 0, 0, 0, 4, 5, 2, 7, 0] , [0, 0, 0, 0, 2, 7, 4, 5, 0] , [0, 0, 0, 0, 4, 5, 2, 7, 0]] $

$$[y_1, 3\,y_1 + 3\,y_2 + 3\,y_3 - 4\,y_5 - y_4, 0, y_2, 2\,y_1 + 2\,y_2 + 2\,y_3 - 3\,y_5, y_3, y_5, y_4, 0]$$

$p' = -s^4 + s^6 \quad p = -s^4 + s^6$

Â» SYNC’D 3645/262144 , 0.01390457153

53 . Coloring, {2, 6, 7}

R: [4, 9, 4, 7, 7, 8, 5, 1, 1] **B**: [2, 4, 5, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-481, -148, -215, -559, -860, -43, -1075, -172, -111] . FixedPtCheck, [481, 148, 215, 559, 860, 43, 1075, 172, 111]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	6 *vs* 8

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[3, 0, 0, 4, 3, 0, 5, 1, 2] , [3, 0, 0, 3, 5, 0, 7, 0, 0] , [0, 0, 0, 3, 7, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

$$[y_1, 0, 0, y_2, y_3, 0, y_4, y_5, 2\, y_5]$$

$p = -\, s^{4} + s^{6}$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}, {3, 5}}, net cycles: 2 . order: 6

$ [[3, 4, 2, 2, 1, 2, 1, 3, 0] , [1, 3, 1, 4, 2, 3, 2, 2, 0] , [2, 1, 2, 3, 1, 2, 3, 4, 0] , [3, 2, 1, 1, 2, 4, 2, 3, 0] , [2, 3, 2, 2, 1, 3, 4, 1, 0] , [4, 2, 1, 3, 2, 1, 3, 2, 0] , [3, 4, 2, 2, 1, 2, 1, 3, 0] , [1, 3, 1, 4, 2, 3, 2, 2, 0]] $

$$[2\, y_1 - y_2 + 3\, y_3 - y_4, 3\, y_1 + 2\, y_3 - y_5 - y_6, y_1, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = -\, s + s^{7}$ $p' = -\, s + s^{7}$

Â» SYNC'D 1176035/33554432 , 0.03504857421

54 . Coloring, {2, 6, 8}

R: [4, 9, 4, 7, 7, 8, 1, 6, 1] **B**: [2, 4, 5, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 - 3τ - τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 5 - 3τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 5 - 3τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [689, 212, 43, 602, 172, 258, 645, 344, 159] . FixedPtCheck, [689, 212, 43, 602, 172, 258, 645, 344, 159]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[4, 0, 0, 4, 0, 2, 5, 1, 2] , [7, 0, 0, 4, 0, 1, 4, 2, 0] , [4, 0, 0, 7, 0, 2, 4, 1, 0] , [4, 0, 0, 4, 0, 1, 7, 2, 0] , [7, 0, 0, 4, 0, 2, 4, 1, 0] , [4, 0, 0, 7, 0, 1, 4, 2, 0]] $

$$[y_5, 0, 0, y_4, 0, y_3, y_2, y_1, -y_5 - y_4 + 5\,y_3 - y_2 + 5\,y_1]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[2, 4, 2, 2, 4, 0, 1, 3, 0] , [3, 2, 4, 4, 3, 0, 0, 2, 0] , [2, 3, 3, 2, 4, 0, 0, 4, 0] , [4, 2, 4, 3, 3, 0, 0, 2, 0] , [2, 4, 3, 2, 4, 0, 0, 3, 0] , [3, 2, 4, 4, 3, 0, 0, 2, 0] , [2, 3, 3, 2, 4, 0, 0, 4, 0]] $

$$[5\,y_1, -16\,y_1 + 33\,y_2 - 16\,y_3 + 33\,y_4 - 5\,y_5, 5\,y_2, 5\,y_3, -7\,y_1 + 16\,y_2 - 7\,y_3 + 16\,y_4, 0, 5\,y_4, 5\,y_5, 0]$$

$p = - s^2 + s^6$ $p' = - s^2 + s^6$

Â» SYNC’D 33201/1048576 , 0.03166294098

55 . Coloring, {2, 6, 9}

R: [4, 9, 4, 7, 7, 8, 1, 1, 2] **B**: [2, 4, 5, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + 3τ - 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + 3τ - 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + 3τ - 3τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [-2079, -1188, -125, -1950, -500, -150, -1875, -600, -891] . FixedPtCheck, [2079, 1188, 125, 1950, 500, 150, 1875, 600, 891]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	7 *vs* 8

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 4, 0, 0, 5, 1, 2] , [6, 2, 0, 5, 0, 0, 4, 0, 1] , [4, 1, 0, 6, 0, 0, 5, 0, 2] , [5, 2, 0, 4, 0, 0, 6, 0, 1] , [6, 1, 0, 5, 0, 0, 4, 0, 2] , [4, 2, 0, 6, 0, 0, 5, 0, 1]] $

$$[5\, y_1 - y_2 - y_5 - y_3 + 5\, y_4, y_1, 0, y_2, 0, 0, y_5, y_3, y_4]$$

$p = s^2 + s^3 - s^5 - s^6$

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 8

$ [[1, 3, 2, 2, 4, 2, 1, 3, 0] , [0, 1, 4, 3, 3, 3, 2, 2, 0] , [0, 0, 3, 1, 6, 2, 3, 3, 0] , [0, 0, 6, 0, 6, 3, 2, 1, 0] , [0, 0, 6, 0, 8, 1, 3, 0, 0] , [0, 0, 8, 0, 9, 0, 1, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[y_1 + y_2 - y_3 - y_4 - y_5 + y_6 + y_7, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

$p = -s^7 + s^8$

Â» SYNC’D 188803/8388608 , 0.02250707150

56 . Coloring, {2, 7, 8}

R: [4, 9, 4, 7, 7, 7, 5, 6, 1] **B**: [2, 4, 5, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-442, -136, -258, -559, -1032, -129, -1290, -172, -102] . FixedPtCheck, [442, 136, 258, 559, 1032, 129, 1290, 172, 102]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 0, 4, 3, 2, 6, 0, 2] , [2, 0, 0, 1, 6, 0, 9, 0, 0] , [0, 0, 0, 2, 9, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

$$[y_1, 0, 0, y_2, y_3, y_5, y_4, 0, y_5]$$

$p = - s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 2 . order: 4

$ [[5, 4, 2, 2, 1, 0, 0, 4, 0] , [4, 5, 1, 4, 2, 0, 0, 2, 0] , [2, 4, 2, 5, 1, 0, 0, 4, 0] , [4, 2, 1, 4, 2, 0, 0, 5, 0] , [5, 4, 2, 2, 1, 0, 0, 4, 0] , [4, 5, 1, 4, 2, 0, 0, 2, 0]] $

$$[y_4, y_3, y_2, -y_4 + 2\,y_2 + 3\,y_1, y_1, 0, 0, -y_3 + 3\,y_2 + 2\,y_1, 0]$$

$p = - s + s^5$ $p' = s - s^5$

Â» SYNC'D 5709/131072 , 0.04355621338

57 . Coloring, {2, 7, 9}

R: [4, 9, 4, 7, 7, 7, 5, 1, 2] **B**: [2, 4, 5, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [294, 168, 150, 375, 600, 25, 750, 100, 126] . FixedPtCheck, [294, 168, 150, 375, 600, 25, 750, 100, 126]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 vs 7	*7 vs 7*	*7 vs 7*	4 *vs* 6	5 *vs 7*

Omega Rank for R : cycles: {{2, 9}, {5, 7}}, net cycles: 1 . order: 4

$ [[2, 1, 0, 4, 3, 0, 6, 0, 2] , [0, 2, 0, 2, 6, 0, 7, 0, 1] , [0, 1, 0, 0, 7, 0, 8, 0, 2] , [0, 2, 0, 0, 8, 0, 7, 0, 1] , [0, 1, 0, 0, 7, 0, 8, 0, 2] , [0, 2, 0, 0, 8, 0, 7, 0, 1]] $

$$[2\,y_1 - y_3 + 3\,y_4, y_1, 0, 3\,y_1 - y_2 + 2\,y_4, y_2, 0, y_3, 0, y_4]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[4, 3, 2, 2, 1, 2, 0, 4, 0] , [0, 4, 1, 3, 2, 4, 0, 4, 0] , [0, 0, 2, 4, 1, 4, 0, 7, 0] , [0, 0, 1, 0, 2, 7, 0, 8, 0] , [0, 0, 2, 0, 1, 8, 0, 7, 0] , [0, 0, 1, 0, 2, 7, 0, 8, 0] , [0, 0, 2, 0, 1, 8, 0, 7, 0]] $

$$[3\,y_1 - y_2 + 2\,y_3 - y_4, 2\,y_1 + 3\,y_3 - y_5, y_1, y_2, y_3, y_4, 0, y_5, 0]$$

$p' = -s^4 + s^6$ $p = -s^4 + s^6$

Â» SYNC'D 1301/131072 , 0.009925842285

58 . Coloring, {2, 8, 9}

R: [4, 9, 4, 7, 7, 7, 1, 6, 2] **B**: [2, 4, 5, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^2 + τ ^3 ‘)‘ , 18‘ (‘ - 5 - τ - 3τ ^2 + τ ^3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^2 ‘ (‘ 5 + 2τ + τ ^2 ‘)‘ , -9‘ (‘ 3 + τ ^2 ‘)‘‘ (‘ 5 + 2τ + τ ^2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^2 + τ ^3 ‘)‘‘]‘

For τ=1/2, [-343, -196, -25, -325, -100, -75, -375, -100, -147] . FixedPtCheck, [343, 196, 25, 325, 100, 75, 375, 100, 147]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[3, 1, 0, 4, 0, 2, 6, 0, 2] , [6, 2, 0, 3, 0, 0, 6, 0, 1] , [6, 1, 0, 6, 0, 0, 3, 0, 2] , [3, 2, 0, 6, 0, 0, 6, 0, 1] , [6, 1, 0, 3, 0, 0, 6, 0, 2] , [6, 2, 0, 6, 0, 0, 3, 0, 1]] $

$$[5\,y_1 - y_2 - y_3 - y_4 + 5\,y_5, y_1, 0, y_2, 0, y_3, y_4, 0, y_5]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 2 . order: 4

$ [[3, 3, 2, 2, 4, 0, 0, 4, 0] , [4, 3, 4, 3, 2, 0, 0, 2, 0] , [2, 4, 2, 3, 4, 0, 0, 3, 0] , [3, 2, 4, 4, 2, 0, 0, 3, 0] , [3, 3, 2, 2, 4, 0, 0, 4, 0] , [4, 3, 4, 3, 2, 0, 0, 2, 0]] $

$$[y_4, y_3, 2\,y_3 - 3\,y_2 + 2\,y_1, -y_4 + 3\,y_3 - 4\,y_2 + 3\,y_1, y_2, 0, 0, y_1, 0]$$

$p = s - s^5$ p’ = $s - s^5$

Â» SYNC’D 15237/524288 , 0.02906227112

59 . Coloring, {3, 4, 5}

R: [4, 4, 5, 8, 3, 7, 1, 1, 1] **B**: [2, 9, 4, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-15, -4, -3, -15, -4, -3, -7, -12, -1] . FixedPtCheck, [15, 4, 3, 15, 4, 3, 7, 12, 1]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	3 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 1 . order: 6

$ [[6, 0, 2, 5, 1, 0, 1, 3, 0] , [4, 0, 1, 6, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 1, 0, 0, 6, 0] , [6, 0, 1, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 6, 1, 0, 0, 5, 0] , [5, 0, 1, 4, 2, 0, 0, 6, 0]] $

$$[y_4, 0, y_1, y_2, y_3, 0, -y_4 + 5\,y_1 - y_2 + 5\,y_3 - y_5, y_5, 0]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{2, 9}, {5, 7}, {6, 8}}, net cycles: 2 . order: 2

$ [[0, 4, 0, 1, 3, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 4, 2, 4] , [0, 4, 0, 0, 4, 2, 5, 1, 2]] $

$$[0, 2\,y_2, 0, -y_1 + y_2 + 2\,y_3, y_1, y_2, 2\,y_2 + y_3, y_3, 2\,y_3]$$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$ $p = -s^2 + s^6$ $p' = -s^2 + s^6$

Â» SYNC'D 4509/2097152 , 0.002150058746

60 . Coloring, {3, 4, 6}

R: [4, 4, 5, 8, 7, 8, 1, 1, 1] **B**: [2, 9, 4, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-1290, -344, -38, -1235, -152, -285, -494, -1140, -86] . FixedPtCheck, [1290, 344, 38, 1235, 152, 285, 494, 1140, 86]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$ [[6, 0, 0, 5, 1, 0, 2, 4, 0] , [6, 0, 0, 6, 0, 0, 1, 5, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[y_2 + y_3, 0, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$p = -s^3 + s^5$ $p = -s^3 + s^4$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 4, 2, 1, 3, 2, 4, 0, 2] , [0, 2, 3, 2, 4, 0, 3, 0, 4] , [0, 4, 4, 3, 3, 0, 2, 0, 2] , [0, 2, 3, 4, 2, 0, 3, 0, 4] , [0, 4, 2, 3, 3, 0, 4, 0, 2] , [0, 2, 3, 2, 4, 0, 3, 0, 4] , [0, 4, 4, 3, 3, 0, 2, 0, 2]] $

$$[0, y_1 + y_4 + y_5 - y_3, y_1 + y_4 + y_5 - y_2, y_1, y_4, y_5, y_2, 0, y_3]$$

$p = -s^2 + s^6$ $p' = -s^2 + s^6$

Â» SYNC'D 2365/262144 , 0.009021759033

61 . Coloring, {3, 4, 7}

R: [4, 4, 5, 8, 7, 7, 5, 1, 1] **B**: [2, 9, 4, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [555, 148, 171, 570, 684, 114, 741, 456, 37] . FixedPtCheck, [555, 148, 171, 570, 684, 114, 741, 456, 37]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 2 . order: 6

$ [[3, 0, 0, 5, 4, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 4, 5, 0] , [5, 0, 0, 3, 4, 0, 3, 3, 0] , [3, 0, 0, 5, 3, 0, 4, 3, 0] , [3, 0, 0, 3, 4, 0, 3, 5, 0]] $

$$[7 y_4, 0, 0, -7 y_4 + 11 y_1 + 11 y_2 - 7 y_3, 7 y_1, 0, 7 y_2, 7 y_3, 0]$$

$p = -s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}, {6, 8}}, net cycles: 1 . order: 6

$ [[3, 4, 2, 1, 0, 2, 3, 1, 2] , [3, 5, 0, 2, 0, 1, 1, 2, 4] , [1, 7, 0, 0, 0, 2, 2, 1, 5] , [2, 6, 0, 0, 0, 1, 0, 2, 7] , [0, 9, 0, 0, 0, 2, 0, 1, 6] , [0, 6, 0, 0, 0, 1, 0, 2, 9] , [0, 9, 0, 0, 0, 2, 0, 1, 6] , [0, 6, 0, 0, 0, 0, 1, 0, 2, 9]] $

$$[-y_2 + y_3 + 4\,y_5 - y_6, -y_1 + 4\,y_3 - y_4 + y_5, y_1, y_2, 0, y_3, y_4, y_5, y_6]$$

p = - s ^{5} + s ^{7} p' = - s ^{5} + s ^{7}

Â» SYNC'D 81607/4194304 , 0.01945662498

62 . Coloring, {3, 4, 8}

R: [4, 4, 5, 8, 7, 7, 1, 6, 1] **B**: [2, 9, 4, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘]‘

For τ=1/2, [-255, -68, -19, -247, -76, -171, -247, -228, -17] . FixedPtCheck, [255, 68, 19, 247, 76, 171, 247, 228, 17]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: 0 . order: 5

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -5/198, -23/198, 13/198, -59/198, 85/198] , [0, -5/198, -23/198, 13/198, -59/198, 85/198] , [1, -59/198, 85/198, -5/198, -23/198, -185/198] , [0, 85/198, -5/198, -23/198, 13/198, -59/198] , [0, 13/198, -59/198, 85/198, -5/198, -23/198] , [0, 13/198, -59/198, 85/198, -5/198, -23/198] , [0, -23/198, 13/198, -59/198, 85/198, -5/198] , [0, -59/198, 85/198, -5/198, -23/198,

13/198] , [0, -23/198, 13/198, -59/198, 85/198, -5/198]] $ x $ [[4, 0, 0, 5, 1, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 3, 5, 0] , [3, 0, 0, 3, 0, 5, 3, 4, 0] , [3, 0, 0, 3, 0, 4, 5, 3, 0] , [5, 0, 0, 3, 0, 3, 4, 3, 0] , [4, 0, 0, 5, 0, 3, 3, 3, 0]] $

Omega Rank for B : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 2, 1, 3, 0, 3, 1, 2] , [1, 4, 3, 2, 3, 0, 1, 0, 4] , [0, 5, 3, 3, 1, 0, 2, 0, 4] , [0, 4, 1, 3, 2, 0, 3, 0, 5] , [0, 5, 2, 1, 3, 0, 3, 0, 4] , [0, 4, 3, 2, 3, 0, 1, 0, 5] , [0, 5, 3, 3, 1, 0, 2, 0, 4] , [0, 4, 1, 3, 2, 0, 3, 0, 5]] $

$$[y_2 + y_3 - y_6, y_1 + y_4 - y_5, y_1, y_2, y_3, 0, y_4, y_5, y_6]$$

p = - s ^ 3 + s ^ 7 p’ = - s ^ 3 + s ^ 7

Â» SYNC’D 155757/16777216 , 0.009283840656

63 . Coloring, {3, 4, 9}

R: [4, 4, 5, 8, 7, 7, 1, 1, 2] **B**: [2, 9, 4, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-429, -132, -17, -425, -68, -85, -221, -340, -33] . FixedPtCheck, [429, 132, 17, 425, 68, 85, 221, 340, 33]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 9	9 *vs* 9	3 *vs* 6	6 *vs* 9

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 5, 1, 0, 3, 3, 0] , [6, 0, 0, 6, 0, 0, 1, 5, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[-y_1 + y_2 + y_3, y_1, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$p = -s^3 + s^4$ $p = -s^3 + s^5$ $p = -s^3 + s^6$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}, {3, 4, 5, 7}}, net cycles: 3 .

$ [[1, 3, 2, 1, 3, 2, 3, 1, 2] , [2, 1, 3, 2, 3, 1, 1, 2, 3] , [3, 2, 3, 3, 1, 2, 2, 1, 1] , [1, 3, 1, 3, 2, 1, 3, 2, 2] , [2, 1, 2, 1, 3, 2, 3, 1, 3] , [3, 2, 3, 2, 3, 1, 1, 2, 1] , [1, 3, 3, 3, 1, 2, 2, 1, 2] , [2, 1, 1, 3, 2, 1, 3, 2, 3] , [3, 2, 2, 1, 3, 2, 3, 1, 1]] $

$$[-y_1 + 2\,y_3 + 2\,y_5 - y_6, y_1, 2\,y_3 - y_4 + y_5, -y_2 + y_3 + 2\,y_5, y_2, y_3, y_4, y_5, y_6]$$

$p' = -1 - s - s^2 + s^4 + s^5 + s^6$ $p' = 1 - s^3 - s^4 + s^7$ $p' = s - s^4 - s^5 + s^8$

Â» SYNC'D 95739/16777216 , 0.005706489086

64 . Coloring, {3, 5, 6}

R: [4, 4, 5, 7, 3, 8, 1, 1, 1] **B**: [2, 9, 4, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-705, -188, -285, -741, -380, -57, -665, -228, -47] . FixedPtCheck, [705, 188, 285, 741, 380, 57, 665, 228, 47]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[6, 0, 2, 5, 1, 0, 3, 1, 0] , [4, 0, 1, 6, 2, 0, 5, 0, 0] , [5, 0, 2, 4, 1, 0, 6, 0, 0] , [6, 0, 1, 5, 2, 0, 4, 0, 0] , [4, 0, 2, 6, 1, 0, 5, 0, 0] , [5, 0, 1, 4, 2, 0, 6, 0, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - y_5, 0, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{2, 9}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 1, 3, 2, 3, 3, 2] , [0, 2, 0, 0, 3, 3, 5, 1, 4] , [0, 4, 0, 0, 5, 1, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2]] $

$$[0, y_3 + y_4 - y_5, 0, y_3 + y_4 - y_1 - y_2, y_1, y_2, y_3, y_4, y_5]$$

$p = -s^4 + s^6$ $p' = -s^4 + s^6$

Â» SYNC'D 4329/524288 , 0.008256912231

65 . Coloring, {3, 5, 7}

R: [4, 4, 5, 7, 3, 7, 5, 1, 1] **B**: [2, 9, 4, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 4 ‘]‘

For τ=1/2, [-165, -44, -513, -285, -684, -19, -399, -76, -11] . FixedPtCheck, [165, 44, 513, 285, 684, 19, 399, 76, 11]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

\+ \ ; - \ ; Δ

$ [[3, 0, 2, 5, 4, 0, 4, 0, 0] , [2, 5, 4, 3, 6, 4, 5, 3, 4] , [14, 10, 6, 7, 9, 5, 9, 9, 3] , [27, 15, 9, 26, 15, 7, 19, 20, 6] , [55, 31, 15, 49, 28, 12, 50, 31, 17] , [94, 56, 28, 103, 65, 33, 97, 67, 33] , [195, 129, 65, 186, 125, 61, 199, 120, 72]] $ $ [[3, 4, 0, 1, 0, 2, 2, 4, 2] , [10, 3, 0, 9, 2, 0, 7, 5, 0] , [10, 6, 2, 17, 7, 3, 15, 7, 5] , [21, 17, 7, 22, 17, 9, 29, 12, 10] , [41, 33, 17, 47, 36, 20, 46, 33, 15] , [98, 72, 36, 89, 63, 31, 95, 61, 31] , [189, 127, 63, 198, 131, 67, 185, 136, 56]] $ $ [[0, -2, 1, 2, 2, -1, 1, -2, -1] , [-4, 1, 2, -3, 2, 2, -1, -1, 2] , [2, 2, 2, -5, 1, 1, -3, 1, -1] , [3, -1, 1, 2, -1, -1, -5, 4, -2] , [7, -1, -1, 1, -4, -4, 2, -1, 1] , [-2, -8, -4, 7, 1, 1, 1, 3, 1] , [3, 1, 1, -6, -3, -3, 7, -8, 8]] $

$$[y_3, -2\,y_3 - 4\,y_1 + y_4 + 2\,y_6 - 2\,y_2 - y_5, -y_4 - y_6, y_2, y_3 + 3\,y_1 - y_4 - 2\,y_6 + y_2, y_1, y_4, y_6, y_5]$$

$p = s^2 - 2s^4 + 8s^5 - 16s^7$

S+ \; S- \; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	4 *vs* 7

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 4

$ [[3, 0, 2, 5, 4, 0, 4, 0, 0] , [0, 0, 4, 3, 6, 0, 5, 0, 0] , [0, 0, 6, 0, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[y_2, 0, y_1, y_4, y_2 + y_1 - y_4 + y_3, 0, y_3, 0, 0]$$

$p = -s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}, {6, 8}}, net cycles: 0 . order: 4

$ [[3, 4, 0, 1, 0, 2, 2, 4, 2] , [2, 5, 0, 0, 0, 4, 0, 3, 4] , [0, 6, 0, 0, 0, 3, 0, 4, 5] , [0, 5, 0, 0, 0, 4, 0, 3, 6] , [0, 6, 0, 0, 0, 3, 0, 4, 5] , [0, 5, 0, 0, 0, 4, 0, 3, 6] , [0, 6, 0, 0, 0, 3, 0, 4, 5]] $

$$[2\,y_1, 9\,y_1 - 15\,y_2 - 11\,y_3 + 9\,y_4, 0, 2\,y_2, 0, 2\,y_3, 4\,y_2, 7\,y_1 - 9\,y_2 - 9\,y_3 + 7\,y_4, 2\,y_4]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$ $p = -s^3 + s^7$

Â» SYNC’D 139/16384 , 0.008483886719

66 . Coloring, {3, 5, 8}

R: [4, 4, 5, 7, 3, 7, 1, 6, 1] **B**: [2, 9, 4, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -27‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 3τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-690, -184, -342, -741, -456, -171, -798, -228, -46] . FixedPtCheck, [690, 184, 342, 741, 456, 171, 798, 228, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[4, 0, 2, 5, 1, 2, 4, 0, 0] , [4, 0, 1, 4, 2, 0, 7, 0, 0] , [7, 0, 2, 4, 1, 0, 4, 0, 0] , [4, 0, 1, 7, 2, 0, 4, 0, 0] , [4, 0, 2, 4, 1, 0, 7, 0, 0] , [7, 0, 1, 4, 2, 0, 4, 0, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - y_5, 0, y_1, y_2, y_3, y_4, y_5, 0, 0]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 0, 1, 3, 0, 2, 4, 2] , [4, 4, 0, 0, 2, 0, 3, 1, 4] , [1, 8, 0, 0, 3, 0, 2, 0, 4] , [0, 5, 0, 0, 2, 0, 3, 0, 8] , [0, 8, 0, 0, 3, 0, 2, 0, 5] , [0, 5, 0, 0, 2, 0, 3, 0, 8] , [0, 8, 0, 0, 3, 0, 2, 0, 5]] $

$$[y_1, -14\,y_1 - 14\,y_2 + 39\,y_3 - y_4 - 14\,y_5, 0, y_2, -5\,y_1 - 5\,y_2 + 14\,y_3 - 5\,y_5, 0, y_3, y_4, y_5]$$

$p = -s^4 + s^6$ $p' = -s^4 + s^6$

Â» SYNC’D 39/2048 , 0.01904296875

67 . Coloring, {3, 5, 9}

R: [4, 4, 5, 7, 3, 7, 1, 1, 2] **B**: [2, 9, 4, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘ , -9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ‘)‘‘]‘

For τ=1/2, [-234, -72, -102, -255, -136, -17, -238, -68, -18] . FixedPtCheck, [234, 72, 102, 255, 136, 17, 238, 68, 18]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 8

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[5, 1, 2, 5, 1, 0, 4, 0, 0] , [4, 0, 1, 6, 2, 0, 5, 0, 0] , [5, 0, 2, 4, 1, 0, 6, 0, 0] , [6, 0, 1, 5, 2, 0, 4, 0, 0] , [4, 0, 2, 6, 1, 0, 5, 0, 0] , [5, 0, 1, 4, 2, 0, 6, 0, 0]] $

$$[-y_1 + 5 y_2 - y_3 + 5 y_4 - y_5, y_1, y_2, y_3, y_4, 0, y_5, 0, 0]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}, {6, 8}}, net cycles: 2 . order: 6

$ [[1, 3, 0, 1, 3, 2, 2, 4, 2] , [2, 1, 0, 0, 2, 4, 3, 3, 3] , [3, 2, 0, 0, 3, 3, 2, 4, 1] , [1, 3, 0, 0, 2, 4, 3, 3, 2] , [2, 1, 0, 0, 3, 3, 2, 4, 3] , [3, 2, 0, 0, 2, 4, 3, 3, 1] , [1, 3, 0, 0, 3, 3, 2, 4, 2] , [2, 1, 0, 0, 2, 4, 3, 3, 3]] $

$$[-y_1 + 6 y_2 + 6 y_3 - 6 y_4 - y_5, y_1, 0, y_2, 5 y_2 + 5 y_3 - 6 y_4, y_3, y_4, 6 y_2 + 6 y_3 - 7 y_4, y_5]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7 p = - s ^ 2 + s ^ 8

Â» SYNC’D 16237/4194304 , 0.003871202469

68 . Coloring, {3, 6, 7}

R: [4, 4, 5, 7, 7, 8, 5, 1, 1] **B**: [2, 9, 4, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘

)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1335, 356, 855, 1482, 3420, 114, 3705, 456, 89] . FixedPtCheck, [1335, 356, 855, 1482, 3420, 114, 3705, 456, 89]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 5	8 *vs* 8

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 4

$$[y_1, 0, 0, y_2, y_3, 0, y_4, y_5, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -2/9, -13/18] , [0, 0, 1, -2/9, -13/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, 5/18, -2/9] , [0, 0, 0, 5/18, -2/9] , [1, -3, 4, 25/9, -85/18] , [0, 0, 0, -2/9, 5/18] , [0, 1, -3, -13/18, 25/9] , [0, 1, -3, -13/18, 25/9]] $ x $ [[3, 0, 0, 5, 4, 0, 5, 1, 0] , [1, 0, 0, 3, 5, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 8

$$[y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7, y_8]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 0, 0, -2/9, 5/18] , [0, 1/2, -1/4, -5/8, 3/16, 27/32, -23/144, -127/288] , [0, 0, 1/2, -1/4, -5/8, 3/16, 29/72, -23/144] , [1/2, -1/4, -5/8, 3/16, 27/32, -45/64, -127/288, 313/576] , [0, 0, 0, 0, 1/2, -1/4, -2/9, 1/36] , [0, 0, 0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 1/2, -1/4, -5/8, 1/36, 29/72] , [0, 0, 0, 0, 0, 0, 5/18, -2/9]] $ x $ [[3, 4, 2, 1, 0, 2, 1, 3, 2] , [1, 5, 0, 2, 0, 3, 2, 1, 4] , [2, 5, 0, 0, 0, 1, 3, 2, 5] , [3, 7, 0, 0, 0, 2, 1, 0, 5] , [1, 8, 0, 0, 0, 0, 2, 0, 7] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

Â» SYNC’D 318899/8388608 , 0.03801572323

69 . Coloring, {3, 6, 8}

R: [4, 4, 5, 7, 7, 8, 1, 6, 1] **B**: [2, 9, 4, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-2055, -548, -171, -1995, -684, -855, -2223, -1140, -137] . FixedPtCheck, [2055, 548, 171, 1995, 684, 855, 2223, 1140, 137]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	8 *vs* 8

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[4, 0, 0, 5, 1, 2, 5, 1, 0] , [5, 0, 0, 4, 0, 1, 6, 2, 0] , [6, 0, 0, 5, 0, 2, 4, 1, 0] , [4, 0, 0, 6, 0, 1, 5, 2, 0] , [5, 0, 0, 4, 0, 2, 6, 1, 0] , [6, 0, 0, 5, 0, 1, 4, 2, 0]] $

$$[-y_1 - y_2 + 5\,y_3 - y_4 + 5\,y_5, 0, 0, y_1, y_2, y_3, y_4, y_5, 0]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 8

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, y_7, y_8]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 1, -3, 7, 25/9, -139/18] , [0, 0, 0, 0, 1, -3, -13/18, 25/9] , [0, 0, 1, -3, 7, -16, -139/18, 169/9] , [1, -3, 7, -16, 34, -70, -751/18, 799/9] , [0, 1, -3, 7, -16, 34, 169/9, -751/18] , [0, 0, 0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 0, 0, 0, 5/18, -2/9]] $ x $ [[2, 4, 2, 1, 3, 0, 1, 3, 2] , [3, 4, 3, 2, 1, 0, 0, 1, 4] , [1, 7, 1, 3, 0, 0, 0, 2, 4] , [2, 5, 0, 1, 0, 0, 0, 3, 7] , [3, 9, 0, 0, 0, 0, 0, 1, 5] , [1, 8, 0, 0, 0, 0, 0, 0, 9] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

Â» SYNC’D 404463/16777216 , 0.02410787344

70 . Coloring, {3, 6, 9}

R: [4, 4, 5, 7, 7, 8, 1, 1, 2] **B**: [2, 9, 4, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘3‘ (‘ 3 + τ 2 ‘)‘ , -6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘ 2 , 3‘ (‘ 3 + τ 2 ‘)‘ , -6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘ 2 , 3‘ (‘ 3 + τ 2 ‘)‘ , -6‘ (‘ - 1 + τ ‘)‘ , 3‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [13, 4, 1, 13, 4, 1, 13, 4, 1] . FixedPtCheck, [13, 4, 1, 13, 4, 1, 13, 4, 1]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[5, 1, 0, 5, 1, 0, 5, 1, 0] , [8, 1, 3, 8, 1, 3, 8, 1, 3] , [10, 7, 7, 10, 7, 7, 10, 7, 7] , [18, 21, 9, 18, 21, 9, 18, 21, 9] , [46, 39, 11, 46, 39, 11, 46, 39, 11] , [106, 61, 25, 106, 61, 25, 106, 61, 25] , [206, 111, 67, 206, 111, 67, 206, 111, 67]] $ $ [[1, 3, 2, 1, 3, 2, 1, 3, 2] , [4, 7, 1, 4, 7, 1, 4, 7, 1] , [14, 9, 1, 14, 9, 1, 14, 9, 1] , [30, 11, 7, 30, 11, 7, 30, 11, 7] , [50, 25, 21, 50, 25, 21, 50, 25, 21] , [86, 67, 39, 86, 67, 39, 86, 67, 39] , [178, 145, 61, 178, 145, 61, 178, 145, 61]] $ $ [[2, -1, -1, 2, -1, -1, 2, -1, -1] , [2, -3, 1, 2, -3, 1, 2, -3, 1] , [-2, -1, 3, -2, -1, 3, -2, -1, 3] , [-6, 5, 1, -6, 5, 1, -6, 5, 1] , [-2, 7, -5, -2, 7, -5, -2, 7, -5] , [10, -3, -7, 10, -3, -7, 10, -3, -7] , [14, -17, 3, 14, -17, 3, 14, -17, 3]] $

$[y_2, -y_1 - y_2, y_1, y_2, -y_1 - y_2, y_1, y_2, -y_1 - y_2, y_1]$

p’ = s + s 4 - 4s 6 p’ = s + s 5 - 6s 6 p’ = - 3s + s 2 + 16s 6 p’ = - s + s 3 + 8s 6 p = s + 5s 2 - 32s 7
S+ \ ; S- \ ; NM
$ [[9, 9, 2, 12, 10, 3, 17, 6, 8] , [14, 7, 2, 12, 12, 4, 12, 7, 6] , [12, 4, 1, 13, 12, 6, 13, 9, 6] , [13, 10, 2, 8, 4, 8, 17, 11, 3] , [12, 12, 6, 12, 1, 4, 14, 13, 2] , [14, 6, 8, 11, 10, 2, 13, 9, 3] , [16, 6, 9, 18, 11, 2, 4, 8, 2] , [12, 7, 4, 14, 13, 4, 12, 6, 4] , [12, 15, 4, 14, 3, 5, 12, 7, 4]] $ $ [[11, 10, 5, 19, 3, 4, 8, 12, 4] , [14, 9, 1, 15, 6, 4, 9, 11, 7] , [12, 5, 7, 16, 14, 2, 10, 6, 4] , [14, 6, 8, 11, 10, 2, 13, 9, 3] , [9, 6, 10, 10, 16, 1, 19, 4, 1] , [13, 10, 2, 8, 4, 8, 17, 11, 3] , [13, 9, 0, 8, 12, 7, 17, 4, 6] , [15, 11, 1, 13, 4, 7, 10, 11, 4] , [13, 10, 4, 14, 7, 3, 11, 8, 6]] $ $ [[3210, 1768, 790, 1605, 1312, 590, 1605, 1200, 760] , [2652, 2140, 777, 1908, 1070, 665, 1860, 1070, 698] , [2370, 1554, 1070, 1650, 1090, 535, 2400, 1636, 535] , [1605, 1272, 550, 3210, 1888, 980, 1605, 1120, 610] , [1968, 1070, 545, 2832, 2140, 890, 1620, 1070, 705] , [1770, 1330, 535, 2940, 1780, 1070, 1710, 1170, 535] , [1605, 1240, 800, 1605, 1080, 570, 3210, 1960, 770] , [1800,

1070, 818, 1680, 1070, 585, 2940, 2140, 737] , [2280, 1396, 535, 1830, 1410, 535, 2310, 1474, 1070]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 7	2 *vs* 9	3 *vs* 9	2 *vs* 6	3 *vs* 9

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[5, 1, 0, 5, 1, 0, 5, 1, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1, y_2, 0, y_1, y_2, 0, y_1, y_2, 0]$$

$p = -s^2 + s^6$ $p = -s^2 + s^4$ $p = -s^2 + s^5$ $p = -s^2 + s^3$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 4, 5, 6, 7, 8}}, net cycles: 2 . order: 6

$ [[1, 3, 2, 1, 3, 2, 1, 3, 2] , [2, 1, 3, 2, 1, 3, 2, 1, 3] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [1, 3, 2, 1, 3, 2, 1, 3, 2] , [2, 1, 3, 2, 1, 3, 2, 1, 3] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [1, 3, 2, 1, 3, 2, 1, 3, 2] , [2, 1, 3, 2, 1, 3, 2, 1, 3] , [3, 2, 1, 3, 2, 1, 3, 2, 1]] $

$$[y_1, y_2, y_3, y_1, y_2, y_3, y_1, y_2, y_3]$$

$p' = -1 + s^3$ $p' = -s + s^7$ $p' = -s^2 + s^8$ $p' = -s + s^4$ $p' = -1 + s^6$ $p' = -s^2 + s^5$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[x_2, x_3, x_4, x_5, -5\,x_2 + x_3 + 3\,x_4 + x_5 - x_1, x_1, -12\,x_2 - 4\,x_3 + 4\,x_4 + 4\,x_5 - 2\,x_1]$
For A+2Δ : $[y_1, -5\,y_1 - 5\,y_2 - y_3 - 5\,y_5 - y_6, 7\,y_1 + 7\,y_2 + 7\,y_5 - y_4 - y_7, y_2, y_3, y_4, y_5, y_6, y_7]$
For A-2Δ : $[-y_1 - y_5, -y_2 - y_4, -y_3 - y_6, y_1, y_2, y_3, y_5, y_4, y_6]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2]$

rank of M is 9 , rank of N is 7

M \; N

$ [[0, 0, 0, 3, 0, 0, 3, 0, 0] , [0, 0, 0, 0, 2, 0, 0, 2, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 1] , [3, 0, 0, 0, 0, 0, 3, 0, 0] , [0, 2, 0, 0, 0, 0, 0, 2, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 1] , [3, 0, 0, 3, 0, 0, 0, 0, 0] , [0, 2, 0, 0, 2, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 1, 0, 0, 0]] $ $ [[0, 186, 280, 535, 414, 480, 535, 470, 310] , [186, 0, 293, 434, 535, 405, 450, 535, 372] , [280, 293, 0, 520, 525, 535, 270, 252, 535] , [535, 434, 520, 0, 126, 90, 535, 510, 460] , [414, 535, 525, 126, 0, 180, 530, 535, 365] , [480, 405, 535, 90, 180, 0, 500, 485, 535] , [535, 450, 270, 535, 530, 500, 0, 90, 300] , [470, 535, 252, 510, 535, 485, 90, 0, 333] , [310, 372, 535, 460, 365, 535, 300, 333, 0]] $

Check is ΩΔN zero? *true*, πΔ= [2, -1, -1, 2, -1, -1, 2, -1, -1]

ker M, [0, 0, 0, 0, 0, 0, 0, 0, 0]
Range M, [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9]

τ= 27 , r'= 2/3

Ranges

Action of R on ranges, [[1], [1], [2]]
Action of B on ranges, [[2], [3], [1]]
β({1, 4, 7}) = 1/2
β({2, 5, 8}) = 1/3
β({3, 6, 9}) = 1/6

ker N, [$-\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2, -\mu_2 - \mu_1, \mu_1, \mu_2$]
Range of N
[$y_1 + y_4 + y_7 - y_2 - y_5, y_1 - y_3 + y_4 - y_6 + y_7, y_1, y_2, y_3, y_4, y_5, y_6, y_7$]

Partitions

Too many possibilities to consider

Sandwich	
Coloring	{3, 6, 9}
Rank	3
R,B	[4, 4, 5, 7, 7, 8, 1, 1, 2], [2, 9, 4, 8, 3, 7, 5, 6, 1]
π_2	[0, 0, 3, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 3, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 0, 0]
$\mathbf{u_2}$	[186, 280, 535, 414, 480, 535, 470, 310, 293, 434, 535, 405, 450, 535, 372, 520, 525, 535, 270, 252, 535, 126, 90, 535, 510, 460, 180, 530, 535, 365, 500, 485, 535, 90, 300, 333] (dim 1)
wpp	[3, 3, 3, 3, 3, 3, 3, 3, 3]
π_3	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$\mathbf{u_3}$	[323, 255, 195, 235, 303, 363, 0, 795, 447, 675, 45, 70, 165, 15, 105, 1605, 1335, 705, 110, 1227, 1047, 685, 1335, 1095, 765, 75, 225, 245, 531, 849, 489, 25, 30, 390, 75, 125, 1047, 1227, 646, 150, 1335, 1605, 606, 855, 1065, 726, 15, 470, 510, 343, 225, 765, 686, 1335, 510, 785, 726, 1065, 705, 606, 1605, 0, 105, 150, 0, 363, 303, 35, 165, 145, 45, 195, 675, 699, 430, 390, 30, 255, 375, 489, 215, 795, 849, 55]

71 . Coloring, {3, 7, 8}

R: [4, 4, 5, 7, 7, 7, 5, 6, 1] **B**: [2, 9, 4, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [-645, -172, -513, -741, -2052, -171, -2223, -228, -43] . FixedPtCheck, [645, 172, 513, 741, 2052, 171, 2223, 228, 43]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[1, 0, 0, 5, 4, 2, 6, 0, 0] , [4, 7, 0, 3, 6, 0, 11, 1, 4] , [12, 8, 2, 15, 11, 1, 9, 13, 1] , [19, 19, 5, 26, 11, 13, 27, 16, 8] , [45, 37, 21, 49, 32, 16, 50, 25, 13] , [98, 70, 32, 93, 71, 25, 97, 63, 27] , [187, 131, 57, 200, 129, 63, 189, 138, 58]] $ $ [[5, 4, 2, 1, 0, 0, 0, 4, 2] , [8, 1, 4, 9, 2, 4, 1, 7, 0] , [12, 8, 6, 9, 5, 7, 15, 3, 7] , [29, 13, 11, 22, 21, 3, 21, 16, 8] , [51, 27, 11, 47, 32, 16, 46, 39, 19] , [94, 58, 32, 99, 57, 39, 95, 65, 37] , [197, 125, 71, 184, 127, 65, 195, 118, 70]] $ $ [[-2, -2, -1, 2, 2, 1, 3, -2, -1] , [-2, 3, -2, -3, 2, -2, 5, -3, 2] , [0, 0, -2, 3, 3, -3, -3, 5, -3] , [-5, 3, -3, 2, -5, 5, 3, 0, 0] , [-3, 5, 5, 1, 0, 0, 2, -7, -3] , [2, 6, 0, -3, 7, -7, 1, -1, -5] , [-5, 3, -7, 8, 1, -1, -3, 10, -6]] $

$[-y_5 - y_6 - y_2 - y_3 - y_1, y_3, y_4, y_5, y_6, y_2, 2\,y_6 + 2\,y_2 + y_3 + y_1, -2\,y_6 - 2\,y_2 - y_3 - y_1 - y_4, y_1]$

$p = s^3 + s^4 - 8s^7$

S+ \ ; S- \ ; NM

$ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11,

4, 11, 6, 5, 14, 10, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true*, *true*, *true*

∆-Rank	**A+(1/2)∆**	**A-(1/2)∆**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	6 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 0, 5, 4, 2, 6, 0, 0] , [0, 0, 0, 1, 6, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

$$[y_1, 0, 0, y_2, y_3, 2\,y_1, y_4, 0, 0]$$

$p = s^{3} - s^{5}$

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5, y_6]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 1/2, -1/4, -7/8, 1/36, 47/72] , [0, 0, 1/2, -1/4, -2/9, 1/36] , [1/2, -1/4, -7/8, -5/16, 47/72, 49/144] , [0, 0, 1/2, -1/4, -2/9, 1/36] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 0, 5/18, -2/9]] $ x $ [[5, 4, 2, 1, 0, 0, 0, 4, 2] , [4, 7, 0, 2, 0, 0, 0, 1, 4] , [1, 8, 0, 0, 0, 0, 0, 2, 7] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

Â» SYNC’D 59/512 , 0.1152343750

72 . Coloring, {3, 7, 9}

R: [4, 4, 5, 7, 7, 7, 5, 1, 2] **B**: [2, 9, 4, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ - 1 + τ ‘)‘ ^{3} , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘

(‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [-13, -4, -9, -15, -36, -1, -39, -4, -1] . FixedPtCheck, [13, 4, 9, 15, 36, 1, 39, 4, 1]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 5	6 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[2, 1, 0, 5, 4, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[2\,y_1, y_1, 0, 3\,y_1 - y_2 + y_3, y_2, 0, y_3, 0, 0]$$

p = s 3 - s 4 p’ = - s 3 + s 4

Omega Rank for B : cycles: {{1, 2, 9}, {6, 8}}, net cycles: 1 . order: 6

$ [[4, 3, 2, 1, 0, 2, 0, 4, 2] , [2, 4, 0, 2, 0, 4, 0, 3, 3] , [3, 2, 0, 0, 0, 3, 0, 6, 4] , [4, 3, 0, 0, 0, 6, 0, 3, 2] , [2, 4, 0, 0, 0, 3, 0, 6, 3] , [3, 2, 0, 0, 0, 6, 0, 3, 4] , [4, 3, 0, 0, 0, 3, 0, 6, 2]] $

$$[y_3, -y_3 + y_1 + y_2 + y_5 + y_6 - y_4, y_1, y_2, 0, y_5, 0, y_6, y_4]$$

p = - s 3 - s 4 + s 6 + s 7

Â» SYNC’D 981/32768 , 0.02993774414

73 . Coloring, {3, 8, 9}

R: [4, 4, 5, 7, 7, 7, 1, 6, 2] **B**: [2, 9, 4, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘5 + 2τ + 8τ 2 - 2τ 3 + 3τ 4 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘‘ (‘5 + 2τ + 8τ 2 - 2τ 3 + 3τ 4 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘5 + 2τ + 8τ 2 - 2τ 3 + 3τ 4 ‘)‘‘]‘

For τ=1/2, [3302, 1016, 306, 3315, 1224, 765, 3978, 1020, 254] . FixedPtCheck, [3302, 1016, 306, 3315, 1224, 765, 3978, 1020, 254]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[3, 1, 0, 5, 1, 2, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0]] $

$$[y_1, y_3, 0, y_2, y_3, 2\,y_3, y_4, 0, 0]$$

p = s ^{2} - s ^{5} p’ = s ^{2} - s ^{5}

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_4, y_5, y_2, y_3, 0, 0, y_7, y_6]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, -11/378, 61/378, -29/378] , [0, 0, 0, 0, -29/378, -11/378, 61/378] , [0, 0, 1/3, -2/9, -11/378, -65/378, 55/378] , [0, 0, 0, 1/3, -29/378, -11/378, -65/378] , [0, 1/3, -2/9, 1/27, -65/378, 55/378, -25/378] , [0, 0, 0, 1/3, -29/378, -11/378, -65/378] , [1/3, -2/9, 1/27, -32/81, 55/378, -25/378, 253/1134] , [0, 0, 0, 0, 61/378, -29/378, -11/378] , [0, 0, 0, 0, 61/378, -29/378, -11/378]] $ x $ [[3, 3, 2, 1, 3, 0, 0, 4, 2] , [6, 3, 3, 2, 0, 0, 0, 1, 3] , [4, 6, 0, 3, 0, 0, 0, 2, 3] , [5, 4, 0, 0, 0, 0, 0, 3, 6] , [9, 5, 0, 0, 0, 0, 0, 0, 4] , [4, 9, 0, 0, 0, 0, 0, 0, 5] , [5, 4, 0, 0, 0, 0, 0, 0, 9]] $

Â» SYNC’D 6075/131072 , 0.04634857178

74 . Coloring, {4, 5, 6}

R: [4, 4, 4, 8, 3, 8, 1, 1, 1] **B**: [2, 9, 5, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-735, -196, -57, -741, -76, -171, -247, -684, -49] . FixedPtCheck, [735, 196, 57, 741, 76, 171, 247, 684, 49]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 2, 6, 0, 0, 0, 4, 0] , [2, 1, 0, 4, 3, 0, 3, 3, 2] , [8, 4, 3, 3, 5, 1, 5, 4, 3] , [12, 5, 5, 15, 8, 4, 15, 4, 4] , [23, 16, 8, 22, 12, 12, 21, 19, 11] , [51, 30, 12, 47, 35, 13, 50, 34, 16] , [100, 61, 35, 93, 66, 30, 97, 60, 34]] $ $ [[0, 4, 0, 0, 4, 2, 6, 0, 2] , [4, 3, 2, 2, 1, 2, 3, 1, 0] , [4, 4, 1, 9, 3, 3, 7, 4, 1] , [12, 11, 3, 9, 8, 4, 9, 12, 4] , [25, 16, 8, 26, 20, 4, 27, 13, 5] , [45, 34, 20, 49, 29, 19, 46, 30, 16] , [92, 67, 29, 99, 62, 34, 95, 68, 30]] $ $ [[3, -2, 1, 3, -2, -1, -3, 2, -1] , [-1, -1, -1, 1, 1, -1, 0, 1, 1] , [2, 0, 1, -3, 1, -1, -1, 0, 1] , [0, -3, 1, 3, 0, 0, 3, -4, 0] , [-1, 0, 0, -2, -4, 4, -3, 3, 3] , [3, -2, -4, -1, 3, -3, 2, 2, 0] , [4, -3, 3, -3, 2, -2, 1, -4, 2]] $

$[-3 y_2 - 3 y_3 + y_4 - y_1, 2 y_2 + 2 y_3 - y_4 - y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$

p = s ^ 3 + s ^ 4 - 8s ^ 7

S+ \ ; S- \ ; NM

$ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 4	3 *vs* 5

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$[y_1, 0, y_2, y_3, 0, 0, 0, y_4, 0]$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 5/27, -4/27, 1/54] , [0, 1/54, 5/27, -4/27] , [1/2, -4/27, 1/54, -17/54] , [0, 1/54, 5/27, -4/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27] , [0, -4/27, 1/54, 5/27]] $ x $ [[6, 0, 2, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0]] $

Omega Rank for B : cycles: {{2, 9}, {5, 7}}, net cycles: 1 . order: 2

$ [[0, 4, 0, 0, 4, 2, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2]] $

$$[0, y_2 - y_3, 0, 0, y_2 - y_1, y_1, y_2, 0, y_3]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$

Â» SYNC'D 51/2048 , 0.02490234375

75 . Coloring, {4, 5, 7}

R: [4, 4, 4, 8, 3, 7, 5, 1, 1] **B**: [2, 9, 5, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1095, 292, 513, 1425, 684, 285, 741, 1140, 73] . FixedPtCheck, [1095, 292, 513, 1425, 684, 285, 741, 1140, 73]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 6

$$[y_4, 0, y_3, y_2, y_1, 0, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 0, 19/54, -17/54, 1/54] , [0, 0, 0, 19/54, -17/54, 1/54] , [0, 0, 0, 19/54, -17/54, 1/54] , [0, 0, 0, 1/54, 19/54, -17/54] , [0, 0, 1, -17/54, 1/54, -35/54] , [1, -3, 7, -35/54, 145/54, -377/54] , [0, 1, -3, 1/54, -35/54, 145/54] , [0, 0, 0, -17/54, 1/54, 19/54] , [0, 0, 0, -17/54, 1/54, 19/54]] $ x $ [[3, 0, 2, 6, 3, 0, 1, 3, 0] , [3, 0, 3, 5, 1, 0, 0, 6, 0] , [6, 0, 1, 6, 0, 0, 0, 5, 0] , [5, 0, 0, 7, 0, 0, 0, 6, 0] , [6, 0, 0, 5, 0, 0, 0, 7, 0] , [7, 0, 0, 6, 0, 0, 0, 5, 0]] $

Omega Rank for B : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[3, 4, 0, 0, 1, 2, 5, 1, 2] , [5, 5, 0, 0, 0, 1, 1, 2, 4] , [1, 9, 0, 0, 0, 2, 0, 1, 5] , [0, 6, 0, 0, 0, 1, 0, 2, 9] , [0, 9, 0, 0, 0, 2, 0, 1, 6] , [0, 6, 0, 0, 0, 1, 0, 2, 9] , [0, 9, 0, 0, 0, 2, 0, 1, 6]] $

$$[y_2 + 4\,y_4 - y_1 - y_5, 4\,y_2 - y_3 + y_4, 0, 0, y_1, y_2, y_3, y_4, y_5]$$

p = - s^4 + s^6 p’ = s^4 - s^6

Â» SYNC’D 138339/4194304 , 0.03298258781

76 . Coloring, {4, 5, 8}

R: [4, 4, 4, 8, 3, 7, 1, 6, 1] **B**: [2, 9, 5, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-1335, -356, -285, -1482, -380, -1026, -1235, -1368, -89] . FixedPtCheck, [1335, 356, 285, 1482, 380, 1026, 1235, 1368, 89]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: 0 . order: 5

$$[y_1, 0, y_2, y_3, 0, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 907/6138, -1019/6138, 529/6138, -371/6138, 295/6138] , [0, 907/6138, -1019/6138, 529/6138, -371/6138, 295/6138] , [0, 907/6138, -1019/6138, 529/6138, -371/6138, 295/6138] , [0, 295/6138, 907/6138, -1019/6138, 529/6138, -371/6138] , [1/2, -1019/6138, 529/6138, -371/6138, 295/6138, -1081/3069] , [0, 529/6138, -371/6138, 295/6138, 907/6138, -1019/6138] , [0, -1019/6138, 529/6138, -371/6138, 295/6138, 907/6138] , [0, -371/6138, 295/6138, 907/6138, -1019/6138, 529/6138] , [0, -1019/6138, 529/6138, -371/6138, 295/6138, 907/6138]] $ x $ [[4, 0, 2, 6, 0, 2, 1, 3, 0] , [1, 0, 0, 6, 0, 3, 2, 6, 0] , [2, 0, 0, 1, 0, 6, 3, 6, 0] , [3, 0, 0, 2, 0, 6, 6, 1, 0] , [6, 0, 0, 3, 0, 1, 6, 2, 0] , [6, 0, 0, 6, 0, 2, 1, 3, 0]] $

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 0, 0, 4, 0, 5, 1, 2] , [1, 4, 0, 0, 5, 0, 4, 0, 4] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5]] $

$$[y_1 - y_4, y_2 - y_3, 0, 0, y_1, 0, y_2, y_3, y_4]$$

p’ = - s ^{3} + s ^{5} p = - s ^{3} + s ^{5}

Â» SYNC’D 351/16384 , 0.02142333984

77 . Coloring, {4, 5, 9}

R: [4, 4, 4, 8, 3, 7, 1, 1, 2] **B**: [2, 9, 5, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-481, -148, -51, -510, -68, -102, -221, -408, -37] . FixedPtCheck, [481, 148, 51, 510, 68, 102, 221, 408, 37]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -2 . order: 3

$ [[5, 1, 2, 6, 0, 0, 1, 3, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0]] $

$$[y_4, y_2, 2\,y_2, y_1, 0, 0, y_2, y_3, 0]$$

p = - s 2 + s 5 p’ = - s 2 + s 5

Omega Rank for B : cycles: {{1, 2, 9}, {6, 8}, {5, 7}}, net cycles: 3 . order: 6

$ [[1, 3, 0, 0, 4, 2, 5, 1, 2] , [2, 1, 0, 0, 5, 1, 4, 2, 3] , [3, 2, 0, 0, 4, 2, 5, 1, 1] , [1, 3, 0, 0, 5, 1, 4, 2, 2] , [2, 1, 0, 0, 4, 2, 5, 1, 3] , [3, 2, 0, 0, 5, 1, 4, 2, 1] , [1, 3, 0, 0, 4, 2, 5, 1, 2]] $

$$[-y_1 + 2\,y_2 + 2\,y_3 - y_4, y_1, 0, 0, y_2 + 2\,y_3, y_2, 2\,y_2 + y_3, y_3, y_4]$$

p = - s - s 2 + s 4 + s 5 p = s - s 3 - s 4 + s 6 p = - s + s 7

Â» SYNC’D 15525/1048576 , 0.01480579376

78 . Coloring, {4, 6, 7}

R: [4, 4, 4, 8, 7, 8, 5, 1, 1] **B**: [2, 9, 5, 7, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘27‘ (‘ 5 + 3τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -27‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 3τ 2 ‘)‘‘]‘

For τ=1/2, [-690, -184, -114, -741, -456, -171, -570, -684, -46] . FixedPtCheck, [690, 184, 114, 741, 456, 171, 570, 684, 46]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 2 . order: 6

$ [[3, 0, 0, 6, 3, 0, 2, 4, 0] , [4, 0, 0, 3, 2, 0, 3, 6, 0] , [6, 0, 0, 4, 3, 0, 2, 3, 0] , [3, 0, 0, 6, 2, 0, 3, 4, 0] , [4, 0, 0, 3, 3, 0, 2, 6, 0]] $

$$[-5\, y_1 + 13\, y_2 + 13\, y_3 - 5\, y_4, 0, 0, 5\, y_1, 5\, y_2, 0, 5\, y_3, 5\, y_4, 0]$$

$p = - s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 1 . order: 4

$ [[3, 4, 2, 0, 1, 2, 4, 0, 2] , [4, 5, 1, 0, 2, 0, 2, 0, 4] , [2, 8, 2, 0, 1, 0, 0, 0, 5] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7]] $

$$[2\, y_1 + 3\, y_2 - y_3 - y_5, 3\, y_1 + 2\, y_2 - y_4, y_1, 0, y_2, y_3, y_4, 0, y_5]$$

$p = - s^4 + s^6$ $p' = - s^4 + s^6$

Â» SYNC’D 7031/524288 , 0.01341056824

79 . Coloring, {4, 6, 8}

R: [4, 4, 4, 8, 7, 8, 1, 6, 1] **B**: [2, 9, 5, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ ‘)‘‘]‘

For τ=1/2, [-405, -108, -19, -399, -76, -513, -285, -684, -27] . FixedPtCheck, [405, 108, 19, 399, 76, 513, 285, 684, 27]

$\det(A + \tau \Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[4, 0, 0, 6, 0, 2, 2, 4, 0] , [1, 2, 2, 2, 3, 2, 0, 4, 2] , [2, 5, 1, 5, 6, 4, 7, 4, 2] , [13, 12, 2, 8, 8, 4, 13, 9, 3] , [23, 16, 8, 27, 17, 9, 28, 12, 4] , [52, 37, 15, 47, 28, 12, 45, 36, 16] , [89, 60, 36, 104, 68, 36, 97, 59, 27]] $ $ [[2, 4, 2, 0, 4, 0, 4, 0, 2] , [5, 2, 0, 4, 1, 0, 6, 0, 0] , [10, 3, 3, 7, 2, 0, 5, 4, 2] , [11, 4, 6, 16, 8, 4, 11, 7, 5] , [25, 16, 8, 21, 15, 7, 20, 20, 12] , [44, 27, 17, 49, 36, 20, 51, 28, 16] , [103, 68, 28, 88, 60, 28, 95, 69, 37]] $ $ [[1, -2, -1, 3, -2, 1, -1, 2, -1] , [-2, 0, 1, -1, 1, 1, -3, 2, 1] , [-4, 1, -1, -1, 2, 2, 1, 0, 0] , [1, 4, -2, -4, 0, 0, 1, 1, -1] , [-1, 0, 0, 3, 1, 1, 4, -4, -4] , [4, 5, -1, -1, -4, -4, -3, 4, 0] , [-7, -4, 4, 8, 4, 4, 1, -5, -5]] $

$[-y_3 - 2 y_6 - y_1 - 3 y_2 + y_4, y_3 + 2 y_6 + 2 y_2 - 2 y_4 - y_5, -y_3 - y_6, y_1, y_2, y_4, y_3, y_6, y_5]$

$p = s^2 - 2s^4 + 8s^5 - 16s^7$

S+ \; S- \; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 5	3 *vs* 6

Omega Rank for R : cycles: {{6, 8}}, net cycles: 0 . order: 4

$[y_1, 0, 0, y_2, 0, y_3, y_4, y_5, 0]$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1/2, -2/9, -2/9] , [0, 0, 1/2, -2/9, -2/9] , [0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 5/18, -2/9] , [1/2, -1, 1/2, 7/9, -13/18] , [0, 0, 0, 5/18, -2/9] , [0, 1/2, -1, -2/9, 7/9] , [0, 0, 0, -2/9, 5/18] , [0, 1/2, -1, -2/9, 7/9]] $ x $ [[4, 0, 0, 6, 0, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 0, 8, 0] , [0, 0, 0, 2, 0, 8, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

Omega Rank for B : cycles: {{2, 9}, {3, 5}}, net cycles: 0 . order: 2

$ [[2, 4, 2, 0, 4, 0, 4, 0, 2] , [0, 4, 4, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 6, 0, 0, 0, 4]] $

$$[-5\,y_3 + 2\,y_1 + 2\,y_2, -4\,y_3 + 2\,y_1 + 2\,y_2, y_1, 0, y_2, 0, -10\,y_3 + 4\,y_1 + 4\,y_2, 0, y_3]$$

p' = $s^2 - s^4$ p' = $-s^3 + s^5$ p = $s^2 - s^4$

Â» SYNC'D 5/512 , 0.009765625000

80 . Coloring, {4, 6, 9}

R: [4, 4, 4, 8, 7, 8, 1, 1, 2] **B**: [2, 9, 5, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ^2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{3} , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ^2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [39, 12, 1, 39, 4, 9, 15, 36, 3] . FixedPtCheck, [39, 12, 1, 39, 4, 9, 15, 36, 3]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	2 *vs* 5	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 6, 0, 0, 2, 4, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[y_1 + y_2, y_1, 0, 2\,y_1 + y_2, 0, 0, 2\,y_1, y_2, 0]$$

p' = $-s^3 + s^4$ p = $s^2 - s^4$ p' = $s^2 - s^3$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[1, 3, 2, 0, 4, 2, 4, 0, 2] , [2, 1, 4, 0, 6, 0, 2, 0, 3] , [3, 2, 6, 0, 6, 0, 0, 0, 1] , [1, 3, 6, 0, 6, 0, 0, 0, 2] , [2, 1, 6, 0, 6, 0, 0, 0, 3] , [3, 2, 6, 0, 6, 0, 0, 0, 1] , [1, 3, 6, 0, 6, 0, 0, 0, 2]] $

$$[y_2, -y_2 + y_1 + y_3 - y_5, y_1, 0, y_4, -y_4 + y_1 + y_3, y_3, 0, y_5]$$

p = s ^ 3 - s ^ 6 p' = s ^ 3 - s ^ 6

Â» SYNC'D 7695/524288 , 0.01467704773

81 . Coloring, {4, 7, 8}

R: [4, 4, 4, 8, 7, 7, 5, 6, 1] **B**: [2, 9, 5, 7, 3, 8, 1, 1, 2]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

'['-9' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')'' (' - 3 + τ ')'' (' 1 + τ ')' , -18' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')' ^ 2 , 9' (' 1 + τ ^ 2 ')'' (' - 5 + τ ^ 2 ')'' (' - 1 + τ ')'' (' 1 + τ ')' , 9' (' - 5 + τ ^ 2 ')'' (' 3 + τ ^ 2 ')'' (' - 1 + τ ')'' (' 1 + τ ')' , -18' (' 1 + τ ^ 2 ')'' (' - 5 + τ ^ 2 ')'' (' 1 + τ ')' , 9' (' - 5 + τ ^ 2 ')'' (' - 1 + τ ')'' (' 1 + τ ')' ^ 3 , 9' (' 1 + τ ^ 2 ')'' (' - 5 + τ ^ 2 ')'' (' - 3 + τ ')'' (' 1 + τ ')' , 18' (' - 5 + τ ^ 2 ')'' (' - 1 + τ ')'' (' 1 + τ ')' ^ 2 , 9' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')' ^ 3 ']'

For τ=1/2, [555, 148, 285, 741, 1140, 513, 1425, 684, 37] . FixedPtCheck, [555, 148, 285, 741, 1140, 513, 1425, 684, 37]

det(A + τ Δ) = 1' (' τ ')' ^ 2 ' (' - 1 + τ ')' ^ 3 ' (' 1 + τ ')' ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -6, 33, 443/72, -2455/72] , [0, 1, -6, 33, 443/72, -2455/72] , [0, 1, -6, 33, 443/72, -2455/72] , [0, 0, 1, -6, -79/72, 443/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 1, 11/72, -79/72] , [1, -6, 33, -182, -2455/72, 13547/72]] $ x $ [[1, 0, 0, 6, 3, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 5, 6, 0] , [0, 0, 0, 0, 5, 6, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

Omega Rank for B : cycles: {{3, 5}, {2, 9}}, net cycles: 0 . order: 4

$ [[5, 4, 2, 0, 1, 0, 3, 1, 2] , [4, 7, 1, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7]] $

$$[y_4, 3\,y_1 + 2\,y_2 - 4\,y_3, y_1, 0, y_2, 0, 3\,y_3, y_3, -y_4 + 2\,y_1 + 3\,y_2]$$

$p = -s^3 + s^5$ $p = -s^3 + s^7$ $p' = -s^3 + s^5$

Â» SYNC'D 213/8192 , 0.02600097656

82 . Coloring, {4, 7, 9}

R: [4, 4, 4, 8, 7, 7, 5, 1, 2] **B**: [2, 9, 5, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-13, -4, -3, -15, -12, -3, -15, -12, -1] . FixedPtCheck, [13, 4, 3, 15, 12, 3, 15, 12, 1]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 8	8 *vs* 8	4 *vs* 6	5 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 1 . order: 6

$ [[2, 1, 0, 6, 3, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 3, 6, 0] , [6, 0, 0, 3, 3, 0, 3, 3, 0] , [3, 0, 0, 6, 3, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 3, 6, 0] , [6, 0, 0, 3, 3, 0, 3, 3, 0]] $

$$[-y_1 - y_2 + 4\,y_3 - y_4, y_1, 0, y_2, y_3, 0, y_3, y_4, 0]$$

$p = -s^2 + s^5$ $p' = -s^2 + s^5$

Omega Rank for B : cycles: {{3, 5}, {6, 8}, {1, 2, 9}}, net cycles: 2 . order: 6

$ [[4, 3, 2, 0, 1, 2, 3, 1, 2] , [5, 4, 1, 0, 2, 1, 0, 2, 3] , [3, 5, 2, 0, 1, 2, 0, 1, 4] , [4, 3, 1, 0, 2, 1, 0, 2, 5] , [5, 4, 2, 0, 1, 2, 0, 1, 3] , [3, 5, 1, 0, 2, 1, 0, 2, 4] , [4, 3, 2, 0, 1, 2, 0, 1, 5] , [5, 4, 1, 0, 2, 1, 0, 2, 3]] $

$$[-y_1 + 4\,y_2 - y_3 + 4\,y_4 - y_5, y_1, y_2, 0, y_4, y_2, y_3, y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$

Â» SYNC'D 210735/16777216 , 0.01256078482

83 . Coloring, {4, 8, 9}

R: [4, 4, 4, 8, 7, 7, 1, 6, 2] **B**: [2, 9, 5, 7, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-1287, -396, -85, -1326, -340, -918, -1275, -1224, -99] . FixedPtCheck, [1287, 396, 85, 1326, 340, 918, 1275, 1224, 99]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	3 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: 0 . order: 5

$$[y_1, y_2, 0, y_3, 0, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -155/1278, 79/1278, -65/1278, -173/1278, 385/1278] , [0, -155/1278, 79/1278, -65/1278, -173/1278, 385/1278] , [0, -155/1278, 79/1278, -65/1278, -173/1278, 385/1278] , [0, 385/1278, -155/1278, 79/1278, -65/1278, -173/1278] , [0, -65/1278, -173/1278, 385/1278, -155/1278, 79/1278] , [0, -65/1278, -173/1278, 385/1278, -155/1278, 79/1278] , [0, 79/1278, -65/1278, -173/1278, 385/1278, -155/1278] , [0, -173/1278, 385/1278, -155/1278, 79/1278, -65/1278] , [1, 79/1278, -65/1278, -173/1278, 385/1278, -1433/1278]] $ x $ [[3, 1, 0, 6, 0, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 2, 6, 0] , [2, 0, 0, 3, 0, 6, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 6, 3, 0] , [6, 0, 0, 3, 0, 3, 4, 2, 0] , [4, 0, 0, 6, 0, 2, 3, 3, 0]] $

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 0 . order: 6

$ [[3, 3, 2, 0, 4, 0, 3, 1, 2] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3]] $

$$[y_2 + y_3, y_2 + y_3, 3\,y_3 - y_1, 0, y_1, 0, 3\,y_2, y_2, y_3]$$

p = - s ^ 2 + s ^ 4 p = - s ^ 2 + s ^ 6 p’ = - s ^ 2 + s ^ 4 p’ = - s ^ 2 + s ^ 6

Â» SYNC’D 90255/4194304 , 0.02151846886

84 . Coloring, {5, 6, 7}

R: [4, 4, 4, 7, 3, 8, 5, 1, 1] **B**: [2, 9, 5, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-375, -100, -513, -741, -684, -57, -741, -228, -25] . FixedPtCheck, [375, 100, 513, 741, 684, 57, 741, 228, 25]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$$[y_1, 0, y_5, y_6, y_4, 0, y_3, y_2, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, -17/72, 19/72, 1/72, 1/72] , [1, -3, 19/72, 1/72, -71/72, 199/72] , [0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 1, -17/72, 19/72, 1/72, -71/72] , [0, 1, -17/72, 19/72, 1/72, -71/72]] $ x $ [[3, 0, 2, 6, 3, 0, 3, 1, 0] , [1, 0, 3, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5,

0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 6

$ [[3, 4, 0, 0, 1, 2, 3, 3, 2] , [3, 5, 0, 0, 0, 3, 3, 0, 4] , [3, 7, 0, 0, 0, 0, 3, 0, 5] , [3, 8, 0, 0, 0, 0, 0, 0, 7] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

$$[y_6, y_5, 0, 0, y_4, y_3, y_1, 3\, y_4, y_2]$$

p = - s ^{5} + s ^{7}

Â» SYNC'D 10359/524288 , 0.01975822449

85 . Coloring, {5, 6, 8}

R: [4, 4, 4, 7, 3, 8, 1, 6, 1] **B**: [2, 9, 5, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-705, -188, -171, -798, -228, -342, -741, -456, -47] . FixedPtCheck, [705, 188, 171, 798, 228, 342, 741, 456, 47]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[4, 0, 2, 6, 0, 2, 3, 1, 0] , [3, 0, 0, 6, 0, 1, 6, 2, 0] , [6, 0, 0, 3, 0, 2, 6, 1, 0] , [6, 0, 0, 6, 0, 1, 3, 2, 0] , [3, 0, 0, 6, 0, 2, 6, 1, 0] , [6, 0, 0, 3, 0, 1, 6, 2, 0]] $

$$[-y_1 - y_2 + 5\, y_3 - y_4 + 5\, y_5, 0, y_1, y_2, 0, y_3, y_4, y_5, 0]$$

p = s ^{2} + s ^{3} - s ^{5} - s ^{6}

Omega Rank for B : cycles: {{2, 9}, {5, 7}}, net cycles: 1 . order: 4

$ [[2, 4, 0, 0, 4, 0, 3, 3, 2] , [3, 4, 0, 0, 3, 0, 4, 0, 4] , [0, 7, 0, 0, 4, 0, 3, 0, 4] , [0, 4, 0, 0, 3, 0, 4, 0, 7] , [0, 7, 0, 0, 4, 0, 3, 0, 4] , [0, 4, 0, 0, 3, 0, 4, 0, 7]] $

$$[5\,y_1, -16\,y_1 + 33\,y_2 - 5\,y_3 - 16\,y_4, 0, 0, -7\,y_1 + 16\,y_2 - 7\,y_4, 0, 5\,y_2, 5\,y_3, 5\,y_4]$$

p = - s ^ 3 + s ^ 5 p’ = - s ^ 3 + s ^ 5

Â» SYNC’D 2439/131072 , 0.01860809326

86 . Coloring, {5, 6, 9}

R: [4, 4, 4, 7, 3, 8, 1, 1, 2] **B**: [2, 9, 5, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1157, 356, 255, 1326, 340, 102, 1105, 408, 89] . FixedPtCheck, [1157, 356, 255, 1326, 340, 102, 1105, 408, 89]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[5, 1, 2, 6, 0, 0, 3, 1, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] $

$$[y_4, y_3, 2\,y_3, y_2, 0, 0, y_1, y_3, 0]$$

p’ = s ^ 2 - s ^ 5 p = s ^ 2 - s ^ 5

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 1 . order: 6

$ [[1, 3, 0, 0, 4, 2, 3, 3, 2] , [2, 1, 0, 0, 3, 3, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2]] $

$$[y_1, -y_1 + y_5 + y_4 - y_2, 0, 0, y_5, y_4, y_3, y_5 + y_4 - y_3, y_2]$$

p' = - s 3 + s 6 p = - s 3 + s 6

Â» SYNC'D 915/32768 , 0.02792358398

87 . Coloring, {5, 7, 8}

R: [4, 4, 4, 7, 3, 7, 5, 6, 1] **B**: [2, 9, 5, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 4 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [-1470, -392, -3078, -3705, -4104, -855, -4446, -1140, -98] . FixedPtCheck, [1470, 392, 3078, 3705, 4104, 855, 4446, 1140, 98]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 2, 6, 3, 2, 4, 0, 0] , [0, 0, 3, 3, 4, 0, 8, 0, 0] , [0, 0, 4, 3, 8, 0, 3, 0, 0] , [0, 0, 8, 4, 3, 0, 3, 0, 0] , [0, 0, 3, 8, 3, 0, 4, 0, 0] , [0, 0, 3, 3, 4, 0, 8, 0, 0]] $

$$[y_1, 0, y_2, y_3, y_4, 2\,y_1, y_5, 0, 0]$$

p = - s 2 + s 6

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 0, 1, 0, 2, 4, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 4\,y_3, y_5]$$

p = - s ^{4} + s ^{6}

Â» SYNC'D 299/4096 , 0.07299804688

88 . Coloring, {5, 7, 9}

R: [4, 4, 4, 7, 3, 7, 5, 1, 2] **B**: [2, 9, 5, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 3τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 3τ ^{2} ‘)‘‘]‘

For τ=1/2, [-598, -184, -918, -1275, -1224, -85, -1326, -340, -46] . FixedPtCheck, [598, 184, 918, 1275, 1224, 85, 1326, 340, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[2, 1, 2, 6, 3, 0, 4, 0, 0] , [0, 0, 3, 5, 4, 0, 6, 0, 0] , [0, 0, 4, 3, 6, 0, 5, 0, 0] , [0, 0, 6, 4, 5, 0, 3, 0, 0] , [0, 0, 5, 6, 3, 0, 4, 0, 0] , [0, 0, 3, 5, 4, 0, 6, 0, 0]] $

$$[2\,y_1, y_1, -3\,y_1 + y_2 + y_3 - y_4, y_2, y_3, 0, y_4, 0, 0]$$

p = - s ^{2} + s ^{3} - s ^{4} + s ^{5} p = - s ^{2} + s ^{6}

Omega Rank for B : cycles: {{1, 2, 9}, {6, 8}}, net cycles: 1 . order: 6

$ [[4, 3, 0, 0, 1, 2, 2, 4, 2] , [4, 4, 0, 0, 0, 4, 1, 2, 3] , [4, 4, 0, 0, 0, 2, 0, 4, 4] , [4, 4, 0, 0, 0, 4, 0, 2, 4] , [4, 4, 0, 0, 0, 2, 0, 4, 4] , [4, 4, 0, 0, 0, 4, 0, 2, 4] , [4, 4, 0, 0, 0, 2, 0, 4, 4]] $

$$[2 y_2 + 2 y_4, -2 y_1 + 2 y_2 + 2 y_4, 0, 0, 2 y_1, 3 y_2 + 3 y_4 - 2 y_3, 2 y_2, 2 y_3, 2 y_4]$$

p = - s ^{3} + s ^{7} p = - s ^{3} + s ^{5} p’ = - s ^{3} + s ^{5}

Â» SYNC’D 903/65536 , 0.01377868652

89 . Coloring, {5, 8, 9}

R: [4, 4, 4, 7, 3, 7, 1, 6, 2] **B**: [2, 9, 5, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [559, 172, 153, 663, 204, 153, 663, 204, 43] . FixedPtCheck, [559, 172, 153, 663, 204, 153, 663, 204, 43]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}}, net cycles: -2 . order: 3

$ [[3, 1, 2, 6, 0, 2, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0] , [6, 0, 0, 8, 0, 0, 4, 0, 0] , [4, 0, 0, 6, 0, 0, 8, 0, 0] , [8, 0, 0, 4, 0, 0, 6, 0, 0]] $

$$[y_4, y_3, 2 y_3, y_2, 0, 2 y_3, y_1, 0, 0]$$

p’ = - s ^{2} + s ^{5} p = - s ^{2} + s ^{5}

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 0, 4, 0, 2, 4, 2] , [6, 3, 0, 0, 2, 0, 4, 0, 3] , [3, 6, 0, 0, 4, 0, 2, 0, 3] , [3, 3, 0, 0, 2, 0, 4, 0, 6] , [6, 3, 0, 0, 4, 0, 2, 0, 3] , [3, 6, 0, 0, 2, 0, 4, 0, 3]] $

$$[y_5, y_4, 0, 0, y_3, 0, y_2, y_1, -y_5 - y_4 + 2\,y_3 + 2\,y_2 - y_1]$$

$p = -s^2 - s^3 + s^5 + s^6$

Â» SYNC'D 3045/65536 , 0.04646301270

90 . Coloring, {6, 7, 8}

R: [4, 4, 4, 7, 7, 8, 5, 6, 1] **B**: [2, 9, 5, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [-15, -4, -9, -21, -36, -9, -45, -12, -1] . FixedPtCheck, [15, 4, 9, 21, 36, 9, 45, 12, 1]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 6, 3, 2, 5, 1, 0] , [0, 0, 0, 1, 5, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0]] $

$$[y_3 - y_4 + 4\,y_2, 0, 0, y_1, -y_1 + 4\,y_3 + y_2, y_3, y_4, y_2, 0]$$

$p' = -s^3 + s^5$ $p = s^3 - s^5$

Omega Rank for B : cycles: {{3, 5}, {2, 9}}, net cycles: 0 . order: 4

$ [[5, 4, 2, 0, 1, 0, 1, 3, 2] , [4, 7, 1, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7]] $

$$[y_4, 3\,y_3 + 2\,y_2 - 4\,y_1, y_3, 0, y_2, 0, y_1, 3\,y_1, -y_4 + 2\,y_3 + 3\,y_2]$$

p' = s ^ 4 - s ^ 6 p' = s ^ 3 - s ^ 5 p = s ^ 3 - s ^ 7

Â» SYNC'D 2865/262144 , 0.01092910767

91 . Coloring, {6, 7, 9}

R: [4, 4, 4, 7, 7, 8, 5, 1, 2] **B**: [2, 9, 5, 8, 3, 7, 1, 6, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

'['-9' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' 3 + τ ^ 2 ')'' (' - 1 + τ ')' , 18' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')' ^ 2 , 9' (' 1 + τ ')'' (' 1 + τ ^ 2 ')'' (' 5 - 2τ + τ ^ 2 ')'' (' - 1 + τ ')' , 9' (' 1 + τ ')'' (' 3 + τ ^ 2 ')'' (' 5 - 2τ + τ ^ 2 ')'' (' - 1 + τ ')' , -18' (' 1 + τ ')'' (' 1 + τ ^ 2 ')'' (' 5 - 2τ + τ ^ 2 ')' , 9' (' 1 + τ ')'' (' 5 - 2τ + τ ^ 2 ')'' (' - 1 + τ ')' ^ 3 , 9' (' 1 + τ ')'' (' 1 + τ ^ 2 ')'' (' 5 - 2τ + τ ^ 2 ')'' (' - 3 + τ ')' , -18' (' 1 + τ ')'' (' 5 - 2τ + τ ^ 2 ')'' (' - 1 + τ ')' ^ 2 , -9' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')' ^ 3 ']'

For τ=1/2, [-481, -148, -255, -663, -1020, -51, -1275, -204, -37] . FixedPtCheck, [481, 148, 255, 663, 1020, 51, 1275, 204, 37]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 8	8 *vs* 8	5 *vs* 6	7 *vs* 8

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[2, 1, 0, 6, 3, 0, 5, 1, 0] , [1, 0, 0, 3, 5, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

$$[y_1, y_5, 0, y_2, y_3, 0, y_4, y_5, 0]$$

p = s ^ 4 - s ^ 6

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 1 . order: 6

$ [[4, 3, 2, 0, 1, 2, 1, 3, 2] , [3, 4, 1, 0, 2, 3, 2, 0, 3] , [5, 3, 2, 0, 1, 0, 3, 0, 4] , [7, 5, 1, 0, 2, 0, 0, 0, 3] , [3, 7, 2, 0, 1, 0, 0, 0, 5] , [5, 3, 1, 0, 2, 0, 0, 0, 7] , [7, 5, 2, 0, 1, 0, 0, 0, 3] , [3, 7, 1, 0, 2, 0, 0, 0, 5]] $

$$[-y_1 + 5\,y_3 + 5\,y_2 - y_5 - y_6 - y_7 - y_4, y_1, y_3, 0, y_2, y_5, y_6, y_7, y_4]$$

$p = -s^4 - s^5 + s^7 + s^8$

Â» SYNC'D 285713/8388608 , 0.03405964375

92 . Coloring, {6, 8, 9}

R: [4, 4, 4, 7, 7, 8, 1, 6, 2] **B**: [2, 9, 5, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 5 - 3τ - τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - 3τ - τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 - 3τ - τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-689, -212, -51, -714, -204, -306, -765, -408, -53] . FixedPtCheck, [689, 212, 51, 714, 204, 306, 765, 408, 53]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 1 . order: 6

$ [[3, 1, 0, 6, 0, 2, 5, 1, 0] , [5, 0, 0, 4, 0, 1, 6, 2, 0] , [6, 0, 0, 5, 0, 2, 4, 1, 0] , [4, 0, 0, 6, 0, 1, 5, 2, 0] , [5, 0, 0, 4, 0, 2, 6, 1, 0] , [6, 0, 0, 5, 0, 1, 4, 2, 0]] $

$$[-y_1 - y_2 + 5\,y_3 - y_4 + 5\,y_5, y_1, 0, y_2, 0, y_3, y_4, y_5, 0]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 0 . order: 6

$ [[3, 3, 2, 0, 4, 0, 1, 3, 2] , [5, 3, 4, 0, 3, 0, 0, 0, 3] , [3, 5, 3, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 3, 0, 0, 0, 5] , [5, 3, 3, 0, 4, 0, 0, 0, 3] , [3, 5, 4, 0, 3, 0, 0, 0, 3] , [3, 3, 3, 0, 4, 0, 0, 0, 5]] $

$$[-7\,y_1 + 11\,y_2 + 11\,y_3 - 10\,y_4 - 7\,y_5, 7\,y_1, 7\,y_2, 0, 7\,y_3, 0, 7\,y_4, 21\,y_4, 7\,y_5]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Â» SYNC’D 181071/4194304 , 0.04317069054

93 . Coloring, {7, 8, 9}

R: [4, 4, 4, 7, 7, 7, 5, 6, 2] **B**: [2, 9, 5, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-26, -8, -18, -39, -72, -9, -90, -12, -2] . FixedPtCheck, [26, 8, 18, 39, 72, 9, 90, 12, 2]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[0, 1, 0, 6, 3, 2, 6, 0, 0] , [0, 0, 0, 1, 6, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

$$[0, y_1, 0, y_3, y_4, 2\,y_1, y_2, 0, 0]$$

$p = s^3 - s^5$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 1 . order: 6

$ [[6, 3, 2, 0, 1, 0, 0, 4, 2] , [6, 6, 1, 0, 2, 0, 0, 0, 3] , [3, 6, 2, 0, 1, 0, 0, 0, 6] , [6, 3, 1, 0, 2, 0, 0, 0, 6] , [6, 6, 2, 0, 1, 0, 0, 0, 3] , [3, 6, 1, 0, 2, 0, 0, 0, 6]] $

$$[y_5, y_4, y_3, 0, y_2, 0, 0, y_1, -y_5 - y_4 + 5\,y_3 + 5\,y_2 - y_1]$$

$p = -s^2 - s^3 + s^5 + s^6$

Â» SYNC’D 59/512 , 0.1152343750

94 . Coloring, {2, 3, 4, 5}

R: [4, 9, 5, 8, 3, 7, 1, 1, 1] **B**: [2, 4, 4, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-741, -228, -129, -645, -172, -129, -301, -516, -171] . FixedPtCheck, [741, 228, 129, 645, 172, 129, 301, 516, 171]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 1 + τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 0 . order: 6

$ [[6, 0, 2, 3, 1, 0, 1, 3, 2] , [6, 0, 1, 6, 2, 0, 0, 3, 0] , [3, 0, 2, 6, 1, 0, 0, 6, 0] , [6, 0, 1, 3, 2, 0, 0, 6, 0] , [6, 0, 2, 6, 1, 0, 0, 3, 0] , [3, 0, 1, 6, 2, 0, 0, 6, 0] , [6, 0, 2, 3, 1, 0, 0, 6, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - 3\,y_4 - y_5, 0, y_1, y_2, y_3, 0, y_4, y_5, 2\,y_4]$$

p = - s 2 - s 3 + s 5 + s 6 p = s 2 - s 4 - s 5 + s 7

Omega Rank for B : cycles: {{5, 7}, {6, 8}}, net cycles: 1 . order: 4

$ [[0, 4, 0, 3, 3, 2, 5, 1, 0] , [0, 0, 0, 4, 5, 1, 6, 2, 0] , [0, 0, 0, 0, 6, 2, 9, 1, 0] , [0, 0, 0, 0, 9, 1, 6, 2, 0] , [0, 0, 0, 0, 6, 2, 9, 1, 0] , [0, 0, 0, 0, 9, 1, 6, 2, 0]] $

$$[0, 4\,y_2 - y_3 + y_4, 0, -y_1 + y_2 + 4\,y_4, y_1, y_2, y_3, y_4, 0]$$

p = - s 3 + s 5 p’ = - s 3 + s 5

Â» SYNC’D 10567/524288 , 0.02015495300

95 . Coloring, {2, 3, 4, 6}

R: [4, 9, 5, 8, 7, 8, 1, 1, 1] **B**: [2, 4, 4, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{3} , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [78, 24, 2, 65, 8, 15, 26, 60, 18] . FixedPtCheck, [78, 24, 2, 65, 8, 15, 26, 60, 18]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 0, 3, 1, 0, 2, 4, 2] , [8, 0, 0, 6, 0, 0, 1, 3, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0]] $

$$[y_1, 0, 0, y_2, y_5, 0, y_3, y_4, 2\, y_5]$$

p = - s ^{3} + s ^{6}

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[0, 4, 2, 3, 3, 2, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0] , [0, 0, 4, 3, 5, 0, 6, 0, 0] , [0, 0, 5, 4, 6, 0, 3, 0, 0] , [0, 0, 6, 5, 3, 0, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0]] $

$$[0, 2\, y_4, y_1, y_2, y_3, y_4, y_5, 0, 0]$$

p = - s ^{2} + s ^{6}

Â» SYNC’D 945/32768 , 0.02883911133

96 . Coloring, {2, 3, 4, 7}

R: [4, 9, 5, 8, 7, 7, 5, 1, 1] **B**: [2, 4, 4, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [481, 148, 129, 430, 516, 86, 559, 344, 111] . FixedPtCheck, [481, 148, 129, 430, 516, 86, 559, 344, 111]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[3, 0, 0, 3, 4, 0, 3, 3, 2] , [5, 0, 0, 3, 3, 0, 4, 3, 0] , [3, 0, 0, 5, 4, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 4, 5, 0] , [5, 0, 0, 3, 4, 0, 3, 3, 0] , [3, 0, 0, 5, 3, 0, 4, 3, 0]] $

$$[7 y_2, 0, 0, -7 y_2 + 11 y_1 + 11 y_3 - 7 y_5 - 7 y_4, 7 y_1, 0, 7 y_3, 7 y_5, 7 y_4]$$

p = s ^ 2 + s ^ 3 - s ^ 5 - s ^ 6

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}}, net cycles: 1 . order: 4

$ [[3, 4, 2, 3, 0, 2, 3, 1, 0] , [3, 3, 0, 6, 0, 1, 3, 2, 0] , [3, 3, 0, 3, 0, 2, 6, 1, 0] , [6, 3, 0, 3, 0, 1, 3, 2, 0] , [3, 6, 0, 3, 0, 2, 3, 1, 0] , [3, 3, 0, 6, 0, 1, 3, 2, 0] , [3, 3, 0, 3, 0, 2, 6, 1, 0]] $

$$[y_4, y_3, y_2, y_1, 0, y_4 + y_1 - 4 y_5, 4 y_4 - y_3 - y_2 + 4 y_1 - 15 y_5, y_5, 0]$$

p = - s ^ 2 + s ^ 6 p’ = - s ^ 2 + s ^ 6

Â» SYNC’D 48475/2097152 , 0.02311468124

97 . Coloring, {2, 3, 4, 8}

R: [4, 9, 5, 8, 7, 7, 1, 6, 1] **B**: [2, 4, 4, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [663, 204, 43, 559, 172, 387, 559, 516, 153] . FixedPtCheck, [663, 204, 43, 559, 172, 387, 559, 516, 153]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: -1 . order: 5

$ [[4, 0, 0, 3, 1, 2, 3, 3, 2] , [5, 0, 0, 4, 0, 3, 3, 3, 0] , [3, 0, 0, 5, 0, 3, 3, 4, 0] , [3, 0, 0, 3, 0, 4, 3, 5, 0] , [3, 0, 0, 3, 0, 5, 4, 3, 0] , [4, 0, 0, 3, 0, 3, 5, 3, 0] , [5, 0, 0, 4, 0, 3, 3, 3, 0]] $

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, 2\, y_3]$$

p = - s ^ 2 + s ^ 7

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$$[y_1, y_5, y_2, y_3, y_4, 0, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -17/72, 19/72, 1/72, -71/72] , [0, 0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 0, -17/72, 19/72, 1/72, 1/72] , [1, -2, 0, 1/72, -71/72, 127/72, 19/72] , [0, 0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 1, -2, 19/72, 1/72, -71/72, 127/72] , [0, 0, 1, -17/72, 19/72, 1/72, -71/72]] $ x $ [[2, 4, 2, 3, 3, 0, 3, 1, 0] , [1, 2, 3, 6, 3, 0, 3, 0, 0] , [0, 1, 3, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0]] $

Â» SYNC’D 306315/16777216 , 0.01825779676

98 . Coloring, {2, 3, 4, 9}

R: [4, 9, 5, 8, 7, 7, 1, 1, 2] **B**: [2, 4, 4, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-693, -396, -25, -625, -100, -125, -325, -500, -297] . FixedPtCheck, [693, 396, 25, 625, 100, 125, 325, 500, 297]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 3, 1, 0, 3, 3, 2] , [6, 2, 0, 5, 0, 0, 1, 3, 1] , [4, 1, 0, 6, 0, 0, 0, 5, 2] , [5, 2, 0, 4, 0, 0, 0, 6, 1] , [6, 1, 0, 5, 0, 0, 0, 4, 2] , [4, 2, 0, 6, 0, 0, 0, 5, 1] , [5, 1, 0, 4, 0, 0, 0, 6, 2]] $

$$[y_2, y_3, 0, -y_2 + 5\,y_3 - y_1 - y_6 - y_4 + 5\,y_5, y_1, 0, y_6, y_4, y_5]$$

p = - s ^ 3 - s ^ 4 + s ^ 6 + s ^ 7

Omega Rank for B : cycles: {{3, 4, 5, 7}, {6, 8}}, net cycles: 1 . order: 4

$ [[1, 3, 2, 3, 3, 2, 3, 1, 0] , [0, 1, 3, 5, 3, 1, 3, 2, 0] , [0, 0, 3, 4, 3, 2, 5, 1, 0] , [0, 0, 3, 3, 5, 1, 4, 2, 0] , [0, 0, 5, 3, 4, 2, 3, 1, 0] , [0, 0, 4, 5, 3, 1, 3, 2, 0] , [0, 0, 3, 4, 3, 2, 5, 1, 0] , [0, 0, 3, 3, 5, 1, 4, 2, 0]] $

$$[-y_1 - y_3 + 2\,y_4 + 3\,y_5, -y_2 + 3\,y_4 - y_6 + 2\,y_5, y_2, y_1, y_3, y_4, y_6, y_5, 0]$$

p = - s ^ 3 + s ^ 7 p’ = - s ^ 3 + s ^ 7

Â» SYNC'D 101475/8388608 , 0.01209676266

99 . Coloring, {2, 3, 5, 6}

R: [4, 9, 5, 7, 3, 8, 1, 1, 1] **B**: [2, 4, 4, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [1833, 564, 645, 1677, 860, 129, 1505, 516, 423] . FixedPtCheck, [1833, 564, 645, 1677, 860, 129, 1505, 516, 423]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	6 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}}, net cycles: 0 . order: 6

$ [[6, 0, 2, 3, 1, 0, 3, 1, 2] , [6, 0, 1, 6, 2, 0, 3, 0, 0] , [3, 0, 2, 6, 1, 0, 6, 0, 0] , [6, 0, 1, 3, 2, 0, 6, 0, 0] , [6, 0, 2, 6, 1, 0, 3, 0, 0] , [3, 0, 1, 6, 2, 0, 6, 0, 0] , [6, 0, 2, 3, 1, 0, 6, 0, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - 3\,y_5, 0, y_1, y_2, y_3, 0, y_4, y_5, 2\,y_5]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[0, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[1/4, -3/16, -3/64, 13/256, 43/576, -197/2304] , [0, 1/4, -3/16, -3/64, -5/144, 43/576] , [0, 1/4, -3/16, -3/64, -5/144, 43/576] , [0, 0, 1/4, -3/16, 1/36, -5/144] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 1/4, -2/9, 1/36] , [1/4, -3/16, -3/64,

13/256, 43/576, -197/2304]] $ x $ [[0, 4, 0, 3, 3, 2, 3, 3, 0] , [0, 0, 0, 4, 3, 3, 5, 3, 0] , [0, 0, 0, 0, 5, 3, 6, 4, 0] , [0, 0, 0, 0, 6, 4, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Â» SYNC'D 9411/262144 , 0.03590011597

100 . Coloring, {2, 3, 5, 7}

R: [4, 9, 5, 7, 3, 7, 5, 1, 1] **B**: [2, 4, 4, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [429, 132, 1161, 645, 1548, 43, 903, 172, 99] . FixedPtCheck, [429, 132, 1161, 645, 1548, 43, 903, 172, 99]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 0, 2, 3, 4, 0, 4, 0, 2] , [4, 3, 4, 7, 6, 4, 3, 5, 0] , [14, 12, 6, 9, 7, 3, 13, 5, 3] , [19, 15, 7, 20, 19, 11, 21, 20, 12] , [59, 33, 19, 45, 28, 12, 44, 33, 15] , [100, 54, 28, 103, 63, 31, 93, 71, 33] , [203, 123, 63, 210, 121, 57, 199, 122, 54]] $ $ [[3, 4, 0, 3, 0, 2, 2, 4, 0] , [8, 5, 0, 5, 2, 0, 9, 3, 4] , [10, 4, 2, 15, 9, 5, 11, 11, 5] , [29, 17, 9, 28, 13, 5, 27, 12, 4] , [37, 31, 13, 51, 36, 20, 52, 31, 17] , [92, 74, 36, 89, 65, 33, 99, 57, 31] , [181, 133, 65, 174, 135, 71, 185, 134, 74]] $ $ [[0, -2, 1, 0, 2, -1, 1, -2, 1] , [-2, -1, 2, 1, 2, 2, -3, 1, -2] , [2, 4, 2, -3, -1, -1, 1, -3, -1] , [-5, -1, -1, -4, 3, 3, -3, 4, 4] , [11, 1, 3, -3, -4, -4, -4, 1, -1] , [4, -10, -4, 7, -1, -1, -3, 7, 1] , [11, -5, -1, 18, -7, -7, 7, -6, -10]] $

$[y_4 + 2\,y_6 - y_1 + y_2 - 3\,y_3 - 2\,y_5, -y_4 - 2\,y_6 - 2\,y_2 + 2\,y_3 + y_5, -y_4 - y_6, y_1, y_2, y_3, y_4, y_6, y_5]$

p = /1\ || |-| || s || $^{\| 3 \|}$ || + || s || $^{\| 4 \|}$ \3/ /8\ || + || |-| || s || $^{\| 7 \|}$ \3/

S+ \ ; S- \ ; NM

$ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[16, 14, 5, 18, 9, 6, 14, 9, 5]

, [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 6

$ [[3, 0, 2, 3, 4, 0, 4, 0, 2] , [2, 0, 4, 3, 6, 0, 3, 0, 0] , [0, 0, 6, 2, 7, 0, 3, 0, 0] , [0, 0, 7, 0, 9, 0, 2, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[-y_1 + y_2 + y_3 - y_4 + y_5, 0, y_1, y_2, y_3, 0, y_4, 0, y_5]$$

$p = - s^5 + s^6$

Omega Rank for B : cycles: {{6, 8}}, net cycles: 0 . order: 6

$$[y_2, y_1, 0, y_3, 0, y_4, y_6, y_5, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1/2, -3/4, -2/9, 19/36] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 0, 5/18, -2/9] , [1/2, -3/4, 1/8, 9/16, -25/72, -5/144] , [0, 0, 0, 0, 5/18, -2/9] , [0, 1/2, -3/4, 1/8, 19/36, -25/72] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 1/2, -3/4, -2/9, 19/36]] $ x $ [[3, 4, 0, 3, 0, 2, 2, 4, 0] , [2, 3, 0, 4, 0, 4, 0, 5, 0] , [0, 2, 0, 3, 0, 5, 0, 8, 0] , [0, 0, 0, 2, 0, 8, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

Â» SYNC’D 4663/131072 , 0.03557586670

101 . Coloring, {2, 3, 5, 8}

R: [4, 9, 5, 7, 3, 7, 1, 6, 1] **B**: [2, 4, 4, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -27‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘‘]‘

For τ=1/2, [598, 184, 258, 559, 344, 129, 602, 172, 138] . FixedPtCheck, [598, 184, 258, 559, 344, 129, 602, 172, 138]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}}, net cycles: 0 . order: 6

$ [[4, 0, 2, 3, 1, 2, 4, 0, 2] , [6, 0, 1, 4, 2, 0, 5, 0, 0] , [5, 0, 2, 6, 1, 0, 4, 0, 0] , [4, 0, 1, 5, 2, 0, 6, 0, 0] , [6, 0, 2, 4, 1, 0, 5, 0, 0] , [5, 0, 1, 6, 2, 0, 4, 0, 0] , [4, 0, 2, 5, 1, 0, 6, 0, 0]] $

$$[y_3, 0, y_4, -y_3 + 5 y_4 + 5 y_1 - 2 y_5 - y_2, y_1, y_5, y_2, 0, y_5]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[2, 4, 0, 3, 3, 0, 2, 4, 0] , [4, 2, 0, 4, 2, 0, 3, 3, 0] , [3, 4, 0, 2, 3, 0, 2, 4, 0] , [4, 3, 0, 4, 2, 0, 3, 2, 0] , [2, 4, 0, 3, 3, 0, 2, 4, 0] , [4, 2, 0, 4, 2, 0, 3, 3, 0]] $

$$[-14 y_1 - y_4 + 39 y_3 - 14 y_2, y_1, 0, y_4, y_3, 0, -5 y_1 + 14 y_3 - 5 y_2, y_2, 0]$$

p = - s + s ^ 5 p’ = - s + s ^ 5

Â» SYNC’D 4653/524288 , 0.008874893188

102 . Coloring, {2, 3, 5, 9}

R: [4, 9, 5, 7, 3, 7, 1, 1, 2] **B**: [2, 4, 4, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘

(‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ‘)‘‘]‘

For τ=1/2, [-378, -216, -150, -375, -200, -25, -350, -100, -162] . FixedPtCheck, [378, 216, 150, 375, 200, 25, 350, 100, 162]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[5, 1, 2, 3, 1, 0, 4, 0, 2] , [4, 3, 1, 8, 4, 4, 6, 5, 1] , [14, 9, 4, 12, 7, 3, 16, 4, 3] , [25, 13, 7, 25, 12, 12, 24, 17, 9] , [48, 32, 12, 53, 31, 15, 57, 27, 13] , [103, 61, 31, 100, 51, 37, 101, 60, 32] , [193, 121, 51, 203, 122, 68, 214, 119, 61]] $ $ [[1, 3, 0, 3, 3, 2, 2, 4, 0] , [8, 5, 3, 4, 4, 0, 6, 3, 3] , [10, 7, 4, 12, 9, 5, 8, 12, 5] , [23, 19, 9, 23, 20, 4, 24, 15, 7] , [48, 32, 20, 43, 33, 17, 39, 37, 19] , [89, 67, 33, 92, 77, 27, 91, 68, 32] , [191, 135, 77, 181, 134, 60, 170, 137, 67]] $ $ [[2, -1, 1, 0, -1, -1, 1, -2, 1] , [-2, -1, -1, 2, 0, 2, 0, 1, -1] , [2, 1, 0, 0, -1, -1, 4, -4, -1] , [1, -3, -1, 1, -4, 4, 0, 1, 1] , [0, 0, -4, 5, -1, -1, 9, -5, -3] , [7, -3, -1, 4, -13, 5, 5, -4, 0] , [1, -7, -13, 11, -6, 4, 22, -9, -3]] $

$[y_2, y_3, y_4, y_5, y_6, y_1, 4 y_2 + y_3 - y_4 + 3 y_6 + 2 y_5 + 2 y_1, -4 y_2 - y_3 - 3 y_6 - 2 y_5 - 2 y_1, -y_2 - y_3 - y_5 - y_6 - y_1]$

$p = s^2 - 8s^4 - 12s^5 + 8s^6 + 16s^7$

S+ \; S- \; NM

$ [[27, 23, 8, 34, 16, 9, 29, 21, 13] , [31, 20, 7, 31, 19, 11, 28, 21, 12] , [27, 16, 12, 32, 25, 8, 31, 19, 10] , [31, 18, 11, 27, 20, 11, 32, 21, 9] , [28, 18, 15, 28, 24, 8, 34, 19, 6] , [31, 18, 11, 27, 20, 11, 32, 21, 9] , [32, 19, 11, 29, 23, 10, 29, 18, 9] , [31, 23, 7, 31, 18, 10, 28, 21, 11] , [32, 24, 9, 31, 16, 13, 27, 18, 10]] $ $ [[27, 22, 9, 34, 17, 10, 29, 20, 12] , [31, 22, 5, 31, 17, 9, 28, 23, 14] , [27, 15, 13, 32, 26, 9, 31, 18, 9] , [31, 19, 10, 27, 19, 10, 32, 22, 10] , [28, 18, 15, 28, 24, 8, 34, 19, 6] , [31, 19, 10, 27, 19, 10, 32, 22, 10] , [32, 18, 12, 29, 24, 11, 29, 17, 8] , [31, 21, 9, 31, 20, 12, 28, 19, 9] , [32, 27, 6, 31, 13, 10, 27, 21, 13]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}, {2, 9}, {3, 5}}, net cycles: 3 . order: 6

$ [[5, 1, 2, 3, 1, 0, 4, 0, 2] , [4, 2, 1, 5, 2, 0, 3, 0, 1] , [3, 1, 2, 4, 1, 0, 5, 0, 2] , [5, 2, 1, 3, 2, 0, 4, 0, 1] , [4, 1, 2, 5, 1, 0, 3, 0, 2] , [3, 2, 1, 4, 2, 0, 5, 0, 1] , [5, 1, 2, 3, 1, 0, 4, 0, 2]] $

$$[y_3, y_1, y_4, -y_3 + 4\,y_1 - y_2 + 4\,y_4, y_1, 0, y_2, 0, y_4]$$

$p = -s - s^2 + s^4 + s^5 \quad p = s - s^3 - s^4 + s^6 \quad p = -s + s^7$

Omega Rank for B : cycles: {{5, 7}, {6, 8}}, net cycles: 1 . order: 4

$$\$ [[1, 3, 0, 3, 3, 2, 2, 4, 0] , [0, 1, 0, 3, 2, 4, 3, 5, 0] , [0, 0, 0, 1, 3, 5, 2, 7, 0] , [0, 0, 0, 0, 2, 7, 3, 6, 0] , [0, 0, 0, 0, 3, 6, 2, 7, 0] , [0, 0, 0, 0, 2, 7, 3, 6, 0] , [0, 0, 0, 0, 3, 6, 2, 7, 0]] \$$$

$$[4\,y_1, 9\,y_1 + 9\,y_2 + 9\,y_3 - 13\,y_4 - 4\,y_5, 0, 4\,y_2, 5\,y_1 + 5\,y_2 + 5\,y_3 - 9\,y_4, 4\,y_3, 4\,y_4, 4\,y_5, 0]$$

$p = -s^4 + s^6 \quad p' = -s^4 + s^6$

Â» SYNC'D 8073/4194304 , 0.001924753189

103 . Coloring, {2, 3, 6, 7}

R: [4, 9, 5, 7, 7, 8, 5, 1, 1] **B**: [2, 4, 4, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [1157, 356, 645, 1118, 2580, 86, 2795, 344, 267] . FixedPtCheck, [1157, 356, 645, 1118, 2580, 86, 2795, 344, 267]

$\det(A + \tau\,\Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 vs 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	*7 vs 7*

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$$\$ [[3, 0, 0, 3, 4, 0, 5, 1, 2] , [3, 0, 0, 3, 5, 0, 7, 0, 0] , [0, 0, 0, 3, 7, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, 0, y_4, y_5, 2\,y_5]$$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -47/756, -29/756, -19/378, 61/756, -83/756, 89/378] , [0, 89/378, -47/756, -29/756, -19/378, 61/756, -83/756] , [0, 89/378, -47/756, -29/756, -19/378, 61/756, -83/756] , [0, -83/756, 89/378, -47/756, -29/756, -19/378, 61/756] , [1/2, -47/756, -29/756, -19/378, 61/756, -83/756, -50/189] , [0, -19/378, 61/756, -83/756, 89/378, -47/756, -29/756] , [0, -29/756, -19/378, 61/756, -83/756, 89/378, -47/756] , [0, 61/756, -83/756, 89/378, -47/756, -29/756, -19/378] , [0, -47/756, -29/756, -19/378, 61/756, -83/756, 89/378]] $ x $ [[3, 4, 2, 3, 0, 2, 1, 3, 0] , [1, 3, 0, 6, 0, 3, 2, 3, 0] , [2, 1, 0, 3, 0, 3, 3, 6, 0] , [3, 2, 0, 1, 0, 6, 3, 3, 0] , [3, 3, 0, 2, 0, 3, 6, 1, 0] , [6, 3, 0, 3, 0, 1, 3, 2, 0] , [3, 6, 0, 3, 0, 2, 1, 3, 0]] $

Â» SYNC'D 97569/2097152 , 0.04652452469

104 . Coloring, {2, 3, 6, 8}

R: [4, 9, 5, 7, 7, 8, 1, 6, 1] **B**: [2, 4, 4, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [1781, 548, 129, 1505, 516, 645, 1677, 860, 411] . FixedPtCheck, [1781, 548, 129, 1505, 516, 645, 1677, 860, 411]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[4, 0, 0, 3, 1, 2, 5, 1, 2] , [7, 0, 0, 4, 0, 1, 4, 2, 0] , [4, 0, 0, 7, 0, 2, 4, 1, 0] , [4, 0, 0, 4, 0, 1, 7, 2, 0] , [7, 0, 0, 4, 0, 2, 4, 1, 0] , [4, 0, 0, 7, 0, 1, 4, 2, 0] , [4, 0, 0, 4, 0, 2, 7, 1, 0]] $

$$[-y_1 - 3\,y_2 + 5\,y_3 - y_4 + 5\,y_5, 0, 0, y_1, y_2, y_3, y_4, y_5, 2\,y_2]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 0, -17/72, 19/72, 1/72, 1/72] , [0, 0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 1, -17/72, 19/72, 1/72, -71/72] , [1, -3, 7, 1/72, -71/72, 199/72, -485/72] , [0, 1, -3, 19/72, 1/72, -71/72, 199/72] , [0, 0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 0, 0, -17/72, 19/72, 1/72, 1/72]] $ x $ [[2, 4, 2, 3, 3, 0, 1, 3, 0] , [3, 2, 3, 6, 1, 0, 0, 3, 0] , [3, 3, 1, 5, 0, 0, 0, 6, 0] , [6, 3, 0, 4, 0, 0, 0, 5, 0] , [5, 6, 0, 3, 0, 0, 0, 4, 0] , [4, 5, 0, 6, 0, 0, 0, 3, 0] , [3, 4, 0, 5, 0, 0, 0, 6, 0]] $

Â» SYNC'D 119025/4194304 , 0.02837777138

105 . Coloring, {2, 3, 6, 9}

R: [4, 9, 5, 7, 7, 8, 1, 1, 2] **B**: [2, 4, 4, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [357, 204, 25, 325, 100, 25, 325, 100, 153] . FixedPtCheck, [357, 204, 25, 325, 100, 25, 325, 100, 153]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 9	9 *vs* 9	5 *vs* 7	7 *vs* 8

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[5, 1, 0, 3, 1, 0, 5, 1, 2] , [6, 2, 0, 5, 0, 0, 4, 0, 1] , [4, 1, 0, 6, 0, 0, 5, 0, 2] , [5, 2, 0, 4, 0, 0, 6, 0, 1] , [6, 1, 0, 5, 0, 0, 4, 0, 2] , [4, 2, 0, 6, 0, 0, 5, 0, 1] , [5, 1, 0, 4, 0, 0, 6, 0, 2]] $

$$[5\, y_1 - y_2 - 2\, y_5 - y_3 + 5\, y_4, y_1, 0, y_2, y_5, 0, y_3, y_5, y_4]$$

p = - s 2 - s 3 + s 5 + s 6 p = s 2 - s 4 - s 5 + s 7

Omega Rank for B : cycles: {{3, 4, 5, 6, 7, 8}}, net cycles: 0 . order: 6

$ [[1, 3, 2, 3, 3, 2, 1, 3, 0] , [0, 1, 3, 5, 1, 3, 2, 3, 0] , [0, 0, 1, 4, 2, 3, 3, 5, 0] , [0, 0, 2, 1, 3, 5, 3, 4, 0] , [0, 0, 3, 2, 3, 4, 5, 1, 0] , [0, 0, 3, 3, 5, 1, 4, 2, 0] , [0, 0, 5, 3, 4, 2, 1, 3, 0] , [0, 0, 4, 5, 1, 3, 2, 3, 0]] $

$$[y_1 + y_2 - y_3 - y_4 - y_5 + y_6 + y_7, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

p = - s 3 + s 4 - s 5 + s 6 - s 7 + s 8

Â» SYNC’D 1366845/67108864 , 0.02036757767

106 . Coloring, {2, 3, 7, 8}

R: [4, 9, 5, 7, 7, 7, 5, 6, 1] **B**: [2, 4, 4, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [13, 4, 9, 13, 36, 3, 39, 4, 3] . FixedPtCheck, [13, 4, 9, 13, 36, 3, 39, 4, 3]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[1, 0, 0, 3, 4, 2, 6, 0, 2] , [6, 5, 0, 7, 6, 0, 9, 3, 0] , [8, 10, 2, 13, 9, 3, 13, 9, 5] , [23, 19, 7, 20, 15, 9, 25, 16, 10] , [49, 31, 17, 45, 32, 16, 44, 35, 19] , [100, 60, 32, 97, 61, 35, 93, 67, 31] , [191, 125, 67, 200, 125, 67, 193, 124, 60]] $ $ [[5, 4, 2, 3, 0, 0, 0, 4, 0] , [6, 3, 4, 5, 2, 4, 3, 5, 4] , [16, 6, 6, 11, 7, 5, 11, 7, 3] , [25, 13, 9, 28, 17, 7, 23, 16, 6] , [47, 33, 15, 51, 32, 16, 52, 29, 13] , [92, 68, 32, 95, 67, 29, 99, 61, 33] , [193, 131, 61, 184, 131, 61, 191, 132, 68]] $ $ [[-2, -2, -1, 0, 2, 1, 3, -2, 1] , [0, 1, -2, 1, 2, -2, 3, -1, -2] , [-4, 2, -2, 1, 1, -1, 1, 1, 1] , [-1, 3, -1, -4, -1, 1, 1, 0, 2] , [1, -1, 1, -3, 0, 0, -4, 3, 3] , [4, -4, 0, 1, -3, 3, -3, 3, -1] , [-1, -3, 3, 8, -3, 3, 1, -4, -4]] $

$[y_2 + y_3 - y_4 - 2\,y_6 - y_1, -2\,y_2 - 2\,y_3 + y_4 + y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$

$p = s^2 + 2s^4 + 8s^5 + 16s^7$

S+ \; S- \; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 0, 3, 4, 2, 6, 0, 2] , [2, 0, 0, 1, 6, 0, 9, 0, 0] , [0, 0, 0, 2, 9, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

$[y_1, 0, 0, y_2, y_3, y_5, y_4, 0, y_5]$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5, 0]$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0,

1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 1/72, -17/72, 19/72] , [0, 19/72, 1/72, 1/72, -17/72] , [0, 19/72, 1/72, 1/72, -17/72] , [0, -17/72, 19/72, 1/72, 1/72] , [1/2, 1/72, 1/72, -17/72, -17/72] , [0, -17/72, 19/72, 1/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, 1/72, -17/72, 19/72]] $ x $ [[5, 4, 2, 3, 0, 0, 0, 4, 0] , [4, 5, 0, 6, 0, 0, 0, 3, 0] , [3, 4, 0, 5, 0, 0, 0, 6, 0] , [6, 3, 0, 4, 0, 0, 0, 5, 0] , [5, 6, 0, 3, 0, 0, 0, 4, 0]] $

Â» SYNC'D 417/8192 , 0.05090332031

107 . Coloring, {2, 3, 7, 9}

R: [4, 9, 5, 7, 7, 7, 5, 1, 2] **B**: [2, 4, 4, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-357, -204, -225, -375, -900, -25, -975, -100, -153] . FixedPtCheck, [357, 204, 225, 375, 900, 25, 975, 100, 153]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 0, 3, 4, 0, 6, 0, 2] , [0, 2, 0, 2, 6, 0, 7, 0, 1] , [0, 1, 0, 0, 7, 0, 8, 0, 2] , [0, 2, 0, 0, 8, 0, 7, 0, 1] , [0, 1, 0, 0, 7, 0, 8, 0, 2] , [0, 2, 0, 0, 8, 0, 7, 0, 1]] $

$$[2\,y_1 - y_2 + 3\,y_3, y_1, 0, 3\,y_1 - y_4 + 2\,y_3, y_4, 0, y_2, 0, y_3]$$

p = - s 3 + s 5 p' = - s 3 + s 5

Omega Rank for B : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[4, 3, 2, 3, 0, 2, 0, 4, 0] , [0, 4, 0, 5, 0, 4, 0, 5, 0] , [0, 0, 0, 4, 0, 5, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[2\, y_3, y_4, y_3, y_2, 0, y_1, 0, -y_4 + y_3 + y_2 + y_1, 0]$$

$p = s^4 - s^5$ $p' = -s^4 + s^5$

Â» SYNC'D 9801/262144 , 0.03738784790

108 . Coloring, {2, 3, 8, 9}

R: [4, 9, 5, 7, 7, 7, 1, 6, 2] **B**: [2, 4, 4, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 3 + τ ‘)‘‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘ , 54‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ 1 + τ ‘)‘‘ (‘5 + 2τ + 8τ ^ 2 - 2τ ^ 3 + 3τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1778, 1016, 150, 1625, 600, 375, 1950, 500, 762] . FixedPtCheck, [1778, 1016, 150, 1625, 600, 375, 1950, 500, 762]

$\det(A + \tau\, \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 7}, {2, 9}}, net cycles: 0 . order: 6

$ [[3, 1, 0, 3, 1, 2, 6, 0, 2] , [6, 2, 0, 3, 0, 0, 6, 0, 1] , [6, 1, 0, 6, 0, 0, 3, 0, 2] , [3, 2, 0, 6, 0, 0, 6, 0, 1] , [6, 1, 0, 3, 0, 0, 6, 0, 2] , [6, 2, 0, 6, 0, 0, 3, 0, 1] , [3, 1, 0, 6, 0, 0, 6, 0, 2]] $

$$[5\, y_1 - y_2 - 3\, y_3 - y_4 + 5\, y_5, y_1, 0, y_2, y_3, 2\, y_3, y_4, 0, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$ $p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[3, 3, 2, 3, 3, 0, 0, 4, 0] , [4, 3, 3, 5, 0, 0, 0, 3, 0] , [3, 4, 0, 6, 0, 0, 0, 5, 0] , [5, 3, 0, 4, 0, 0, 0, 6, 0] , [6, 5, 0, 3, 0, 0, 0, 4, 0] , [4, 6, 0, 5, 0, 0, 0, 3, 0]] $

$$[y_3, y_4, y_5, y_1, y_2, 0, 0, y_3 - y_4 - y_5 + y_1 + y_2, 0]$$

$p = s^3 - s^4 + s^5 - s^6$

Â» SYNC'D 7235/262144 , 0.02759933472

109 . Coloring, {2, 4, 5, 6}

R: [4, 9, 4, 8, 3, 8, 1, 1, 1] **B**: [2, 4, 5, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1911, 588, 129, 1677, 172, 387, 559, 1548, 441] . FixedPtCheck, [1911, 588, 129, 1677, 172, 387, 559, 1548, 441]

$\det(A + \tau \Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[6, 0, 2, 4, 0, 0, 0, 4, 2] , [3, 0, 0, 6, 3, 0, 4, 2, 0] , [6, 5, 3, 7, 4, 2, 3, 6, 0] , [9, 10, 4, 12, 10, 2, 11, 9, 5] , [25, 18, 10, 19, 17, 7, 24, 14, 10] , [48, 29, 17, 49, 30, 18, 53, 26, 18] , [97, 62, 30, 100, 58, 38, 95, 67, 29]] $ $ [[0, 4, 0, 2, 4, 2, 6, 0, 0] , [3, 4, 2, 0, 1, 2, 2, 2, 2] , [6, 3, 1, 5, 4, 2, 9, 2, 4] , [15, 6, 4, 12, 6, 6, 13, 7, 3] , [23, 14, 6, 29, 15, 9, 24, 18, 6] , [48, 35, 15, 47, 34, 14, 43, 38, 14] , [95, 66, 34, 92, 70, 26, 97, 61, 35]] $ $ [[3, -2, 1, 1, -2, -1, -3, 2, 1] , [0, -2, -1, 3, 1, -1, 1, 0, -1] , [0, 1, 1, 1, 0, 0, -3, 2, -2] , [-3, 2, 0, 0, 2, -2, -1, 1, 1] , [1, 2, 2, -5, 1, -1, 0, -2, 2] , [0, -3, 1, 1, -2, 2, 5, -6, 2] , [1, -2, -2, 4, -6, 6, -1, 3, -3]] $

$$[-3 y_2 - 3 y_3 + y_4 - 2 y_6 - y_1, 2 y_2 + 2 y_3 - y_4 + y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$$

$p = s^{2} + 2s^{4} + 8s^{5} + 16s^{7}$

S+ \; S- \; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[6, 0, 2, 4, 0, 0, 0, 4, 2] , [6, 0, 0, 8, 0, 0, 0, 4, 0] , [4, 0, 0, 6, 0, 0, 0, 8, 0] , [8, 0, 0, 4, 0, 0, 0, 6, 0] , [6, 0, 0, 8, 0, 0, 0, 4, 0]] $

$$[y_1, 0, y_4, y_2, 0, 0, 0, y_3, y_4]$$

$p = s^{2} - s^{5}$

Omega Rank for B : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[0, 4, 0, 2, 4, 2, 6, 0, 0] , [0, 0, 0, 4, 6, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

$$[0, 2\,y_2, 0, y_1, y_4, y_2, y_3, 0, 0]$$

$p = -s^{3} + s^{5}$

Â» SYNC’D 9/128 , 0.07031250000

110 . Coloring, {2, 4, 5, 7}

R: [4, 9, 4, 8, 3, 7, 5, 1, 1] **B**: [2, 4, 5, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [-949, -292, -387, -1075, -516, -215, -559, -860, -219] . FixedPtCheck, [949, 292, 387, 1075, 516, 215, 559, 860, 219]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 6

$ [[3, 0, 2, 4, 3, 0, 1, 3, 2] , [5, 0, 3, 5, 1, 0, 0, 4, 0] , [4, 0, 1, 8, 0, 0, 0, 5, 0] , [5, 0, 0, 5, 0, 0, 0, 8, 0] , [8, 0, 0, 5, 0, 0, 0, 5, 0] , [5, 0, 0, 8, 0, 0, 0, 5, 0] , [5, 0, 0, 5, 0, 0, 0, 8, 0]] $

$$[y_1, 0, y_2, y_3, y_4, 0, y_5, y_6, 2\, y_5]$$

p = - s ^ 4 + s ^ 7

Omega Rank for B : cycles: {{6, 8}, {1, 2, 4, 7}}, net cycles: 1 . order: 4

$ [[3, 4, 0, 2, 1, 2, 5, 1, 0] , [5, 3, 0, 4, 0, 1, 3, 2, 0] , [3, 5, 0, 3, 0, 2, 4, 1, 0] , [4, 3, 0, 5, 0, 1, 3, 2, 0] , [3, 4, 0, 3, 0, 2, 5, 1, 0] , [5, 3, 0, 4, 0, 1, 3, 2, 0] , [3, 5, 0, 3, 0, 2, 4, 1, 0]] $

$$[y_3 + 4\, y_5 - y_1 - y_2, 4\, y_3 - y_4 + y_5, 0, y_1, y_2, y_3, y_4, y_5, 0]$$

p’ = - s ^ 2 + s ^ 6 p = s ^ 2 - s ^ 6

Â» SYNC’D 507/32768 , 0.01547241211

111 . Coloring, {2, 4, 5, 8}

R: [4, 9, 4, 8, 3, 7, 1, 6, 1] **B**: [2, 4, 5, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-3471, -1068, -645, -3354, -860, -2322, -2795, -3096, -801] . FixedPtCheck, [3471, 1068, 645, 3354, 860, 2322, 2795, 3096, 801]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	6 *vs* 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: -1 . order: 5

$ [[4, 0, 2, 4, 0, 2, 1, 3, 2] , [3, 0, 0, 6, 0, 3, 2, 4, 0] , [2, 0, 0, 3, 0, 4, 3, 6, 0] , [3, 0, 0, 2, 0, 6, 4, 3, 0] , [4, 0, 0, 3, 0, 3, 6, 2, 0] , [6, 0, 0, 4, 0, 2, 3, 3, 0] , [3, 0, 0, 6, 0, 3, 2, 4, 0]] $

$$[y_6, 0, y_5, y_1, 0, y_2, y_3, y_4, y_5]$$

p = - s ^ 2 + s ^ 7

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -2, -13/18, 16/9] , [0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 5/18, -2/9] , [1, -2, 0, 6, -13/18, -38/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 1, -2, 0, 16/9, -13/18] , [0, 0, 1, -2, -13/18, 16/9]] $ x $ [[2, 4, 0, 2, 4, 0, 5, 1, 0] , [1, 2, 0, 4, 5, 0, 6, 0, 0] , [0, 1, 0, 2, 6, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Â» SYNC’D 20385/524288 , 0.03888130188

112 . Coloring, {2, 4, 5, 9}

R: [4, 9, 4, 8, 3, 7, 1, 1, 2] **B**: [2, 4, 5, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-777, -444, -75, -750, -100, -150, -325, -600, -333] . FixedPtCheck, [777, 444, 75, 750, 100, 150, 325, 600, 333]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 1 + τ ‘)‘ 2

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[5, 1, 2, 4, 0, 0, 1, 3, 2] , [4, 3, 0, 10, 5, 1, 6, 6, 1] , [15, 9, 5, 9, 10, 2, 6, 13, 3] , [24, 12, 10, 27, 21, 3, 23, 15, 9] , [45, 33, 21, 54, 31, 17, 35, 40, 12] , [95, 63, 31, 97, 72, 24, 92, 69, 33] , [192, 130, 72, 191, 133, 59, 175, 137, 63]] $ $ [[1, 3, 0, 2, 4, 2, 5, 1, 0] , [8, 5, 4, 2, 3, 3, 6, 2, 3] , [9, 7, 3, 15, 6, 6, 18, 3, 5] , [24, 20, 6, 21, 11, 13, 25, 17, 7] , [51, 31, 11, 42, 33, 15, 61, 24, 20] , [97, 65, 33, 95, 56, 40, 100, 59, 31] , [192, 126, 56, 193, 123, 69, 209, 119, 65]] $ $ [[2, -1, 1, 1, -2, -1, -2, 1, 1] , [-2, -1, -2, 4, 1, -1, 0, 2, -1] , [3, 1, 1, -3, 2, -2, -6, 5, -1] , [0, -4, 2, 3, 5, -5, -1, -1, 1] , [-3, 1, 5, 6, -1, 1, -13, 8, -4] , [-1, -1, -1, 1, 8, -8, -4, 5, 1] , [0, 2, 8, -1, 5, -5, -17, 9, -1]] $

$[y_6, y_2, y_3, y_4, y_5, y_1, -y_3 - 3\,y_2 - y_5 - 2\,y_6 - 2\,y_4, 3\,y_2 + y_5 + 2\,y_6 + 2\,y_4, -y_6 - y_2 - y_4 - y_1 - y_5]$

p = s 2 - 4s 5 - 8s 6 + 16s 7

S+ \ ; S- \ ; NM

$ [[19, 14, 6, 20, 11, 7, 18, 14, 7] , [21, 14, 5, 20, 12, 8, 15, 14, 7] , [20, 9, 6, 20, 18, 6, 19, 11, 7] , [19, 14, 8, 18, 11, 7, 20, 14, 5] , [17, 13, 10, 18, 13, 5, 23, 13, 4] , [19, 13, 7, 18, 12, 6, 22, 13, 6] , [20, 11, 6, 20, 16, 6, 19, 11, 7] , [10, 6, 2, 10, 7, 3, 10, 6, 4] , [19, 15, 5, 20, 10, 6, 20, 13, 8]] $ $ [[19, 13, 5, 20, 12, 6, 20, 13, 8] , [21, 12, 3, 20, 14, 6, 19, 12, 9] , [20, 10, 7, 20, 17, 7, 17, 12, 6] , [19, 13, 7, 18, 12, 6, 22, 13, 6] , [17, 13, 10, 18, 13, 5, 23, 13, 4] , [19, 14, 8, 18, 11, 7, 20, 14, 5] , [20, 12, 7, 20, 15, 7, 17, 12, 6] , [10, 7, 3, 10, 6, 4, 8, 7, 3] , [19, 16, 6, 20, 9, 7, 18, 14, 7]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 0 . order: 6

$ [[5, 1, 2, 4, 0, 0, 1, 3, 2] , [4, 2, 0, 7, 0, 0, 0, 4, 1] , [4, 1, 0, 4, 0, 0, 0, 7, 2] , [7, 2, 0, 4, 0, 0, 0, 4, 1] , [4, 1, 0, 7, 0, 0, 0, 4, 2] , [4, 2, 0, 4, 0, 0, 0, 7, 1] , [7, 1, 0, 4, 0, 0, 0, 4, 2]] $

$$[5\,y_1 - y_2 - 3\,y_5 - y_3 + 5\,y_4, y_1, 2\,y_5, y_2, 0, 0, y_5, y_3, y_4]$$

p' = - s 2 - s 3 + s 5 + s 6 p = - s 2 - s 3 + s 5 + s 6

Omega Rank for B : cycles: {{5, 7}, {6, 8}}, net cycles: 1 . order: 4

$ [[1, 3, 0, 2, 4, 2, 5, 1, 0] , [0, 1, 0, 3, 5, 1, 6, 2, 0] , [0, 0, 0, 1, 6, 2, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 7, 2, 0] , [0, 0, 0, 0, 7, 2, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 7, 2, 0] , [0, 0, 0, 0, 7, 2, 8, 1, 0]] $

$$[y_3, 3\,y_1 - y_4 + 2\,y_5, 0, y_2, -y_3 - y_2 + 2\,y_1 + 3\,y_5, y_1, y_4, y_5, 0]$$

p' = - s 4 + s 6 p = s 4 - s 6

Â» SYNC'D 111537/8388608 , 0.01329624653

113 . Coloring, {2, 4, 6, 7}

R: [4, 9, 4, 8, 7, 8, 5, 1, 1] **B**: [2, 4, 5, 7, 3, 7, 1, 6, 2]

' See graph

' ' See pair graph

'

Ω for A+τΔ :
'['27' (' 5 + 3τ 2 ')'' (' 3 + τ 2 ')' , -54' (' 5 + 3τ 2 ')'' (' - 1 + τ ')' , -9' (' - 1 + τ ')'' (' 5 - τ + 3τ 2 + τ 3 ')' , 9' (' 5 - τ + 3τ 2 + τ 3 ')'' (' 3 + τ 2 ')' , 18' (' 5 - τ + 3τ 2 + τ 3 ')' , -9' (' 1 + τ ')'' (' - 1 + τ ')'' (' 5 - τ + 3τ 2 + τ 3 ')' , -9' (' 5 - τ + 3τ 2 + τ 3 ')'' (' - 3 + τ ')' , 18' (' 5 - τ + 3τ 2 + τ 3 ')'' (' 1 + τ ')' , -27' (' 5 + 3τ 2 ')'' (' - 1 + τ ')'' (' 1 + τ ')'']'

For τ=1/2, [598, 184, 86, 559, 344, 129, 430, 516, 138] . FixedPtCheck, [598, 184, 86, 559, 344, 129, 430, 516, 138]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[3, 0, 0, 4, 3, 0, 2, 4, 2] , [6, 0, 0, 3, 2, 0, 3, 4, 0] , [4, 0, 0, 6, 3, 0, 2, 3, 0] , [3, 0, 0, 4, 2, 0, 3, 6, 0] , [6, 0, 0, 3, 3, 0, 2, 4, 0] , [4, 0, 0, 6, 2, 0, 3, 3, 0]] $

$$[5\,y_2, 0, 0, -5\,y_2 + 13\,y_1 + 13\,y_3 - 5\,y_5 - 5\,y_4, 5\,y_1, 0, 5\,y_3, 5\,y_5, 5\,y_4]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {3, 5}}, net cycles: 1 . order: 4

$ [[3, 4, 2, 2, 1, 2, 4, 0, 0] , [4, 3, 1, 4, 2, 0, 4, 0, 0] , [4, 4, 2, 3, 1, 0, 4, 0, 0] , [4, 4, 1, 4, 2, 0, 3, 0, 0] , [3, 4, 2, 4, 1, 0, 4, 0, 0] , [4, 3, 1, 4, 2, 0, 4, 0, 0] , [4, 4, 2, 3, 1, 0, 4, 0, 0]] $

$$[y_3, 3\,y_2 + 2\,y_1 - y_5, y_2, -y_3 + 2\,y_2 + 3\,y_1 - y_4, y_1, y_4, y_5, 0, 0]$$

$p' = -s^2 + s^6 \quad p = -s^2 + s^6$

Â» SYNC’D 10669/1048576 , 0.01017475128

114 . Coloring, {2, 4, 6, 8}

R: [4, 9, 4, 8, 7, 8, 1, 6, 1] **B**: [2, 4, 5, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ ‘)‘‘]‘

For τ=1/2, [1053, 324, 43, 903, 172, 1161, 645, 1548, 243] . FixedPtCheck, [1053, 324, 43, 903, 172, 1161, 645, 1548, 243]

$\det(A + \tau\,\Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[4, 0, 0, 4, 0, 2, 2, 4, 2] , [2, 1, 2, 4, 3, 2, 1, 3, 0] , [2, 6, 1, 7, 5, 3, 5, 6, 1] , [8, 13, 3, 5, 10, 6, 11, 10, 6] , [23, 18, 6, 14, 18, 10, 31, 11, 13] , [65, 28, 14, 43, 27, 11, 58, 24, 18] , [116, 45, 37, 115, 56, 24, 101, 54, 28]] $ $ [[2, 4, 2, 2, 4, 0, 4, 0, 0] , [4, 3, 0, 2, 1, 0, 5, 1, 2] , [10, 2, 3, 5, 3, 1, 7, 2, 3] , [16, 3, 5, 19, 6, 2, 13, 6, 2] , [25, 14, 10, 34, 14, 6, 17, 21, 3] , [31, 36, 18, 53, 37, 21, 38, 40, 14] , [76, 83, 27, 77, 72, 40, 91, 74, 36]] $ $ [[1, -2, -1, 1, -2, 1, -1, 2, 1] , [-1, -1, 1, 1, 1, 1, -2, 1, -1] , [-4, 2, -1, 1, 1, 1, -1, 2, -1] , [-4, 5, -1, -7, 2, 2, -1, 2, 2] , [-1, 2, -2, -10, 2, 2, 7, -5, 5] , [17, -4, -2, -5, -5, -5, 10, -8, 2] , [20, -19, 5, 19, -8, -8, 5, -10, -4]] $

$$[-3\,y_6 + y_2 - 2\,y_5 - y_3 - 2\,y_4 - y_1, 2\,y_6 - 2\,y_2 + y_5 + y_3 + 2\,y_4, -y_3 - y_4, y_1, y_6, y_2, y_3, y_4, y_5]$$

$p = s^3 + 3s^4 + 8s^7$

S+ \; S- \; NM

$ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[4, 0, 0, 4, 0, 2, 2, 4, 2] , [4, 0, 0, 4, 0, 4, 0, 6, 0] , [0, 0, 0, 4, 0, 6, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0]] $

$$[y_1, 0, 0, y_2, 0, y_3, y_5, y_4, y_5]$$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/2, -1, 3/2, 23/18, -20/9] , [0, 0, 1/2, -1, -13/18, 23/18] , [0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 1/2, 5/18, -13/18] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 1/2, 5/18, -13/18] , [0, 0, 0, 0, -2/9, 5/18] , [1/2, -1, 3/2, -3, -20/9, 77/18] , [0, 1/2, -1, 3/2, 23/18, -20/9]] $ x $ [[2, 4, 2, 2, 4, 0, 4, 0, 0] , [0, 2, 4, 4, 6, 0, 2, 0, 0] , [0, 0, 6, 2, 6, 0, 4, 0, 0] , [0, 0, 6, 0, 10, 0, 2, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0] , [0, 0,

8, 0, 10, 0, 0, 0, 0]] $

Â» SYNC'D 1473/65536 , 0.02247619629

115 . Coloring, {2, 4, 6, 9}

R: [4, 9, 4, 8, 7, 8, 1, 1, 2] **B**: [2, 4, 5, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [1071, 612, 25, 975, 100, 225, 375, 900, 459] . FixedPtCheck, [1071, 612, 25, 975, 100, 225, 375, 900, 459]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 4, 0, 0, 2, 4, 2] , [6, 2, 0, 5, 0, 0, 0, 4, 1] , [4, 1, 0, 6, 0, 0, 0, 5, 2] , [5, 2, 0, 4, 0, 0, 0, 6, 1] , [6, 1, 0, 5, 0, 0, 0, 4, 2] , [4, 2, 0, 6, 0, 0, 0, 5, 1]] $

$$[y_5, y_4, 0, y_3, 0, 0, y_2, -y_5 + 5\,y_4 - y_3 - y_2 + 5\,y_1, y_1]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{3, 5}}, net cycles: -1 . order: 6

$ [[1, 3, 2, 2, 4, 2, 4, 0, 0] , [0, 1, 4, 3, 6, 0, 4, 0, 0] , [0, 0, 6, 1, 8, 0, 3, 0, 0] , [0, 0, 8, 0, 9, 0, 1, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[y_1, 3\,y_1 - y_2 + y_5 + y_4 - y_3, y_2, y_5, y_4, 2\,y_1, y_3, 0, 0]$$

$p = s^5 - s^6$ p' = $-s^5 + s^6$

Â» SYNC'D 2001/131072 , 0.01526641846

116 . Coloring, {2, 4, 7, 8}

R: [4, 9, 4, 8, 7, 7, 5, 6, 1] **B**: [2, 4, 5, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-481, -148, -215, -559, -860, -387, -1075, -516, -111] . FixedPtCheck, [481, 148, 215, 559, 860, 387, 1075, 516, 111]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ - 1 + τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, y_7]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/2, -1/4, -7/8, 11/72, 19/36] , [1/2, -1/4, -7/8, 3/16, 49/32, -5/144, -289/288] , [0, 0, 1/2, -1/4, -7/8, 11/72, 19/36] , [0, 0, 0, 1/2, -1/4, -25/72, 11/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 1/2, -7/72, -25/72] , [0, 1/2, -1/4, -7/8, 3/16, 19/36, -5/144]] $ x $ [[1, 0, 0, 4, 3, 2, 3, 3, 2] , [2, 0, 0, 1, 3, 3, 5, 4, 0] , [0, 0, 0, 2, 5, 4, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 4, 7}, {3, 5}}, net cycles: 1 . order: 4

$ [[5, 4, 2, 2, 1, 0, 3, 1, 0] , [4, 5, 1, 4, 2, 0, 2, 0, 0] , [2, 4, 2, 5, 1, 0, 4, 0, 0] , [4, 2, 1, 4, 2, 0, 5, 0, 0] , [5, 4, 2, 2, 1, 0, 4, 0, 0] , [4, 5, 1, 4, 2, 0, 2, 0, 0] , [2, 4, 2, 5, 1, 0, 4, 0, 0]] $

$$[2\,y_1 - y_2 + 3\,y_5, 3\,y_1 + 2\,y_5 - y_3 - y_4, y_1, y_2, y_5, 0, y_3, y_4, 0]$$

$p = -s^2 + s^6$ $p' = -s^2 + s^6$

Â» SYNC'D 52005/2097152 , 0.02479791641

117 . Coloring, {2, 4, 7, 9}

R: [4, 9, 4, 8, 7, 7, 5, 1, 2] **B**: [2, 4, 5, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘ , -18‘ (‘ 5 - 2τ + τ ² ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ² ‘)‘ , 9‘ (‘ 5 + 2τ + τ ² ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 2τ + τ ² ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ² ‘)‘ , 9‘ (‘ 5 + 2τ + τ ² ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 2τ + τ ² ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘‘]‘

For τ=1/2, [-119, -68, -25, -125, -100, -25, -125, -100, -51] . FixedPtCheck, [119, 68, 25, 125, 100, 25, 125, 100, 51]

$\det(A + \tau\,\Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[2, 1, 0, 4, 3, 0, 3, 3, 2] , [6, 6, 1, 5, 5, 1, 5, 6, 1] , [16, 7, 3, 9, 8, 2, 13, 8, 6] , [21, 14, 8, 28, 18, 8, 25, 15, 7] , [47, 34, 14, 47, 33, 17, 46, 36, 14] , [104, 63, 31, 91, 64, 28, 99, 62, 34] , [185, 122, 64, 200, 132, 66, 193, 127, 63]] $ $ [[4, 3, 2, 2, 1, 2, 3, 1, 0] , [6, 2, 3, 7, 3, 3, 7, 2, 3] , [8, 9, 5, 15, 8, 6, 11, 8, 2] , [27, 18, 8, 20, 14, 8, 23, 17, 9] , [49, 30, 18, 49, 31, 15, 50, 28, 18] , [88, 65, 33, 101, 64, 36, 93, 66, 30] , [199, 134, 64, 184, 124, 62, 191, 129, 65]] $ $ [[-1, -1, -1, 1, 1, -1, 0, 1, 1] , [0, 2, -1, -1, 1, -1, -1, 2, -1] , [4, -1, -1, -3, 0, -2, 1, 0, 2] , [-3, -2, 0, 4, 2, 0, 1, -1, -1] , [-1, 2, -2, -1, 1, 1, -2, 4, -2] , [8, -1, -1, -5, 0, -4, 3, -2, 2] , [-7, -6, 0, 8, 4, 2, 1, -1, -1]] $

$$[-y_1 + y_3, y_2, -y_3 - y_4, y_1, -y_2 - y_6 - y_3 - y_5, y_6, y_3, y_4, y_5]$$

$p = s - 8s^3 - 12s^4 + 32s^6 + 32s^7$

S+ \ ; S- \ ; NM

$ [[48, 13, 13, 42, 7, 7, 10, 27, 3] , [11, 33, 3, 17, 24, 14, 42, 9, 17] , [42, 9, 19, 43, 12, 4, 15, 26, 0] , [39, 8, 14, 47, 9, 3, 14, 30, 6] , [12, 24, 8, 13, 35, 13, 45, 7, 13] , [9, 28, 4, 17, 29, 13, 44, 10, 16] , [13, 26, 6, 11, 31, 13, 46, 10, 14] , [47, 9, 13, 40, 7, 7, 13, 30, 4] , [19, 30, 0, 10, 26, 16, 41, 11, 17]] $ $ [[18, 33, 3, 12, 27, 17, 40, 7, 13] , [41, 13, 13, 47, 4, 4, 12, 29, 7] , [12, 29, 9, 13, 32, 14, 45, 6, 10] , [9, 28, 4, 17,

29, 13, 44, 10, 16] , [42, 4, 18, 43, 15, 3, 15, 27, 3] , [39, 8, 14, 47, 9, 3, 14, 30, 6] , [43, 6, 16, 41, 11, 3, 16, 30, 4] , [17, 29, 3, 10, 27, 17, 43, 10, 14] , [49, 10, 10, 40, 6, 6, 11, 31, 7]] $ $ [[570, 0, 190, 570, 0, 0, 0, 380, 0] , [0, 380, 0, 0, 380, 190, 570, 0, 190] , [570, 0, 190, 570, 0, 0, 0, 380, 0] , [570, 0, 190, 570, 0, 0, 0, 380, 0] , [0, 380, 0, 0, 380, 190, 570, 0, 190] , [0, 380, 0, 0, 380, 190, 570, 0, 190] , [0, 380, 0, 0, 380, 190, 570, 0, 190] , [570, 0, 190, 570, 0, 0, 0, 380, 0] , [0, 380, 0, 0, 380, 190, 570, 0, 190]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 8	7 *vs* 8	4 *vs* 7	4 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}, {2, 9}}, net cycles: 3 . order: 6

$ [[2, 1, 0, 4, 3, 0, 3, 3, 2] , [3, 2, 0, 2, 3, 0, 3, 4, 1] , [4, 1, 0, 3, 3, 0, 3, 2, 2] , [2, 2, 0, 4, 3, 0, 3, 3, 1] , [3, 1, 0, 2, 3, 0, 3, 4, 2] , [4, 2, 0, 3, 3, 0, 3, 2, 1] , [2, 1, 0, 4, 3, 0, 3, 3, 2]] $

$$[-y_1 + 3\,y_3 - y_4, y_3 - y_2, 0, y_1, y_3, 0, y_3, y_4, y_2]$$

$p = s - s^3 - s^4 + s^6$ $p = -s + s^7$ $p = -s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}, {3, 5}}, net cycles: 3 . order: 4

$ [[4, 3, 2, 2, 1, 2, 3, 1, 0] , [3, 4, 1, 3, 2, 1, 2, 2, 0] , [2, 3, 2, 4, 1, 2, 3, 1, 0] , [3, 2, 1, 3, 2, 1, 4, 2, 0] , [4, 3, 2, 2, 1, 2, 3, 1, 0] , [3, 4, 1, 3, 2, 1, 2, 2, 0] , [2, 3, 2, 4, 1, 2, 3, 1, 0] , [3, 2, 1, 3, 2, 1, 4, 2, 0]] $

$$[-y_1 + 2\,y_2 + 2\,y_4, 2\,y_2 - y_3 + 2\,y_4, y_2, y_1, y_4, y_2, y_3, y_4, 0]$$

$p' = -s^3 + s^7$ $p' = -s + s^5$ $p = -s + s^5$ $p' = -s^2 + s^6$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[x_1, 0, -8\,x_1, -12\,x_1, 0, 32\,x_1, 32\,x_1]$
For A+2Δ : $[-26\,y_1 - 27\,y_2, 8\,y_1 + 9\,y_2, y_1, -2\,y_1 - 3\,y_2, 8\,y_1 + 9\,y_2, 80\,y_1 + 81\,y_2, -y_1, y_1, y_2]$
For A-2Δ : $[2\,y_1, -8\,y_1 - 6\,y_2, -y_1 - 3\,y_2, 26\,y_1 + 24\,y_2, -8\,y_1 - 6\,y_2, 2\,y_2, y_1 + 3\,y_2, -y_1 - 3\,y_2, -80\,y_1 - 78\,y_2]$

Range of $\{\Omega\Delta^i\}$: $[\mu_2, -\mu_4 - \mu_1 - \mu_6 - \mu_5, -\mu_4 - \mu_3, -\mu_2 + \mu_4, \mu_1, \mu_6, \mu_4, \mu_3, \mu_5]$

rank of M is 8 , rank of N is 2

M \; N

$ [[0, 142, 0, 0, 104, 87, 191, 0, 46] , [142, 0, 52, 94, 0, 0, 0, 92, 0] , [0, 52, 0, 0, 0, 57, 81, 0, 0] , [0, 94, 0, 0, 162, 46, 171, 0, 97] , [104, 0, 0, 162, 0, 0, 0, 114, 0] , [87, 0, 57, 46, 0, 0, 0, 0, 0] , [191, 0, 81, 171, 0, 0, 0, 127, 0] , [0, 92, 0, 0, 114, 0, 127, 0, 47] , [46, 0, 0, 97, 0, 0, 0, 47, 0]] $ $ [[0, 1, 0, 0, 1, 1, 1, 0, 1] , [1, 0, 1, 1, 0, 0, 0, 1, 0] , [0, 1, 0, 0, 1, 1, 1, 0, 1] , [0, 1, 0, 0, 1, 1, 1, 0, 1] , [1, 0, 1, 1, 0, 0, 0, 1, 0] , [1, 0, 1,

1, 0, 0, 0, 1, 0] , [1, 0, 1, 1, 0, 0, 0, 1, 0] , [0, 1, 0, 0, 1, 1, 1, 0, 1] , [1, 0, 1, 1, 0, 0, 0, 1, 0]] $

Check is ΩΔN zero? *true,* πΔ= [-1, -1, -1, 1, 1, -1, 0, 1, 1]

ker M, [0, -21054 λ_1, 0, 0, 25083 λ_1, 9544 λ_1, 6800 λ_1, 0, -38002 λ_1]
Range M, [6800 x_1, 6800 x_2, 6800 x_3, 6800 x_4, 6800 x_5, 6800 x_6, 21054 x_2 - 25083 x_5 - 9544 x_6 + 38002 x_8, 6800 x_7, 6800 x_8]

τ= 41 , r'= 1/2

Ranges

Action of R on ranges, [[14], [13], [13], [11], [7], [14], [17], [5], [13], [11], [16], [16], [15], [8], [4], [2], [1]]
Action of B on ranges, [[7], [6], [8], [1], [1], [11], [13], [12], [15], [2], [10], [16], [4], [4], [9], [3], [3]]
β({1, 2}) = 71/855
β({1, 5}) = 52/855
β({1, 6}) = 29/570
β({1, 7}) = 191/1710
β({1, 9}) = 23/855
β({2, 3}) = 26/855
β({2, 4}) = 47/855
β({2, 8}) = 46/855
β({3, 6}) = 1/30
β({3, 7}) = 9/190
β({4, 5}) = 9/95
β({4, 6}) = 23/855
β({4, 7}) = 1/10
β({4, 9}) = 97/1710
β({5, 8}) = 1/15
β({7, 8}) = 127/1710
β({8, 9}) = 47/1710

ker N, [-μ_1 - μ_2 - μ_6, -μ_3 - μ_4 - μ_5 - μ_7, μ_1, μ_2, μ_3, μ_4, μ_5, μ_6, μ_7]
Range of N
[y_2, y_1, y_2, y_2, y_1, y_1, y_1, y_2, y_1]

Partitions
α([{1, 3, 4, 8}, {2, 5, 6, 7, 9}]) = 1/1
b1 = {1, 3, 4, 8} ' , ' b2 = {2, 5, 6, 7, 9}

Action of R and B on the blocks of the partitions: $ [[1, 1] , [1, 1]] $ = $ [[1, 0] , [0, 1]] $ + $ [[0, 1] , [1, 0]] $
['1', '2'], ['2', '1'] with invariant measure [1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Right Group	
Coloring	{2, 4, 7, 9}
Rank	2
R,B	[4, 9, 4, 8, 7, 7, 5, 1, 2], [2, 4, 5, 7, 3, 8, 1, 6, 1]
π_2	[142, 0, 0, 104, 87, 191, 0, 46, 52, 94, 0, 0, 0, 92, 0, 0, 0, 57, 81, 0, 0, 162, 46, 171, 0, 97, 0, 0, 114, 0, 0, 0, 0, 127, 0, 47]
$\mathbf{u}_2$	[1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1] (dim 1)
wpp	[4, 5, 4, 4, 5, 5, 5, 4, 5]

118 . Coloring, {2, 4, 8, 9}

R: [4, 9, 4, 8, 7, 7, 1, 6, 2] **B**: [2, 4, 5, 7, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ - 5 + 3τ - 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-2079, -1188, -125, -1950, -500, -1350, -1875, -1800, -891] . FixedPtCheck, [2079, 1188, 125, 1950, 500, 1350, 1875, 1800, 891]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3}

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 6, 7, 8}}, net cycles: 2 .

$ [[3, 1, 0, 4, 0, 2, 3, 3, 2] , [3, 2, 0, 3, 0, 3, 2, 4, 1] , [2, 1, 0, 3, 0, 4, 3, 3, 2] , [3, 2, 0, 2, 0, 3, 4, 3, 1] , [4, 1, 0, 3, 0, 3, 3, 2, 2] , [3, 2, 0, 4, 0, 2, 3, 3, 1] , [3, 1, 0, 3, 0, 3, 2, 4, 2]] $

$$[y_3, y_4, 0, y_5, 0, y_6, -y_3 + 5\,y_4 - y_5 - y_6 - y_1 + 5\,y_2, y_1, y_2]$$

$p = -s - s^2 + s^6 + s^7$

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 6

$ [[3, 3, 2, 2, 4, 0, 3, 1, 0] , [1, 3, 4, 3, 5, 0, 2, 0, 0] , [0, 1, 5, 3, 6, 0, 3, 0, 0] , [0, 0, 6, 1, 8, 0, 3, 0, 0] , [0, 0, 8, 0, 9, 0, 1, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[y_1 + y_2 - y_3 - y_4 + y_5 + y_6, y_1, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

$p = -s^6 + s^7$

Â» SYNC’D 407263/33554432 , 0.01213738322

119 . Coloring, {2, 5, 6, 7}

R: [4, 9, 4, 7, 3, 8, 5, 1, 1] **B**: [2, 4, 5, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 387, 559, 516, 43, 559, 172, 75] . FixedPtCheck, [325, 100, 387, 559, 516, 43, 559, 172, 75]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[3, 0, 2, 4, 3, 0, 3, 1, 2] , [3, 0, 3, 5, 3, 0, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0] , [0, 0, 4, 3, 5, 0, 6, 0, 0] , [0, 0, 5, 4, 6, 0, 3, 0, 0] , [0, 0, 6, 5, 3, 0, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0]] $

$$[y_1, 0, y_2, y_3, y_4, 0, y_5, y_6, 2\, y_6]$$

$p = - s^3 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 0 . order: 6

$$[y_1, y_2, 0, y_3, y_4, y_5, y_6, y_7, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 7/27, 5/54, -2/27, -13/54, -11/27, 23/54] , [0, 23/54, 7/27, 5/54, -2/27, -13/54, -11/27] , [1, -13/54, -11/27, 23/54, 7/27, 5/54, -29/27] , [0, -11/27, 23/54, 7/27, 5/54, -2/27, -13/54] , [0, -2/27, -13/54, -11/27, 23/54, 7/27, 5/54] , [0, -2/27, -13/54, -11/27, 23/54, 7/27, 5/54] , [0, 5/54, -2/27, -13/54, -11/27, 23/54, 7/27] , [0, -13/54, -11/27, 23/54, 7/27, 5/54, -2/27] , [0, 7/27, 5/54, -2/27, -13/54, -11/27, 23/54]] $ x $ [[3, 4, 0, 2, 1, 2, 3, 3, 0] , [3, 3, 0, 4, 0, 3, 3, 2, 0] , [3, 3, 0, 3, 0, 2, 3, 4, 0] , [3, 3, 0, 3, 0, 4, 2, 3, 0] , [2, 3, 0, 3, 0, 3, 4, 3, 0] , [4, 2, 0, 3, 0, 3, 3, 3, 0] , [3, 4, 0, 2, 0, 3, 3, 3, 0]] $

Â» SYNC'D 140385/8388608 , 0.01673519611

120 . Coloring, {2, 5, 6, 8}

R: [4, 9, 4, 7, 3, 8, 1, 6, 1] **B**: [2, 4, 5, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [-611, -188, -129, -602, -172, -258, -559, -344, -141] . FixedPtCheck, [611, 188, 129, 602, 172, 258, 559, 344, 141]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[4, 0, 2, 4, 0, 2, 3, 1, 2] , [5, 0, 0, 6, 0, 1, 4, 2, 0] , [4, 0, 0, 5, 0, 2, 6, 1, 0] , [6, 0, 0, 4, 0, 1, 5, 2, 0] , [5, 0, 0, 6, 0, 2, 4, 1, 0] , [4, 0, 0, 5, 0, 1, 6, 2, 0] , [6, 0, 0, 4, 0, 2, 5, 1, 0]] $

$$[-2\,y_5 - y_1 + 5\,y_2 - y_3 + 5\,y_4, 0, y_5, y_1, 0, y_2, y_3, y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[2, 4, 0, 2, 4, 0, 3, 3, 0] , [3, 2, 0, 4, 3, 0, 4, 2, 0] , [2, 3, 0, 2, 4, 0, 3, 4, 0] , [4, 2, 0, 3, 3, 0, 4, 2, 0] , [2, 4, 0, 2, 4, 0, 3, 3, 0] , [3, 2, 0, 4, 3, 0, 4, 2, 0]] $

$$[5\,y_3, 5\,y_4, 0, -5\,y_3 - 16\,y_4 + 33\,y_1 - 16\,y_2, 5\,y_1, 0, -7\,y_4 + 16\,y_1 - 7\,y_2, 5\,y_2, 0]$$

$p' = -s + s^5 \quad p = -s + s^5$

Â» SYNC’D 3949/262144 , 0.01506423950

121 . Coloring, {2, 5, 6, 9}

R: [4, 9, 4, 7, 3, 8, 1, 1, 2] **B**: [2, 4, 5, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1869, 1068, 375, 1950, 500, 150, 1625, 600, 801] . FixedPtCheck, [1869, 1068, 375, 1950, 500, 150, 1625, 600, 801]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[5, 1, 2, 4, 0, 0, 3, 1, 2] , [4, 2, 0, 7, 0, 0, 4, 0, 1] , [4, 1, 0, 4, 0, 0, 7, 0, 2] , [7, 2, 0, 4, 0, 0, 4, 0, 1] , [4, 1, 0, 7, 0, 0, 4, 0, 2] , [4, 2, 0, 4, 0, 0, 7, 0, 1] , [7, 1, 0, 4, 0, 0, 4, 0, 2]] $

$$[5\,y_1 - y_2 - y_3 - 3\,y_5 + 5\,y_4, y_1, 2\,y_5, y_2, 0, 0, y_3, y_5, y_4]$$

$p = -s^2 - s^3 + s^5 + s^6$ $p = -s^2 + s^4 + s^5 - s^7$

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$ [[1, 3, 0, 2, 4, 2, 3, 3, 0] , [0, 1, 0, 3, 3, 3, 6, 2, 0] , [0, 0, 0, 1, 6, 2, 6, 3, 0] , [0, 0, 0, 0, 6, 3, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 - y_2 - y_3 - y_4 + y_5 + y_6, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = -s^6 + s^7$

Â» SYNC’D 154171/4194304 , 0.03675723076

122 . Coloring, {2, 5, 7, 8}

R: [4, 9, 4, 7, 3, 7, 5, 6, 1] **B**: [2, 4, 5, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 - τ - 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [1274, 392, 2322, 2795, 3096, 645, 3354, 860, 294] . FixedPtCheck, [1274, 392, 2322, 2795, 3096, 645, 3354, 860, 294]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[1, 0, 2, 4, 3, 2, 4, 0, 2] , [8, 5, 3, 7, 4, 0, 7, 2, 0] , [11, 8, 4, 14, 8, 2, 11, 9, 5] , [25, 16, 8, 23, 15, 9, 24, 16, 8] , [48, 31, 15, 49, 32, 16, 49, 32, 16] , [95, 64, 32, 96, 66, 32, 97, 63, 31] , [191, 130, 66, 191, 129, 63, 190, 128, 64]] $ $ [[5, 4, 0, 2, 1, 0, 2, 4, 0] , [4, 3, 1, 5, 4, 4, 5, 6, 4] , [13, 8, 4, 10, 8, 6, 13, 7, 3] , [23, 16, 8, 25, 17, 7, 24, 16, 8] , [48, 33, 17, 47, 32, 16, 47, 32, 16] , [97, 64, 32, 96, 62, 32, 95, 65, 33] , [193, 126, 62, 193, 127, 65, 194, 128, 64]] $ $ [[-2, -2, 1, 1, 1, 1, 1, -2, 1] , [2, 1, 1, 1, 0, -2, 1, -2, -2] , [-1, 0, 0, 2, 0, -2, -1, 1, 1] , [1, 0, 0, -1, -1, 1, 0, 0, 0] , [0, -1, -1, 1, 0, 0, 1, 0, 0] , [-1, 0, 0, 0, 2, 0, 1, -1, -1] , [-1, 2, 2, -1, 1, -1, -2, 0, 0]] $

$$[y_3 - y_1 + y_2, y_3, y_4, y_5, y_1, -y_3 - y_5, y_2, -y_4 - y_2, -y_3 - y_2]$$

$p = s^3 - 16s^5 + 8s^6 - 32s^7$

S+ \; S- \; NM

$ [[37, 29, 12, 40, 20, 12, 36, 27, 15] , [42, 20, 7, 41, 36, 16, 27, 23, 16] , [46, 13, 4, 43, 40, 14, 30, 20, 18] , [34, 28, 16, 33, 21, 11, 48, 29, 8] , [29, 32, 27, 32, 13, 9, 52, 28, 6] , [34, 28, 16, 33, 21, 11, 48, 29, 8] , [43, 20, 9, 41, 36, 14, 30, 21, 14] , [43, 22, 6, 41, 25, 15, 35, 23, 18] , [34, 36, 17, 38, 16, 12, 36, 28, 11]] $ $ [[36, 25, 9, 40, 26, 12, 39, 27, 14] , [47, 22, 4, 41, 24, 16, 30, 23, 21] , [42, 15, 10, 43, 46, 14, 24, 20, 14] , [35, 32, 19, 33, 15, 11, 45, 29, 9] , [28, 28, 24, 32, 19, 9, 55, 28, 5] , [35, 32, 19, 33, 15, 11, 45, 29, 9] , [43, 20, 9, 41, 36, 14, 30, 21, 14] , [39, 24, 12, 41, 31, 15, 29, 23, 14] , [37, 30, 8, 38, 16, 12, 45, 28, 14]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

$p' = s^3 + 4s^4 + 8s^6$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
5 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	6 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[1, 0, 2, 4, 3, 2, 4, 0, 2] , [2, 0, 3, 3, 4, 0, 6, 0, 0] , [0, 0, 4, 5, 6, 0, 3, 0, 0] , [0, 0, 6, 4, 3, 0, 5, 0, 0] , [0, 0, 3, 6, 5, 0, 4, 0, 0] , [0, 0, 5, 3, 4, 0, 6, 0, 0] , [0, 0, 4, 5, 6, 0, 3, 0, 0]] $

$$[y_6, 0, y_5, y_4, y_3, y_2, y_1, 0, y_2]$$

$p = s^3 - s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0,

1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 19/72, -17/72, 1/72, 1/72] , [0, 0, 1/72, 19/72, -17/72, 1/72] , [1, -2, 1/72, 19/72, -89/72, 145/72] , [0, 0, 1/72, 1/72, 19/72, -17/72] , [0, 1, 1/72, 1/72, 19/72, -89/72] , [0, 0, 1/72, 1/72, 19/72, -17/72] , [0, 0, -17/72, 1/72, 1/72, 19/72] , [0, 0, -17/72, 1/72, 1/72, 19/72] , [0, 0, 19/72, -17/72, 1/72, 1/72]] $ x $ [[5, 4, 0, 2, 1, 0, 2, 4, 0] , [6, 5, 0, 4, 0, 0, 1, 2, 0] , [3, 6, 0, 5, 0, 0, 0, 4, 0] , [4, 3, 0, 6, 0, 0, 0, 5, 0] , [5, 4, 0, 3, 0, 0, 0, 6, 0] , [6, 5, 0, 4, 0, 0, 0, 3, 0]] $

Â» SYNC'D 735/32768 , 0.02243041992

123 . Coloring, {2, 5, 7, 9}

R: [4, 9, 4, 7, 3, 7, 5, 1, 2] **B**: [2, 4, 5, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 3τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 54‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 27‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 3τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-966, -552, -1350, -1875, -1800, -125, -1950, -500, -414] . FixedPtCheck, [966, 552, 1350, 1875, 1800, 125, 1950, 500, 414]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 2, 4, 3, 0, 4, 0, 2] , [0, 2, 3, 4, 4, 0, 4, 0, 1] , [0, 1, 4, 3, 4, 0, 4, 0, 2] , [0, 2, 4, 4, 4, 0, 3, 0, 1] , [0, 1, 4, 4, 3, 0, 4, 0, 2] , [0, 2, 3, 4, 4, 0, 4, 0, 1] , [0, 1, 4, 3, 4, 0, 4, 0, 2]] $

$$[2\,y_1 - y_2 - y_4 + 3\,y_5, y_1, y_2, 3\,y_1 - y_3 + 2\,y_5, y_3, 0, y_4, 0, y_5]$$

p = - s ^{2} + s ^{6} p' = - s ^{2} + s ^{6}

Omega Rank for B : cycles: {{6, 8}}, net cycles: 0 . order: 6

$ [[4, 3, 0, 2, 1, 2, 2, 4, 0] , [2, 4, 0, 3, 0, 4, 1, 4, 0] , [1, 2, 0, 4, 0, 4, 0, 7, 0] , [0, 1, 0, 2, 0, 7, 0, 8, 0] , [0, 0, 0, 1, 0, 8, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_1 - y_2 - y_3 - y_4 + y_5 + y_6, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = - s^6 + s^7$

Â» SYNC'D 14193/524288 , 0.02707099915

124 . Coloring, {2, 5, 8, 9}

R: [4, 9, 4, 7, 3, 7, 1, 6, 2] **B**: [2, 4, 5, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [301, 172, 75, 325, 100, 75, 325, 100, 129] . FixedPtCheck, [301, 172, 75, 325, 100, 75, 325, 100, 129]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[3, 1, 2, 4, 0, 2, 4, 0, 2] , [4, 2, 0, 5, 0, 0, 6, 0, 1] , [6, 1, 0, 4, 0, 0, 5, 0, 2] , [5, 2, 0, 6, 0, 0, 4, 0, 1] , [4, 1, 0, 5, 0, 0, 6, 0, 2] , [6, 2, 0, 4, 0, 0, 5, 0, 1] , [5, 1, 0, 6, 0, 0, 4, 0, 2]] $

$$[5 y_1 - y_2 - 2 y_3 - y_4 + 5 y_5, y_1, y_3, y_2, 0, y_3, y_4, 0, y_5]$$

$p = - s^2 - s^3 + s^5 + s^6$ $p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[3, 3, 0, 2, 4, 0, 2, 4, 0] , [4, 3, 0, 3, 2, 0, 4, 2, 0] , [2, 4, 0, 3, 4, 0, 2, 3, 0] , [3, 2, 0, 4, 2, 0, 4, 3, 0] , [3, 3, 0, 2, 4, 0, 2, 4, 0] , [4, 3, 0, 3, 2, 0, 4, 2, 0]] $

$$[y_1, 3\,y_1 + 3\,y_4 - 4\,y_2 - y_3, 0, y_4, 2\,y_1 + 2\,y_4 - 3\,y_2, 0, y_2, y_3, 0]$$

p = s - s ^{5} p' = s - s ^{5}

Â» SYNC'D 595/65536 , 0.009078979492

125 . Coloring, {2, 6, 7, 8}

R: [4, 9, 4, 7, 7, 8, 5, 6, 1] **B**: [2, 4, 5, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-247, -76, -129, -301, -516, -129, -645, -172, -57] . FixedPtCheck, [247, 76, 129, 301, 516, 129, 645, 172, 57]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 4, 3, 2, 5, 1, 2] , [2, 0, 0, 1, 5, 1, 7, 2, 0] , [0, 0, 0, 2, 7, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0]] $

$$[y_2 - y_5 + 4\,y_3, 0, 0, -y_1 + 4\,y_2 + y_3 - y_4, y_1, y_2, y_5, y_3, y_4]$$

p = - s ^{4} + s ^{6} p' = - s ^{4} + s ^{6}

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[5, 4, 2, 2, 1, 0, 1, 3, 0] , [4, 5, 1, 4, 2, 0, 0, 2, 0] , [2, 4, 2, 5, 1, 0, 0, 4, 0] , [4, 2, 1, 4, 2, 0, 0, 5, 0] , [5, 4, 2, 2, 1, 0, 0, 4, 0] , [4, 5, 1, 4, 2, 0, 0, 2, 0] , [2, 4, 2, 5, 1, 0, 0, 4, 0]] $

$$[2\,y_1 - y_2 + 3\,y_3, 3\,y_1 + 2\,y_3 - y_4 - y_5, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

p' = - s ^ 2 + s ^ 6 p = - s ^ 2 + s ^ 6

Â» SYNC'D 30183/2097152 , 0.01439237595

126 . Coloring, {2, 6, 7, 9}

R: [4, 9, 4, 7, 7, 8, 5, 1, 2] **B**: [2, 4, 5, 8, 3, 7, 1, 6, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

'['-9' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' 3 + τ ')'' (' - 1 + τ ')' , -18' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')' , 9' (' 1 + τ ^ 2 ')'' (' - 1 + τ ')'' (' 5 + 2τ + τ ^ 2 ')' , 9' (' - 1 + τ ')'' (' 3 + τ ^ 2 ')'' (' 5 + 2τ + τ ^ 2 ')' , -18' (' 1 + τ ^ 2 ')'' (' 5 + 2τ + τ ^ 2 ')' , 9' (' - 1 + τ ')' ^ 3 ' (' 5 + 2τ + τ ^ 2 ')' , 9' (' 1 + τ ^ 2 ')'' (' 5 + 2τ + τ ^ 2 ')'' (' - 3 + τ ')' , -18' (' - 1 + τ ')' ^ 2 ' (' 5 + 2τ + τ ^ 2 ')' , -9' (' 1 + τ ')'' (' - 5 + τ - τ ^ 2 + τ ^ 3 ')'' (' - 1 + τ ')'']'

For τ=1/2, [-259, -148, -125, -325, -500, -25, -625, -100, -111] . FixedPtCheck, [259, 148, 125, 325, 500, 25, 625, 100, 111]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	6 *vs* 8

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 0, 4, 3, 0, 5, 1, 2] , [1, 2, 0, 2, 5, 0, 7, 0, 1] , [0, 1, 0, 1, 7, 0, 7, 0, 2] , [0, 2, 0, 0, 7, 0, 8, 0, 1] , [0, 1, 0, 0, 8, 0, 7, 0, 2] , [0, 2, 0, 0, 7, 0, 8, 0, 1] , [0, 1, 0, 0, 8, 0, 7, 0, 2]] $

$$[3\,y_1 - y_3 + 2\,y_2, y_1, 0, y_5, y_4, 0, y_3, 2\,y_1 - y_5 - y_4 + 3\,y_2, y_2]$$

p = - s ^ 4 + s ^ 6 p' = - s ^ 4 + s ^ 6

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 6, 7, 8}}, net cycles: 2 . order: 6

$ [[4, 3, 2, 2, 1, 2, 1, 3, 0] , [1, 4, 1, 3, 2, 3, 2, 2, 0] , [2, 1, 2, 4, 1, 2, 3, 3, 0] , [3, 2, 1, 1, 2, 3, 2, 4, 0] , [2, 3, 2, 2, 1, 4, 3, 1, 0] , [3, 2, 1, 3, 2, 1, 4, 2, 0] , [4, 3, 2, 2, 1, 2, 1, 3, 0] , [1, 4, 1, 3, 2, 3, 2, 2, 0]] $

$$[3\,y_1 - y_2 + 2\,y_6 - y_5, 2\,y_1 + 3\,y_6 - y_3 - y_4, y_1, y_2, y_6, y_5, y_3, y_4, 0]$$

p = - s + s ^{7} p' = - s + s ^{7}

Â» SYNC'D 162285/8388608 , 0.01934587955

127 . Coloring, {2, 6, 8, 9}

R: [4, 9, 4, 7, 7, 8, 1, 6, 2] **B**: [2, 4, 5, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘ , 18‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 - 3τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-371, -212, -25, -350, -100, -150, -375, -200, -159] . FixedPtCheck, [371, 212, 25, 350, 100, 150, 375, 200, 159]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {2, 9}, {1, 4, 7}}, net cycles: 3 . order: 6

$ [[3, 1, 0, 4, 0, 2, 5, 1, 2] , [5, 2, 0, 3, 0, 1, 4, 2, 1] , [4, 1, 0, 5, 0, 2, 3, 1, 2] , [3, 2, 0, 4, 0, 1, 5, 2, 1] , [5, 1, 0, 3, 0, 2, 4, 1, 2] , [4, 2, 0, 5, 0, 1, 3, 2, 1] , [3, 1, 0, 4, 0, 2, 5, 1, 2]] $

$$[4\,y_2 - y_4 - y_1 + 4\,y_3, y_2, 0, y_4, 0, y_3, y_1, y_2, y_3]$$

p = - s - s ^{2} + s ^{4} + s ^{5} p = s - s ^{3} - s ^{4} + s ^{6} p = - s + s ^{7}

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[3, 3, 2, 2, 4, 0, 1, 3, 0] , [3, 3, 4, 3, 3, 0, 0, 2, 0] , [2, 3, 3, 3, 4, 0, 0, 3, 0] , [3, 2, 4, 3, 3, 0, 0, 3, 0] , [3, 3, 3, 2, 4, 0, 0, 3, 0] , [3, 3, 4, 3, 3, 0, 0, 2, 0] , [2, 3, 3, 3, 4, 0, 0, 3, 0]] $

$$[2\,y_1, 9\,y_1 - 11\,y_2 + 9\,y_3 - 11\,y_4 - 2\,y_5, 2\,y_2, 2\,y_3, 7\,y_1 - 9\,y_2 + 7\,y_3 - 9\,y_4, 0, 2\,y_4, 2\,y_5, 0]$$

$p = -s^2 + s^6$ $p' = -s^2 + s^6$

Â» SYNC'D 111573/33554432 , 0.003325134516

128 . Coloring, {2, 7, 8, 9}

R: [4, 9, 4, 7, 7, 7, 5, 6, 2] **B**: [2, 4, 5, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-238, -136, -150, -325, -600, -75, -750, -100, -102] . FixedPtCheck, [238, 136, 150, 325, 600, 75, 750, 100, 102]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	3 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 0 . order: 2

$ [[0, 1, 0, 4, 3, 2, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1]] $

$$[0, y_1 + 3 y_2 - 4 y_3, 0, 2 y_2, y_1, y_2, 4 y_1 + 12 y_2 - 15 y_3, 0, y_3]$$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$ $p = -s^2 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 2 . order: 4

$ [[6, 3, 2, 2, 1, 0, 0, 4, 0] , [4, 6, 1, 3, 2, 0, 0, 2, 0] , [2, 4, 2, 6, 1, 0, 0, 3, 0] , [3, 2, 1, 4, 2, 0, 0, 6, 0] , [6, 3, 2, 2, 1, 0, 0, 4, 0] , [4, 6, 1, 3, 2, 0, 0, 2, 0]] $

$$[3 y_1 - y_2 + 2 y_3, 2 y_1 + 3 y_3 - y_4, y_1, y_2, y_3, 0, 0, y_4, 0]$$

$p = s - s^5 \quad p' = s - s^5$

Â» SYNC'D 795/32768 , 0.02426147461

129 . Coloring, {3, 4, 5, 6}

R: [4, 4, 5, 8, 3, 8, 1, 1, 1] **B**: [2, 9, 4, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-750, -200, -114, -741, -152, -171, -266, -684, -50] . FixedPtCheck, [750, 200, 114, 741, 152, 171, 266, 684, 50]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	3 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}}, net cycles: 2 . order: 6

$ [[6, 0, 2, 5, 1, 0, 0, 4, 0] , [4, 0, 1, 6, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 1, 0, 0, 6, 0] , [6, 0, 1, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 6, 1, 0, 0, 5, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4, 0, y_1, y_2, y_3, 0, 0, y_4, 0]$$

$p = - s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 0 . order: 2

$ [[0, 4, 0, 1, 3, 2, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 6, 0, 4]] $

$$[0, y_2 - y_3, 0, y_1, -3\,y_1 + y_2, 2\,y_1, y_2, 0, y_3]$$

$p' = - s^3 + s^5 \quad p = s^2 - s^4 \quad p' = s^2 - s^4$

Â» SYNC'D 121/8192 , 0.01477050781

130 . Coloring, {3, 4, 5, 7}

R: [4, 4, 5, 8, 3, 7, 5, 1, 1] **B**: [2, 9, 4, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-465, -124, -513, -570, -684, -114, -399, -456, -31] . FixedPtCheck, [465, 124, 513, 570, 684, 114, 399, 456, 31]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[3, 0, 2, 5, 4, 0, 1, 3, 0] , [3, 0, 4, 3, 3, 0, 0, 5, 0] , [5, 0, 3, 3, 4, 0, 0, 3, 0] , [3, 0, 4, 5, 3, 0, 0, 3, 0] , [3, 0, 3, 3, 4, 0, 0, 5, 0] , [5, 0, 4, 3, 3, 0, 0, 3, 0]] $

$$[11\,y_1 - 7\,y_2 + 11\,y_3 + 11\,y_4 - 7\,y_5, 0, 7\,y_1, 7\,y_2, 7\,y_3, 0, 7\,y_4, 7\,y_5, 0]$$

$p = s^2 + s^3 - s^5 - s^6$

Omega Rank for B : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[3, 4, 0, 1, 0, 2, 5, 1, 2] , [5, 5, 0, 0, 0, 1, 1, 2, 4] , [1, 9, 0, 0, 0, 2, 0, 1, 5] , [0, 6, 0, 0, 0, 1, 0, 2, 9] , [0, 9, 0, 0, 0, 2, 0, 1, 6] , [0, 6, 0, 0, 0, 1, 0, 2, 9] , [0, 9, 0, 0, 0, 2, 0, 1, 6]] $

$$[y_5, y_4, 0, y_3, 0, y_2, y_1, y_4 - 4\,y_2 + y_1, -y_5 + 4\,y_4 - y_3 - 15\,y_2 + 4\,y_1]$$

$p' = s^4 - s^6$ $p = s^4 - s^6$

Â» SYNC'D 51985/2097152 , 0.02478837967

131 . Coloring, {3, 4, 5, 8}

R: [4, 4, 5, 8, 3, 7, 1, 6, 1] **B**: [2, 9, 4, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-705, -188, -285, -741, -380, -513, -665, -684, -47] . FixedPtCheck, [705, 188, 285, 741, 380, 513, 665, 684, 47]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}, {3, 5}}, net cycles: 2 .

$ [[4, 0, 2, 5, 1, 2, 1, 3, 0] , [1, 0, 1, 4, 2, 3, 2, 5, 0] , [2, 0, 2, 1, 1, 5, 3, 4, 0] , [3, 0, 1, 2, 2, 4, 5, 1, 0] , [5, 0, 2, 3, 1, 1, 4, 2, 0] , [4, 0, 1, 5, 2, 2, 1, 3, 0] , [1, 0, 2, 4, 1, 3, 2, 5, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_6 - y_5 - y_4, 0, y_1, y_2, y_3, y_6, y_5, y_4, 0]$$

p = - s - s ^ 2 + s ^ 6 + s ^ 7

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 0 . order: 4

$ [[2, 4, 0, 1, 3, 0, 5, 1, 2] , [1, 4, 0, 0, 5, 0, 4, 0, 4] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5] , [0, 5, 0, 0, 4, 0, 5, 0, 4]] $

$$[y_3 + y_1 - y_4, -y_3 + y_2, 0, y_3, y_1, 0, y_2, y_3, y_4]$$

p = - s ^ 3 + s ^ 5 p’ = - s ^ 3 + s ^ 5 p = - s ^ 3 + s ^ 7

Â» SYNC’D 2821/262144 , 0.01076126099

132 . Coloring, {3, 4, 5, 9}

R: [4, 4, 5, 8, 3, 7, 1, 1, 2] **B**: [2, 9, 4, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-247, -76, -51, -255, -68, -51, -119, -204, -19] . FixedPtCheck, [247, 76, 51, 255, 68, 51, 119, 204, 19]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 0 . order: 6

$ [[5, 1, 2, 5, 1, 0, 1, 3, 0] , [4, 0, 1, 6, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 1, 0, 0, 6, 0] , [6, 0, 1, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 6, 1, 0, 0, 5, 0] , [5, 0, 1, 4, 2, 0, 0, 6, 0] , [6, 0, 2, 5, 1, 0, 0, 4, 0]] $

$$[-2\,y_5 + 5\,y_2 - y_1 + 5\,y_3 - y_4, y_5, y_2, y_1, y_3, 0, y_5, y_4, 0]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{6, 8}, {5, 7}, {1, 2, 9}}, net cycles: 2 . order: 6

$ [[1, 3, 0, 1, 3, 2, 5, 1, 2] , [2, 1, 0, 0, 5, 1, 4, 2, 3] , [3, 2, 0, 0, 4, 2, 5, 1, 1] , [1, 3, 0, 0, 5, 1, 4, 2, 2] , [2, 1, 0, 0, 4, 2, 5, 1, 3] , [3, 2, 0, 0, 5, 1, 4, 2, 1] , [1, 3, 0, 0, 4, 2, 5, 1, 2] , [2, 1, 0, 0, 5, 1, 4, 2, 3]] $

$$[-y_1 + 2\,y_3 + 2\,y_5 - y_4, y_1, 0, -y_2 + y_3 + 2\,y_5, y_2, y_3, 2\,y_3 + y_5, y_5, y_4]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7 p = - s ^ 2 + s ^ 8

Â» SYNC’D 119533/16777216 , 0.007124722004

133 . Coloring, {3, 4, 6, 7}

R: [4, 4, 5, 8, 7, 8, 5, 1, 1] **B**: [2, 9, 4, 7, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-735, -196, -171, -741, -684, -171, -741, -684, -49] . FixedPtCheck, [735, 196, 171, 741, 684, 171, 741, 684, 49]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 0, 0, 5, 4, 0, 2, 4, 0] , [8, 5, 0, 5, 2, 0, 7, 5, 4] , [14, 4, 6, 17, 7, 3, 13, 5, 3] , [19, 15, 9, 20, 19, 11, 19, 20, 12] , [61, 33, 13, 41, 28, 12, 52, 31, 17] , [92, 50, 36, 113, 65, 33, 103, 53, 31] , [173, 133, 63, 170, 139, 75, 175, 146, 78]] $ $ [[3, 4, 2, 1, 0, 2, 4, 0, 2] , [4, 3, 4, 7, 6, 4, 5, 3, 0] , [10, 12, 2, 7, 9, 5, 11, 11, 5] , [29, 17, 7, 28, 13, 5, 29, 12, 4] , [35, 31, 19, 55, 36, 20, 44, 33, 15] , [100, 78, 28, 79, 63, 31, 89, 75, 33] , [211, 123, 65, 214, 117, 53, 209, 110, 50]] $ $ [[0, -2, -1, 2, 2, -1, -1, 2, -1] , [2, 1, -2, -1, -2, -2, 1, 1, 2] , [2, -4, 2, 5, -1, -1, 1, -3, -1] , [-5, -1, 1, -4, 3, 3, -5, 4, 4] , [13, 1, -3, -7, -4, -4, 4, -1, 1] , [-4, -14, 4, 17, 1, 1, 7, -11, -1] , [-19, 5, -1, -22, 11, 11, -17, 18, 14]] $

$[-2 y_5 - y_4 - y_1 + y_2 - 3 y_3, 2 y_5 + y_4 - 2 y_2 + 2 y_3 - y_6, -y_5 - y_4, y_1, y_2, y_3, y_4, y_5, y_6]$

p = s ^ 3 - 3s ^ 4 + 8s ^ 7

S+ \ ; S- \ ; NM

$ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[16, 14, 5, 18, 9, 6, 14, 9, 5] , [15, 11, 3, 19, 10, 5, 14, 11, 8] , [15, 9, 6, 19, 15, 5, 14, 8, 5] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [14, 10, 9, 14, 11, 4, 20, 11, 3] , [13, 10, 6, 16, 9, 4, 19, 13, 6] , [19, 8, 5, 14, 14, 6, 15, 10, 5] , [19, 11, 4, 15, 11, 7, 14, 10, 5] , [20, 13, 4, 13, 8, 7, 15, 11, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true*, *true*, *true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 2 . order: 6

$ [[3, 0, 0, 5, 4, 0, 2, 4, 0] , [4, 0, 0, 3, 2, 0, 4, 5, 0] , [5, 0, 0, 4, 4, 0, 2, 3, 0] , [3, 0, 0, 5, 2, 0, 4, 4, 0] , [4, 0, 0, 3, 4, 0, 2, 5, 0]] $

$$[-y_1 + 2\,y_2 + 2\,y_3 - y_4, 0, 0, y_1, y_2, 0, y_3, y_4, 0]$$

$p = -s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 6

$ [[3, 4, 2, 1, 0, 2, 4, 0, 2] , [4, 5, 0, 2, 0, 0, 3, 0, 4] , [3, 8, 0, 0, 0, 0, 2, 0, 5] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

$$[y_1, y_2, y_4, y_3, 0, y_4, y_5, 0, y_6]$$

$p = -s^5 + s^7$

Â» SYNC'D 2725/65536 , 0.04158020020

134 . Coloring, {3, 4, 6, 8}

R: [4, 4, 5, 8, 7, 8, 1, 6, 1] **B**: [2, 9, 4, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 3τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 3τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-690, -184, -38, -665, -152, -855, -494, -1140, -46] . FixedPtCheck, [690, 184, 38, 665, 152, 855, 494, 1140, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}}, net cycles: 0 . order: 6

$ [[4, 0, 0, 5, 1, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 1, 7, 0] , [1, 0, 0, 2, 0, 7, 0, 8, 0] , [0, 0, 0, 1, 0, 8, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_1 - y_2 + y_5 + y_3 - y_4, 0, 0, y_1, y_2, y_5, y_3, y_4, 0]$$

$p = - s^5 + s^6$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 2, 1, 3, 0, 4, 0, 2] , [0, 4, 3, 2, 4, 0, 1, 0, 4] , [0, 4, 4, 3, 1, 0, 2, 0, 4] , [0, 4, 1, 4, 2, 0, 3, 0, 4] , [0, 4, 2, 1, 3, 0, 4, 0, 4] , [0, 4, 3, 2, 4, 0, 1, 0, 4] , [0, 4, 4, 3, 1, 0, 2, 0, 4]] $

$$[2\,y_5, 2\,y_4, 2\,y_3, 2\,y_2, 2\,y_1, 0, -2\,y_3 - 2\,y_2 - 2\,y_1 + 5\,y_4, 0, -2\,y_5 + 2\,y_4]$$

$p = s^2 - s^6$ p’ = $s^2 - s^6$

Â» SYNC’D 15171/1048576 , 0.01446819305

135 . Coloring, {3, 4, 6, 9}

R: [4, 4, 5, 8, 7, 8, 1, 1, 2] **B**: [2, 9, 4, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1118, 344, 34, 1105, 136, 255, 442, 1020, 86] . FixedPtCheck, [1118, 344, 34, 1105, 136, 255, 442, 1020, 86]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 6	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[5, 1, 0, 5, 1, 0, 2, 4, 0] , [6, 0, 0, 6, 0, 0, 1, 5, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] $

$$[-y_1 + y_2 + y_3, y_1, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$p = -s^3 + s^5$ $p = -s^3 + s^6$ $p = -s^3 + s^4$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 4, 5, 7}}, net cycles: 1 .

$ [[1, 3, 2, 1, 3, 2, 4, 0, 2] , [2, 1, 3, 2, 4, 0, 3, 0, 3] , [3, 2, 4, 3, 3, 0, 2, 0, 1] , [1, 3, 3, 4, 2, 0, 3, 0, 2] , [2, 1, 2, 3, 3, 0, 4, 0, 3] , [3, 2, 3, 2, 4, 0, 3, 0, 1] , [1, 3, 4, 3, 3, 0, 2, 0, 2] , [2, 1, 3, 4, 2, 0, 3, 0, 3]] $

$$[y_1, -y_1 + y_2 + y_3 + y_4 - y_6, -y_5 + y_2 + y_3 + y_4, y_2, y_3, y_4, y_5, 0, y_6]$$

$p' = -s^2 - s^4 + s^5 + s^7$ $p = -s^2 - s^4 + s^5 + s^7$

Â» SYNC’D 24189/4194304 , 0.005767107010

136 . Coloring, {3, 4, 7, 8}

R: [4, 4, 5, 8, 7, 7, 5, 6, 1] **B**: [2, 9, 4, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘]‘

For τ=1/2, [1335, 356, 855, 1482, 3420, 1026, 3705, 1368, 89] . FixedPtCheck, [1335, 356, 855, 1482, 3420, 1026, 3705, 1368, 89]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_1, 0, 0, y_2, y_3, y_6, y_4, y_5, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -5, 22, 371/72, -1663/72] , [0, 1, -5, 22, 371/72, -1663/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 1, -5, -79/72, 371/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 1, 11/72, -79/72] , [1, -5, 22, -97, -1663/72, 7355/72]] $ x $ [[1, 0, 0, 5, 4, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 6, 5, 0] , [0, 0, 0, 0, 6, 5, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 6

$ [[5, 4, 2, 1, 0, 0, 3, 1, 2] , [4, 7, 0, 2, 0, 0, 1, 0, 4] , [1, 8, 0, 0, 0, 0, 2, 0, 7] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

$$[y_1, y_2, 2\,y_5, y_3, 0, 0, y_4, y_5, y_6]$$

p = - s ^{5} + s ^{7}

Â» SYNC’D 6991/65536 , 0.1066741943

137 . Coloring, {3, 4, 7, 9}

R: [4, 4, 5, 8, 7, 7, 5, 1, 2] **B**: [2, 9, 4, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘ , -18‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ - τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-481, -148, -153, -510, -612, -102, -663, -408, -37] . FixedPtCheck, [481, 148, 153, 510, 612, 102, 663, 408, 37]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{4}

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	7 *vs* 8

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[2, 1, 0, 5, 4, 0, 3, 3, 0] , [3, 0, 0, 3, 3, 0, 4, 5, 0] , [5, 0, 0, 3, 4, 0, 3, 3, 0] , [3, 0, 0, 5, 3, 0, 4, 3, 0] , [3, 0, 0, 3, 4, 0, 3, 5, 0] , [5, 0, 0, 3, 3, 0, 4, 3, 0]] $

$$[7\,y_5, 7\,y_4, 0, 7\,y_3, 7\,y_2, 0, 7\,y_1, -7\,y_5 - 7\,y_4 - 7\,y_3 + 11\,y_2 + 11\,y_1, 0]$$

$p = s^2 + s^3 - s^5 - s^6$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[4, 3, 2, 1, 0, 2, 3, 1, 2] , [5, 4, 0, 2, 0, 1, 1, 2, 3] , [4, 5, 0, 0, 0, 2, 2, 1, 4] , [6, 4, 0, 0, 0, 1, 0, 2, 5] , [5, 6, 0, 0, 0, 2, 0, 1, 4] , [4, 5, 0, 0, 0, 1, 0, 2, 6] , [6, 4, 0, 0, 0, 2, 0, 1, 5] , [5, 6, 0, 0, 0, 1, 0, 2, 4]] $

$$[y_7, y_6, y_5, y_4, 0, y_3, y_2, y_1, -y_7 - y_6 - y_5 - y_4 + 5\,y_3 - y_2 + 5\,y_1]$$

$p = s^4 + s^5 - s^7 - s^8$

Â» SYNC’D 1537181/33554432 , 0.04581156373

138 . Coloring, {3, 4, 8, 9}

R: [4, 4, 5, 8, 7, 7, 1, 6, 2] **B**: [2, 9, 4, 7, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [13, 4, 1, 13, 4, 9, 13, 12, 1] . FixedPtCheck, [13, 4, 1, 13, 4, 9, 13, 12, 1]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	5 *vs* 8

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: -1 . order: 5

$ [[3, 1, 0, 5, 1, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 3, 5, 0] , [3, 0, 0, 3, 0, 5, 3, 4, 0] , [3, 0, 0, 3, 0, 4, 5, 3, 0] , [5, 0, 0, 3, 0, 3, 4, 3, 0] , [4, 0, 0, 5, 0, 3, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 3, 5, 0]] $

$$[y_1, y_3, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = - s^2 + s^7$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {1, 2, 9}}, net cycles: 1 .

$ [[3, 3, 2, 1, 3, 0, 3, 1, 2] , [3, 3, 3, 2, 3, 0, 1, 0, 3] , [3, 3, 3, 3, 1, 0, 2, 0, 3] , [3, 3, 1, 3, 2, 0, 3, 0, 3] , [3, 3, 2, 1, 3, 0, 3, 0, 3] , [3, 3, 3, 2, 3, 0, 1, 0, 3] , [3, 3, 3, 3, 1, 0, 2, 0, 3] , [3, 3, 1, 3, 2, 0, 3, 0, 3]] $

$$[y_4 + y_5, y_4 + y_5, 3\,y_4 + 3\,y_5 - y_1 - y_2 - y_3, y_1, y_2, 0, y_3, y_4, y_5]$$

$p = - s^2 + s^6$ $p' = - s^2 + s^6$ $p' = - s^3 + s^7$

Â» SYNC’D 449775/67108864 , 0.006702169776

139 . Coloring, {3, 5, 6, 7}

R: [4, 4, 5, 7, 3, 8, 5, 1, 1] **B**: [2, 9, 4, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 4 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [-885, -236, -2565, -1482, -3420, -114, -1995, -456, -59] . FixedPtCheck, [885, 236, 2565, 1482, 3420, 114, 1995, 456, 59]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 6	7 *vs* 7

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[y_2, 0, y_1, y_6, y_5, 0, y_3, y_4, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -3, -13/18, 25/9] , [0, 0, 1, -3, -13/18, 25/9] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 0, -2/9, 5/18] , [1, -3, 4, 0, -85/18, 25/9] , [0, 0, 0, 0, 5/18, -2/9] , [0, 1, -3, 4, 25/9, -85/18] , [0, 1, -3, 4, 25/9, -85/18]] $ x $ [[3, 0, 2, 5, 4, 0, 3, 1, 0] , [1, 0, 4, 3, 5, 0, 5, 0, 0] , [0, 0, 5, 1, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 8, 0, 1, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, 0, y_5, 0, y_4, y_3, y_7, y_6]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, -2/9, 5/18] , [0, 0, 0, 0, 0, 5/18, -2/9] , [1, -3, 7, -18, 46, 383/18, -488/9] , [0, 1, -3, 7, -18, -74/9, 383/18] , [0, 0, 0, 1, -3, -11/9, 59/18] , [0, 0, 0, 1, -3, -11/9, 59/18] , [0, 0, 0, 0, 1, 5/18, -11/9] , [0, 0, 1, -3, 7, 59/18, -74/9] , [0, 0, 0, 0, 0, -2/9, 5/18]] $ x $ [[3, 4, 0, 1, 0, 2, 3, 3, 2] , [3, 5, 0, 0, 0, 3, 2, 1, 4] , [2, 7, 0, 0, 0, 1, 3, 0, 5] , [3, 7, 0, 0, 0, 0, 1, 0, 7] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC’D 50733/2097152 , 0.02419137955

140 . Coloring, {3, 5, 6, 8}

R: [4, 4, 5, 7, 3, 8, 1, 6, 1] **B**: [2, 9, 4, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-375, -100, -171, -399, -228, -171, -399, -228, -25] . FixedPtCheck, [375, 100, 171, 399, 228, 171, 399, 228, 25]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}, {3, 5}}, net cycles: 3 . order: 6

$ [[4, 0, 2, 5, 1, 2, 3, 1, 0] , [3, 0, 1, 4, 2, 1, 5, 2, 0] , [5, 0, 2, 3, 1, 2, 4, 1, 0] , [4, 0, 1, 5, 2, 1, 3, 2, 0] , [3, 0, 2, 4, 1, 2, 5, 1, 0] , [5, 0, 1, 3, 2, 1, 4, 2, 0] , [4, 0, 2, 5, 1, 2, 3, 1, 0]] $

$$[-y_1 + 4 y_2 - y_3 + 4 y_4, 0, y_2, y_1, y_4, y_2, y_3, y_4, 0]$$

p = s - s 3 - s 4 + s 6 p = - s - s 2 + s 4 + s 5 p = - s + s 7

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 4, 0, 1, 3, 0, 3, 3, 2] , [3, 4, 0, 0, 3, 0, 3, 1, 4] , [1, 7, 0, 0, 3, 0, 3, 0, 4] , [0, 5, 0, 0, 3, 0, 3, 0, 7] , [0, 7, 0, 0, 3, 0, 3, 0, 5] , [0, 5, 0, 0, 3, 0, 3, 0, 7] , [0, 7, 0, 0, 3, 0, 3, 0, 5]] $

$$[-y_1 - y_2 + 4 y_3 - y_4 - y_5, y_1, 0, y_2, y_3, 0, y_3, y_4, y_5]$$

p = - s 4 + s 6 p’ = - s 4 + s 6

Â» SYNC’D 4455/2097152 , 0.002124309540

141 . Coloring, {3, 5, 6, 9}

R: [4, 4, 5, 7, 3, 8, 1, 1, 2] **B**: [2, 9, 4, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [611, 188, 255, 663, 340, 51, 595, 204, 47] . FixedPtCheck, [611, 188, 255, 663, 340, 51, 595, 204, 47]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	6 *vs* 8

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}}, net cycles: 0 . order: 6

$ [[5, 1, 2, 5, 1, 0, 3, 1, 0] , [4, 0, 1, 6, 2, 0, 5, 0, 0] , [5, 0, 2, 4, 1, 0, 6, 0, 0] , [6, 0, 1, 5, 2, 0, 4, 0, 0] , [4, 0, 2, 6, 1, 0, 5, 0, 0] , [5, 0, 1, 4, 2, 0, 6, 0, 0] , [6, 0, 2, 5, 1, 0, 4, 0, 0]] $

$$[y_2, y_5, y_1, -y_2 - 2\,y_5 + 5\,y_1 + 5\,y_3 - y_4, y_3, 0, y_4, y_5, 0]$$

p = s 2 - s 4 - s 5 + s 7 p = - s 2 - s 3 + s 5 + s 6

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[1, 3, 0, 1, 3, 2, 3, 3, 2] , [2, 1, 0, 0, 3, 3, 5, 1, 3] , [3, 2, 0, 0, 5, 1, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3]] $

$$[y_3, y_4, 0, y_5, -y_5 - y_1 + y_3 + y_4 + y_6, y_1, y_3 + y_4 - y_2 + y_6, y_2, y_6]$$

p’ = s 4 - s 7 p = s 4 - s 7

Â» SYNC’D 45045/4194304 , 0.01073956490

142 . Coloring, {3, 5, 7, 8}

R: [4, 4, 5, 7, 3, 7, 5, 6, 1] **B**: [2, 9, 4, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 4 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 4 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-375, -100, -1539, -741, -2052, -171, -1197, -228, -25] . FixedPtCheck, [375, 100, 1539, 741, 2052, 171, 1197, 228, 25]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[1, 0, 2, 5, 4, 2, 4, 0, 0] , [6, 7, 4, 1, 6, 0, 7, 1, 4] , [16, 6, 6, 13, 11, 1, 3, 15, 1] , [23, 15, 11, 24, 9, 15, 19, 18, 10] , [53, 31, 9, 43, 30, 18, 62, 25, 17] , [90, 58, 30, 107, 71, 25, 95, 67, 33] , [191, 133, 71, 182, 125, 67, 189, 124, 70]] $ $ [[5, 4, 0, 1, 0, 0, 2, 4, 2] , [6, 1, 0, 11, 2, 4, 5, 7, 0] , [8, 10, 2, 11, 5, 7, 21, 1, 7] , [25, 17, 5, 24, 23, 1, 29, 14, 6] , [43, 33, 23, 53, 34, 14, 34, 39, 15] , [102, 70, 34, 85, 57, 39, 97, 61, 31] , [193, 123, 57, 202, 131, 61, 195, 132, 58]] $ $ [[-2, -2, 1, 2, 2, 1, 1, -2, -1] , [0, 3, 2, -5, 2, -2, 1, -3, 2] , [4, -2, 2, 1, 3, -3, -9, 7, -3] , [-1, -1, 3, 0, -7, 7, -5, 2, 2] , [5, -1, -7, -5, -2, 2, 14, -7, 1] , [-6, -6, -2, 11, 7, -7, -1, 3, 1] , [-1, 5, 7, -10, -3, 3, -3, -4, 6]] $

$[y_6, y_5, y_4, y_3, y_2, y_1, -y_6 - 2\,y_4 - y_3 + y_2 + y_1, y_4 + y_6 + y_3 - y_2 - y_1, -y_6 - y_5 - y_3 - y_2 - y_1]$

$p = s^2 + 2s^4 - 16s^7$

S+ \; S- \; NM

$ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 4

$ [[1, 0, 2, 5, 4, 2, 4, 0, 0] , [0, 0, 4, 1, 6, 0, 7, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0]] $

$[y_1, 0, y_5, y_2, y_3, 2\,y_1, y_4, 0, 0]$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 1, 0, 0, 2, 4, 2] , [6, 7, 0, 0, 0, 0, 0, 1, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, y_3, 0, 0, 2\, y_3, y_4, y_5]$$

$p = -s^4 + s^6$

Â» SYNC'D 13/128 , 0.1015625000

143 . Coloring, {3, 5, 7, 9}

R: [4, 4, 5, 7, 3, 7, 5, 1, 2] **B**: [2, 9, 4, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 4 ‘]‘

For τ=1/2, [143, 44, 459, 255, 612, 17, 357, 68, 11] . FixedPtCheck, [143, 44, 459, 255, 612, 17, 357, 68, 11]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 4

$ [[2, 1, 2, 5, 4, 0, 4, 0, 0] , [0, 0, 4, 3, 6, 0, 5, 0, 0] , [0, 0, 6, 0, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[2\, y_4, y_4, y_3, y_2, y_1, 0, -3\, y_4 - y_3 + y_2 + y_1, 0, 0]$$

$p = -s^4 + s^5$ $p = -s^4 + s^6$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}}, net cycles: 0 . order: 6

$ [[4, 3, 0, 1, 0, 2, 2, 4, 2] , [4, 4, 0, 0, 0, 4, 0, 3, 3] , [3, 4, 0, 0, 0, 3, 0, 4, 4] , [4, 3, 0, 0, 0, 4, 0, 3, 4] , [4, 4, 0, 0, 0, 3, 0, 4, 3] , [3, 4, 0, 0, 0, 4, 0, 3, 4] , [4, 3, 0, 0, 0, 3, 0, 4, 4]] $

$$[3\,y_1, 3\,y_2, 0, -7\,y_1 - 7\,y_2 + 11\,y_3 + 11\,y_5 - 7\,y_4, 0, 3\,y_3, -14\,y_1 - 14\,y_2 + 22\,y_3 + 22\,y_5 - 14\,y_4, 3\,y_5, 3\,y_4]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Â» SYNC'D 4245/262144 , 0.01619338989

144 . Coloring, {3, 5, 8, 9}

R: [4, 4, 5, 7, 3, 7, 1, 6, 2] **B**: [2, 9, 4, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -54‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [598, 184, 306, 663, 408, 153, 714, 204, 46] . FixedPtCheck, [598, 184, 306, 663, 408, 153, 714, 204, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}}, net cycles: 0 . order: 6

$ [[3, 1, 2, 5, 1, 2, 4, 0, 0] , [4, 0, 1, 4, 2, 0, 7, 0, 0] , [7, 0, 2, 4, 1, 0, 4, 0, 0] , [4, 0, 1, 7, 2, 0, 4, 0, 0] , [4, 0, 2, 4, 1, 0, 7, 0, 0] , [7, 0, 1, 4, 2, 0, 4, 0, 0] , [4, 0, 2, 7, 1, 0, 4, 0, 0]] $

$$[y_2, y_3, y_4, -y_2 - 3\,y_3 + 5\,y_4 + 5\,y_1 - y_5, y_1, 2\,y_3, y_5, 0, 0]$$

$p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 1, 3, 0, 2, 4, 2] , [6, 3, 0, 0, 2, 0, 3, 1, 3] , [4, 6, 0, 0, 3, 0, 2, 0, 3] , [3, 4, 0, 0, 2, 0, 3, 0, 6] , [6, 3, 0, 0, 3, 0, 2, 0, 4] , [4, 6, 0, 0, 2, 0, 3, 0, 3] , [3, 4, 0, 0, 3, 0, 2, 0, 6]] $

$$[-5\,y_1 - 5\,y_2 + 13\,y_3 + 13\,y_4 - 5\,y_5 - 5\,y_6, 5\,y_1, 0, 5\,y_2, 5\,y_3, 0, 5\,y_4, 5\,y_5, 5\,y_6]$$

$p = -s^3 - s^4 + s^6 + s^7$

Â» SYNC'D 86953/2097152 , 0.04146242142

145 . Coloring, {3, 6, 7, 8}

R: [4, 4, 5, 7, 7, 8, 5, 6, 1] **B**: [2, 9, 4, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [705, 188, 513, 798, 2052, 342, 2223, 456, 47] . FixedPtCheck, [705, 188, 513, 798, 2052, 342, 2223, 456, 47]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 5, 4, 2, 5, 1, 0] , [0, 0, 0, 1, 5, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0]] $

$$[y_1 - y_2 + 4\,y_3, 0, 0, -y_4 + 4\,y_1 + y_3, y_4, y_1, y_2, y_3, 0]$$

$p' = s^3 - s^5$ $p = -s^3 + s^5$

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 6

$ [[5, 4, 2, 1, 0, 0, 1, 3, 2] , [4, 7, 0, 2, 0, 0, 0, 1, 4] , [1, 8, 0, 0, 0, 0, 0, 2, 7] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

$$[y_1, y_2, 2\, y_4, y_3, 0, 0, y_4, y_5, y_6]$$

$p = - s^5 + s^7$

Â» SYNC'D 41/1024 , 0.04003906250

146 . Coloring, {3, 6, 7, 9}

R: [4, 4, 5, 7, 7, 8, 5, 1, 2] **B**: [2, 9, 4, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-1157, -356, -765, -1326, -3060, -102, -3315, -408, -89] . FixedPtCheck, [1157, 356, 765, 1326, 3060, 102, 3315, 408, 89]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 4 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 6	8 *vs* 8

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[2, 1, 0, 5, 4, 0, 5, 1, 0] , [1, 0, 0, 3, 5, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

$$[y_1, y_5, 0, y_2, y_3, 0, y_4, y_5, 0]$$

$p = - s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7, y_8]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, -11/54, 7/54, 7/54] , [0, 0, 0, 0, 0, 7/54, -11/54, 7/54] , [0, 1/2, -1/4, -5/8, 3/16, 41/108, 91/216, -241/432] , [0, 0, 1/2, -1/4, -5/8, -10/27, 41/108, 91/216] , [1/2, -1/4, -5/8, 3/16, 27/32, 91/216, -241/432, -401/864] , [0, 0, 0, 0, 1/2, 7/54, -11/54, -10/27] , [0, 0, 0, 0, 0, 7/54, 7/54, -11/54] , [0, 0, 0, 1/2, -1/4, -11/54, -10/27, 41/108] , [0, 0, 0, 0, 0, 7/54, 7/54, -11/54]] $ x $ [[4, 3, 2, 1, 0, 2, 1, 3, 2] , [3, 4, 0, 2, 0, 3, 2, 1, 3] , [5, 3, 0, 0, 0, 1, 3, 2, 4] , [7, 5, 0, 0, 0, 2, 1, 0, 3] , [4, 7, 0, 0, 0, 0, 2, 0, 5] , [7, 4, 0, 0, 0, 0, 0, 0, 7] , [7, 7, 0, 0, 0, 0, 0, 0, 4] , [4, 7, 0, 0, 0, 0, 0, 0, 7]] $

Â» SYNC'D 2262579/67108864 , 0.03371505439

147 . Coloring, {3, 6, 8, 9}

R: [4, 4, 5, 7, 7, 8, 1, 6, 2] **B**: [2, 9, 4, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1781, 548, 153, 1785, 612, 765, 1989, 1020, 137] . FixedPtCheck, [1781, 548, 153, 1785, 612, 765, 1989, 1020, 137]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	8 *vs* 8

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[3, 1, 0, 5, 1, 2, 5, 1, 0] , [5, 0, 0, 4, 0, 1, 6, 2, 0] , [6, 0, 0, 5, 0, 2, 4, 1, 0] , [4, 0, 0, 6, 0, 1, 5, 2, 0] , [5, 0, 0, 4, 0, 2, 6, 1, 0] , [6, 0, 0, 5, 0, 1, 4, 2, 0] , [4, 0, 0, 6, 0, 2, 5, 1, 0]] $

$$[-2\,y_2 - y_1 + 5\,y_3 - y_5 + 5\,y_4, y_2, 0, y_1, y_2, y_3, y_5, y_4, 0]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, y_7, y_8]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 0, 13/54, -5/54, -5/54] , [0, 0, 0, 0, 0, -5/54, 13/54, -5/54] , [0, 0, 0, 1, -3, 13/54, -59/54, 157/54] , [0, 0, 0, 0, 1, -5/54, 13/54, -59/54] , [0, 0, 1, -3, 7, -59/54, 157/54, -365/54] , [1, -3, 7, -16, 34, -365/54, 805/54, -1679/54] , [0, 1, -3, 7, -16, 157/54, -365/54, 805/54] , [0, 0, 0, 0, 0, -5/54, -5/54, 13/54] , [0, 0, 0, 0, 0, -5/54, -5/54, 13/54]] $ x $ [[3, 3, 2, 1, 3, 0, 1, 3, 2] , [5, 3, 3, 2, 1, 0, 0, 1, 3] , [4, 5, 1, 3, 0, 0, 0, 2, 3] , [5, 4, 0, 1, 0, 0, 0, 3, 5] , [8, 5, 0, 0, 0, 0, 0, 1, 4] , [5, 8, 0, 0, 0, 0, 0, 0, 5] , [5, 5, 0, 0, 0, 0, 0, 0, 8] , [8, 5, 0, 0, 0, 0, 0, 0, 5]] $

Â» SYNC'D 947699/33554432 , 0.02824363112

148 . Coloring, {3, 7, 8, 9}

R: [4, 4, 5, 7, 7, 7, 5, 6, 2] **B**: [2, 9, 4, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-559, -172, -459, -663, -1836, -153, -1989, -204, -43] . FixedPtCheck, [559, 172, 459, 663, 1836, 153, 1989, 204, 43]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	6 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[0, 1, 0, 5, 4, 2, 6, 0, 0] , [0, 0, 0, 1, 6, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

$$[0, y_1, 0, y_2, y_3, 2\,y_1, y_4, 0, 0]$$

$p = -s^3 + s^5$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_4, y_2, y_3, y_1, 0, 0, 0, y_5, y_6]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, -4/27, 5/27, 1/54] , [0, 0, 0, 1/54, -4/27, 5/27] , [0, 1/2, -1/4, -4/27, -17/54, 29/108] , [0, 0, 1/2, 1/54, -4/27, -17/54] , [1/2, -1/4, -7/8, -17/54, 29/108, 157/216] , [0, 0, 1/2, 1/54, -4/27, -17/54] , [0, 0, 0, 5/27, 1/54, -4/27] , [0, 0, 0, 5/27, 1/54, -4/27] , [0, 0, 0, 5/27, 1/54, -4/27]] $ x $ [[6, 3, 2, 1, 0, 0, 0, 4, 2] , [6, 6, 0, 2, 0, 0, 0, 1, 3] , [4, 6, 0, 0, 0, 0, 0, 2, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC’D 537/4096 , 0.1311035156

149 . Coloring, {4, 5, 6, 7}

R: [4, 4, 4, 8, 3, 8, 5, 1, 1] **B**: [2, 9, 5, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 1 + τ ‘)‘‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -27‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘]‘

For τ=1/2, [-3090, -824, -1026, -3705, -1368, -855, -1482, -3420, -206] . FixedPtCheck, [3090, 824, 1026, 3705, 1368, 855, 1482, 3420, 206]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: 0 . order: 3

$$[y_1, 0, y_2, y_3, y_4, 0, 0, y_5, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 19/54, 1/54, -17/54] , [0, 0, 19/54, 1/54, -17/54] , [0, 0, 19/54, 1/54, -17/54] , [0, 0, -17/54, 19/54, 1/54] , [0, 1/3, 1/54, -17/54, 1/54] , [0, 0, -17/54, 19/54, 1/54] , [1/3, -2/9, -17/54, 1/54, 13/54] , [0, 0, 1/54, -17/54, 19/54] , [0, 0, 1/54, -17/54, 19/54]] $ x $ [[3, 0, 2, 6, 3, 0, 0, 4, 0] , [4, 0, 3, 5, 0, 0, 0, 6, 0] , [6, 0, 0, 7, 0, 0, 0, 5, 0] , [5, 0, 0, 6, 0, 0, 0, 7, 0] , [7, 0, 0, 5, 0, 0, 0, 6, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[3, 4, 0, 0, 1, 2, 6, 0, 2] , [6, 5, 0, 0, 0, 0, 3, 0, 4] , [3, 10, 0, 0, 0, 0, 0, 0, 5] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_2, y_1, 0, 0, y_3, 2\,y_3, y_4, 0, y_5]$$

$p = -s^4 + s^6$

Â» SYNC'D 19/256 , 0.07421875000

150 . Coloring, {4, 5, 6, 8}

R: [4, 4, 4, 8, 3, 8, 1, 6, 1] **B**: [2, 9, 5, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , 9‘ (‘ 1 + τ ‘)‘ ³ ‘ (‘ - 5 + τ ² ‘)‘ , -9‘ (‘ - 5 + τ ² ‘)‘‘ (‘ 3 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ² ‘ (‘ - 5 + τ ² ‘)‘ , 9‘ (‘ 5 + 2τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ ³ ‘]‘

For τ=1/2, [-375, -100, -57, -399, -76, -513, -247, -684, -25] . FixedPtCheck, [375, 100, 57, 399, 76, 513, 247, 684, 25]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[4, 0, 2, 6, 0, 2, 0, 4, 0] , [0, 2, 0, 3, 3, 2, 2, 4, 2] , [4, 6, 3, 2, 6, 4, 4, 5, 2] , [9, 10, 6, 13, 9, 5, 12, 6, 2] , [24, 21, 9, 25, 14, 6, 21, 18, 6] , [41, 34, 14, 54, 34, 18, 51, 31, 11] , [95, 76, 34, 89, 63, 31, 86, 72, 30]] $ $ [[2, 4, 0, 0, 4, 0, 6, 0, 2] , [6, 2, 2, 3, 1, 0, 4, 0, 0] , [8, 2, 1, 10, 2, 0, 8, 3, 2] , [15, 6, 2, 11, 7, 3, 12, 10, 6] , [24, 11, 7, 23, 18, 10, 27, 14, 10] , [55, 30, 18, 42, 30, 14, 45, 33, 21] , [97, 52, 30, 103, 65, 33, 106, 56, 34]] $ $ [[1, -2, 1, 3, -2, 1, -3, 2, -1] , [-3, 0, -1, 0, 1, 1, -1, 2, 1] , [-2, 2, 1, -4, 2, 2, -2, 1, 0] , [-3, 2, 2, 1, 1, 1, 0, -2, -2] , [0, 5, 1, 1, -2, -2, -3, 2, -2] , [-7, 2, -2, 6, 2, 2, 3, -1, -5] , [-1, 12, 2, -7, -1, -1, -10, 8, -2]] $

$[-y_2 - 3 y_1 + y_4 + y_3, 2 y_1 - y_4 - 2 y_3 - y_6, -y_4 - y_5, y_2, y_1, y_3, y_4, y_5, y_6]$

$p = s^3 - s^4 + 4s^5 - 8s^7$

S+ \; S- \; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[4, 0, 2, 6, 0, 2, 0, 4, 0] , [0, 0, 0, 6, 0, 4, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0]] $

$[2 y_1, 0, y_1, y_2, 0, y_3, 0, y_4, 0]$

$p = -s^3 + s^5$

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 2

$ [[2, 4, 0, 0, 4, 0, 6, 0, 2] , [0, 4, 0, 0, 6, 0, 4, 0, 4] , [0, 4, 0, 0, 4, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 4, 0, 4] , [0, 4, 0, 0, 4, 0, 6, 0, 4]] $

$$[2\,y_1 - 2\,y_3, 2\,y_1, 0, 0, 5\,y_1 - 2\,y_2, 0, 2\,y_2, 0, 2\,y_3]$$

p = s ^ 2 - s ^ 4 p' = s ^ 2 - s ^ 4

Â» SYNC'D 1/64 , 0.01562500000

151 . Coloring, {4, 5, 6, 9}

R: [4, 4, 4, 8, 3, 8, 1, 1, 2] **B**: [2, 9, 5, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-637, -196, -51, -663, -68, -153, -221, -612, -49] . FixedPtCheck, [637, 196, 51, 663, 68, 153, 221, 612, 49]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 5	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[5, 1, 2, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0] , [6, 0, 0, 4, 0, 0, 0, 8, 0] , [8, 0, 0, 6, 0, 0, 0, 4, 0] , [4, 0, 0, 8, 0, 0, 0, 6, 0]] $

$$[y_1, y_2, 2\,y_2, y_3, 0, 0, 0, y_4, 0]$$

p = - s ^ 2 + s ^ 5

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[1, 3, 0, 0, 4, 2, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1]] $

$$[-y_1 + y_3 - y_4, y_1, 0, 0, y_3 - y_2, y_2, y_3, 0, y_4]$$

p = - s ^ 2 + s ^ 5 p' = - s ^ 2 + s ^ 5

Â» SYNC'D 4005/131072 , 0.03055572510

152 . Coloring, {4, 5, 7, 8}

R: [4, 4, 4, 8, 3, 7, 5, 6, 1] **B**: [2, 9, 5, 7, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-375, -100, -513, -741, -684, -513, -741, -684, -25] . FixedPtCheck, [375, 100, 513, 741, 684, 513, 741, 684, 25]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 6, 7, 8}}, net cycles: 0 . order: 6

$$[y_7, 0, y_6, y_5, y_4, y_3, y_2, y_1, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, 1853/6696, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696] , [0, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696, -1081/6696] , [0,

-1027/6696, -73/6696, 1853/6696, -1081/6696, 197/6696, 503/6696] , [0, 503/6696, -1027/6696, -73/6696, 1853/6696, -1081/6696, 197/6696] , [0, -73/6696, 1853/6696, -1081/6696, 197/6696, 503/6696, -1027/6696] , [1, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696, -7777/6696]] $ x $ [[1, 0, 2, 6, 3, 2, 1, 3, 0] , [0, 0, 3, 3, 1, 3, 2, 6, 0] , [0, 0, 1, 3, 2, 6, 3, 3, 0] , [0, 0, 2, 1, 3, 3, 6, 3, 0] , [0, 0, 3, 2, 6, 3, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 1, 3, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 0, 1, 0, 5, 1, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, 0, y_4, 0, y_3, y_4, y_5]$$

$p = - s^4 + s^6$

Â» SYNC'D 5415/65536 , 0.08262634277

153 . Coloring, {4, 5, 7, 9}

R: [4, 4, 4, 8, 3, 7, 5, 1, 2] **B**: [2, 9, 5, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-949, -292, -459, -1275, -612, -255, -663, -1020, -73] . FixedPtCheck, [949, 292, 459, 1275, 612, 255, 663, 1020, 73]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 6

$ [[2, 1, 2, 6, 3, 0, 1, 3, 0] , [3, 0, 3, 5, 1, 0, 0, 6, 0] , [6, 0, 1, 6, 0, 0, 0, 5, 0] , [5, 0, 0, 7, 0, 0, 0, 6, 0] , [6, 0, 0, 5, 0, 0, 0, 7, 0] , [7, 0, 0, 6, 0, 0, 0, 5, 0] , [5, 0, 0, 7, 0, 0, 0, 6, 0]] $

$$[y_1, y_5, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

$p = -s^4 + s^7$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[4, 3, 0, 0, 1, 2, 5, 1, 2] , [7, 4, 0, 0, 0, 1, 1, 2, 3] , [4, 7, 0, 0, 0, 2, 0, 1, 4] , [4, 4, 0, 0, 0, 1, 0, 2, 7] , [7, 4, 0, 0, 0, 2, 0, 1, 4] , [4, 7, 0, 0, 0, 1, 0, 2, 4] , [4, 4, 0, 0, 0, 2, 0, 1, 7]] $

$$[y_1, -y_1 - y_5 + 5\,y_3 - y_4 + 5\,y_2 - y_6, 0, 0, y_5, y_3, y_4, y_2, y_6]$$

$p = -s^3 - s^4 + s^6 + s^7$

Â» SYNC'D 125277/2097152 , 0.05973672867

154 . Coloring, {4, 5, 8, 9}

R: [4, 4, 4, 8, 3, 7, 1, 6, 2] **B**: [2, 9, 5, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1157, 356, 255, 1326, 340, 918, 1105, 1224, 89] . FixedPtCheck, [1157, 356, 255, 1326, 340, 918, 1105, 1224, 89]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	3 *vs* 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}}, net cycles: -1 . order: 5

$ [[3, 1, 2, 6, 0, 2, 1, 3, 0] , [1, 0, 0, 6, 0, 3, 2, 6, 0] , [2, 0, 0, 1, 0, 6, 3, 6, 0] , [3, 0, 0, 2, 0, 6, 6, 1, 0] , [6, 0, 0, 3, 0, 1, 6, 2, 0] , [6, 0, 0, 6, 0, 2, 1, 3, 0] , [1, 0, 0, 6, 0, 3, 2, 6, 0]] $

$$[y_1, y_2, 2\,y_2, y_3, 0, y_4, y_5, y_6, 0]$$

$p = -s^2 + s^7$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 0, 4, 0, 5, 1, 2] , [3, 3, 0, 0, 5, 0, 4, 0, 3] , [3, 3, 0, 0, 4, 0, 5, 0, 3] , [3, 3, 0, 0, 5, 0, 4, 0, 3] , [3, 3, 0, 0, 4, 0, 5, 0, 3] , [3, 3, 0, 0, 5, 0, 4, 0, 3]] $

$$[y_2, y_2, 0, 0, -y_1 + 3\,y_2, 0, y_1, y_2 - y_3, y_3]$$

$p = -s^2 + s^4$ p' $= -s^2 + s^4$ $p = -s^2 + s^6$

Â» SYNC'D 58695/2097152 , 0.02798795700

155 . Coloring, {4, 6, 7, 8}

R: [4, 4, 4, 8, 7, 8, 5, 6, 1] **B**: [2, 9, 5, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , -9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} ‘]‘

For τ=1/2, [-330, -88, -114, -399, -456, -513, -570, -684, -22] . FixedPtCheck, [330, 88, 114, 399, 456, 513, 570, 684, 22]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 6, 3, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0]] $

$$[y_4, 0, 0, y_3, y_2, y_1, -5\,y_3 + 14\,y_2 - 5\,y_1, -y_4 - 14\,y_3 + 39\,y_2 - 14\,y_1, 0]$$

$p = s^3 - s^5 \quad p' = s^3 - s^5$

Omega Rank for B : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 4

$ [[5, 4, 2, 0, 1, 0, 4, 0, 2] , [4, 7, 1, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8]] $

$$[y_4, y_3, y_2, 0, y_1, 0, -y_3 + 3 y_2 + 2 y_1, 0, -y_4 + 2 y_2 + 3 y_1]$$

$p' = - s^3 + s^5 \quad p = - s^3 + s^5$

Â» SYNC'D 233/32768 , 0.007110595703

156 . Coloring, {4, 6, 7, 9}

R: [4, 4, 4, 8, 7, 8, 5, 1, 2] **B**: [2, 9, 5, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -54‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [598, 184, 102, 663, 408, 153, 510, 612, 46] . FixedPtCheck, [598, 184, 102, 663, 408, 153, 510, 612, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}}, net cycles: 1 . order: 6

$ [[2, 1, 0, 6, 3, 0, 2, 4, 0] , [4, 0, 0, 3, 2, 0, 3, 6, 0] , [6, 0, 0, 4, 3, 0, 2, 3, 0] , [3, 0, 0, 6, 2, 0, 3, 4, 0] , [4, 0, 0, 3, 3, 0, 2, 6, 0] , [6, 0, 0, 4, 2, 0, 3, 3, 0]] $

$$[-5 y_1 - 5 y_2 + 13 y_3 + 13 y_4 - 5 y_5, 5 y_1, 0, 5 y_2, 5 y_3, 0, 5 y_4, 5 y_5, 0]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[4, 3, 2, 0, 1, 2, 4, 0, 2] , [6, 4, 1, 0, 2, 0, 2, 0, 3] , [5, 6, 2, 0, 1, 0, 0, 0, 4] , [4, 5, 1, 0, 2, 0, 0, 0, 6] , [6, 4, 2, 0, 1, 0, 0, 0, 5] , [5, 6, 1, 0, 2, 0, 0, 0, 4] , [4, 5, 2, 0, 1, 0, 0, 0, 6]] $

$$[y_6, -y_6 + 5\,y_1 + 5\,y_2 - y_3 - y_4 - y_5, y_1, 0, y_2, y_3, y_4, 0, y_5]$$

$p = -s^3 - s^4 + s^6 + s^7$

Â» SYNC'D 18465/524288 , 0.03521919250

157 . Coloring, {4, 6, 8, 9}

R: [4, 4, 4, 8, 7, 8, 1, 6, 2] **B**: [2, 9, 5, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [351, 108, 17, 357, 68, 459, 255, 612, 27] . FixedPtCheck, [351, 108, 17, 357, 68, 459, 255, 612, 27]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[3, 1, 0, 6, 0, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 0, 8, 0] , [0, 0, 0, 2, 0, 8, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0]] $

$$[y_3, y_1, 0, y_2, 0, y_4, 2\,y_1, y_5, 0]$$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[3, 3, 2, 0, 4, 0, 4, 0, 2] , [2, 3, 4, 0, 6, 0, 0, 0, 3] , [3, 2, 6, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 6, 0, 0, 0, 2] , [2, 3, 6, 0, 4, 0, 0, 0, 3] , [3, 2, 4, 0, 6, 0, 0, 0, 3]] $

$$[4\,y_1, 4\,y_2, 4\,y_5, 0, 5\,y_1 + 5\,y_2 - 4\,y_5 - 4\,y_4 + 5\,y_3, 0, 4\,y_4, 0, 4\,y_3]$$

$p = -s^2 - s^3 + s^5 + s^6$

Â» SYNC'D 105/4096 , 0.02563476562

158 . Coloring, {4, 7, 8, 9}

R: [4, 4, 4, 8, 7, 7, 5, 6, 2] **B**: [2, 9, 5, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [-481, -148, -255, -663, -1020, -459, -1275, -612, -37] . FixedPtCheck, [481, 148, 255, 663, 1020, 459, 1275, 612, 37]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[0, y_1, 0, y_2, y_3, y_6, y_4, y_5, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -6, 33, 443/72, -2455/72] , [0, 1, -6, 33, 443/72, -2455/72] , [0, 1, -6, 33, 443/72, -2455/72] , [0, 0, 1, -6, -79/72, 443/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 1, 11/72, -79/72] , [1, -6, 33, -182, -2455/72, 13547/72]] $ x $ [

[0, 1, 0, 6, 3, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 5, 6, 0] , [0, 0, 0, 0, 5, 6, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 0 . order: 6

$ [[6, 3, 2, 0, 1, 0, 3, 1, 2] , [6, 6, 1, 0, 2, 0, 0, 0, 3] , [3, 6, 2, 0, 1, 0, 0, 0, 6] , [6, 3, 1, 0, 2, 0, 0, 0, 6] , [6, 6, 2, 0, 1, 0, 0, 0, 3] , [3, 6, 1, 0, 2, 0, 0, 0, 6] , [6, 3, 2, 0, 1, 0, 0, 0, 6]] $

$$[-y_1 + 5\,y_2 + 5\,y_3 - 4\,y_4 - y_5, y_1, y_2, 0, y_3, 0, 3\,y_4, y_4, y_5]$$

$p = s^2 + s^3 - s^5 - s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Â» SYNC'D 34245/524288 , 0.06531715393

159 . Coloring, {5, 6, 7, 8}

R: [4, 4, 4, 7, 3, 8, 5, 6, 1] **B**: [2, 9, 5, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 4 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-885, -236, -1539, -1995, -2052, -855, -2223, -1140, -59] . FixedPtCheck, [885, 236, 1539, 1995, 2052, 855, 2223, 1140, 59]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 4, 5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 2, 6, 3, 2, 3, 1, 0] , [0, 0, 3, 3, 3, 1, 6, 2, 0] , [0, 0, 3, 3, 6, 2, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 3, 1, 0] , [0, 0, 3, 3, 3, 1, 6, 2, 0] , [0, 0, 3, 3, 6, 2, 3, 1, 0]] $

$$[-y_1 + y_3 - y_4 + 4\,y_5, 0, y_1, 4\,y_3 + y_5 - y_2, y_2, y_3, y_4, y_5, 0]$$

$p = -s^2 + s^6 \quad p' = -s^2 + s^6$

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$$\$ [[5, 4, 0, 0, 1, 0, 3, 3, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 3\,y_3, y_5]$$

$p = -s^4 + s^6$

Â» SYNC'D 7689/131072 , 0.05866241455

160 . Coloring, {5, 6, 7, 9}

R: [4, 4, 4, 7, 3, 8, 5, 1, 2] **B**: [2, 9, 5, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 459, 663, 612, 51, 663, 204, 25] . FixedPtCheck, [325, 100, 459, 663, 612, 51, 663, 204, 25]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$$\$ [[2, 1, 2, 6, 3, 0, 3, 1, 0] , [1, 0, 3, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0]] \$$$

$$[y_1, y_6, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

$p = -s^3 + s^7$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 6

$ [[4, 3, 0, 0, 1, 2, 3, 3, 2] , [5, 4, 0, 0, 0, 3, 3, 0, 3] , [6, 5, 0, 0, 0, 0, 3, 0, 4] , [7, 6, 0, 0, 0, 0, 0, 0, 5] , [5, 7, 0, 0, 0, 0, 0, 0, 6] , [6, 5, 0, 0, 0, 0, 0, 0, 7] , [7, 6, 0, 0, 0, 0, 0, 0, 5]] $

$$[y_1, y_2, 0, 0, y_3, y_4, y_5, 3\,y_3, y_6]$$

p = - s ^{4} + s ^{7}

Â» SYNC'D 64197/4194304 , 0.01530575752

161 . Coloring, {5, 6, 8, 9}

R: [4, 4, 4, 7, 3, 8, 1, 6, 2] **B**: [2, 9, 5, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [611, 188, 153, 714, 204, 306, 663, 408, 47] . FixedPtCheck, [611, 188, 153, 714, 204, 306, 663, 408, 47]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ - 1 + τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}}, net cycles: 0 . order: 6

$ [[3, 1, 2, 6, 0, 2, 3, 1, 0] , [3, 0, 0, 6, 0, 1, 6, 2, 0] , [6, 0, 0, 3, 0, 2, 6, 1, 0] , [6, 0, 0, 6, 0, 1, 3, 2, 0] , [3, 0, 0, 6, 0, 2, 6, 1, 0] , [6, 0, 0, 3, 0, 1, 6, 2, 0] , [6, 0, 0, 6, 0, 2, 3, 1, 0]] $

$$[y_5, y_4, 2\,y_4, y_3, 0, y_2, -y_5 - 3\,y_4 - y_3 + 5\,y_2 + 5\,y_1, y_1, 0]$$

p = s ^{2} - s ^{4} - s ^{5} + s ^{7} p = s ^{2} + s ^{3} - s ^{5} - s ^{6}

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 0, 4, 0, 3, 3, 2] , [5, 3, 0, 0, 3, 0, 4, 0, 3] , [3, 5, 0, 0, 4, 0, 3, 0, 3] , [3, 3, 0, 0, 3, 0, 4, 0, 5] , [5, 3, 0, 0, 4, 0, 3, 0, 3] , [3, 5, 0, 0, 3, 0, 4, 0, 3]] $

$$[7\, y_1, -7\, y_1 + 11\, y_4 + 11\, y_5 - 7\, y_2 - 7\, y_3, 0, 0, 7\, y_4, 0, 7\, y_5, 7\, y_2, 7\, y_3]$$

$p = - s^2 - s^3 + s^5 + s^6$

Â» SYNC'D 85825/2097152 , 0.04092454910

162 . Coloring, {5, 7, 8, 9}

R: [4, 4, 4, 7, 3, 7, 5, 6, 2] **B**: [2, 9, 5, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 4 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [1274, 392, 2754, 3315, 3672, 765, 3978, 1020, 98] . FixedPtCheck, [1274, 392, 2754, 3315, 3672, 765, 3978, 1020, 98]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 vs 7	*7 vs 7*	*7 vs 7*	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[0, 1, 2, 6, 3, 2, 4, 0, 0] , [0, 0, 3, 3, 4, 0, 8, 0, 0] , [0, 0, 4, 3, 8, 0, 3, 0, 0] , [0, 0, 8, 4, 3, 0, 3, 0, 0] , [0, 0, 3, 8, 3, 0, 4, 0, 0] , [0, 0, 3, 3, 4, 0, 8, 0, 0]] $

$$[0, y_1, y_2, y_3, y_4, 2\, y_1, y_5, 0, 0]$$

$p = - s^2 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 0, 1, 0, 2, 4, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_3, y_1, 0, 0, y_2, 0, y_4, 4\, y_2, y_5]$$

$p = - s^3 + s^6$

Â» SYNC'D 4005/65536 , 0.06111145020

163 . Coloring, {6, 7, 8, 9}

R: [4, 4, 4, 7, 7, 8, 5, 6, 2] **B**: [2, 9, 5, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-247, -76, -153, -357, -612, -153, -765, -204, -19] . FixedPtCheck, [247, 76, 153, 357, 612, 153, 765, 204, 19]

$\det(A + \tau\, \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 6, 3, 2, 5, 1, 0] , [0, 0, 0, 1, 5, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0]] $

$$[0, y_1 - y_2 + 4\, y_4, 0, y_3, -y_3 + 4\, y_1 + y_4, y_1, y_2, y_4, 0]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}}, net cycles: 0 . order: 6

$ [[6, 3, 2, 0, 1, 0, 1, 3, 2] , [6, 6, 1, 0, 2, 0, 0, 0, 3] , [3, 6, 2, 0, 1, 0, 0, 0, 6] , [6, 3, 1, 0, 2, 0, 0, 0, 6] , [6, 6, 2, 0, 1, 0, 0, 0, 3] , [3, 6, 1, 0, 2, 0, 0, 0, 6] , [6, 3, 2, 0, 1, 0, 0, 0, 6]] $

$$[-y_1 + 5\,y_2 + 5\,y_3 - 4\,y_4 - y_5, y_1, y_2, 0, y_3, 0, y_4, 3\,y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$

Â» SYNC'D 1265/32768 , 0.03860473633

164 . Coloring, {2, 3, 4, 5, 6}

R: [4, 9, 5, 8, 3, 8, 1, 1, 1] **B**: [2, 4, 4, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [1950, 600, 258, 1677, 344, 387, 602, 1548, 450] . FixedPtCheck, [1950, 600, 258, 1677, 344, 387, 602, 1548, 450]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 5

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[6, 0, 2, 3, 1, 0, 0, 4, 2] , [6, 0, 1, 6, 2, 0, 0, 3, 0] , [3, 0, 2, 6, 1, 0, 0, 6, 0] , [6, 0, 1, 3, 2, 0, 0, 6, 0] , [6, 0, 2, 6, 1, 0, 0, 3, 0] , [3, 0, 1, 6, 2, 0, 0, 6, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - y_5, 0, y_1, y_2, y_3, 0, 0, y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[0, 4, 0, 3, 3, 2, 6, 0, 0] , [0, 0, 0, 4, 6, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

$$[0, 2\,y_2, 0, y_3, y_1, y_2, y_4, 0, 0]$$

$p = s^3 - s^5$

Â» SYNC'D 269/4096 , 0.06567382812

165 . Coloring, {2, 3, 4, 5, 7}

R: [4, 9, 5, 8, 3, 7, 5, 1, 1] **B**: [2, 4, 4, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - 3τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - 3τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - 3τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-403, -124, -387, -430, -516, -86, -301, -344, -93] . FixedPtCheck, [403, 124, 387, 430, 516, 86, 301, 344, 93]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}}, net cycles: 0 . order: 6

$ [[3, 0, 2, 3, 4, 0, 1, 3, 2] , [5, 0, 4, 3, 3, 0, 0, 3, 0] , [3, 0, 3, 5, 4, 0, 0, 3, 0] , [3, 0, 4, 3, 3, 0, 0, 5, 0] , [5, 0, 3, 3, 4, 0, 0, 3, 0] , [3, 0, 4, 5, 3, 0, 0, 3, 0] , [3, 0, 3, 3, 4, 0, 0, 5, 0]] $

$$[3\,y_1, 0, 3\,y_2, 3\,y_3, 3\,y_4, 0, -7\,y_1 + 11\,y_2 - 7\,y_3 + 11\,y_4 - 7\,y_5, 3\,y_5, -14\,y_1 + 22\,y_2 - 14\,y_3 + 22\,y_4 - 14\,y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$ $p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 4, 7}}, net cycles: 2 . order: 4

$ [[3, 4, 0, 3, 0, 2, 5, 1, 0] , [5, 3, 0, 4, 0, 1, 3, 2, 0] , [3, 5, 0, 3, 0, 2, 4, 1, 0] , [4, 3, 0, 5, 0, 1, 3, 2, 0] , [3, 4, 0, 3, 0, 2, 5, 1, 0] , [5, 3, 0, 4, 0, 1, 3, 2, 0]] $

$$[y_1, 4\,y_4 + y_3 - y_2, 0, -y_1 + y_4 + 4\,y_3, 0, y_4, y_2, y_3, 0]$$

p' = - s + s ^{5} p = - s + s ^{5}

Â» SYNC'D 8581/524288 , 0.01636695862

166 . Coloring, {2, 3, 4, 5, 8}

R: [4, 9, 5, 8, 3, 7, 1, 6, 1] **B**: [2, 4, 4, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [1833, 564, 645, 1677, 860, 1161, 1505, 1548, 423] . FixedPtCheck, [1833, 564, 645, 1677, 860, 1161, 1505, 1548, 423]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{4}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 8	6 *vs* 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}, {3, 5}}, net cycles: 1 .

$ [[4, 0, 2, 3, 1, 2, 1, 3, 2] , [3, 0, 1, 4, 2, 3, 2, 3, 0] , [2, 0, 2, 3, 1, 3, 3, 4, 0] , [3, 0, 1, 2, 2, 4, 3, 3, 0] , [3, 0, 2, 3, 1, 3, 4, 2, 0] , [4, 0, 1, 3, 2, 2, 3, 3, 0] , [3, 0, 2, 4, 1, 3, 2, 3, 0] , [2, 0, 1, 3, 2, 3, 3, 4, 0]] $

$$[5\,y_1 - y_2 + 5\,y_3 - y_4 - y_5 - y_6 - y_7, 0, y_1, y_2, y_3, y_4, y_5, y_6, y_7]$$

p = - s ^{2} - s ^{3} + s ^{7} + s ^{8}

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 1, -2, -13/18, 16/9] , [0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 1, -2/9, -13/18] , [0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, 5/18, -2/9] , [1, -2, 0, 5, -13/18, -29/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 1, -2, 0, 16/9, -13/18] , [0, 0, 1, -2, -13/18, 16/9]] $ x $ [[2, 4, 0, 3, 3, 0, 5, 1, 0] , [1, 2, 0, 4, 5, 0, 6, 0, 0] , [0, 1, 0, 2, 6, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] $

Â» SYNC'D 223785/8388608 , 0.02667725086

167 . Coloring, {2, 3, 4, 5, 9}

R: [4, 9, 5, 8, 3, 7, 1, 1, 2] **B**: [2, 4, 4, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-399, -228, -75, -375, -100, -75, -175, -300, -171] . FixedPtCheck, [399, 228, 75, 375, 100, 75, 175, 300, 171]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	5 *vs* 7

Omega Rank for R : cycles: {{3, 5}, {2, 9}, {1, 4, 8}}, net cycles: 2 . order: 6

$ [[5, 1, 2, 3, 1, 0, 1, 3, 2] , [4, 2, 1, 5, 2, 0, 0, 3, 1] , [3, 1, 2, 4, 1, 0, 0, 5, 2] , [5, 2, 1, 3, 2, 0, 0, 4, 1] , [4, 1, 2, 5, 1, 0, 0, 3, 2] , [3, 2, 1, 4, 2, 0, 0, 5, 1] , [5, 1, 2, 3, 1, 0, 0, 4, 2] , [4, 2, 1, 5, 2, 0, 0, 3, 1]] $

$$[y_5, y_4, y_3, y_2, y_4, 0, y_1, -y_5 + 4\,y_4 + 4\,y_3 - y_2 - y_1, y_3]$$

$p' = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 + s^3 - s^5 - s^6 \quad p' = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 3, 0, 3, 3, 2, 5, 1, 0] , [0, 1, 0, 3, 5, 1, 6, 2, 0] , [0, 0, 0, 1, 6, 2, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 7, 2, 0] , [0, 0, 0, 0, 7, 2, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 7, 2, 0] , [0, 0, 0, 0, 7, 2, 8, 1, 0]] $

$$[-y_1 - y_2 + 2y_3 + 3y_5, 3y_3 - y_4 + 2y_5, 0, y_1, y_2, y_3, y_4, y_5, 0]$$

$p' = s^4 - s^6 \quad p = -s^4 + s^6$

Â» SYNC'D 53229/16777216 , 0.003172695637

168 . Coloring, {2, 3, 4, 6, 7}

R: [4, 9, 5, 8, 7, 8, 5, 1, 1] **B**: [2, 4, 4, 7, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [637, 196, 129, 559, 516, 129, 559, 516, 147] . FixedPtCheck, [637, 196, 129, 559, 516, 129, 559, 516, 147]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

\+ \ ; - \ ; Δ

$ [[3, 0, 0, 3, 4, 0, 2, 4, 2] , [10, 3, 0, 9, 2, 0, 9, 3, 0] , [6, 6, 6, 19, 9, 5, 9, 9, 3] , [27, 23, 7, 18, 15, 7, 17, 24, 6] , [61, 31, 17, 45, 24, 8, 54, 25, 23] , [90, 44, 40, 109, 71, 39, 99, 53, 31] , [177, 135, 57, 198, 139, 75, 179, 148, 44]] $ $ [[3, 4, 2, 3, 0, 2, 4, 0, 0] , [2, 5, 4, 3, 6, 4, 3, 5, 4] , [18, 10, 2, 5, 7, 3, 15, 7, 5] , [21, 9, 9, 30, 17, 9, 31, 8, 10] , [35, 33, 15, 51, 40, 24, 42, 39, 9] , [102, 84, 24, 83, 57, 25, 93, 75, 33] , [207, 121, 71, 186, 117, 53, 205, 108, 84]] $ $ [[0, -2, -1, 0, 2, -1, -1, 2, 1] , [4, -1, -2, 3, -2, -2, 3, -1, -2] , [-6, -2, 2, 7, 1, 1, -3, 1, -1] , [3, 7, -1, -6, -1, -1, -7, 8, -2] , [13, -1, 1, -3, -8, -8, 6, -7, 7] , [-6, -20, 8, 13, 7, 7, 3, -11, -1] , [-15, 7, -7, 6, 11, 11, -13, 20, -20]] $

$$[y_5, y_3 + 2\,y_4 - 2\,y_1 + 2\,y_2 + y_6, -y_3 - y_4, -y_5 - y_3 - 2\,y_4 + y_1 - 3\,y_2 - 2\,y_6, y_1, y_2, y_3, y_4, y_6]$$

$$p = s^2 + 2s^4 - 8s^5 - 16s^7$$

S+ \; S- \; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[3, 0, 0, 3, 4, 0, 2, 4, 2] , [6, 0, 0, 3, 2, 0, 4, 3, 0] , [3, 0, 0, 6, 4, 0, 2, 3, 0] , [3, 0, 0, 3, 2, 0, 4, 6, 0] , [6, 0, 0, 3, 4, 0, 2, 3, 0] , [3, 0, 0, 6, 2, 0, 4, 3, 0]] $

$$[y_2, 0, 0, -y_2 + 2\,y_1 + 2\,y_5 - y_4 - y_3, y_1, 0, y_5, y_4, y_3]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[3, 4, 2, 3, 0, 2, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0] , [5, 4, 0, 3, 0, 0, 6, 0, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0]] $

$$[y_1, y_2, y_4, y_3, 0, y_4, y_5, 0, 0]$$

$$p = s^2 - s^6$$

Â» SYNC’D 165/4096 , 0.04028320312

169 . Coloring, {2, 3, 4, 6, 8}

R: [4, 9, 5, 8, 7, 8, 1, 6, 1] **B**: [2, 4, 4, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1794, 552, 86, 1505, 344, 1935, 1118, 2580, 414] . FixedPtCheck, [1794, 552, 86, 1505, 344, 1935, 1118, 2580, 414]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	6 *vs* 6

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 6

$ [[4, 0, 0, 3, 1, 2, 2, 4, 2] , [4, 0, 0, 4, 0, 4, 1, 5, 0] , [1, 0, 0, 4, 0, 5, 0, 8, 0] , [0, 0, 0, 1, 0, 8, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_3, 0, 0, y_4, y_5, y_3 - y_4 - y_5 - y_1 + y_2, y_1, y_2, 2\, y_5]$$

p = - s ^ 5 + s ^ 6 p = - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/2, 1/72, -17/72, 19/72, -35/72] , [0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 1/72, 1/72, -17/72, 19/72] , [0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 0, 1/72, -17/72, 19/72, 1/72] , [0, 0, 19/72, 1/72, 1/72, -17/72] , [0, 0, -17/72, 19/72, 1/72, 1/72] , [1/2, -1, -17/72, 19/72, -35/72, 73/72] , [0, 1/2, 1/72, -17/72, 19/72, -35/72]] $ x $ [[2, 4, 2, 3, 3, 0, 4, 0, 0] , [0, 2, 3, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0]] $

Â» SYNC’D 1059/32768 , 0.03231811523

170 . Coloring, {2, 3, 4, 6, 9}

R: [4, 9, 5, 8, 7, 8, 1, 1, 2] **B**: [2, 4, 4, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [1806, 1032, 50, 1625, 200, 375, 650, 1500, 774] . FixedPtCheck, [1806, 1032, 50, 1625, 200, 375, 650, 1500, 774]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[5, 1, 0, 3, 1, 0, 2, 4, 2] , [6, 2, 0, 5, 0, 0, 1, 3, 1] , [4, 1, 0, 6, 0, 0, 0, 5, 2] , [5, 2, 0, 4, 0, 0, 0, 6, 1] , [6, 1, 0, 5, 0, 0, 0, 4, 2] , [4, 2, 0, 6, 0, 0, 0, 5, 1] , [5, 1, 0, 4, 0, 0, 0, 6, 2]] $

$$[5\, y_1 - y_2 - y_3 - y_4 - y_5 + 5\, y_6, y_1, 0, y_2, y_3, 0, y_4, y_5, y_6]$$

p = - s ^ 3 - s ^ 4 + s ^ 6 + s ^ 7

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: -1 . order: 4

$ [[1, 3, 2, 3, 3, 2, 4, 0, 0] , [0, 1, 3, 5, 4, 0, 5, 0, 0] , [0, 0, 4, 4, 5, 0, 5, 0, 0] , [0, 0, 5, 4, 5, 0, 4, 0, 0] , [0, 0, 5, 5, 4, 0, 4, 0, 0] , [0, 0, 4, 5, 4, 0, 5, 0, 0] , [0, 0, 4, 4, 5, 0, 5, 0, 0]] $

$$[y_2, y_3, y_4, y_5, -3\, y_2 + y_3 + y_4 - y_5 + y_1, 2\, y_2, y_1, 0, 0]$$

p = - s ^ 3 + s ^ 7 p = - s ^ 3 + s ^ 4 - s ^ 5 + s ^ 6

Â» SYNC’D 27669/2097152 , 0.01319360733

171 . Coloring, {2, 3, 4, 7, 8}

R: [4, 9, 5, 8, 7, 7, 5, 6, 1] **B**: [2, 4, 4, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [1157, 356, 645, 1118, 2580, 774, 2795, 1032, 267] . FixedPtCheck, [1157, 356, 645, 1118, 2580, 774, 2795, 1032, 267]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	7 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[y_7, 0, 0, y_6, y_5, y_3, y_4, y_2, y_1]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/2, -1/4, -5/8, 11/72, 5/18] , [1/2, -1/4, -5/8, -1/16, 27/32, 31/144, -163/288] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 1/2, -1/4, -25/72, 11/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 1/2, -7/72, -25/72] , [0, 1/2, -1/4, -5/8, -1/16, 5/18, 31/144]] $ x $ [[1, 0, 0, 3, 4, 2, 3, 3, 2] , [2, 0, 0, 1, 3, 3, 6, 3, 0] , [0, 0, 0, 2, 6, 3, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[5, 4, 2, 3, 0, 0, 3, 1, 0] , [4, 5, 0, 6, 0, 0, 3, 0, 0] , [3, 4, 0, 5, 0, 0, 6, 0, 0] , [6, 3, 0, 4, 0, 0, 5, 0, 0] , [5, 6, 0, 3, 0, 0, 4, 0, 0] , [4, 5, 0, 6, 0, 0, 3, 0, 0]] $

$$[y_1, y_2, 2\,y_5, y_3, 0, 0, y_4, y_5, 0]$$

$p = -s^2 + s^6$

Â» SYNC'D 6933/131072 , 0.05289459229

172 . Coloring, {2, 3, 4, 7, 9}

R: [4, 9, 5, 8, 7, 7, 5, 1, 2] **B**: [2, 4, 4, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 + τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-259, -148, -75, -250, -300, -50, -325, -200, -111] . FixedPtCheck, [259, 148, 75, 250, 300, 50, 325, 200, 111]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {2, 9}, {1, 4, 8}}, net cycles: 3 . order: 6

$ [[2, 1, 0, 3, 4, 0, 3, 3, 2] , [3, 2, 0, 2, 3, 0, 4, 3, 1] , [3, 1, 0, 3, 4, 0, 3, 2, 2] , [2, 2, 0, 3, 3, 0, 4, 3, 1] , [3, 1, 0, 2, 4, 0, 3, 3, 2] , [3, 2, 0, 3, 3, 0, 4, 2, 1] , [2, 1, 0, 3, 4, 0, 3, 3, 2]] $

$$[-8 y_1 - 2 y_2 + 8 y_3 - 2 y_4, 2 y_1, 0, 2 y_2, -7 y_1 + 5 y_3, 0, 2 y_3, 2 y_4, -5 y_1 + 3 y_3]$$

$p = -s - s^2 + s^4 + s^5$ $p = s - s^3 - s^4 + s^6$ $p = -s + s^7$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 4, 7}}, net cycles: 1 . order: 4

$ [[4, 3, 2, 3, 0, 2, 3, 1, 0] , [3, 4, 0, 5, 0, 1, 3, 2, 0] , [3, 3, 0, 4, 0, 2, 5, 1, 0] , [5, 3, 0, 3, 0, 1, 4, 2, 0] , [4, 5, 0, 3, 0, 2, 3, 1, 0] , [3, 4, 0, 5, 0, 1, 3, 2, 0] , [3, 3, 0, 4, 0, 2, 5, 1, 0]] $

$$[-y_2 + 2 y_3 + 3 y_5, -y_1 + 3 y_3 - y_4 + 2 y_5, y_1, y_2, 0, y_3, y_4, y_5, 0]$$

$p = -s^2 + s^6$ $p' = -s^2 + s^6$

Â» SYNC’D 55359/16777216 , 0.003299653530

173 . Coloring, {2, 3, 4, 8, 9}

R: [4, 9, 5, 8, 7, 7, 1, 6, 2] **B**: [2, 4, 4, 7, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [357, 204, 25, 325, 100, 225, 325, 300, 153] . FixedPtCheck, [357, 204, 25, 325, 100, 225, 325, 300, 153]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	3 *vs* 8	6 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 6, 7, 8}}, net cycles: 1 .

$ [[3, 1, 0, 3, 1, 2, 3, 3, 2] , [3, 2, 0, 3, 0, 3, 3, 3, 1] , [3, 1, 0, 3, 0, 3, 3, 3, 2] , [3, 2, 0, 3, 0, 3, 3, 3, 1] , [3, 1, 0, 3, 0, 3, 3, 3, 2] , [3, 2, 0, 3, 0, 3, 3, 3, 1] , [3, 1, 0, 3, 0, 3, 3, 3, 2] , [3, 2, 0, 3, 0, 3, 3, 3, 1]] $

$$[y_2 + y_3, y_2, 0, y_2 + y_3, y_2 + y_3 - y_1, y_1, y_2 + y_3, y_2 + y_3, y_3]$$

p = - s 2 + s 4 p’ = - s 2 + s 4 p = - s 2 + s 6 p’ = - s 2 + s 6 p = - s 2 + s 8

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$ [[3, 3, 2, 3, 3, 0, 3, 1, 0] , [1, 3, 3, 5, 3, 0, 3, 0, 0] , [0, 1, 3, 6, 3, 0, 5, 0, 0] , [0, 0, 3, 4, 5, 0, 6, 0, 0] , [0, 0, 5, 3, 6, 0, 4, 0, 0] , [0, 0, 6, 5, 4, 0, 3, 0, 0] , [0, 0, 4, 6, 3, 0, 5, 0, 0]] $

$$[y_6 + y_1 - y_2 - y_3 + y_4 + y_5, y_6, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

p = - s 4 + s 5 - s 6 + s 7

Â» SYNC’D 598125/33554432 , 0.01782551408

174 . Coloring, {2, 3, 5, 6, 7}

R: [4, 9, 5, 7, 3, 8, 5, 1, 1] **B**: [2, 4, 4, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-767, -236, -1935, -1118, -2580, -86, -1505, -344, -177] . FixedPtCheck, [767, 236, 1935, 1118, 2580, 86, 1505, 344, 177]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 6

$ [[3, 0, 2, 3, 4, 0, 3, 1, 2] , [3, 0, 4, 3, 5, 0, 3, 0, 0] , [0, 0, 5, 3, 7, 0, 3, 0, 0] , [0, 0, 7, 0, 8, 0, 3, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0]] $

$$[y_1, 0, y_3, y_2, y_4, 0, y_6, y_5, 2\, y_5]$$

p = - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 1 . order: 6

$ [[3, 4, 0, 3, 0, 2, 3, 3, 0] , [3, 3, 0, 4, 0, 3, 2, 3, 0] , [2, 3, 0, 3, 0, 3, 3, 4, 0] , [3, 2, 0, 3, 0, 4, 3, 3, 0] , [3, 3, 0, 2, 0, 3, 4, 3, 0] , [4, 3, 0, 3, 0, 3, 3, 2, 0]] $

$$[y_4, y_3, 0, y_2, 0, y_1, y_3 - y_2 + y_1, -y_4 + y_3 + y_1, 0]$$

p’ = s - s ^ 2 + s ^ 4 - s ^ 5 p = s - s ^ 3 + s ^ 4 - s ^ 6

Â» SYNC’D 4725/262144 , 0.01802444458

175 . Coloring, {2, 3, 5, 6, 8}

R: [4, 9, 5, 7, 3, 8, 1, 6, 1] **B**: [2, 4, 4, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [325, 100, 129, 301, 172, 129, 301, 172, 75] . FixedPtCheck, [325, 100, 129, 301, 172, 129, 301, 172, 75]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 4 ‘ (‘ - 1 + τ ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}, {3, 5}}, net cycles: 2 . order: 6

$ [[4, 0, 2, 3, 1, 2, 3, 1, 2] , [5, 0, 1, 4, 2, 1, 3, 2, 0] , [3, 0, 2, 5, 1, 2, 4, 1, 0] , [4, 0, 1, 3, 2, 1, 5, 2, 0] , [5, 0, 2, 4, 1, 2, 3, 1, 0] , [3, 0, 1, 5, 2, 1, 4, 2, 0] , [4, 0, 2, 3, 1, 2, 5, 1, 0] , [5, 0, 1, 4, 2, 1, 3, 2, 0]] $

$$[4\,y_2 - y_1 + 4\,y_4 - y_3 - y_5, 0, y_2, y_1, y_4, y_2, y_3, y_4, y_5]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7 p = - s ^ 2 + s ^ 8

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[2, 4, 0, 3, 3, 0, 3, 3, 0] , [3, 2, 0, 4, 3, 0, 3, 3, 0] , [3, 3, 0, 2, 3, 0, 3, 4, 0] , [4, 3, 0, 3, 3, 0, 3, 2, 0] , [2, 4, 0, 3, 3, 0, 3, 3, 0] , [3, 2, 0, 4, 3, 0, 3, 3, 0]] $

$$[-y_1 - y_2 + 4\,y_3 - y_4, y_1, 0, y_2, y_3, 0, y_3, y_4, 0]$$

p’ = - s + s ^ 5 p = - s + s ^ 5

Â» SYNC’D 15975/8388608 , 0.001904368401

176 . Coloring, {2, 3, 5, 6, 9}

R: [4, 9, 5, 7, 3, 8, 1, 1, 2] **B**: [2, 4, 4, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [987, 564, 375, 975, 500, 75, 875, 300, 423] . FixedPtCheck, [987, 564, 375, 975, 500, 75, 875, 300, 423]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}, {2, 9}}, net cycles: 2 . order: 6

$ [[5, 1, 2, 3, 1, 0, 3, 1, 2] , [4, 2, 1, 5, 2, 0, 3, 0, 1] , [3, 1, 2, 4, 1, 0, 5, 0, 2] , [5, 2, 1, 3, 2, 0, 4, 0, 1] , [4, 1, 2, 5, 1, 0, 3, 0, 2] , [3, 2, 1, 4, 2, 0, 5, 0, 1] , [5, 1, 2, 3, 1, 0, 4, 0, 2] , [4, 2, 1, 5, 2, 0, 3, 0, 1]] $

$$[4\,y_2 - y_1 - y_3 - y_5 + 4\,y_4, y_2, y_4, y_1, y_2, 0, y_3, y_5, y_4]$$

p = - s ^ 2 - s ^ 3 + s ^ 5 + s ^ 6 p = s ^ 2 - s ^ 4 - s ^ 5 + s ^ 7 p = - s ^ 2 + s ^ 8

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$ [[1, 3, 0, 3, 3, 2, 3, 3, 0] , [0, 1, 0, 3, 3, 3, 5, 3, 0] , [0, 0, 0, 1, 5, 3, 6, 3, 0] , [0, 0, 0, 0, 6, 3, 8, 1, 0] , [0, 0, 0, 0, 8, 1, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 - y_2 - y_3 - y_4 + y_5 + y_6, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

p = - s ^ 6 + s ^ 7

Â» SYNC’D 114885/16777216 , 0.006847679615

177 . Coloring, {2, 3, 5, 7, 8}

R: [4, 9, 5, 7, 3, 7, 5, 6, 1] **B**: [2, 4, 4, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 1161, 559, 1548, 129, 903, 172, 75] . FixedPtCheck, [325, 100, 1161, 559, 1548, 129, 903, 172, 75]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[1, 0, 2, 3, 4, 2, 4, 0, 2] , [8, 5, 4, 5, 6, 0, 5, 3, 0] , [12, 8, 6, 11, 9, 3, 7, 11, 5] , [27, 15, 9, 22, 13, 11, 21, 18, 8] , [49, 29, 13, 51, 30, 18, 52, 31, 15] , [92, 64, 30, 103, 65, 31, 103, 59, 29] , [187, 135, 65, 190, 133, 59, 197, 122, 64]] $ $ [[5, 4, 0, 3, 0, 0, 2, 4, 0] , [4, 3, 0, 7, 2, 4, 7, 5, 4] , [12, 8, 2, 13, 7, 5, 17, 5, 3] , [21, 17, 7, 26, 19, 5, 27, 14, 8] , [47, 35, 19, 45, 34, 14, 44, 33, 17] , [100, 64, 34, 89, 63, 33, 89, 69, 35] , [197, 121, 63, 194, 123, 69, 187, 134, 64]] $ $ [[-2, -2, 1, 0, 2, 1, 1, -2, 1] , [2, 1, 2, -1, 2, -2, -1, -1, -2] , [0, 0, 2, -1, 1, -1, -5, 3, 1] , [3, -1, 1, -2, -3, 3, -3, 2, 0] , [1, -3, -3, 3, -2, 2, 4, -1, -1] , [-4, 0, -2, 7, 1, -1, 7, -5, -3] , [-5, 7, 1, -2, 5, -5, 5, -6, 0]] $

$[-y_4 + y_3 + 2\,y_5 - 2\,y_6 + y_1 + y_2, -y_3 - 2\,y_5 + y_6 - 2\,y_1 - 2\,y_2, -y_3 - y_5, y_4, y_1, y_2, y_3, y_5, y_6]$

p = s 3 + s 4 + 4s 5 + 8s 7

S+ \ ; S- \ ; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	6 *vs* 7	5 *vs* 5

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 6

$ [[1, 0, 2, 3, 4, 2, 4, 0, 2] , [2, 0, 4, 1, 6, 0, 5, 0, 0] , [0, 0, 6, 2, 9, 0, 1, 0, 0] , [0, 0, 9, 0, 7, 0, 2, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0]] $

$$[y_1, 0, y_2, y_3, y_4, y_6, y_5, 0, y_6]$$

$p = - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, 0, 0, y_4, y_5, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 19/72, -17/72, 1/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, -17/72, 1/72, 1/72, 19/72] , [1/2, -17/72, 1/72, 1/72, -17/72] , [0, -17/72, 1/72, 1/72, 19/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 1/72, 19/72, -17/72, 1/72]] $ x $ [[5, 4, 0, 3, 0, 0, 2, 4, 0] , [6, 5, 0, 4, 0, 0, 0, 3, 0] , [3, 6, 0, 5, 0, 0, 0, 4, 0] , [4, 3, 0, 6, 0, 0, 0, 5, 0] , [5, 4, 0, 3, 0, 0, 0, 6, 0]] $

Â» SYNC’D 2441/65536 , 0.03724670410

178 . Coloring, {2, 3, 5, 7, 9}

R: [4, 9, 5, 7, 3, 7, 5, 1, 2] **B**: [2, 4, 4, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [231, 132, 675, 375, 900, 25, 525, 100, 99] . FixedPtCheck, [231, 132, 675, 375, 900, 25, 525, 100, 99]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 2, 3, 4, 0, 4, 0, 2] , [0, 2, 4, 2, 6, 0, 3, 0, 1] , [0, 1, 6, 0, 7, 0, 2, 0, 2] , [0, 2, 7, 0, 8, 0, 0, 0, 1] , [0, 1, 8, 0, 7, 0, 0, 0, 2] , [0, 2, 7, 0, 8, 0, 0, 0, 1] , [0, 1, 8, 0, 7, 0, 0, 0, 2]] $

$$[y_2, y_3, y_4, 3\,y_3 - y_5 + 2\,y_1, y_5, 0, -y_2 + 2\,y_3 - y_4 + 3\,y_1, 0, y_1]$$

p = - s ^ 4 + s ^ 6 p’ = - s ^ 4 + s ^ 6

Omega Rank for B : cycles: {{6, 8}}, net cycles: 0 . order: 6

$ [[4, 3, 0, 3, 0, 2, 2, 4, 0] , [2, 4, 0, 3, 0, 4, 0, 5, 0] , [0, 2, 0, 4, 0, 5, 0, 7, 0] , [0, 0, 0, 2, 0, 7, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, 0, y_3, y_4, y_5, 0]$$

p = s ^ 5 - s ^ 6

Â» SYNC’D 8917/524288 , 0.01700782776

179 . Coloring, {2, 3, 5, 8, 9}

R: [4, 9, 5, 7, 3, 7, 1, 6, 2] **B**: [2, 4, 4, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘ , 54‘ (‘ 5 + 3τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [322, 184, 150, 325, 200, 75, 350, 100, 138] . FixedPtCheck, [322, 184, 150, 325, 200, 75, 350, 100, 138]

$\det(A + \tau\,\Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 8	4 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {2, 9}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[3, 1, 2, 3, 1, 2, 4, 0, 2] , [4, 2, 1, 3, 2, 0, 5, 0, 1] , [5, 1, 2, 4, 1, 0, 3, 0, 2] , [3, 2, 1, 5, 2, 0, 4, 0, 1] , [4, 1, 2, 3, 1, 0, 5, 0, 2] , [5, 2, 1, 4, 2, 0, 3, 0, 1] , [3, 1, 2, 5, 1, 0, 4, 0, 2] , [4, 2, 1, 3, 2, 0, 5, 0, 1]] $

$$[4\,y_5 + 4\,y_4 - y_1 - y_2 - y_3, y_5, y_4, y_1, y_5, y_2, y_3, 0, y_4]$$

$p = -s^2 - s^3 + s^5 + s^6$ $p = s^2 - s^4 - s^5 + s^7$ $p = -s^2 + s^8$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[3, 3, 0, 3, 3, 0, 2, 4, 0] , [4, 3, 0, 3, 2, 0, 3, 3, 0] , [3, 4, 0, 3, 3, 0, 2, 3, 0] , [3, 3, 0, 4, 2, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 2, 4, 0] , [4, 3, 0, 3, 2, 0, 3, 3, 0]] $

$$[4\,y_4, 4\,y_3, 0, -4\,y_4 + 9\,y_3 - 13\,y_2 + 9\,y_1, 4\,y_2, 0, 5\,y_3 - 9\,y_2 + 5\,y_1, 4\,y_1, 0]$$

$p' = -s + s^5$ $p = -s + s^5$

Â» SYNC’D 9855/4194304 , 0.002349615097

180 . Coloring, {2, 3, 6, 7, 8}

R: [4, 9, 5, 7, 7, 8, 5, 6, 1] **B**: [2, 4, 4, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [611, 188, 387, 602, 1548, 258, 1677, 344, 141] . FixedPtCheck, [611, 188, 387, 602, 1548, 258, 1677, 344, 141]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 3, 4, 2, 5, 1, 2] , [2, 0, 0, 1, 5, 1, 7, 2, 0] , [0, 0, 0, 2, 7, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0]] $

$$[y_4, 0, 0, y_3, y_2, y_1, -y_4 - 15\, y_1 + 4\, y_3 + 4\, y_2 + 4\, y_5, y_3 + y_2 - 4\, y_1 + y_5, y_5]$$

$p = s^4 - s^6$ $p' = s^4 - s^6$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: -1 . order: 4

$ [[5, 4, 2, 3, 0, 0, 1, 3, 0] , [4, 5, 0, 6, 0, 0, 0, 3, 0] , [3, 4, 0, 5, 0, 0, 0, 6, 0] , [6, 3, 0, 4, 0, 0, 0, 5, 0] , [5, 6, 0, 3, 0, 0, 0, 4, 0] , [4, 5, 0, 6, 0, 0, 0, 3, 0]] $

$$[y_1, y_2, 2\, y_4, y_3, 0, 0, y_4, y_5, 0]$$

$p = s^2 - s^6$

Â» SYNC'D 3367/131072 , 0.02568817139

181 . Coloring, {2, 3, 6, 7, 9}

R: [4, 9, 5, 7, 7, 8, 5, 1, 2] **B**: [2, 4, 4, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-623, -356, -375, -650, -1500, -50, -1625, -200, -267] . FixedPtCheck, [623, 356, 375, 650, 1500, 50, 1625, 200, 267]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ τ ‘)‘ ^{2}

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 0, 3, 4, 0, 5, 1, 2] , [1, 2, 0, 2, 5, 0, 7, 0, 1] , [0, 1, 0, 1, 7, 0, 7, 0, 2] , [0, 2, 0, 0, 7, 0, 8, 0, 1] , [0, 1, 0, 0, 8, 0, 7, 0, 2] , [0, 2, 0, 0, 7, 0, 8, 0, 1] , [0, 1, 0, 0, 8, 0, 7, 0, 2]] $

$$[3\,y_1 - y_3 + 2\,y_5, y_1, 0, 2\,y_1 - y_2 - y_4 + 3\,y_5, y_2, 0, y_3, y_4, y_5]$$

p’ = s ^{4} - s ^{6} p = - s ^{4} + s ^{6}

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 0 . order: 6

$ [[4, 3, 2, 3, 0, 2, 1, 3, 0] , [1, 4, 0, 5, 0, 3, 2, 3, 0] , [2, 1, 0, 4, 0, 3, 3, 5, 0] , [3, 2, 0, 1, 0, 5, 3, 4, 0] , [3, 3, 0, 2, 0, 4, 5, 1, 0] , [5, 3, 0, 3, 0, 1, 4, 2, 0] , [4, 5, 0, 3, 0, 2, 1, 3, 0]] $

$$[y_2 + y_1 - y_3 - y_4 + y_5 + y_6, y_2, y_1, y_3, 0, y_4, y_5, y_6, 0]$$

p = - s ^{2} + s ^{3} - s ^{4} + s ^{5} - s ^{6} + s ^{7}

Â» SYNC’D 1971851/67108864 , 0.02938286960

182 . Coloring, {2, 3, 6, 8, 9}

R: [4, 9, 5, 7, 7, 8, 1, 6, 2] **B**: [2, 4, 4, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 4τ + 6τ ^{2} + τ ^{4} ‘)‘ , 18‘ (‘ 5 + 4τ + 6τ ^{2} + τ ^{4} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + 4τ + 6τ ^{2} + τ ^{4} ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [959, 548, 75, 875, 300, 375, 975, 500, 411] . FixedPtCheck, [959, 548, 75, 875, 300, 375, 975, 500, 411]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2}

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 7}, {6, 8}, {2, 9}}, net cycles: 2 . order: 6

$ [[3, 1, 0, 3, 1, 2, 5, 1, 2] , [5, 2, 0, 3, 0, 1, 4, 2, 1] , [4, 1, 0, 5, 0, 2, 3, 1, 2] , [3, 2, 0, 4, 0, 1, 5, 2, 1] , [5, 1, 0, 3, 0, 2, 4, 1, 2] , [4, 2, 0, 5, 0, 1, 3, 2, 1] , [3, 1, 0, 4, 0, 2, 5, 1, 2] , [5, 2, 0, 3, 0, 1, 4, 2, 1]] $

$$[4\,y_4 - y_1 - y_2 + 4\,y_5 - y_3, y_4, 0, y_1, y_2, y_5, y_3, y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = -s^2 + s^8 \quad p = s^2 - s^4 - s^5 + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[3, 3, 2, 3, 3, 0, 1, 3, 0] , [3, 3, 3, 5, 1, 0, 0, 3, 0] , [3, 3, 1, 6, 0, 0, 0, 5, 0] , [5, 3, 0, 4, 0, 0, 0, 6, 0] , [6, 5, 0, 3, 0, 0, 0, 4, 0] , [4, 6, 0, 5, 0, 0, 0, 3, 0] , [3, 4, 0, 6, 0, 0, 0, 5, 0]] $

$$[y_1 + y_2 - y_3 - y_4 + y_5 + y_6, y_1, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

$p = -s^4 + s^5 - s^6 + s^7$

Â» SYNC’D 599877/134217728 , 0.004469431937

183 . Coloring, {2, 3, 7, 8, 9}

R: [4, 9, 5, 7, 7, 7, 5, 6, 2] **B**: [2, 4, 4, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-301, -172, -225, -325, -900, -75, -975, -100, -129] . FixedPtCheck, [301, 172, 225, 325, 900, 75, 975, 100, 129]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 6	4 *vs* 5

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 0 . order: 2

$ [[0, 1, 0, 3, 4, 2, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1]] $

$$[0, 2\,y_1 + 5\,y_2 - 8\,y_3, 0, 3\,y_2, 2\,y_1, 2\,y_2, 8\,y_1 + 20\,y_2 - 30\,y_3, 0, 2\,y_3]$$

p’ = $s^3 - s^5$ p = $s^2 - s^6$ p’ = $s^2 - s^4$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[6, 3, 2, 3, 0, 0, 0, 4, 0] , [4, 6, 0, 5, 0, 0, 0, 3, 0] , [3, 4, 0, 6, 0, 0, 0, 5, 0] , [5, 3, 0, 4, 0, 0, 0, 6, 0] , [6, 5, 0, 3, 0, 0, 0, 4, 0]] $

$$[y_1 + y_2 - y_3 + y_4, y_1, y_2, y_3, 0, 0, 0, y_4, 0]$$

p = $-s^2 + s^3 - s^4 + s^5$

Â» SYNC’D 39/1024 , 0.03808593750

184 . Coloring, {2, 4, 5, 6, 7}

R: [4, 9, 4, 8, 3, 8, 5, 1, 1] **B**: [2, 4, 5, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘5 - 2τ + 8τ^2 + 2τ^3 + 3τ^4 ‘)‘‘ (‘ 3 + τ^2 ‘)‘ , 54‘ (‘5 - 2τ + 8τ^2 + 2τ^3 + 3τ^4 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ^2 ‘)‘‘ (‘ 3 + τ^2 ‘)‘ , 18‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 3 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 27‘ (‘5 - 2τ + 8τ^2 + 2τ^3 + 3τ^4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-2678, -824, -774, -2795, -1032, -645, -1118, -2580, -618] . FixedPtCheck, [2678, 824, 774, 2795, 1032, 645, 1118, 2580, 618]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[3, 0, 2, 4, 3, 0, 0, 4, 2] , [6, 0, 3, 5, 0, 0, 0, 4, 0] , [4, 0, 0, 9, 0, 0, 0, 5, 0] , [5, 0, 0, 4, 0, 0, 0, 9, 0] , [9, 0, 0, 5, 0, 0, 0, 4, 0] , [4, 0, 0, 9, 0, 0, 0, 5, 0]] $

$$[2\,y_1, 0, 2\,y_2, 2\,y_3, 3\,y_5, 0, 0, 2\,y_4, 2\,y_5]$$

$p = -s^3 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[3, 4, 0, 2, 1, 2, 6, 0, 0] , [6, 3, 0, 4, 0, 0, 5, 0, 0] , [5, 6, 0, 3, 0, 0, 4, 0, 0] , [4, 5, 0, 6, 0, 0, 3, 0, 0] , [3, 4, 0, 5, 0, 0, 6, 0, 0] , [6, 3, 0, 4, 0, 0, 5, 0, 0]] $

$$[y_3, y_4, 0, y_1, y_2, 2\,y_2, y_5, 0, 0]$$

$p = -s^2 + s^6$

Â» SYNC’D 2685/65536 , 0.04096984863

185 . Coloring, {2, 4, 5, 6, 8}

R: [4, 9, 4, 8, 3, 8, 1, 6, 1] **B**: [2, 4, 5, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [975, 300, 129, 903, 172, 1161, 559, 1548, 225] . FixedPtCheck, [975, 300, 129, 903, 172, 1161, 559, 1548, 225]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[4, 0, 2, 4, 0, 2, 0, 4, 2] , [1, 1, 0, 5, 3, 2, 3, 3, 0] , [4, 7, 3, 4, 5, 3, 2, 7, 1] , [4, 11, 5, 8, 11, 7, 12, 7, 7] , [28, 21, 11, 14, 15, 7, 22, 15, 11] , [50, 25, 15, 50, 31, 15, 60, 21, 21] , [124, 57, 31, 104, 53, 21, 96, 65, 25]] $ $ [[2, 4, 0, 2, 4, 0, 6, 0, 0] , [5, 3, 2, 1, 1, 0, 3, 1, 2] , [8, 1, 1, 8, 3, 1, 10, 1, 3] , [20, 5, 3, 16, 5, 1, 12, 9, 1] , [20, 11, 5, 34, 17, 9, 26, 17, 5] , [46, 39, 17, 46, 33, 17, 36, 43, 11] , [68, 71, 33, 88, 75, 43, 96, 63, 39]] $ $ [[1, -2, 1, 1, -2, 1, -3, 2, 1] , [-2, -1, -1, 2, 1, 1, 0, 1, -1] , [-2, 3, 1, -2, 1, 1, -4, 3, -1] , [-8, 3, 1, -4, 3, 3, 0, -1, 3] , [4, 5, 3, -10, -1, -1, -2, -1, 3] , [2, -7, -1, 2, -1, -1, 12, -11, 5] , [28, -7, -1, 8, -11, -11, 0, 1, -7]] $

$$[-3\,y_2 + y_3 + y_4 - 2\,y_6 - y_1, 2\,y_2 - 2\,y_3 - y_4 + y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$$

$p = s^2 + 6s^4 + 16s^7$

S+ \; S- \; NM

$ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[4, 0, 2, 4, 0, 2, 0, 4, 2] , [2, 0, 0, 6, 0, 4, 0, 6, 0] , [0, 0, 0, 2, 0, 6, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

$$[y_1, 0, y_5, y_2, 0, y_3, 0, y_4, y_5]$$

$p = s^4 - s^6$

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/2, -1, -13/18, 23/18] , [0, 0, 1/2, 5/18, -13/18] , [0, 0, 0,

5/18, -2/9] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, 5/18, -2/9] , [1/2, -1, 3/2, 23/18, -20/9] , [0, 1/2, -1, -13/18, 23/18]] $ x $ [[2, 4, 0, 2, 4, 0, 6, 0, 0] , [0, 2, 0, 4, 6, 0, 6, 0, 0] , [0, 0, 0, 2, 6, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

Â» SYNC'D 391/8192 , 0.04772949219

186 . Coloring, {2, 4, 5, 6, 9}

R: [4, 9, 4, 8, 3, 8, 1, 1, 2] **B**: [2, 4, 5, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-1029, -588, -75, -975, -100, -225, -325, -900, -441] . FixedPtCheck, [1029, 588, 75, 975, 100, 225, 325, 900, 441]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[5, 1, 2, 4, 0, 0, 0, 4, 2] , [4, 2, 0, 7, 0, 0, 0, 4, 1] , [4, 1, 0, 4, 0, 0, 0, 7, 2] , [7, 2, 0, 4, 0, 0, 0, 4, 1] , [4, 1, 0, 7, 0, 0, 0, 4, 2] , [4, 2, 0, 4, 0, 0, 0, 7, 1]] $

$$[5\,y_1 - y_2 - y_3 - y_4 + 5\,y_5, y_1, y_2, y_3, 0, 0, 0, y_4, y_5]$$

p = - s ^{2} - s ^{3} + s ^{5} + s ^{6}

Omega Rank for B : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 3, 0, 2, 4, 2, 6, 0, 0] , [0, 1, 0, 3, 6, 0, 8, 0, 0] , [0, 0, 0, 1, 8, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1, 3\,y_1 + y_2 + y_3 - y_4, 0, y_2, y_3, 2\,y_1, y_4, 0, 0]$$

$p = -s^4 + s^6$ $p = -s^4 + s^5$

Â» SYNC'D 4893/131072 , 0.03733062744

187 . Coloring, {2, 4, 5, 7, 8}

R: [4, 9, 4, 8, 3, 7, 5, 6, 1] **B**: [2, 4, 5, 7, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [325, 100, 387, 559, 516, 387, 559, 516, 75] . FixedPtCheck, [325, 100, 387, 559, 516, 387, 559, 516, 75]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 4

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	8 *vs* 8	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 6, 7, 8}}, net cycles: 0 . order: 6

$$[y_4, 0, y_1, y_2, y_3, y_6, y_7, y_8, y_5]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 527/2808, -427/2808, -265/2808, 149/2808, 167/2808, 5/2808] , [1/2, -1/4, -265/2808, 149/2808, 167/2808, 5/2808, -877/2808, 275/2808] , [0, 0, 527/2808, -427/2808, -265/2808, 149/2808, 167/2808, 5/2808] , [0, 0, 5/2808, 527/2808, -427/2808, -265/2808, 149/2808, 167/2808] , [0, 0, -427/2808, -265/2808, 149/2808, 167/2808, 5/2808, 527/2808] , [0, 0, 149/2808, 167/2808, 5/2808, 527/2808, -427/2808, -265/2808] , [0, 0, -265/2808, 149/2808, 167/2808, 5/2808, 527/2808, -427/2808] , [0, 0, 167/2808, 5/2808, 527/2808, -427/2808, -265/2808, 149/2808] , [0, 1/2, -427/2808, -265/2808, 149/2808, 167/2808, 5/2808, -877/2808]] $ x $ [[1, 0, 2, 4, 3, 2, 1, 3, 2] , [2, 0, 3, 3, 1, 3, 2, 4, 0] , [0, 0, 1, 5, 2, 4, 3, 3, 0] , [0, 0, 2, 1, 3, 3, 4, 5, 0] , [0, 0, 3, 2, 4, 5, 3, 1, 0] , [0, 0, 4, 3,

3, 1, 5, 2, 0] , [0, 0, 3, 4, 5, 2, 1, 3, 0] , [0, 0, 5, 3, 1, 3, 2, 4, 0]] $

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 2, 1, 0, 5, 1, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0] , [5, 4, 0, 3, 0, 0, 6, 0, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0]] $

$$[y_3, y_2, 0, y_1, y_5, 0, y_4, y_5, 0]$$

$p = s^2 - s^6$

Â» SYNC'D 455085/16777216 , 0.02712517977

188 . Coloring, {2, 4, 5, 7, 9}

R: [4, 9, 4, 8, 3, 7, 5, 1, 2] **B**: [2, 4, 5, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘ , -18‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 4τ + 6τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [-511, -292, -225, -625, -300, -125, -325, -500, -219] . FixedPtCheck, [511, 292, 225, 625, 300, 125, 325, 500, 219]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	7 *vs* 8	5 *vs* 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[2, 1, 2, 4, 3, 0, 1, 3, 2] , [3, 2, 3, 4, 1, 0, 0, 4, 1] , [4, 1, 1, 6, 0, 0, 0, 4, 2] , [4, 2, 0, 5, 0, 0, 0, 6, 1] , [6, 1, 0, 4, 0, 0, 0, 5, 2] , [5, 2, 0, 6, 0, 0, 0, 4, 1] , [4, 1, 0, 5, 0, 0, 0, 6, 2] , [6, 2, 0, 4, 0, 0, 0, 5, 1]] $

$$[5\,y_1 - y_2 - y_3 - y_4 - y_5 - y_6 + 5\,y_7, y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7]$$

$p = s^4 + s^5 - s^7 - s^8$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 4, 7}}, net cycles: 1 . order: 4

$ [[4, 3, 0, 2, 1, 2, 5, 1, 0] , [5, 4, 0, 3, 0, 1, 3, 2, 0] , [3, 5, 0, 4, 0, 2, 3, 1, 0] , [3, 3, 0, 5, 0, 1, 4, 2, 0] , [4, 3, 0, 3, 0, 2, 5, 1, 0] , [5, 4, 0, 3, 0, 1, 3, 2, 0] , [3, 5, 0, 4, 0, 2, 3, 1, 0]] $

$$[y_5, y_4, 0, -y_5 - y_3 + 2\,y_1 + 3\,y_2, y_3, y_1, -y_4 + 3\,y_1 + 2\,y_2, y_2, 0]$$

$p' = -s^2 + s^6 \quad p = s^2 - s^6$

Â» SYNC'D 184063/16777216 , 0.01097100973

189 . Coloring, {2, 4, 5, 8, 9}

R: [4, 9, 4, 8, 3, 7, 1, 6, 2] **B**: [2, 4, 5, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘]‘

For τ=1/2, [1869, 1068, 375, 1950, 500, 1350, 1625, 1800, 801] . FixedPtCheck, [1869, 1068, 375, 1950, 500, 1350, 1625, 1800, 801]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 4

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 8	5 *vs* 6

Omega Rank for R : cycles: {{2, 9}, {1, 4, 6, 7, 8}}, net cycles: 1 .

$ [[3, 1, 2, 4, 0, 2, 1, 3, 2] , [1, 2, 0, 5, 0, 3, 2, 4, 1] , [2, 1, 0, 1, 0, 4, 3, 5, 2] , [3, 2, 0, 2, 0, 5, 4, 1, 1] , [4, 1, 0, 3, 0, 1, 5, 2, 2] , [5, 2, 0, 4, 0, 2, 1, 3, 1] , [1, 1, 0, 5, 0, 3, 2, 4, 2] , [2, 2, 0, 1, 0, 4, 3, 5, 1]] $

$$[5\,y_7 - y_4 - y_5 - y_6 - y_2 - y_3 + 5\,y_1, y_7, y_4, y_5, 0, y_6, y_2, y_3, y_1]$$

$p = -s^2 - s^3 + s^7 + s^8$

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$ [[3, 3, 0, 2, 4, 0, 5, 1, 0] , [1, 3, 0, 3, 5, 0, 6, 0, 0] , [0, 1, 0, 3, 6, 0, 8, 0, 0] , [0, 0, 0, 1, 8, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_3, y_4, 0, y_5, -y_3 + y_4 - y_5 + y_1 + y_2, 0, y_1, y_2, 0]$$

$p = -s^5 + s^6$

Â» SYNC'D 45069/1048576 , 0.04298114777

190 . Coloring, {2, 4, 6, 7, 8}

R: [4, 9, 4, 8, 7, 8, 5, 6, 1] **B**: [2, 4, 5, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [286, 88, 86, 301, 344, 387, 430, 516, 66] . FixedPtCheck, [286, 88, 86, 301, 344, 387, 430, 516, 66]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 4, 3, 2, 2, 4, 2] , [2, 0, 0, 1, 2, 4, 3, 6, 0] , [0, 0, 0, 2, 3, 6, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0]] $

$$[y_1, 0, 0, -14\,y_1 - y_2 + 39\,y_3 - 14\,y_4 - y_5, -5\,y_1 + 14\,y_3 - 5\,y_4, y_2, y_3, y_4, y_5]$$

p' = $-s^4 + s^6$ p = $-s^4 + s^6$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 7}}, net cycles: 2 . order: 4

$ [[5, 4, 2, 2, 1, 0, 4, 0, 0] , [4, 5, 1, 4, 2, 0, 2, 0, 0] , [2, 4, 2, 5, 1, 0, 4, 0, 0] , [4, 2, 1, 4, 2, 0, 5, 0, 0] , [5, 4, 2, 2, 1, 0, 4, 0, 0] , [4, 5, 1, 4, 2, 0, 2, 0, 0]] $

$$[2\, y_1 - y_2 + 3\, y_3, 3\, y_1 + 2\, y_3 - y_4, y_1, y_2, y_3, 0, y_4, 0, 0]$$

$p = -s + s^5$ $p' = -s + s^5$

Â» SYNC'D 2267/262144 , 0.008647918701

191 . Coloring, {2, 4, 6, 7, 9}

R: [4, 9, 4, 8, 7, 8, 5, 1, 2] **B**: [2, 4, 5, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘27‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘ , 54‘ (‘ 5 + 3τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ^ 2 ‘)‘‘]‘

For τ=1/2, [322, 184, 50, 325, 200, 75, 250, 300, 138] . FixedPtCheck, [322, 184, 50, 325, 200, 75, 250, 300, 138]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 vs 7	*7 vs 7*	*7 vs 7*	*4 vs 7*	*5 vs 7*

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}, {2, 9}}, net cycles: 3 . order: 6

$ [[2, 1, 0, 4, 3, 0, 2, 4, 2] , [4, 2, 0, 2, 2, 0, 3, 4, 1] , [4, 1, 0, 4, 3, 0, 2, 2, 2] , [2, 2, 0, 4, 2, 0, 3, 4, 1] , [4, 1, 0, 2, 3, 0, 2, 4, 2] , [4, 2, 0, 4, 2, 0, 3, 2, 1] , [2, 1, 0, 4, 3, 0, 2, 4, 2]] $

$$[-y_1 + 10\, y_2 - y_3 - 10\, y_4, 3\, y_2 - 4\, y_4, 0, y_1, y_2, 0, 4\, y_2 - 5\, y_4, y_3, y_4]$$

$p = -s - s^2 + s^4 + s^5$ $p' = -s - s^2 + s^4 + s^5$ $p = -s + s^7$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 7}}, net cycles: 1 . order: 4

$ [[4, 3, 2, 2, 1, 2, 4, 0, 0] , [4, 4, 1, 3, 2, 0, 4, 0, 0] , [4, 4, 2, 4, 1, 0, 3, 0, 0] , [3, 4, 1, 4, 2, 0, 4, 0, 0] , [4, 3, 2, 4, 1, 0, 4, 0, 0] , [4, 4, 1, 3, 2, 0, 4, 0, 0] , [4, 4, 2, 4, 1, 0, 3, 0, 0]] $

$$[y_1, y_2, y_3, y_4, y_5, -y_1 + 3\,y_3 - y_4 + 2\,y_5, -y_2 + 2\,y_3 + 3\,y_5, 0, 0]$$

p' = $s^2 - s^6$ p = $-s^2 + s^6$

Â» SYNC'D 877/262144 , 0.003345489502

192 . Coloring, {2, 4, 6, 8, 9}

R: [4, 9, 4, 8, 7, 8, 1, 6, 2] **B**: [2, 4, 5, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 18‘ (‘ - 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} ‘]‘

For τ=1/2, [567, 324, 25, 525, 100, 675, 375, 900, 243] . FixedPtCheck, [567, 324, 25, 525, 100, 675, 375, 900, 243]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[3, 1, 0, 4, 0, 2, 2, 4, 2] , [2, 2, 0, 3, 0, 4, 0, 6, 1] , [0, 1, 0, 2, 0, 6, 0, 7, 2] , [0, 2, 0, 0, 0, 7, 0, 8, 1] , [0, 1, 0, 0, 0, 8, 0, 7, 2] , [0, 2, 0, 0, 0, 7, 0, 8, 1] , [0, 1, 0, 0, 0, 8, 0, 7, 2]] $

$$[3\,y_1 - y_4 + 2\,y_5, y_1, 0, 2\,y_1 - y_2 - y_3 + 3\,y_5, 0, y_2, y_3, y_4, y_5]$$

p = $-s^4 + s^6$ p' = $-s^4 + s^6$

Omega Rank for B : cycles: {{3, 5}}, net cycles: 0 . order: 6

$ [[3, 3, 2, 2, 4, 0, 4, 0, 0] , [0, 3, 4, 3, 6, 0, 2, 0, 0] , [0, 0, 6, 3, 6, 0, 3, 0, 0] , [0, 0, 6, 0, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] $

$$[y_1 + y_2 - y_3 - y_4 + y_5, y_1, y_2, y_3, y_4, 0, y_5, 0, 0]$$

$p = s^5 - s^6$

Â» SYNC'D 4545/524288 , 0.008668899536

193 . Coloring, {2, 4, 7, 8, 9}

R: [4, 9, 4, 8, 7, 7, 5, 6, 2] **B**: [2, 4, 5, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ - τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-259, -148, -125, -325, -500, -225, -625, -300, -111] . FixedPtCheck, [259, 148, 125, 325, 500, 225, 625, 300, 111]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 4, 3, 2, 3, 3, 2] , [0, 2, 0, 0, 3, 3, 5, 4, 1] , [0, 1, 0, 0, 5, 4, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2]] $

$$[0, y_1 + y_2 + y_3 - 4\,y_5, 0, y_1, y_2, y_3, 4\,y_1 + 4\,y_2 + 4\,y_3 - 15\,y_5 - y_4, y_4, y_5]$$

$p = -s^4 + s^6$ $p' = -s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {3, 5}}, net cycles: 1 . order: 4

$ [[6, 3, 2, 2, 1, 0, 3, 1, 0] , [4, 6, 1, 3, 2, 0, 2, 0, 0] , [2, 4, 2, 6, 1, 0, 3, 0, 0] , [3, 2, 1, 4, 2, 0, 6, 0, 0] , [6, 3, 2, 2, 1, 0, 4, 0, 0] , [4, 6, 1, 3, 2, 0, 2, 0, 0] , [2, 4, 2, 6, 1, 0, 3, 0, 0]] $

$$[3\,y_2 - y_1 + 2\,y_3, 2\,y_2 + 3\,y_3 - y_4 - y_5, y_2, y_1, y_3, 0, y_4, y_5, 0]$$

p = - s ^ 2 + s ^ 6 p' = - s ^ 2 + s ^ 6

Â» SYNC'D 6423/524288 , 0.01225090027

194 . Coloring, {2, 5, 6, 7, 8}

R: [4, 9, 4, 7, 3, 8, 5, 6, 1] **B**: [2, 4, 5, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [767, 236, 1161, 1505, 1548, 645, 1677, 860, 177] . FixedPtCheck, [767, 236, 1161, 1505, 1548, 645, 1677, 860, 177]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 4

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 8	6 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 4, 5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 2, 4, 3, 2, 3, 1, 2] , [2, 0, 3, 3, 3, 1, 4, 2, 0] , [0, 0, 3, 5, 4, 2, 3, 1, 0] , [0, 0, 4, 3, 3, 1, 5, 2, 0] , [0, 0, 3, 4, 5, 2, 3, 1, 0] , [0, 0, 5, 3, 3, 1, 4, 2, 0] , [0, 0, 3, 5, 4, 2, 3, 1, 0] , [0, 0, 4, 3, 3, 1, 5, 2, 0]] $

$$[-y_1 + y_3 - y_4 + 4\,y_5, 0, y_1, -y_2 + 4\,y_3 + y_5 - y_6, y_2, y_3, y_4, y_5, y_6]$$

p = - s ^ 3 + s ^ 7 p' = - s ^ 3 + s ^ 7

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, y_6, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 19/72, -17/72, 1/72, 1/72] , [0, 0, 1/72, 19/72, -17/72, 1/72] , [1, -3, 1/72, 19/72, -89/72, 217/72] , [0, 0, 1/72, 1/72, 19/72, -17/72] , [0, 1, 1/72, 1/72, 19/72, -89/72] , [0, 1, 1/72, 1/72, 19/72, -89/72] , [0, 0, -17/72, 1/72, 1/72, 19/72] , [0, 0, -17/72, 1/72, 1/72, 19/72] , [0, 0, 19/72, -17/72, 1/72, 1/72]] $ x $ [[5, 4, 0, 2, 1, 0, 3, 3, 0] , [6, 5, 0, 4, 0, 0, 1, 2, 0] , [3, 6, 0, 5, 0, 0, 0, 4, 0] , [4, 3, 0, 6, 0, 0, 0, 5, 0] , [5, 4, 0, 3, 0, 0, 0, 6, 0] , [6, 5, 0, 4, 0, 0, 0, 3, 0]] $

Â» SYNC'D 104409/4194304 , 0.02489304543

195 . Coloring, {2, 5, 6, 7, 9}

R: [4, 9, 4, 7, 3, 8, 5, 1, 2] **B**: [2, 4, 5, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [7, 4, 9, 13, 12, 1, 13, 4, 3] . FixedPtCheck, [7, 4, 9, 13, 12, 1, 13, 4, 3]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 8	6 *vs* 7

Omega Rank for R : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[2, 1, 2, 4, 3, 0, 3, 1, 2] , [1, 2, 3, 4, 3, 0, 4, 0, 1] , [0, 1, 3, 4, 4, 0, 4, 0, 2] , [0, 2, 4, 3, 4, 0, 4, 0, 1] , [0, 1, 4, 4, 4, 0, 3, 0, 2] , [0, 2, 4, 4, 3, 0, 4, 0, 1] , [0, 1, 3, 4, 4, 0, 4, 0, 2] , [0, 2, 4, 3, 4, 0, 4, 0, 1]] $

$$[y_6, y_5, y_4, y_3, y_2, 0, -y_6 + 3\,y_5 - y_4 + 2\,y_1, 2\,y_5 - y_3 - y_2 + 3\,y_1, y_1]$$

p' = s ^{3} - s ^{7} p = s ^{3} - s ^{7}

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 0 . order: 6

$ [[4, 3, 0, 2, 1, 2, 3, 3, 0] , [3, 4, 0, 3, 0, 3, 3, 2, 0] , [3, 3, 0, 4, 0, 2, 3, 3, 0] , [3, 3, 0, 3, 0, 3, 2, 4, 0] , [2, 3, 0, 3, 0, 4, 3, 3, 0] , [3, 2, 0, 3, 0, 3, 4, 3, 0] , [4, 3, 0, 2, 0, 3, 3, 3, 0]] $

$$[y_1, y_1 + y_2 + y_3 + y_4 - y_5 - y_6, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

$p = s^2 - s^3 + s^4 - s^5 + s^6 - s^7$

Â» SYNC'D 47451/4194304 , 0.01131320000

196 . Coloring, {2, 5, 6, 8, 9}

R: [4, 9, 4, 7, 3, 8, 1, 6, 2] **B**: [2, 4, 5, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [329, 188, 75, 350, 100, 150, 325, 200, 141] . FixedPtCheck, [329, 188, 75, 350, 100, 150, 325, 200, 141]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 4 ‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {2, 9}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[3, 1, 2, 4, 0, 2, 3, 1, 2] , [3, 2, 0, 5, 0, 1, 4, 2, 1] , [4, 1, 0, 3, 0, 2, 5, 1, 2] , [5, 2, 0, 4, 0, 1, 3, 2, 1] , [3, 1, 0, 5, 0, 2, 4, 1, 2] , [4, 2, 0, 3, 0, 1, 5, 2, 1] , [5, 1, 0, 4, 0, 2, 3, 1, 2] , [3, 2, 0, 5, 0, 1, 4, 2, 1]] $

$$[-y_1 - y_2 + 4\,y_5 - y_3 + 4\,y_4, y_4, y_1, y_2, 0, y_5, y_3, y_4, y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$ $p = s^2 - s^4 - s^5 + s^7$ $p = -s^2 + s^8$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$ [[3, 3, 0, 2, 4, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 4, 2, 0] , [2, 3, 0, 3, 4, 0, 3, 3, 0] , [3, 2, 0, 3, 3, 0, 4, 3, 0] , [3, 3, 0, 2, 4, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 4, 2, 0]] $

$$[9\,y_1 - 2\,y_2 - 11\,y_3 + 9\,y_4, 2\,y_1, 0, 2\,y_2, 2\,y_3, 0, 7\,y_1 - 9\,y_3 + 7\,y_4, 2\,y_4, 0]$$

p = - s + s ^{5} p' = - s + s ^{5}

Â» SYNC'D 8523/4194304 , 0.002032041550

197 . Coloring, {2, 5, 7, 8, 9}

R: [4, 9, 4, 7, 3, 7, 5, 6, 2] **B**: [2, 4, 5, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [686, 392, 1350, 1625, 1800, 375, 1950, 500, 294] . FixedPtCheck, [686, 392, 1350, 1625, 1800, 375, 1950, 500, 294]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 1, 2, 4, 3, 2, 4, 0, 2] , [0, 2, 3, 2, 4, 0, 6, 0, 1] , [0, 1, 4, 3, 6, 0, 2, 0, 2] , [0, 2, 6, 4, 2, 0, 3, 0, 1] , [0, 1, 2, 6, 3, 0, 4, 0, 2] , [0, 2, 3, 2, 4, 0, 6, 0, 1] , [0, 1, 4, 3, 6, 0, 2, 0, 2]] $

$$[0, -4\,y_5 + y_1 + y_2 + y_3, -15\,y_5 + 4\,y_1 + 4\,y_2 + 4\,y_3 - y_4, y_1, y_2, y_3, y_4, 0, y_5]$$

p' = - s ^{2} + s ^{6} p = - s ^{2} + s ^{6}

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 2, 1, 0, 2, 4, 0] , [6, 6, 0, 3, 0, 0, 1, 2, 0] , [3, 6, 0, 6, 0, 0, 0, 3, 0] , [3, 3, 0, 6, 0, 0, 0, 6, 0] , [6, 3, 0, 3, 0, 0, 0, 6, 0] , [6, 6, 0, 3, 0, 0, 0, 3, 0]] $

$$[y_1, y_1 + y_2 + y_3 - y_4 - y_5, 0, y_2, y_3, 0, y_4, y_5, 0]$$

$p = -s^3 + s^4 - s^5 + s^6$

Â» SYNC'D 17723/524288 , 0.03380393982

198 . Coloring, {2, 6, 7, 8, 9}

R: [4, 9, 4, 7, 7, 8, 5, 6, 2] **B**: [2, 4, 5, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-133, -76, -75, -175, -300, -75, -375, -100, -57] . FixedPtCheck, [133, 76, 75, 175, 300, 75, 375, 100, 57]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 7	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 1, 0, 4, 3, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2]] $

$$[0, y_3, 0, -y_2 + y_3 + 3\,y_1, y_2, y_1, 3\,y_3 + y_1, y_3, y_1]$$

$p' = -s^3 + s^5$ $p' = -s^2 + s^6$ $p' = -s^2 + s^4$ $p = s^2 - s^4$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[6, 3, 2, 2, 1, 0, 1, 3, 0] , [4, 6, 1, 3, 2, 0, 0, 2, 0] , [2, 4, 2, 6, 1, 0, 0, 3, 0] , [3, 2, 1, 4, 2, 0, 0, 6, 0] , [6, 3, 2, 2, 1, 0, 0, 4, 0] , [4, 6, 1, 3, 2, 0, 0, 2, 0] , [2, 4, 2, 6, 1, 0, 0, 3, 0]] $

$$[3\,y_1 - y_2 + 2\,y_3, 2\,y_1 + 3\,y_3 - y_4 - y_5, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$p = -s^2 + s^6 \quad p' = -s^2 + s^6$

Â» SYNC'D 13779/4194304 , 0.003285169601

199 . Coloring, {3, 4, 5, 6, 7}

R: [4, 4, 5, 8, 3, 8, 5, 1, 1] **B**: [2, 9, 4, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘]‘

For τ=1/2, [-645, -172, -513, -741, -684, -171, -399, -684, -43] . FixedPtCheck, [645, 172, 513, 741, 684, 171, 399, 684, 43]

$\det(A + \tau\,\Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

\+ \; - \; Δ

$ [[3, 0, 2, 5, 4, 0, 0, 4, 0] , [10, 5, 4, 3, 2, 0, 3, 5, 4] , [18, 2, 2, 15, 7, 3, 19, 3, 3] , [11, 11, 7, 26, 21, 13, 23, 18, 14] , [57, 39, 21, 31, 30, 14, 36, 39, 21] , [120, 50, 30, 107, 57, 25, 117, 45, 25] , [145, 111, 57, 204, 147, 83, 195, 132, 78]] $ $ [[3, 4, 0, 1, 0, 2, 6, 0, 2] , [2, 3, 0, 9, 6, 4, 9, 3, 0] , [6, 14, 6, 9, 9, 5, 5, 13, 5] , [37, 21, 9, 22, 11, 3, 25, 14, 2] , [39, 25, 11, 65, 34, 18, 60, 25, 11] , [72, 78, 34, 85, 71, 39, 75, 83, 39] , [239, 145, 71, 180, 109, 45, 189, 124, 50]] $ $ [[0, -2, 1, 2, 2, -1, -3, 2, -1] , [4, 1, 2, -3, -2, -2, -3, 1, 2] , [6, -6, -2, 3, -1, -1, 7, -5, -1] , [-13, -5, -1, 2, 5, 5, -1, 2, 6] , [9, 7, 5, -17, -2, -2, -12, 7, 5] , [24, -14, -2, 11, -7, -7, 21, -19, -7] , [-47, -17, -7, 12, 19, 19, 3, 4, 14]] $

$[-y_1 + y_2 - 3\,y_3 + y_4, -2\,y_2 + 2\,y_3 - y_4 - y_6, -y_4 - y_5, y_1, y_2, y_3, y_4, y_5, y_6]$

$p = s^2 - 6s^4 + 16s^7$

S+ \; S- \; NM

$ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10, 4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[13, 13, 5, 19, 8, 5, 14, 9, 6] , [14, 11, 3, 17, 9, 4, 15, 12, 7] , [12, 9, 7, 18, 12, 4, 16, 9, 5] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [15, 9, 7, 14, 13, 4, 17, 10, 3] , [15, 8, 5, 14, 10, 5, 17, 12, 6] , [18, 9, 6, 13, 12, 6, 15, 9, 4] , [17, 12, 4, 15, 10, 6, 14, 10,

4] , [19, 13, 4, 14, 8, 7, 13, 9, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}}, net cycles: 2 . order: 6

$ [[3, 0, 2, 5, 4, 0, 0, 4, 0] , [4, 0, 4, 3, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 4, 0, 0, 3, 0] , [3, 0, 4, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 3, 4, 0, 0, 5, 0]] $

$$[2\,y_1 - y_2 + 2\,y_3 - y_4, 0, y_1, y_2, y_3, 0, 0, y_4, 0]$$

$p = - s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[3, 4, 0, 1, 0, 2, 6, 0, 2] , [6, 5, 0, 0, 0, 0, 3, 0, 4] , [3, 10, 0, 0, 0, 0, 0, 0, 5] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, y_3, 0, 2\,y_3, y_4, 0, y_5]$$

$p = - s^4 + s^6$

Â» SYNC’D 1297/16384 , 0.07916259766

200 . Coloring, {3, 4, 5, 6, 8}

R: [4, 4, 5, 8, 3, 8, 1, 6, 1] **B**: [2, 9, 4, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘27‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 54‘ (‘ 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , -27‘ (‘ 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [390, 104, 114, 399, 152, 513, 266, 684, 26] . FixedPtCheck, [390, 104, 114, 399, 152, 513, 266, 684, 26]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[4, 0, 2, 5, 1, 2, 0, 4, 0] , [0, 0, 1, 4, 2, 4, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0]] $

$$[3 y_1 + 2 y_2 - y_4, 0, y_1, 2 y_1 + 3 y_2 - y_3, y_2, y_3, 0, y_4, 0]$$

p = - s ^ 3 + s ^ 5 p’ = - s ^ 3 + s ^ 5

Omega Rank for B : cycles: {{5, 7}, {2, 9}}, net cycles: 0 . order: 2

$ [[2, 4, 0, 1, 3, 0, 6, 0, 2] , [0, 4, 0, 0, 6, 0, 4, 0, 4] , [0, 4, 0, 0, 4, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 4, 0, 4] , [0, 4, 0, 0, 4, 0, 6, 0, 4] , [0, 4, 0, 0, 6, 0, 4, 0, 4]] $

$$[4 y_1, 4 y_1 + 2 y_3, 0, 2 y_1, 8 y_1 + 5 y_3 - 2 y_2, 0, 2 y_2, 0, 2 y_3]$$

p = - s ^ 2 + s ^ 4 p’ = - s ^ 2 + s ^ 4 p = - s ^ 2 + s ^ 6

Â» SYNC’D 155/32768 , 0.004730224609

201 . Coloring, {3, 4, 5, 6, 9}

R: [4, 4, 5, 8, 3, 8, 1, 1, 2] **B**: [2, 9, 4, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [650, 200, 102, 663, 136, 153, 238, 612, 50] . FixedPtCheck, [650, 200, 102, 663, 136, 153, 238, 612, 50]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 1 . order: 6

$ [[5, 1, 2, 5, 1, 0, 0, 4, 0] , [4, 0, 1, 6, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 1, 0, 0, 6, 0] , [6, 0, 1, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 6, 1, 0, 0, 5, 0] , [5, 0, 1, 4, 2, 0, 0, 6, 0]] $

$$[-y_1 + 5 y_2 - y_3 + 5 y_4 - y_5, y_1, y_2, y_3, y_4, 0, 0, y_5, 0]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 0 . order: 6

$ [[1, 3, 0, 1, 3, 2, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2]] $

$$[-y_1 + y_3 - y_4, y_1, 0, y_2, -3 y_2 + y_3, 2 y_2, y_3, 0, y_4]$$

$p = - s^2 + s^5$ p’ = $- s^2 + s^5$ p’ = $- s^3 + s^6$

Â» SYNC’D 2839/131072 , 0.02165985107

202 . Coloring, {3, 4, 5, 7, 8}

R: [4, 4, 5, 8, 3, 7, 5, 6, 1] **B**: [2, 9, 4, 7, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘ ^{4} ‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-885, -236, -2565, -1482, -3420, -1026, -1995, -1368, -59] . FixedPtCheck, [885, 236, 2565, 1482, 3420, 1026, 1995, 1368, 59]

$\det(A + \tau \Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	7 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[y_1, 0, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -5, 22, -97, -1663/72, 7355/72] , [0, 1, -5, 22, -97, -1663/72, 7355/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 1, -5, 22, 371/72, -1663/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 1, 11/72, -79/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 1, -5, -79/72, 371/72] , [1, -5, 22, -97, 428, 7355/72, -32479/72]] $ x $ [[1, 0, 2, 5, 4, 2, 1, 3, 0] , [0, 0, 4, 1, 3, 3, 2, 5, 0] , [0, 0, 3, 0, 6, 5, 3, 1, 0] , [0, 0, 6, 0, 6, 1, 5, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0]] $

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 1, 0, 0, 5, 1, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, y_4, 0, 0, y_3, y_4, y_5]$$

$p = - s^4 + s^6$

Â» SYNC'D 1711/16384 , 0.1044311523

203 . Coloring, {3, 4, 5, 7, 9}

R: [4, 4, 5, 8, 3, 7, 5, 1, 2] **B**: [2, 9, 4, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ² ‘)‘‘ (‘ 5 - 3τ + τ ² + τ ³ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 3τ + τ ² + τ ³ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ³ ‘ (‘ 5 - 2τ + τ ² ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ² ‘ (‘ 5 - 2τ + τ ² ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ² ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ² ‘ (‘ 5 - 3τ + τ ² + τ ³ ‘)‘‘]‘

For τ=1/2, [403, 124, 459, 510, 612, 102, 357, 408, 31] . FixedPtCheck, [403, 124, 459, 510, 612, 102, 357, 408, 31]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 0 . order: 6

$ [[2, 1, 2, 5, 4, 0, 1, 3, 0] , [3, 0, 4, 3, 3, 0, 0, 5, 0] , [5, 0, 3, 3, 4, 0, 0, 3, 0] , [3, 0, 4, 5, 3, 0, 0, 3, 0] , [3, 0, 3, 3, 4, 0, 0, 5, 0] , [5, 0, 4, 3, 3, 0, 0, 3, 0] , [3, 0, 3, 5, 4, 0, 0, 3, 0]] $

$$[4 y_5, 7 y_5 - 11 y_3 + 7 y_4 - 11 y_2 + 7 y_1, 4 y_3, 4 y_4, 4 y_2, 0, 7 y_5 - 11 y_3 + 7 y_4 - 11 y_2 + 7 y_1, 4 y_1, 0]$$

$p = - s^2 - s^3 + s^5 + s^6$ p’ = $- s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[4, 3, 0, 1, 0, 2, 5, 1, 2] , [7, 4, 0, 0, 0, 1, 1, 2, 3] , [4, 7, 0, 0, 0, 2, 0, 1, 4] , [4, 4, 0, 0, 0, 1, 0, 2, 7] , [7, 4, 0, 0, 0, 2, 0, 1, 4] , [4, 7, 0, 0, 0, 1, 0, 2, 4] , [4, 4, 0, 0, 0, 2, 0, 1, 7]] $

$$[-y_1 - y_4 + 5 y_2 - y_3 + 5 y_5 - y_6, y_1, 0, y_4, 0, y_2, y_3, y_5, y_6]$$

$p = - s^3 - s^4 + s^6 + s^7$

Â» SYNC’D 256761/4194304 , 0.06121659279

204 . Coloring, {3, 4, 5, 8, 9}

R: [4, 4, 5, 8, 3, 7, 1, 6, 2] **B**: [2, 9, 4, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [611, 188, 255, 663, 340, 459, 595, 612, 47] . FixedPtCheck, [611, 188, 255, 663, 340, 459, 595, 612, 47]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 8	3 *vs* 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}, {3, 5}}, net cycles: 1 .

$ [[3, 1, 2, 5, 1, 2, 1, 3, 0] , [1, 0, 1, 4, 2, 3, 2, 5, 0] , [2, 0, 2, 1, 1, 5, 3, 4, 0] , [3, 0, 1, 2, 2, 4, 5, 1, 0] , [5, 0, 2, 3, 1, 1, 4, 2, 0] , [4, 0, 1, 5, 2, 2, 1, 3, 0] , [1, 0, 2, 4, 1, 3, 2, 5, 0] , [2, 0, 1, 1, 2, 5, 3, 4, 0]] $

$$[y_4, y_5, y_6, y_7, y_1, -y_4 - y_5 + 5 y_6 - y_7 + 5 y_1 - y_2 - y_3, y_2, y_3, 0]$$

p = - s 2 - s 3 + s 7 + s 8

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 0 . order: 6

$ [[3, 3, 0, 1, 3, 0, 5, 1, 2] , [3, 3, 0, 0, 5, 0, 4, 0, 3] , [3, 3, 0, 0, 4, 0, 5, 0, 3] , [3, 3, 0, 0, 5, 0, 4, 0, 3] , [3, 3, 0, 0, 4, 0, 5, 0, 3] , [3, 3, 0, 0, 5, 0, 4, 0, 3] , [3, 3, 0, 0, 4, 0, 5, 0, 3]] $

$$[y_2 + y_1, y_2 + y_1, 0, y_2, 2 y_2 + 3 y_1 - y_3, 0, y_3, y_2, y_1]$$

p’ = - s 4 + s 6 p = s 2 - s 4 p’ = - s 3 + s 5 p’ = s 2 - s 4

Â» SYNC’D 604887/33554432 , 0.01802703738

205 . Coloring, {3, 4, 6, 7, 8}

R: [4, 4, 5, 8, 7, 8, 5, 6, 1] **B**: [2, 9, 4, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 5 + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-375, -100, -171, -399, -684, -513, -741, -684, -25] . FixedPtCheck, [375, 100, 171, 399, 684, 513, 741, 684, 25]

$\det(A + \tau \Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[1, 0, 0, 5, 4, 2, 2, 4, 0] , [4, 7, 0, 3, 2, 4, 5, 7, 4] , [12, 8, 6, 15, 5, 7, 11, 7, 1] , [23, 19, 11, 22, 17, 7, 15, 22, 8] , [51, 33, 15, 47, 26, 22, 52, 29, 13] , [92, 64, 38, 101, 67, 29, 85, 69, 31] , [197, 133, 61, 182, 123, 69, 193, 130, 64]] $ $ [[5, 4, 2, 1, 0, 0, 4, 0, 2] , [8, 1, 4, 9, 6, 0, 7, 1, 0] , [12, 8, 2, 9, 11, 1, 13, 9, 7] , [25, 13, 5, 26, 15, 9, 33, 10, 8] , [45, 31, 17, 49, 38, 10, 44, 35, 19] , [100, 64, 26, 91, 61, 35, 107, 59, 33] , [187, 123, 67, 202, 133, 59, 191, 126, 64]] $ $ [[-2, -2, -1, 2, 2, 1, -1, 2, -1] , [-2, 3, -2, -3, -2, 2, -1, 3, 2] , [0, 0, 2, 3, -3, 3, -1, -1, -3] , [-1, 3, 3, -2, 1, -1, -9, 6, 0] , [3, 1, -1, -1, -6, 6, 4, -3, -3] , [-4, 0, 6, 5, 3, -3, -11, 5, -1] , [5, 5, -3, -10, -5, 5, 1, 2, 0]] $

$[y_5, y_4, y_3, y_2, y_1, -y_5 - y_4 - y_2 - y_1 - y_6, 2 y_5 + 2 y_2 + y_4 + y_6 - 2 y_3, y_3 - 2 y_5 - 2 y_2 - y_4 - y_6, y_6]$

$p = s^3 - s^4 - 4s^5 + 8s^7$

S+ \; S- \; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 5, 4, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0]] $

$[3 y_2 - 4 y_1 + 3 y_3 - y_4, 0, 0, y_2, y_1, y_3, 2 y_2 - 3 y_1 + 2 y_3, y_4, 0]$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 6

$$[y_6, y_5, y_4, y_3, 0, 0, y_2, 0, y_1]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 5/18, -2/9] , [0, 0, 0, 0, -2/9, 5/18] , [0, 1/2, -1/4, -7/8, 1/36, 47/72] , [0, 0, 1/2, -1/4, -2/9, 1/36] , [1/2, -1/4, -7/8, -5/16, 47/72, 49/144] , [0, 0, 1/2, -1/4, -2/9, 1/36] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 1/2, -2/9, -2/9] , [0, 0, 0, 0, 5/18, -2/9]] $ x $ [[5, 4, 2, 1, 0, 0, 4, 0, 2] , [4, 7, 0, 2, 0, 0, 1, 0, 4] , [1, 8, 0, 0, 0, 0, 2, 0, 7] , [2, 8, 0, 0, 0, 0, 0, 0, 8] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

Â» SYNC'D 329/8192 , 0.04016113281

206 . Coloring, {3, 4, 6, 7, 9}

R: [4, 4, 5, 8, 7, 8, 5, 1, 2] **B**: [2, 9, 4, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-637, -196, -153, -663, -612, -153, -663, -612, -49] . FixedPtCheck, [637, 196, 153, 663, 612, 153, 663, 612, 49]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[2, 1, 0, 5, 4, 0, 2, 4, 0] , [10, 4, 0, 5, 2, 0, 7, 5, 3] , [11, 5, 6, 18, 7, 3, 13, 5, 4] , [20, 17, 9, 18, 19, 11, 18, 21, 11] , [56, 39, 13, 44, 27, 11, 54, 29, 15] , [88, 55, 37, 114, 67, 35, 100, 55, 25] , [186, 129, 61, 170, 137, 73, 174, 149, 73]] $ $ [[4, 3, 2, 1, 0, 2, 4, 0, 2] , [2, 4, 4, 7, 6, 4, 5, 3, 1] , [13, 11, 2, 6, 9, 5, 11, 11, 4] , [28, 15, 7, 30, 13, 5, 30, 11, 5] , [40, 25, 19, 52, 37, 21, 42, 35, 17] , [104, 73, 27, 78, 61, 29, 92, 73, 39] , [198, 127, 67, 214, 119, 55, 210, 107, 55]] $ $ [[-1, -1, -1, 2, 2, -1, -1, 2, -1] , [4, 0, -2, -1, -2, -2, 1, 1, 1] , [-1, -3, 2, 6, -1, -1, 1, -3, 0] , [-4, 1, 1, -6, 3, 3, -6, 5, 3] , [8, 7, -3, -4, -5, -5, 6, -3, -1] , [-8, -9, 5, 18,

3, 3, 4, -9, -7] , [-6, 1, -3, -22, 9, 9, -18, 21, 9]] $

$$[y_2, y_3, y_4, y_5, -y_4 - y_2, -y_3 + y_4 - y_5 - y_1, y_4 + y_2 - y_1, -y_2 - 2\,y_4 + y_1, y_1]$$

$p = s^2 + 4s^4 + 4s^5 + 8s^6$

S+ \; S- \; NM

$ [[11, 8, 3, 12, 6, 4, 11, 8, 5] , [10, 11, 4, 10, 4, 3, 13, 9, 4] , [8, 8, 7, 10, 6, 3, 17, 9, 0] , [13, 6, 3, 11, 10, 5, 9, 6, 5] , [14, 3, 3, 13, 14, 3, 8, 6, 4] , [13, 6, 3, 11, 10, 5, 9, 6, 5] , [10, 8, 6, 11, 6, 3, 14, 8, 2] , [10, 10, 3, 11, 6, 4, 13, 9, 2] , [13, 8, 2, 13, 6, 4, 8, 7, 7]] $ $ [[11, 8, 3, 12, 6, 4, 11, 8, 5] , [11, 10, 3, 11, 4, 3, 13, 10, 3] , [8, 8, 9, 9, 5, 2, 16, 8, 3] , [14, 5, 2, 12, 10, 5, 9, 7, 4] , [12, 5, 3, 12, 15, 4, 9, 5, 3] , [14, 5, 2, 12, 10, 5, 9, 7, 4] , [9, 9, 7, 10, 6, 3, 14, 7, 3] , [11, 9, 4, 11, 5, 3, 12, 9, 4] , [12, 9, 1, 13, 7, 5, 9, 7, 5]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

$p' = s^2 + 4s^4 + 4s^5 + 8s^6$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
5 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	4 *vs* 7

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}}, net cycles: 1 . order: 6

$ [[2, 1, 0, 5, 4, 0, 2, 4, 0] , [4, 0, 0, 3, 2, 0, 4, 5, 0] , [5, 0, 0, 4, 4, 0, 2, 3, 0] , [3, 0, 0, 5, 2, 0, 4, 4, 0] , [4, 0, 0, 3, 4, 0, 2, 5, 0] , [5, 0, 0, 4, 2, 0, 4, 3, 0]] $

$$[y_3, y_4, 0, y_5, y_2, 0, y_1, -y_3 - y_4 - y_5 + 2\,y_2 + 2\,y_1, 0]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 6

$ [[4, 3, 2, 1, 0, 2, 4, 0, 2] , [6, 4, 0, 2, 0, 0, 3, 0, 3] , [6, 6, 0, 0, 0, 0, 2, 0, 4] , [6, 6, 0, 0, 0, 0, 0, 0, 6] , [6, 6, 0, 0, 0, 0, 0, 0, 6] , [6, 6, 0, 0, 0, 0, 0, 0, 6] , [6, 6, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_4, y_4 - y_3, y_2, y_3, 0, y_2, y_1, 0, y_4 + y_2 - y_1]$$

$p = s^4 - s^6$ $p' = s^4 - s^5$ $p' = -s^5 + s^6$

Â» SYNC’D 6495/131072 , 0.04955291748

207 . Coloring, {3, 4, 6, 8, 9}

R: [4, 4, 5, 8, 7, 8, 1, 6, 2] **B**: [2, 9, 4, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 3τ ^{2} ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 3τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 3τ ^{2} ‘)‘‘]‘

For τ=1/2, [-598, -184, -34, -595, -136, -765, -442, -1020, -46] . FixedPtCheck, [598, 184, 34, 595, 136, 765, 442, 1020, 46]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	6 *vs* 7

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 6

$ [[3, 1, 0, 5, 1, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 1, 7, 0] , [1, 0, 0, 2, 0, 7, 0, 8, 0] , [0, 0, 0, 1, 0, 8, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0]] $

$$[y_5, y_2, 0, y_1, y_2, y_3, y_4, -y_5 - 2\,y_2 + y_1 + y_3 + y_4, 0]$$

p = - s ^{5} + s ^{6} p = - s ^{5} + s ^{7}

Omega Rank for B : cycles: {{3, 4, 5, 7}, {1, 2, 9}}, net cycles: 2 .

$ [[3, 3, 2, 1, 3, 0, 4, 0, 2] , [2, 3, 3, 2, 4, 0, 1, 0, 3] , [3, 2, 4, 3, 1, 0, 2, 0, 3] , [3, 3, 1, 4, 2, 0, 3, 0, 2] , [2, 3, 2, 1, 3, 0, 4, 0, 3] , [3, 2, 3, 2, 4, 0, 1, 0, 3] , [3, 3, 4, 3, 1, 0, 2, 0, 2]] $

$$[4\,y_1, 4\,y_2, 5\,y_1 + 5\,y_2 - 4\,y_3 - 4\,y_4 - 4\,y_5 + 5\,y_6, 4\,y_3, 4\,y_4, 0, 4\,y_5, 0, 4\,y_6]$$

p = - s - s ^{2} - s ^{3} + s ^{5} + s ^{6} + s ^{7}

Â» SYNC’D 48415/4194304 , 0.01154303551

208 . Coloring, {3, 4, 7, 8, 9}

R: [4, 4, 5, 8, 7, 7, 5, 6, 2] **B**: [2, 9, 4, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-1157, -356, -765, -1326, -3060, -918, -3315, -1224, -89] . FixedPtCheck, [1157, 356, 765, 1326, 3060, 918, 3315, 1224, 89]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{5, 7}}, net cycles: 0 . order: 6

$$[0, y_1, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -5, 22, 371/72, -1663/72] , [0, 1, -5, 22, 371/72, -1663/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 1, -5, -79/72, 371/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 1, 11/72, -79/72] , [1, -5, 22, -97, -1663/72, 7355/72]] $ x $ [[0, 1, 0, 5, 4, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 6, 5, 0] , [0, 0, 0, 0, 6, 5, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 6

$ [[6, 3, 2, 1, 0, 0, 3, 1, 2] , [6, 6, 0, 2, 0, 0, 1, 0, 3] , [4, 6, 0, 0, 0, 0, 2, 0, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8] , [8, 4, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 2\,y_5, y_3, 0, 0, y_4, y_5, y_6]$$

p = s ^ 4 - s ^ 7

Â» SYNC’D 7785/65536 , 0.1187896729

209 . Coloring, {3, 5, 6, 7, 8}

R: [4, 4, 5, 7, 3, 8, 5, 6, 1] **B**: [2, 9, 4, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 4 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 4 ‘]‘

For τ=1/2, [-435, -116, -1539, -798, -2052, -342, -1197, -456, -29] . FixedPtCheck, [435, 116, 1539, 798, 2052, 342, 1197, 456, 29]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[1, 0, 2, 5, 4, 2, 3, 1, 0] , [0, 0, 4, 1, 5, 1, 5, 2, 0] , [0, 0, 5, 0, 9, 2, 1, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0]] $

$$[-y_1 + y_4 - y_5 + 4\,y_3, 0, y_1, -y_2 + 4\,y_4 + y_3, y_2, y_4, y_5, y_3, 0]$$

p = - s ^ 4 + s ^ 6 p’ = - s ^ 4 + s ^ 6

Omega Rank for B : cycles: {{2, 9}}, net cycles: -1 . order: 4

$ [[5, 4, 0, 1, 0, 0, 3, 3, 2] , [6, 7, 0, 0, 0, 0, 0, 1, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] $

$$[y_1, y_2, 0, y_3, 0, 0, 3\,y_3, y_4, y_5]$$

p = - s ^ 4 + s ^ 6

Â» SYNC’D 141/4096 , 0.03442382812

210 . Coloring, {3, 5, 6, 7, 9}

R: [4, 4, 5, 7, 3, 8, 5, 1, 2] **B**: [2, 9, 4, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 4 ‘]‘

For τ=1/2, [767, 236, 2295, 1326, 3060, 102, 1785, 408, 59] . FixedPtCheck, [767, 236, 2295, 1326, 3060, 102, 1785, 408, 59]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 6

$ [[2, 1, 2, 5, 4, 0, 3, 1, 0] , [1, 0, 4, 3, 5, 0, 5, 0, 0] , [0, 0, 5, 1, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 8, 0, 1, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0]] $

$$[y_1, y_6, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

p = - s ^ 5 + s ^ 7

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, 0, y_3, 0, y_4, y_5, y_6, y_7]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, 0, 19/54, 1/54, -17/54] , [0, 0, 0, 0, -17/54, 19/54, 1/54] , [1, -3, 7, -18, 145/54, -359/54, 919/54] , [0, 1, -3, 7, -53/54, 145/54, -359/54] , [0, 0, 0, 1, -17/54, 19/54, -53/54] , [0, 0, 0, 1, -17/54, 19/54, -53/54] , [0, 0, 0, 0, 1/54, -17/54, 19/54] , [0, 0, 1, -3, 19/54, -53/54, 145/54] , [0, 0, 0, 0, 1/54, -17/54, 19/54]] $ x $ [[4, 3, 0, 1, 0, 2, 3, 3, 2] , [5, 4, 0, 0, 0, 3, 2, 1, 3] , [5, 5, 0, 0, 0, 1, 3, 0, 4] , [7, 5, 0, 0, 0, 0, 1, 0, 5] , [6, 7, 0, 0, 0, 0, 0, 0, 5] , [5, 6, 0, 0, 0, 0, 0, 0, 7] , [7, 5, 0, 0, 0, 0, 0, 0, 6]] $

Â» SYNC’D 52455/2097152 , 0.02501249313

211 . Coloring, {3, 5, 6, 8, 9}

R: [4, 4, 5, 7, 3, 8, 1, 6, 2] **B**: [2, 9, 4, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 153, 357, 204, 153, 357, 204, 25] . FixedPtCheck, [325, 100, 153, 357, 204, 153, 357, 204, 25]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 9	9 *vs* 9	5 *vs* 8	5 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {3, 5}, {1, 4, 7}}, net cycles: 2 . order: 6

$ [[3, 1, 2, 5, 1, 2, 3, 1, 0] , [3, 0, 1, 4, 2, 1, 5, 2, 0] , [5, 0, 2, 3, 1, 2, 4, 1, 0] , [4, 0, 1, 5, 2, 1, 3, 2, 0] , [3, 0, 2, 4, 1, 2, 5, 1, 0] , [5, 0, 1, 3, 2, 1, 4, 2, 0] , [4, 0, 2, 5, 1, 2, 3, 1, 0] , [3, 0, 1, 4, 2, 1, 5, 2, 0]] $

$$[-y_1 + 4\,y_3 - y_2 + 4\,y_5 - y_4, y_1, y_3, y_2, y_5, y_3, y_4, y_5, 0]$$

p = - s 2 - s 3 + s 5 + s 6 p = - s 2 + s 8 p = s 2 - s 4 - s 5 + s 7

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 1, 3, 0, 3, 3, 2] , [5, 3, 0, 0, 3, 0, 3, 1, 3] , [4, 5, 0, 0, 3, 0, 3, 0, 3] , [3, 4, 0, 0, 3, 0, 3, 0, 5] , [5, 3, 0, 0, 3, 0, 3, 0, 4] , [4, 5, 0, 0, 3, 0, 3, 0, 3] , [3, 4, 0, 0, 3, 0, 3, 0, 5]] $

$$[-y_1 - y_2 + 4\,y_3 - y_4 - y_5, y_1, 0, y_2, y_3, 0, y_3, y_4, y_5]$$

p = - s 3 + s 6 p’ = - s 3 + s 6

Â» SYNC’D 867735/134217728 , 0.006465129554

212 . Coloring, {3, 5, 7, 8, 9}

R: [4, 4, 5, 7, 3, 7, 5, 6, 2] **B**: [2, 9, 4, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 4 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 4 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 1377, 663, 1836, 153, 1071, 204, 25] . FixedPtCheck, [325, 100, 1377, 663, 1836, 153, 1071, 204, 25]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: -1 . order: 4

$ [[0, 1, 2, 5, 4, 2, 4, 0, 0] , [0, 0, 4, 1, 6, 0, 7, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0]] $

$$[0, y_1, y_2, y_3, y_4, 2\,y_1, y_5, 0, 0]$$

p = - s 4 + s 6

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 1, 0, 0, 2, 4, 2] , [8, 6, 0, 0, 0, 0, 0, 1, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 0, y_5, 0, 0, 2\,y_5, y_3, y_4]$$

p = - s 3 + s 6

Â» SYNC’D 933/8192 , 0.1138916016

213 . Coloring, {3, 6, 7, 8, 9}

R: [4, 4, 5, 7, 7, 8, 5, 6, 2] **B**: [2, 9, 4, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-611, -188, -459, -714, -1836, -306, -1989, -408, -47] . FixedPtCheck, [611, 188, 459, 714, 1836, 306, 1989, 408, 47]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ ^ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 6	6 *vs* 7

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 5, 4, 2, 5, 1, 0] , [0, 0, 0, 1, 5, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0]] $

$$[0, y_1, 0, y_2, y_3, y_4, -y_1 - 15\, y_4 + 4\, y_2 + 4\, y_3, y_2 + y_3 - 4\, y_4, 0]$$

p’ = - s ^ 3 + s ^ 5 p = - s ^ 3 + s ^ 5

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 6

$ [[6, 3, 2, 1, 0, 0, 1, 3, 2] , [6, 6, 0, 2, 0, 0, 0, 1, 3] , [4, 6, 0, 0, 0, 0, 0, 2, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8] , [8, 4, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_4, y_5, 2\, y_1, y_6, 0, 0, y_1, y_2, y_3]$$

p = - s ^ 4 + s ^ 7

Â» SYNC’D 7935/131072 , 0.06053924561

214 . Coloring, {4, 5, 6, 7, 8}

R: [4, 4, 4, 8, 3, 8, 5, 6, 1] **B**: [2, 9, 5, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 5 + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-1290, -344, -1026, -1995, -1368, -2565, -1482, -3420, -86] . FixedPtCheck, [1290, 344, 1026, 1995, 1368, 2565, 1482, 3420, 86]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[1, 0, 2, 6, 3, 2, 0, 4, 0] , [0, 0, 3, 3, 0, 4, 0, 8, 0] , [0, 0, 0, 3, 0, 8, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0]] $

$$[y_4, 0, y_5, y_3, 3\,y_4, y_2, 0, y_1, 0]$$

p = - s ^ 4 + s ^ 6

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 0, y_5]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, -2/9, 5/18] , [0, 0, 0, 5/18, -2/9] , [1, -6, 31, 113/18, -290/9] , [0, 1, -6, -11/9, 113/18] , [0, 1, -6, -11/9, 113/18] , [0, 1, -6, -11/9, 113/18] , [0, 0, 1, 5/18, -11/9] , [0, 0, 1, 5/18, -11/9] , [0, 0, 0, -2/9, 5/18]] $ x $ [[5, 4, 0, 0, 1, 0, 6, 0, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC’D 25/256 , 0.09765625000

215 . Coloring, {4, 5, 6, 7, 9}

R: [4, 4, 4, 8, 3, 8, 5, 1, 2] **B**: [2, 9, 5, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘27‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘]‘

For τ=1/2, [2678, 824, 918, 3315, 1224, 765, 1326, 3060, 206] . FixedPtCheck, [2678, 824, 918, 3315, 1224, 765, 1326, 3060, 206]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}}, net cycles: -1 . order: 3

$ [[2, 1, 2, 6, 3, 0, 0, 4, 0] , [4, 0, 3, 5, 0, 0, 0, 6, 0] , [6, 0, 0, 7, 0, 0, 0, 5, 0] , [5, 0, 0, 6, 0, 0, 0, 7, 0] , [7, 0, 0, 5, 0, 0, 0, 6, 0] , [6, 0, 0, 7, 0, 0, 0, 5, 0]] $

$$[y_1, y_2, y_3, y_4, 3\,y_2, 0, 0, y_5, 0]$$

p = s ^ 3 - s ^ 6

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[4, 3, 0, 0, 1, 2, 6, 0, 2] , [8, 4, 0, 0, 0, 0, 3, 0, 3] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4]] $

$$[y_1, y_2, 0, 0, y_3, 2\,y_3, y_4, 0, y_5]$$

$p = - s^{3} + s^{6}$

Â» SYNC’D 4293/65536 , 0.06550598145

216 . Coloring, {4, 5, 6, 8, 9}

R: [4, 4, 4, 8, 3, 8, 1, 6, 2] **B**: [2, 9, 5, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-325, -100, -51, -357, -68, -459, -221, -612, -25] . FixedPtCheck, [325, 100, 51, 357, 68, 459, 221, 612, 25]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -2 . order: 4

$ [[3, 1, 2, 6, 0, 2, 0, 4, 0] , [0, 0, 0, 6, 0, 4, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] $

$$[3\,y_1, y_1, 2\,y_1, y_2, 0, y_3, 0, y_4, 0]$$

$p' = - s^{3} + s^{5}$ $p = - s^{3} + s^{5}$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 2 . order: 6

$ [[3, 3, 0, 0, 4, 0, 6, 0, 2] , [2, 3, 0, 0, 6, 0, 4, 0, 3] , [3, 2, 0, 0, 4, 0, 6, 0, 3] , [3, 3, 0, 0, 6, 0, 4, 0, 2] , [2, 3, 0, 0, 4, 0, 6, 0, 3]] $

$$[4\,y_1, 4\,y_2, 0, 0, 5\,y_1 + 5\,y_2 - 4\,y_3 + 5\,y_4, 0, 4\,y_3, 0, 4\,y_4]$$

$p = - s - s^{2} + s^{4} + s^{5}$

Â» SYNC’D 25/1024 , 0.02441406250

217 . Coloring, {4, 5, 7, 8, 9}

R: [4, 4, 4, 8, 3, 7, 5, 6, 2] **B**: [2, 9, 5, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 459, 663, 612, 459, 663, 612, 25] . FixedPtCheck, [325, 100, 459, 663, 612, 459, 663, 612, 25]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	7 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 6, 7, 8}}, net cycles: 0 . order: 6

$$[0, y_1, y_2, y_7, y_3, y_4, y_5, y_6, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696] , [0, 1853/6696, -1081/6696, 197/6696, 503/6696, -1027/6696, -73/6696] , [0, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696, -1081/6696] , [0, -1027/6696, -73/6696, 1853/6696, -1081/6696, 197/6696, 503/6696] , [0, 503/6696, -1027/6696, -73/6696, 1853/6696, -1081/6696, 197/6696] , [0, -73/6696, 1853/6696, -1081/6696, 197/6696, 503/6696, -1027/6696] , [1, 197/6696, 503/6696, -1027/6696, -73/6696, 1853/6696, -7777/6696]] $ x $ [[0, 1, 2, 6, 3, 2, 1, 3, 0] , [0, 0, 3, 3, 1, 3, 2, 6, 0] , [0, 0, 1, 3, 2, 6, 3, 3, 0] , [0, 0, 2, 1, 3, 3, 6, 3, 0] , [0, 0, 3, 2, 6, 3, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 1, 3, 0]] $

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 0, 1, 0, 5, 1, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 0, 0, y_4, 0, y_3, y_4, y_5]$$

p = - s ^{3} + s ^{6}

Â» SYNC'D 298755/4194304 , 0.07122874260

218 . Coloring, {4, 6, 7, 8, 9}

R: [4, 4, 4, 8, 7, 8, 5, 6, 2] **B**: [2, 9, 5, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{3} ‘]‘

For τ=1/2, [-286, -88, -102, -357, -408, -459, -510, -612, -22] . FixedPtCheck, [286, 88, 102, 357, 408, 459, 510, 612, 22]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 6, 3, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0]] $

$$[0, -14\, y_1 + 39\, y_2 - 14\, y_3 - y_4, 0, y_1, y_2, y_3, -5\, y_1 + 14\, y_2 - 5\, y_3, y_4, 0]$$

p = s ^{3} - s ^{5} p' = s ^{3} - s ^{5}

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[6, 3, 2, 0, 1, 0, 4, 0, 2] , [6, 6, 1, 0, 2, 0, 0, 0, 3] , [3, 6, 2, 0, 1, 0, 0, 0, 6] , [6, 3, 1, 0, 2, 0, 0, 0, 6] , [6, 6, 2, 0, 1, 0, 0, 0, 3] , [3, 6, 1, 0, 2, 0, 0, 0, 6]] $

$$[-y_1 + 5\,y_2 + 5\,y_5 - y_3 - y_4, y_1, y_2, 0, y_5, 0, y_3, 0, y_4]$$

$p = - s^2 - s^3 + s^5 + s^6$

Â» SYNC'D 3125/131072 , 0.02384185791

219 . Coloring, {5, 6, 7, 8, 9}

R: [4, 4, 4, 7, 3, 8, 5, 6, 2] **B**: [2, 9, 5, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 4 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [767, 236, 1377, 1785, 1836, 765, 1989, 1020, 59] . FixedPtCheck, [767, 236, 1377, 1785, 1836, 765, 1989, 1020, 59]

$\det(A + \tau\,\Delta) = 0$

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 4, 5, 7}}, net cycles: 1 . order: 4

$ [[0, 1, 2, 6, 3, 2, 3, 1, 0] , [0, 0, 3, 3, 3, 1, 6, 2, 0] , [0, 0, 3, 3, 6, 2, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 3, 1, 0] , [0, 0, 3, 3, 3, 1, 6, 2, 0] , [0, 0, 3, 3, 6, 2, 3, 1, 0]] $

$$[0, -y_1 + y_5 + 4\,y_4 - y_3, y_1, -y_2 + 4\,y_5 + y_4, y_2, y_5, y_3, y_4, 0]$$

$p' = - s^2 + s^6$ $p = - s^2 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 0, 1, 0, 3, 3, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 3\,y_3, y_5]$$

$p = s^{3} - s^{6}$

Â» SYNC'D 4347/65536 , 0.06632995605

220 . Coloring, {2, 3, 4, 5, 6, 7}

R: [4, 9, 5, 8, 3, 8, 5, 1, 1] **B**: [2, 4, 4, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ² ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ² , 9‘ (‘ 3 + τ ² ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [13, 4, 9, 13, 12, 3, 7, 12, 3] . FixedPtCheck, [13, 4, 9, 13, 12, 3, 7, 12, 3]

$\det(A + \tau\,\Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 0, 2, 3, 4, 0, 0, 4, 2] , [12, 3, 4, 7, 2, 0, 5, 3, 0] , [10, 4, 2, 17, 9, 5, 15, 7, 3] , [19, 19, 9, 28, 17, 9, 17, 22, 4] , [57, 41, 17, 39, 26, 10, 42, 37, 19] , [110, 52, 26, 95, 59, 27, 117, 49, 41] , [165, 105, 59, 224, 143, 79, 203, 122, 52]] $ $ [[3, 4, 0, 3, 0, 2, 6, 0, 0] , [0, 5, 0, 5, 6, 4, 7, 5, 4] , [14, 12, 6, 7, 7, 3, 9, 9, 5] , [29, 13, 7, 20, 15, 7, 31, 10, 12] , [39, 23, 15, 57, 38, 22, 54, 27, 13] , [82, 76, 38, 97, 69, 37, 75, 79, 23] , [219, 151, 69, 160, 113, 49, 181, 134, 76]] $ $ [[0, -2, 1, 0, 2, -1, -3, 2, 1] , [6, -1, 2, 1, -2, -2, -1, -1, -2] , [-2, -4, -2, 5, 1, 1, 3, -1, -1] , [-5, 3, 1, 4, 1, 1, -7, 6, -4] , [9, 9, 1, -9, -6, -6, -6, 5, 3] , [14, -12, -6, -1, -5, -5, 21, -15, 9] , [-27, -23, -5, 32, 15, 15, 11, -6, -12]] $

$$[y_2 - 3\,y_1 + y_5 - 2\,y_6 - y_3, -2\,y_2 + 2\,y_1 - y_5 + y_6, -y_5 - y_4, y_3, y_2, y_1, y_5, y_4, y_6]$$

$p = s^{3} + s^{4} - 4s^{5} - 8s^{7}$

S+ \ ; S- \ ; NM

$ [[18, 14, 5, 20, 11, 6, 18, 13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[18, 14, 5, 20, 11, 6, 18,

13, 7] , [22, 10, 2, 21, 15, 8, 13, 11, 10] , [22, 6, 3, 22, 23, 7, 12, 9, 8] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [13, 15, 14, 15, 7, 4, 28, 14, 2] , [16, 15, 9, 16, 8, 5, 24, 15, 4] , [22, 9, 4, 20, 19, 7, 14, 10, 7] , [21, 11, 4, 20, 14, 8, 15, 11, 8] , [18, 17, 6, 18, 7, 6, 20, 14, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 1 . order: 6

$ [[3, 0, 2, 3, 4, 0, 0, 4, 2] , [6, 0, 4, 3, 2, 0, 0, 3, 0] , [3, 0, 2, 6, 4, 0, 0, 3, 0] , [3, 0, 4, 3, 2, 0, 0, 6, 0] , [6, 0, 2, 3, 4, 0, 0, 3, 0] , [3, 0, 4, 6, 2, 0, 0, 3, 0]] $

$$[2\,y_3 - y_4 + 2\,y_5 - y_2 - y_1, 0, y_3, y_4, y_5, 0, 0, y_2, y_1]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, 0, y_4, y_5, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, -17/72, 19/72, 1/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, 1/72, -17/72, 19/72] , [0, 1/72, 1/72, -17/72, 19/72] , [0, 1/72, 1/72, -17/72, 19/72] , [0, 19/72, 1/72, 1/72, -17/72] , [1/2, 1/72, -17/72, 19/72, -35/72] , [0, -17/72, 19/72, 1/72, 1/72]] $ x $ [[3, 4, 0, 3, 0, 2, 6, 0, 0] , [6, 3, 0, 4, 0, 0, 5, 0, 0] , [5, 6, 0, 3, 0, 0, 4, 0, 0] , [4, 5, 0, 6, 0, 0, 3, 0, 0] , [3, 4, 0, 5, 0, 0, 6, 0, 0]] $

Â» SYNC’D 509/8192 , 0.06213378906

221 . Coloring, {2, 3, 4, 5, 6, 8}

R: [4, 9, 5, 8, 3, 8, 1, 6, 1] **B**: [2, 4, 4, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-27‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 54‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , -9‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 27‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 3τ ‘)‘‘]‘

For τ=1/2, [1014, 312, 258, 903, 344, 1161, 602, 1548, 234] . FixedPtCheck, [1014, 312, 258, 903, 344, 1161, 602, 1548, 234]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[4, 0, 2, 3, 1, 2, 0, 4, 2] , [2, 0, 1, 4, 2, 4, 0, 5, 0] , [0, 0, 2, 2, 1, 5, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0]] $

$$[3\,y_1 + 2\,y_2 - y_4, 0, y_1, 2\,y_1 + 3\,y_2 - y_3 - y_5, y_2, y_3, 0, y_4, y_5]$$

p = s ^{4} - s ^{6} p’ = s ^{4} - s ^{6}

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/2, -1, -13/18, 23/18] , [0, 0, 1/2, 5/18, -13/18] , [0, 0, 1/2, 5/18, -13/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, -2/9, 5/18] , [0, 0, 0, 5/18, -2/9] , [1/2, -1, 5/4, 23/18, -71/36] , [0, 1/2, -1, -13/18, 23/18]] $ x $ [[2, 4, 0, 3, 3, 0, 6, 0, 0] , [0, 2, 0, 4, 6, 0, 6, 0, 0] , [0, 0, 0, 2, 6, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] $

Â» SYNC’D 579/32768 , 0.01766967773

222 . Coloring, {2, 3, 4, 5, 6, 9}

R: [4, 9, 5, 8, 3, 8, 1, 1, 2] **B**: [2, 4, 4, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [42, 24, 6, 39, 8, 9, 14, 36, 18] . FixedPtCheck, [42, 24, 6, 39, 8, 9, 14, 36, 18]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}, {2, 9}}, net cycles: 3 . order: 6

$ [[5, 1, 2, 3, 1, 0, 0, 4, 2] , [4, 2, 1, 5, 2, 0, 0, 3, 1] , [3, 1, 2, 4, 1, 0, 0, 5, 2] , [5, 2, 1, 3, 2, 0, 0, 4, 1] , [4, 1, 2, 5, 1, 0, 0, 3, 2] , [3, 2, 1, 4, 2, 0, 0, 5, 1] , [5, 1, 2, 3, 1, 0, 0, 4, 2]] $

$$[4\,y_2 + 4\,y_4 - y_1 - y_3, y_2, y_4, y_1, y_2, 0, 0, y_3, y_4]$$

p = s - s ^{3} - s ^{4} + s ^{6} p = - s + s ^{7} p = - s - s ^{2} + s ^{4} + s ^{5}

Omega Rank for B : cycles: {{5, 7}}, net cycles: -1 . order: 4

$ [[1, 3, 0, 3, 3, 2, 6, 0, 0] , [0, 1, 0, 3, 6, 0, 8, 0, 0] , [0, 0, 0, 1, 8, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1, 3\,y_1 + y_2 + y_3 - y_4, 0, y_2, y_3, 2\,y_1, y_4, 0, 0]$$

p = - s ^{4} + s ^{5} p = - s ^{4} + s ^{6}

Â» SYNC’D 28791/2097152 , 0.01372861862

223 . Coloring, {2, 3, 4, 5, 7, 8}

R: [4, 9, 5, 8, 3, 7, 5, 6, 1] **B**: [2, 4, 4, 7, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [-767, -236, -1935, -1118, -2580, -774, -1505, -1032, -177] . FixedPtCheck, [767, 236, 1935, 1118, 2580, 774, 1505, 1032, 177]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	8 *vs* 8	5 *vs* 5

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 8

$$[y_1, 0, y_2, y_3, y_4, y_5, y_6, y_7, y_8]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/2, -1/4, -5/8, -1/16, 5/18, 31/144] , [1/2, -1/4, -5/8, -1/16, 27/32, 39/64, -163/288, -227/576] , [0, 0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 1/2, -1/4, -5/8, 11/72, 5/18] , [0, 0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 0, 0, 0, 1/2, -7/72, -25/72] , [0, 0, 0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 1/2, -1/4, -25/72, 11/72] , [0, 1/2, -1/4, -5/8, -1/16, 27/32, 31/144, -163/288]] $ x $ [[1, 0, 2, 3, 4, 2, 1, 3, 2] , [2, 0, 4, 1, 3, 3, 2, 3, 0] , [0, 0, 3, 2, 6, 3, 3, 1, 0] , [0, 0, 6, 0, 6, 1, 3, 2, 0] , [0, 0, 6, 0, 9, 2, 1, 0, 0] , [0, 0, 9, 0, 7, 0, 2, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, 0, 0, y_4, y_5, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 19/72, -17/72, 1/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, -17/72, 1/72, 1/72, 19/72] , [0, -17/72, 1/72, 1/72, 19/72] , [1, -17/72, 1/72, 1/72, -53/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 1/72, 19/72, -17/72, 1/72]] $ x $ [[5, 4, 0, 3, 0, 0, 5, 1, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0] , [5, 4, 0, 3, 0, 0, 6, 0, 0]] $

Â» SYNC’D 45091/1048576 , 0.04300212860

224 . Coloring, {2, 3, 4, 5, 7, 9}

R: [4, 9, 5, 8, 3, 7, 5, 1, 2] **B**: [2, 4, 4, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘ , 18‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 3τ + τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [217, 124, 225, 250, 300, 50, 175, 200, 93] . FixedPtCheck, [217, 124, 225, 250, 300, 50, 175, 200, 93]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ τ ‘)‘ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 8	4 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {2, 9}, {1, 4, 8}}, net cycles: 2 . order: 6

$ [[2, 1, 2, 3, 4, 0, 1, 3, 2] , [3, 2, 4, 2, 3, 0, 0, 3, 1] , [3, 1, 3, 3, 4, 0, 0, 2, 2] , [2, 2, 4, 3, 3, 0, 0, 3, 1] , [3, 1, 3, 2, 4, 0, 0, 3, 2] , [3, 2, 4, 3, 3, 0, 0, 2, 1] , [2, 1, 3, 3, 4, 0, 0, 3, 2] , [3, 2, 4, 2, 3, 0, 0, 3, 1]] $

$$[2\,y_5, 3\,y_3 - 5\,y_1, 2\,y_4, 2\,y_2, 2\,y_3, 0, -2\,y_4 + 5\,y_3 - 7\,y_1, -2\,y_5 - 2\,y_2 + 8\,y_3 - 8\,y_1, 2\,y_1]$$

p = s 2 - s 4 - s 5 + s 7 p = - s 2 + s 8 p = - s 2 - s 3 + s 5 + s 6

Omega Rank for B : cycles: {{6, 8}, {1, 2, 4, 7}}, net cycles: 2 . order: 4

$ [[4, 3, 0, 3, 0, 2, 5, 1, 0] , [5, 4, 0, 3, 0, 1, 3, 2, 0] , [3, 5, 0, 4, 0, 2, 3, 1, 0] , [3, 3, 0, 5, 0, 1, 4, 2, 0] , [4, 3, 0, 3, 0, 2, 5, 1, 0] , [5, 4, 0, 3, 0, 1, 3, 2, 0]] $

$$[-y_3 + 2\,y_1 + 3\,y_2, y_4, 0, y_3, 0, y_1, -y_4 + 3\,y_1 + 2\,y_2, y_2, 0]$$

p’ = - s + s 5 p = - s + s 5

Â» SYNC’D 84291/16777216 , 0.005024135113

225 . Coloring, {2, 3, 4, 5, 8, 9}

R: [4, 9, 5, 8, 3, 7, 1, 6, 2] **B**: [2, 4, 4, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + τ + τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [987, 564, 375, 975, 500, 675, 875, 900, 423] . FixedPtCheck, [987, 564, 375, 975, 500, 675, 875, 900, 423]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	8 *vs* 9	6 *vs* 9	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}, {3, 5}, {2, 9}}, net cycles: 3 .

$ [[3, 1, 2, 3, 1, 2, 1, 3, 2] , [1, 2, 1, 3, 2, 3, 2, 3, 1] , [2, 1, 2, 1, 1, 3, 3, 3, 2] , [3, 2, 1, 2, 2, 3, 3, 1, 1] , [3, 1, 2, 3, 1, 1, 3, 2, 2] , [3, 2, 1, 3, 2, 2, 1, 3, 1] , [1, 1, 2, 3, 1, 3, 2, 3, 2] , [2, 2, 1, 1, 2, 3, 3, 3, 1] , [3, 1, 2, 2, 1, 3, 3, 1, 2]] $

$$[4 y_2 + 4 y_5 - y_1 - y_3 - y_6 - y_4, y_2, y_5, y_1, y_2, y_3, y_6, y_4, y_5]$$

p’ = - 1 + s 2 + s 5 - s 7 p’ = - s - s 2 + s 6 + s 7 p’ = - s 2 - s 3 + s 7 + s 8

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 6

$ [[3, 3, 0, 3, 3, 0, 5, 1, 0] , [1, 3, 0, 3, 5, 0, 6, 0, 0] , [0, 1, 0, 3, 6, 0, 8, 0, 0] , [0, 0, 0, 1, 8, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, y_3, 0, y_4, y_5, 0]$$

p = - s 5 + s 6

Â» SYNC’D 399/32768 , 0.01217651367

226 . Coloring, {2, 3, 4, 6, 7, 8}

R: [4, 9, 5, 8, 7, 8, 5, 6, 1] **B**: [2, 4, 4, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘‘]‘

For τ=1/2, [325, 100, 129, 301, 516, 387, 559, 516, 75] . FixedPtCheck, [325, 100, 129, 301, 516, 387, 559, 516, 75]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[1, 0, 0, 3, 4, 2, 2, 4, 2] , [6, 5, 0, 7, 2, 4, 7, 5, 0] , [8, 10, 6, 13, 7, 5, 7, 11, 5] , [27, 19, 9, 16, 13, 11, 21, 18, 10] , [51, 27, 19, 47, 30, 18, 50, 27, 19] , [102, 58, 34, 101, 69, 27, 93, 65, 27] , [189, 127, 59, 202, 127, 65, 197, 128, 58]] $ $ [[5, 4, 2, 3, 0, 0, 4, 0, 0] , [6, 3, 4, 5, 6, 0, 5, 3, 4] , [16, 6, 2, 11, 9, 3, 17, 5, 3] , [21, 13, 7, 32, 19, 5, 27, 14, 6] , [45, 37, 13, 49, 34, 14, 46, 37, 13] , [90, 70, 30, 91, 59, 37, 99, 63, 37] , [195, 129, 69, 182, 129, 63, 187, 128, 70]] $ $ [[-2, -2, -1, 0, 2, 1, -1, 2, 1] , [0, 1, -2, 1, -2, 2, 1, 1, -2] , [-4, 2, 2, 1, -1, 1, -5, 3, 1] , [3, 3, 1, -8, -3, 3, -3, 2, 2] , [3, -5, 3, -1, -2, 2, 2, -5, 3] , [6, -6, 2, 5, 5, -5, -3, 1, -5] , [-3, -1, -5, 10, -1, 1, 5, 0, -6]] $

$[-y_4 - 2\,y_6 + y_2 + y_3 - 2\,y_5 - y_1, y_4 + 2\,y_6 - 2\,y_2 - 2\,y_3 + y_5, -y_4 - y_6, y_1, y_2, y_3, y_4, y_6, y_5]$

p = s 2 - 2s 4 - 16s 7

S+ \ ; S- \ ; NM

$ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[11, 10, 4, 15, 7, 4, 12, 8, 5] , [13, 8, 2, 14, 9, 4, 11, 9, 6] , [12, 6, 4, 15, 12, 4, 11, 7, 5] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [11, 9, 7, 11, 8, 3, 16, 9, 2] , [12, 8, 5, 11, 7, 4, 15, 10, 4] , [15, 7, 4, 12, 11, 5, 11, 7, 4] , [14, 9, 3, 13, 9, 5, 11, 8, 4] , [14, 11, 4, 12, 6, 5, 12, 8, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] ,

[0, 0, 0, 0, 0, 0, 0, 0, 0]] $
CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 7	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[1, 0, 0, 3, 4, 2, 2, 4, 2] , [2, 0, 0, 1, 2, 4, 4, 5, 0] , [0, 0, 0, 2, 4, 5, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0]] $

$$[y_1, 0, 0, 3\,y_1 - y_2 - 4\,y_3 + 3\,y_4 - y_5, 2\,y_1 - 3\,y_3 + 2\,y_4, y_2, y_3, y_4, y_5]$$

$p = s^4 - s^6$ $p' = s^4 - s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, 0, y_5, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 1/72, -17/72, 19/72] , [0, 19/72, 1/72, 1/72, -17/72] , [0, 19/72, 1/72, 1/72, -17/72] , [0, -17/72, 19/72, 1/72, 1/72] , [1/2, 1/72, 1/72, -17/72, -17/72] , [0, -17/72, 19/72, 1/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, -17/72, 19/72, 1/72] , [0, 1/72, 1/72, -17/72, 19/72]] $ x $ [[5, 4, 2, 3, 0, 0, 4, 0, 0] , [4, 5, 0, 6, 0, 0, 3, 0, 0] , [3, 4, 0, 5, 0, 0, 6, 0, 0] , [6, 3, 0, 4, 0, 0, 5, 0, 0] , [5, 6, 0, 3, 0, 0, 4, 0, 0]] $

Â» SYNC’D 111/4096 , 0.02709960938

227 . Coloring, {2, 3, 4, 6, 7, 9}

R: [4, 9, 5, 8, 7, 8, 5, 1, 2] **B**: [2, 4, 4, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 5 - τ - 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-343, -196, -75, -325, -300, -75, -325, -300, -147] . FixedPtCheck, [343, 196, 75, 325, 300, 75, 325, 300, 147]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {5, 7}, {2, 9}}, net cycles: 3 . order: 6

$ [[2, 1, 0, 3, 4, 0, 2, 4, 2] , [4, 2, 0, 2, 2, 0, 4, 3, 1] , [3, 1, 0, 4, 4, 0, 2, 2, 2] , [2, 2, 0, 3, 2, 0, 4, 4, 1] , [4, 1, 0, 2, 4, 0, 2, 3, 2] , [3, 2, 0, 4, 2, 0, 4, 2, 1] , [2, 1, 0, 3, 4, 0, 2, 4, 2]] $

$$[3\,y_1 - y_2 - y_3 + 3\,y_4, y_1, 0, y_2, 2\,y_4, 0, 2\,y_1, y_3, y_4]$$

$p = -s - s^2 + s^4 + s^5$ $p = s - s^3 - s^4 + s^6$ $p = -s + s^7$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[4, 3, 2, 3, 0, 2, 4, 0, 0] , [4, 4, 0, 5, 0, 0, 5, 0, 0] , [5, 4, 0, 4, 0, 0, 5, 0, 0] , [5, 5, 0, 4, 0, 0, 4, 0, 0] , [4, 5, 0, 5, 0, 0, 4, 0, 0] , [4, 4, 0, 5, 0, 0, 5, 0, 0]] $

$$[y_1, y_2, y_3, y_4, 0, y_3, y_1 - y_2 + y_4, 0, 0]$$

$p = -s^2 + s^6$ $p = -s^2 + s^3 - s^4 + s^5$

Â» SYNC’D 16725/2097152 , 0.007975101471

228 . Coloring, {2, 3, 4, 6, 8, 9}

R: [4, 9, 5, 8, 7, 8, 1, 6, 2] **B**: [2, 4, 4, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 54‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 27‘ (‘ 5 + 3τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [-966, -552, -50, -875, -200, -1125, -650, -1500, -414] . FixedPtCheck, [966, 552, 50, 875, 200, 1125, 650, 1500, 414]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 8	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[3, 1, 0, 3, 1, 2, 2, 4, 2] , [2, 2, 0, 3, 0, 4, 1, 5, 1] , [1, 1, 0, 2, 0, 5, 0, 7, 2] , [0, 2, 0, 1, 0, 7, 0, 7, 1] , [0, 1, 0, 0, 0, 7, 0, 8, 2] , [0, 2, 0, 0, 0, 8, 0, 7, 1] , [0, 1, 0, 0, 0, 7, 0, 8, 2] , [0, 2, 0, 0, 0, 8, 0, 7, 1]] $

$$[2 y_1 - y_2 - y_5 + 3 y_6, y_1, 0, 3 y_1 - y_3 - y_4 + 2 y_6, y_2, y_3, y_4, y_5, y_6]$$

p’ = - s ^{5} + s ^{7} p = - s ^{5} + s ^{7}

Omega Rank for B : cycles: {{3, 4, 5, 7}}, net cycles: 0 . order: 4

$ [[3, 3, 2, 3, 3, 0, 4, 0, 0] , [0, 3, 3, 5, 4, 0, 3, 0, 0] , [0, 0, 4, 6, 3, 0, 5, 0, 0] , [0, 0, 3, 4, 5, 0, 6, 0, 0] , [0, 0, 5, 3, 6, 0, 4, 0, 0] , [0, 0, 6, 5, 4, 0, 3, 0, 0]] $

$$[y_1 + y_2 - y_3 - y_4 + y_5, y_1, y_2, y_3, y_4, 0, y_5, 0, 0]$$

p = - s ^{3} + s ^{4} - s ^{5} + s ^{6}

Â» SYNC’D 69969/4194304 , 0.01668190956

229 . Coloring, {2, 3, 4, 7, 8, 9}

R: [4, 9, 5, 8, 7, 7, 5, 6, 2] **B**: [2, 4, 4, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-623, -356, -375, -650, -1500, -450, -1625, -600, -267] . FixedPtCheck, [623, 356, 375, 650, 1500, 450, 1625, 600, 267]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{5, 7}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 3, 4, 2, 3, 3, 2] , [0, 2, 0, 0, 3, 3, 6, 3, 1] , [0, 1, 0, 0, 6, 3, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2] , [0, 2, 0, 0, 6, 0, 9, 0, 1] , [0, 1, 0, 0, 9, 0, 6, 0, 2]] $

$$[0, y_1, 0, y_5, -15\, y_1 - y_5 - y_4 + 4\, y_2 + 4\, y_3, y_4, y_2, y_3, -4\, y_1 + y_2 + y_3]$$

p’ = - s 4 + s 6 p = s 4 - s 6

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[6, 3, 2, 3, 0, 0, 3, 1, 0] , [4, 6, 0, 5, 0, 0, 3, 0, 0] , [3, 4, 0, 6, 0, 0, 5, 0, 0] , [5, 3, 0, 4, 0, 0, 6, 0, 0] , [6, 5, 0, 3, 0, 0, 4, 0, 0] , [4, 6, 0, 5, 0, 0, 3, 0, 0]] $

$$[y_1 - y_2 + y_3 + 3\, y_4, y_1, 2\, y_4, y_2, 0, 0, y_3, y_4, 0]$$

p = s 2 - s 6 p’ = - s 2 + s 3 - s 4 + s 5

Â» SYNC’D 16285/524288 , 0.03106117249

230 . Coloring, {2, 3, 5, 6, 7, 8}

R: [4, 9, 5, 7, 3, 8, 5, 6, 1] **B**: [2, 4, 4, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 5 + 4τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 5 + 4τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 , -9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 5 + 4τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-377, -116, -1161, -602, -1548, -258, -903, -344, -87] . FixedPtCheck, [377, 116, 1161, 602, 1548, 258, 903, 344, 87]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	6 *vs* 8	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 6

$ [[1, 0, 2, 3, 4, 2, 3, 1, 2] , [2, 0, 4, 1, 5, 1, 3, 2, 0] , [0, 0, 5, 2, 7, 2, 1, 1, 0] , [0, 0, 7, 0, 6, 1, 2, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0]] $

$$[-y_1 + y_3 - y_6 + 4 y_4, 0, y_1, -y_2 + 4 y_3 + y_4 - y_5, y_2, y_3, y_6, y_4, y_5]$$

p = s ^{5} - s ^{7} p’ = - s ^{5} + s ^{7}

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$$[y_2, y_1, 0, y_3, 0, 0, y_4, y_5, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 19/72, -17/72, 1/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, 1/72, 1/72, 19/72, -17/72] , [0, -17/72, 1/72, 1/72, 19/72] , [1/3, -17/72, 1/72, 1/72, -5/72] , [1/3, -17/72, 1/72, 1/72, -5/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 1/72, 19/72, -17/72, 1/72]] $ x $ [[5, 4, 0, 3, 0, 0, 3, 3, 0] , [6, 5, 0, 4, 0, 0, 0, 3, 0] , [3, 6, 0, 5, 0, 0, 0, 4, 0] , [4, 3, 0, 6, 0, 0, 0, 5, 0] , [5, 4, 0, 3, 0, 0, 0, 6, 0]] $

Â» SYNC’D 18381/1048576 , 0.01752948761

231 . Coloring, {2, 3, 5, 6, 7, 9}

R: [4, 9, 5, 7, 3, 8, 5, 1, 2] **B**: [2, 4, 4, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘

(‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [413, 236, 1125, 650, 1500, 50, 875, 200, 177] . FixedPtCheck, [413, 236, 1125, 650, 1500, 50, 875, 200, 177]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 8	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 6

$ [[2, 1, 2, 3, 4, 0, 3, 1, 2] , [1, 2, 4, 2, 5, 0, 3, 0, 1] , [0, 1, 5, 1, 7, 0, 2, 0, 2] , [0, 2, 7, 0, 7, 0, 1, 0, 1] , [0, 1, 7, 0, 8, 0, 0, 0, 2] , [0, 2, 8, 0, 7, 0, 0, 0, 1] , [0, 1, 7, 0, 8, 0, 0, 0, 2] , [0, 2, 8, 0, 7, 0, 0, 0, 1]] $

$$[3\,y_1 - y_2 - y_4 + 2\,y_6, y_1, y_2, 2\,y_1 - y_3 - y_5 + 3\,y_6, y_3, 0, y_4, y_5, y_6]$$

p’ = s ^ 5 - s ^ 7 p = s ^ 5 - s ^ 7

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}}, net cycles: 1 . order: 6

$ [[4, 3, 0, 3, 0, 2, 3, 3, 0] , [3, 4, 0, 3, 0, 3, 2, 3, 0] , [2, 3, 0, 4, 0, 3, 3, 3, 0] , [3, 2, 0, 3, 0, 3, 3, 4, 0] , [3, 3, 0, 2, 0, 4, 3, 3, 0] , [3, 3, 0, 3, 0, 3, 4, 2, 0]] $

$$[y_5, y_4, 0, y_3, 0, y_2, y_1, y_5 - y_4 + y_3 + y_2 - y_1, 0]$$

p = - s + s ^ 2 - s ^ 3 + s ^ 4 - s ^ 5 + s ^ 6

Â» SYNC’D 365379/33554432 , 0.01088914275

232 . Coloring, {2, 3, 5, 6, 8, 9}

R: [4, 9, 5, 7, 3, 8, 1, 6, 2] **B**: [2, 4, 4, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘3‘ (‘ 3 + τ ‘)‘ , 6 , 3‘ (‘ 1 + τ ‘)‘ , 3‘ (‘ 3 + τ ‘)‘ , 6 , 3‘ (‘ 1 + τ ‘)‘ , 3‘ (‘ 3 + τ ‘)‘ , 6 , 3‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [7, 4, 3, 7, 4, 3, 7, 4, 3] . FixedPtCheck, [7, 4, 3, 7, 4, 3, 7, 4, 3]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[3, 1, 2, 3, 1, 2, 3, 1, 2] , [6, 5, 1, 6, 5, 1, 6, 5, 1] , [12, 7, 5, 12, 7, 5, 12, 7, 5] , [24, 17, 7, 24, 17, 7, 24, 17, 7] , [48, 31, 17, 48, 31, 17, 48, 31, 17] , [96, 65, 31, 96, 65, 31, 96, 65, 31] , [192, 127, 65, 192, 127, 65, 192, 127, 65]] $ $ [[3, 3, 0, 3, 3, 0, 3, 3, 0] , [6, 3, 3, 6, 3, 3, 6, 3, 3] , [12, 9, 3, 12, 9, 3, 12, 9, 3] , [24, 15, 9, 24, 15, 9, 24, 15, 9] , [48, 33, 15, 48, 33, 15, 48, 33, 15] , [96, 63, 33, 96, 63, 33, 96, 63, 33] , [192, 129, 63, 192, 129, 63, 192, 129, 63]] $ $ [[0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, 1, -1, 0, 1, -1, 0, 1, -1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, 1, -1, 0, 1, -1, 0, 1, -1] , [0, -1, 1, 0, -1, 1, 0, -1, 1] , [0, 1, -1, 0, 1, -1, 0, 1, -1] , [0, -1, 1, 0, -1, 1, 0, -1, 1]] $

$[0, -y_1, y_1, 0, -y_1, y_1, 0, -y_1, y_1]$

p = s - 64s 7

S+ \ ; S- \ ; NM

$ [[0, 17, 7, 1, 13, 7, 0, 13, 7] , [23, 0, 0, 25, 0, 0, 16, 0, 1] , [24, 0, 0, 27, 1, 0, 13, 0, 0] , [0, 16, 10, 0, 9, 5, 1, 18, 6] , [16, 0, 1, 17, 0, 0, 31, 0, 0] , [17, 0, 0, 19, 0, 0, 28, 1, 0] , [1, 10, 4, 0, 21, 9, 0, 12, 8] , [25, 0, 0, 22, 0, 1, 17, 0, 0] , [23, 1, 0, 18, 0, 0, 23, 0, 0]] $ $ [[20, 1, 0, 24, 0, 0, 20, 0, 0] , [0, 11, 3, 1, 18, 8, 0, 13, 11] , [0, 9, 3, 1, 24, 8, 0, 10, 10] , [17, 0, 0, 19, 0, 0, 28, 1, 0] , [0, 18, 14, 0, 7, 5, 1, 17, 3] , [0, 16, 10, 0, 9, 5, 1, 18, 6] , [27, 0, 0, 21, 1, 0, 16, 0, 0] , [1, 13, 5, 0, 17, 9, 0, 12, 8] , [1, 18, 8, 0, 10, 8, 0, 15, 5]] $ $ [[198, 108, 54, 153, 108, 54, 153, 108, 54] , [162, 132, 61, 162, 102, 56, 162, 102, 51] , [162, 122, 66, 162, 102, 51, 162, 112, 51] , [153, 108, 54, 198, 108, 54, 153, 108, 54] , [162, 102, 51, 162, 132, 61, 162, 102, 56] , [162, 112, 51, 162, 122, 66, 162, 102, 51] , [153, 108, 54, 153, 108, 54, 198, 108, 54] , [162, 102, 56, 162, 102, 51, 162, 132, 61] , [162, 102, 51, 162, 112, 51, 162, 122, 66]] $

CmmCk *true, true, true*

p’ = s + 32s 6 p’ = s 2 - 16s 6 p’ = s 3 + 8s 6 p’ = s 4 - 4s 6 p’ = s 5 + 2s 6

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
1 *vs* 7	2 *vs* 9	2 *vs* 9	2 *vs* 9	1 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}, {3, 5}, {2, 9}}, net cycles: 4 . order: 6

$ [[3, 1, 2, 3, 1, 2, 3, 1, 2] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [3, 1, 2, 3, 1, 2, 3, 1, 2] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [3, 1, 2, 3, 1, 2, 3, 1, 2] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [3, 1, 2, 3, 1, 2, 3, 1, 2] , [3, 2, 1, 3, 2, 1, 3, 2, 1] , [3, 1, 2, 3, 1, 2, 3, 1, 2]] $

$[y_1 + y_2, y_1, y_2, y_1 + y_2, y_1, y_2, y_1 + y_2, y_1, y_2]$

$p' = -s^2 + s^8$ $p' = -s + s^7$ $p' = -s^2 + s^6$ $p' = -s + s^5$ $p' = -s^2 + s^4$ $p' = -s + s^3$ $p' = 1 - s^2$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}}, net cycles: 2 . order: 4

$$\$ [[3, 3, 0, 3, 3, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 3, 3, 0] , [3, 3, 0, 3, 3, 0, 3, 3, 0]] \$$$

$$[y_1, y_1, 0, y_1, y_1, 0, y_1, y_1, 0]$$

$p = -s + s^3$ $p = -s + s^4$ $p = -s + s^5$ $p = -s + s^6$ $p = -s + s^2$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[x_1, x_2, x_3, x_4, x_5, x_6, -64\,x_1 + 32\,x_2 - 16\,x_3 + 8\,x_4 - 4\,x_5 + 2\,x_6]$
For A+2Δ : $[-y_1 - y_2 - y_4 - y_5 - y_7, y_1 + y_4 + y_7 - y_3 - y_6, y_1, y_2, y_3, y_4, y_5, y_6, y_7]$
For A-2Δ : $[-y_1 - y_2 - y_3 - y_5 - y_6, y_1, y_1 + y_3 + y_6 - y_4 - y_7, y_2, y_3, y_4, y_5, y_6, y_7]$

Range of $\{\Omega\Delta^i\}$: $[0, -\mu_1, \mu_1, 0, -\mu_1, \mu_1, 0, -\mu_1, \mu_1]$

rank of M is 9 , rank of N is 8

M \; N

$$\$ [[0, 8, 4, 15, 8, 4, 15, 8, 4] , [8, 0, 0, 8, 10, 0, 8, 10, 0] , [4, 0, 0, 4, 0, 5, 4, 0, 5] , [15, 8, 4, 0, 8, 4, 15, 8, 4] , [8, 10, 0, 8, 0, 0, 8, 10, 0] , [4, 0, 5, 4, 0, 0, 4, 0, 5] , [15, 8, 4, 15, 8, 4, 0, 8, 4] , [8, 10, 0, 8, 10, 0, 8, 0, 0] , [4, 0, 5, 4, 0, 5, 4, 0, 0]] \$ \quad \$ [[0, 3, 3, 3, 3, 3, 3, 3, 3] , [3, 0, 1, 3, 3, 2, 3, 3, 3] , [3, 1, 0, 3, 3, 3, 3, 2, 3] , [3, 3, 3, 0, 3, 3, 3, 3, 3] , [3, 3, 3, 3, 0, 1, 3, 3, 2] , [3, 2, 3, 3, 1, 0, 3, 3, 3] , [3, 3, 3, 3, 3, 3, 0, 3, 3] , [3, 3, 2, 3, 3, 3, 3, 0, 1] , [3, 3, 3, 3, 2, 3, 3, 1, 0]] \$$$

Check is ΩΔN zero? *true*, $\pi\Delta = [0, -1, 1, 0, -1, 1, 0, -1, 1]$

ker M, [0, 0, 0, 0, 0, 0, 0, 0, 0]
Range M, $[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9]$

$\tau = 15$, r'= 5/6

Ranges

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[1], [1]]
β({1, 2, 4, 5, 7, 8}) = 2/3
β({1, 3, 4, 6, 7, 9}) = 1/3

ker N, $[0, -\mu_1, \mu_1, 0, -\mu_1, \mu_1, 0, -\mu_1, \mu_1]$
Range of N
$[y_1, y_7 - y_2 + y_3 - y_5 + y_6, y_7, y_8, y_2, y_3, y_4, y_5, y_6]$

Partitions

Action of R on partitions, [[2], [1]]
Action of B on partitions, [[1], [1]]

α([{1}, {8, 9}, {5, 6}, {4}, {7}, {2, 3}]) = 2/3
α([{1}, {5, 9}, {4}, {3, 8}, {7}, {2, 6}]) = 1/3
b1 = {1} ‘ , ‘ b2 = {8, 9} ‘ , ‘ b3 = {5, 6} ‘ , ‘ b4 = {5, 9} ‘ , ‘ b5 = {4} ‘ , ‘ b6 = {3, 8} ‘ , ‘ b7 = {7} ‘ , ‘ b8 = {2, 3} ‘ , ‘ b9 = {2, 6}

Action of R and B on the blocks of the partitions: $ [[0, 1, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 1, 0, 0, 0, 0, 0] , [1, 1, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ + $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $
[‘7‘, ‘9‘, ‘6‘, ‘8‘, ‘1‘, ‘3‘, ‘5‘, ‘4‘, ‘2‘], [‘2‘, ‘5‘, ‘7‘, ‘7‘, ‘8‘, ‘5‘, ‘3‘, ‘1‘, ‘1‘] with invariant measure [3, 2, 2, 1, 3, 1, 3, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-6 partition graph.

‘

Sandwich	
Coloring	{2, 3, 5, 6, 8, 9}
Rank	6
R,B	[4, 9, 5, 7, 3, 8, 1, 6, 2], [2, 4, 4, 8, 7, 7, 5, 1, 1]
π_2	[8, 4, 15, 8, 4, 15, 8, 4, 0, 8, 10, 0, 8, 10, 0, 4, 0, 5, 4, 0, 5, 8, 4, 15, 8, 4, 0, 8, 10, 0, 4, 0, 5, 8, 4, 0]
u_2	[3, 3, 3, 3, 3, 3, 3, 3, 1, 3, 3, 2, 3, 3, 3, 3, 3, 3, 3, 2, 3, 3, 3, 3, 3, 3, 1, 3, 3, 2, 3, 3, 3, 3, 3, 1] (dim 2)
wpp	[1, 2, 2, 1, 2, 2, 1, 2, 2]
π_6	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
u_6	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 2, 0, 2, 2, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 2, 2, 0, 2, 3, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

233 . Coloring, {2, 3, 5, 7, 8, 9}

R: [4, 9, 5, 7, 3, 7, 5, 6, 2] **B**: [2, 4, 4, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 18‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [7, 4, 27, 13, 36, 3, 21, 4, 3] . FixedPtCheck, [7, 4, 27, 13, 36, 3, 21, 4, 3]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	4 *vs* 5

Omega Rank for R : cycles: {{3, 5}, {2, 9}}, net cycles: 0 . order: 4

$ [[0, 1, 2, 3, 4, 2, 4, 0, 2] , [0, 2, 4, 0, 6, 0, 5, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2]] $

$$[0, 2\,y_1 + 5\,y_4 - 8\,y_2, 8\,y_1 + 20\,y_4 - 30\,y_2 - 2\,y_3, 3\,y_4, 2\,y_1, 2\,y_4, 2\,y_3, 0, 2\,y_2]$$

p = - s 3 + s 5 p’ = - s 3 + s 5 p = - s 3 + s 7

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 3, 0, 0, 2, 4, 0] , [6, 6, 0, 3, 0, 0, 0, 3, 0] , [3, 6, 0, 6, 0, 0, 0, 3, 0] , [3, 3, 0, 6, 0, 0, 0, 6, 0] , [6, 3, 0, 3, 0, 0, 0, 6, 0]] $

$$[y_3, y_4, 0, y_2, 0, 0, y_3 - y_4 + y_2 - y_1, y_1, 0]$$

p = - s 2 + s 3 - s 4 + s 5

Â» SYNC’D 481/16384 , 0.02935791016

234 . Coloring, {2, 3, 6, 7, 8, 9}

R: [4, 9, 5, 7, 7, 8, 5, 6, 2] **B**: [2, 4, 4, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [-329, -188, -225, -350, -900, -150, -975, -200, -141] . FixedPtCheck, [329, 188, 225, 350, 900, 150, 975, 200, 141]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^ 2 ‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
7 *vs* 7	9 *vs* 9	9 *vs* 9	3 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 1, 0, 3, 4, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2] , [0, 2, 0, 0, 5, 1, 7, 2, 1] , [0, 1, 0, 0, 7, 2, 5, 1, 2]] $

$$[0, y_2, 0, y_2 - y_1 + 3 y_3, y_1, y_3, 3 y_2 + y_3, y_2, y_3]$$

p = - s ^ 2 + s ^ 4 p’ = - s ^ 2 + s ^ 4 p = - s ^ 2 + s ^ 6 p’ = - s ^ 2 + s ^ 6

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: -1 . order: 4

$ [[6, 3, 2, 3, 0, 0, 1, 3, 0] , [4, 6, 0, 5, 0, 0, 0, 3, 0] , [3, 4, 0, 6, 0, 0, 0, 5, 0] , [5, 3, 0, 4, 0, 0, 0, 6, 0] , [6, 5, 0, 3, 0, 0, 0, 4, 0] , [4, 6, 0, 5, 0, 0, 0, 3, 0]] $

$$[y_1 - y_2 + 3 y_3 + y_4, y_1, 2 y_3, y_2, 0, 0, y_3, y_4, 0]$$

p = s ^ 2 - s ^ 3 + s ^ 4 - s ^ 5 p = - s ^ 2 + s ^ 6

Â» SYNC’D 29799/4194304 , 0.007104635239

235 . Coloring, {2, 4, 5, 6, 7, 8}

R: [4, 9, 4, 8, 3, 8, 5, 6, 1] **B**: [2, 4, 5, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [26, 8, 18, 35, 24, 45, 26, 60, 6] . FixedPtCheck, [26, 8, 18, 35, 24, 45, 26, 60, 6]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 7	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[1, 0, 2, 4, 3, 2, 0, 4, 2] , [2, 0, 3, 3, 0, 4, 0, 6, 0] , [0, 0, 0, 5, 0, 6, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0]] $

$$[y_1, 0, y_2, y_4, -9\,y_1 + 6\,y_2, y_3, 0, y_5, -6\,y_1 + 4\,y_2]$$

p’ = - s 4 + s 6 p = s 4 - s 6

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, 0, 0]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1/72, 19/72, -17/72, 1/72] , [0, 1/72, 1/72, 19/72, -17/72] , [1, 1/72, 1/72, 19/72, -89/72] , [0, -17/72, 1/72, 1/72, 19/72] , [0, -17/72, 1/72, 1/72, 19/72] , [0, -17/72, 1/72, 1/72, 19/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 19/72, -17/72, 1/72, 1/72] , [0, 1/72, 19/72, -17/72, 1/72]] $ x $ [[5, 4, 0, 2, 1, 0, 6, 0, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0] , [5, 4, 0, 3, 0, 0, 6, 0, 0]] $

Â» SYNC’D 365/8192 , 0.04455566406

236 . Coloring, {2, 4, 5, 6, 7, 9}

R: [4, 9, 4, 8, 3, 8, 5, 1, 2] **B**: [2, 4, 5, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 3 + τ ‘)‘ , 54‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 27‘ (‘5 - 2τ + 8τ ^ 2 + 2τ ^ 3 + 3τ ^ 4 ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [1442, 824, 450, 1625, 600, 375, 650, 1500, 618] . FixedPtCheck, [1442, 824, 450, 1625, 600, 375, 650, 1500, 618]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	6 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}}, net cycles: 1 . order: 6

$ [[2, 1, 2, 4, 3, 0, 0, 4, 2] , [4, 2, 3, 4, 0, 0, 0, 4, 1] , [4, 1, 0, 7, 0, 0, 0, 4, 2] , [4, 2, 0, 4, 0, 0, 0, 7, 1] , [7, 1, 0, 4, 0, 0, 0, 4, 2] , [4, 2, 0, 7, 0, 0, 0, 4, 1] , [4, 1, 0, 4, 0, 0, 0, 7, 2]] $

$$[y_3, y_4, y_5, y_6, y_2, 0, 0, -y_3 + 5\,y_4 - y_5 - y_6 - y_2 + 5\,y_1, y_1]$$

p = - s ^ 3 - s ^ 4 + s ^ 6 + s ^ 7

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[4, 3, 0, 2, 1, 2, 6, 0, 0] , [6, 4, 0, 3, 0, 0, 5, 0, 0] , [5, 6, 0, 4, 0, 0, 3, 0, 0] , [3, 5, 0, 6, 0, 0, 4, 0, 0] , [4, 3, 0, 5, 0, 0, 6, 0, 0] , [6, 4, 0, 3, 0, 0, 5, 0, 0]] $

$$[y_1 - y_2 - 3\,y_3 + y_4, y_1, 0, y_2, y_3, 2\,y_3, y_4, 0, 0]$$

p = - s ^ 2 + s ^ 6 p = - s ^ 2 + s ^ 3 - s ^ 4 + s ^ 5

Â» SYNC’D 4543/131072 , 0.03466033936

237 . Coloring, {2, 4, 5, 6, 8, 9}

R: [4, 9, 4, 8, 3, 8, 1, 6, 2] **B**: [2, 4, 5, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [-21, -12, -3, -21, -4, -27, -13, -36, -9] . FixedPtCheck, [21, 12, 3, 21, 4, 27, 13, 36, 9]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {2, 9}}, net cycles: 0 . order: 4

$ [[3, 1, 2, 4, 0, 2, 0, 4, 2] , [0, 2, 0, 5, 0, 4, 0, 6, 1] , [0, 1, 0, 0, 0, 6, 0, 9, 2] , [0, 2, 0, 0, 0, 9, 0, 6, 1] , [0, 1, 0, 0, 0, 6, 0, 9, 2] , [0, 2, 0, 0, 0, 9, 0, 6, 1] , [0, 1, 0, 0, 0, 6, 0, 9, 2]] $

$$[3\,y_1, 5\,y_1 + 2\,y_3 - 8\,y_4, 2\,y_1, 20\,y_1 + 8\,y_3 - 30\,y_4 - 2\,y_2, 0, 2\,y_2, 0, 2\,y_3, 2\,y_4]$$

p = - s ^{3} + s ^{5} p’ = - s ^{3} + s ^{5} p = - s ^{3} + s ^{7}

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 3, 0, 2, 4, 0, 6, 0, 0] , [0, 3, 0, 3, 6, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 - y_4 - y_2 + y_3, y_1, 0, y_4, y_2, 0, y_3, 0, 0]$$

p = - s ^{4} + s ^{5}

Â» SYNC’D 1771/65536 , 0.02702331543

238 . Coloring, {2, 4, 5, 7, 8, 9}

R: [4, 9, 4, 8, 3, 7, 5, 6, 2] **B**: [2, 4, 5, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ 3 + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [7, 4, 9, 13, 12, 9, 13, 12, 3] . FixedPtCheck, [7, 4, 9, 13, 12, 9, 13, 12, 3]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	7 *vs* 8	6 *vs* 8	4 *vs* 6

Omega Rank for R : cycles: {{2, 9}, {3, 4, 5, 6, 7, 8}}, net cycles: 2 . order: 6

$ [[0, 1, 2, 4, 3, 2, 1, 3, 2] , [0, 2, 3, 2, 1, 3, 2, 4, 1] , [0, 1, 1, 3, 2, 4, 3, 2, 2] , [0, 2, 2, 1, 3, 2, 4, 3, 1] , [0, 1, 3, 2, 4, 3, 2, 1, 2] , [0, 2, 4, 3, 2, 1, 3, 2, 1] , [0, 1, 2, 4, 3, 2, 1, 3, 2] , [0, 2, 3, 2, 1, 3, 2, 4, 1]] $

$$[0, y_1 + y_6 + y_2 - 4\,y_5, 4\,y_1 + 4\,y_6 + 4\,y_2 - 15\,y_5 - y_3 - y_4, y_1, y_6, y_2, y_3, y_4, y_5]$$

p = - s + s ^{7} p’ = - s + s ^{7}

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: -1 . order: 4

$ [[6, 3, 0, 2, 1, 0, 5, 1, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0] , [6, 3, 0, 3, 0, 0, 6, 0, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0]] $

$$[y_1, y_2, 0, -y_1 + y_2 + y_3, y_4, 0, y_3, y_4, 0]$$

p = - s ^{2} + s ^{3} - s ^{4} + s ^{5} p = - s ^{2} + s ^{6}

Â» SYNC’D 1431675/33554432 , 0.04266723990

239 . Coloring, {2, 4, 6, 7, 8, 9}

R: [4, 9, 4, 8, 7, 8, 5, 6, 2] **B**: [2, 4, 5, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-154, -88, -50, -175, -200, -225, -250, -300, -66] . FixedPtCheck, [154, 88, 50, 175, 200, 225, 250, 300, 66]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	3 *vs* 7	4 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 1, 0, 4, 3, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 6, 1] , [0, 1, 0, 0, 3, 6, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 6, 1] , [0, 1, 0, 0, 3, 6, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 3, 6, 1] , [0, 1, 0, 0, 3, 6, 2, 4, 2]] $

$$[0, y_1, 0, -10\, y_1 - y_2 + 8\, y_3, -5\, y_1 + 4\, y_3, y_2, y_3, 2\, y_3, -4\, y_1 + 3\, y_3]$$

p’ = - s ^{2} + s ^{4} p = - s ^{2} + s ^{6} p = - s ^{2} + s ^{4} p’ = - s ^{2} + s ^{6}

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 7}}, net cycles: 2 . order: 4

$ [[6, 3, 2, 2, 1, 0, 4, 0, 0] , [4, 6, 1, 3, 2, 0, 2, 0, 0] , [2, 4, 2, 6, 1, 0, 3, 0, 0] , [3, 2, 1, 4, 2, 0, 6, 0, 0] , [6, 3, 2, 2, 1, 0, 4, 0, 0] , [4, 6, 1, 3, 2, 0, 2, 0, 0]] $

$$[3\, y_1 - y_2 + 2\, y_3, 2\, y_1 + 3\, y_3 - y_4, y_1, y_2, y_3, 0, y_4, 0, 0]$$

p = - s + s ^{5} p’ = - s + s ^{5}

Â» SYNC’D 447/262144 , 0.001705169678

240 . Coloring, {2, 5, 6, 7, 8, 9}

R: [4, 9, 4, 7, 3, 8, 5, 6, 2] **B**: [2, 4, 5, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [413, 236, 675, 875, 900, 375, 975, 500, 177] . FixedPtCheck, [413, 236, 675, 875, 900, 375, 975, 500, 177]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	7 *vs* 8	4 *vs* 8	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {2, 9}, {3, 4, 5, 7}}, net cycles: 3 . order: 4

$ [[0, 1, 2, 4, 3, 2, 3, 1, 2] , [0, 2, 3, 2, 3, 1, 4, 2, 1] , [0, 1, 3, 3, 4, 2, 2, 1, 2] , [0, 2, 4, 3, 2, 1, 3, 2, 1] , [0, 1, 2, 4, 3, 2, 3, 1, 2] , [0, 2, 3, 2, 3, 1, 4, 2, 1] , [0, 1, 3, 3, 4, 2, 2, 1, 2] , [0, 2, 4, 3, 2, 1, 3, 2, 1]] $

$$[0, y_3, 3\,y_3 + y_4 - y_2, y_3 - y_1 + 3\,y_4, y_1, y_4, y_2, y_3, y_4]$$

p’ = - s ^ 3 + s ^ 7 p = - s + s ^ 5 p’ = - s + s ^ 5 p’ = - s ^ 2 + s ^ 6

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 2, 1, 0, 3, 3, 0] , [6, 6, 0, 3, 0, 0, 1, 2, 0] , [3, 6, 0, 6, 0, 0, 0, 3, 0] , [3, 3, 0, 6, 0, 0, 0, 6, 0] , [6, 3, 0, 3, 0, 0, 0, 6, 0] , [6, 6, 0, 3, 0, 0, 0, 3, 0]] $

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, y_3, 0, y_4, y_5, 0]$$

p = - s ^ 3 + s ^ 4 - s ^ 5 + s ^ 6

Â» SYNC’D 469899/33554432 , 0.01400408149

241 . Coloring, {3, 4, 5, 6, 7, 8}

R: [4, 4, 5, 8, 3, 8, 5, 6, 1] **B**: [2, 9, 4, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘ 3 ‘]‘

For τ=1/2, [15, 4, 27, 21, 36, 27, 21, 36, 1] . FixedPtCheck, [15, 4, 27, 21, 36, 27, 21, 36, 1]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \ ; - \ ; Δ

$ [[1, 0, 2, 5, 4, 2, 0, 4, 0] , [6, 7, 4, 1, 2, 4, 1, 7, 4] , [16, 6, 2, 13, 5, 7, 17, 5, 1] , [19, 15, 5, 28, 19, 5, 23, 20, 10] , [47, 35, 19, 45, 28, 20, 44, 33, 17] , [100, 64, 28, 95, 63, 33, 99, 65, 29] , [185, 127, 63, 200, 127, 65, 193, 128, 64]] $ $ [[5, 4, 0, 1, 0, 0, 6, 0, 2] , [6, 1, 0, 11, 6, 0, 11, 1, 0] , [8, 10, 6, 11, 11, 1, 7, 11, 7] , [29, 17, 11, 20, 13, 11, 25, 12, 6] , [49, 29, 13, 51, 36, 12, 52, 31, 15] , [92, 64, 36, 97, 65, 31, 93, 63, 35] , [199, 129, 65, 184, 129, 63, 191, 128, 64]] $ $ [[-2, -2, 1, 2, 2, 1, -3, 2, -1] , [0, 3, 2, -5, -2, 2, -5, 3, 2] , [4, -2, -2, 1, -3, 3, 5, -3, -3] , [-5, -1, -3, 4, 3, -3, -1, 4, 2] , [-1, 3, 3, -3, -4, 4, -4, 1, 1] , [4, 0, -4, -1, -1, 1, 3, 1, -3] , [-7, -1, -1, 8, -1, 1, 1, 0, 0]] $

$[y_1, -2 y_1 - 2 y_6 + y_4 - y_3, -y_4 - y_2, y_6, y_1 + y_6 - y_5 - y_4, y_5, y_4, y_2, y_3]$

$p = s^2 - 2s^4 - 8s^5 + 16s^7$

S+ \ ; S- \ ; NM

$ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[15, 13, 5, 21, 10, 5, 18, 13, 8] , [21, 10, 2, 19, 14, 7, 14, 12, 9] , [19, 6, 4, 21, 20, 6, 14, 10, 8] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [14, 14, 12, 15, 9, 4, 25, 13, 2] , [18, 13, 8, 14, 9, 6, 22, 14, 4] , [21, 10, 5, 19, 17, 7, 14, 9, 6] , [19, 12, 4, 20, 13, 7, 15, 11, 7] , [17, 17, 6, 19, 7, 6, 18, 12, 6]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[1, 0, 2, 5, 4, 2, 0, 4, 0] , [0, 0, 4, 1, 2, 4, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0]] $

$$[y_4, 0, 2\,y_3 - 3\,y_2 + 2\,y_1, y_3, y_2, y_1, 0, -y_4 + 3\,y_3 - 4\,y_2 + 3\,y_1, 0]$$

p' = $s^3 - s^5$ p = $s^3 - s^5$

Omega Rank for B : cycles: {{2, 9}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, 0, 0, y_4, 0, y_5]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, -2/9, 5/18] , [0, 0, 0, 5/18, -2/9] , [1, -6, 31, 113/18, -290/9] , [0, 1, -6, -11/9, 113/18] , [0, 1, -6, -11/9, 113/18] , [0, 1, -6, -11/9, 113/18] , [0, 0, 1, 5/18, -11/9] , [0, 0, 1, 5/18, -11/9] , [0, 0, 0, -2/9, 5/18]] $ x $ [[5, 4, 0, 1, 0, 0, 6, 0, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC'D 35/1024 , 0.03417968750

242 . Coloring, {3, 4, 5, 6, 7, 9}

R: [4, 4, 5, 8, 3, 8, 5, 1, 2] **B**: [2, 9, 4, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [559, 172, 459, 663, 612, 153, 357, 612, 43] . FixedPtCheck, [559, 172, 459, 663, 612, 153, 357, 612, 43]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}}, net cycles: 1 . order: 6

$ [[2, 1, 2, 5, 4, 0, 0, 4, 0] , [4, 0, 4, 3, 2, 0, 0, 5, 0] , [5, 0, 2, 4, 4, 0, 0, 3, 0] , [3, 0, 4, 5, 2, 0, 0, 4, 0] , [4, 0, 2, 3, 4, 0, 0, 5, 0] , [5, 0, 4, 4, 2, 0, 0, 3, 0]] $

$$[-y_1 + 2\,y_2 - y_3 + 2\,y_4 - y_5, y_1, y_2, y_3, y_4, 0, 0, y_5, 0]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[4, 3, 0, 1, 0, 2, 6, 0, 2] , [8, 4, 0, 0, 0, 0, 3, 0, 3] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4]] $

$$[y_1, y_2, 0, y_3, 0, 2\,y_3, y_4, 0, y_5]$$

$p = - s^3 + s^6$

Â» SYNC’D 1521/16384 , 0.09283447266

243 . Coloring, {3, 4, 5, 6, 8, 9}

R: [4, 4, 5, 8, 3, 8, 1, 6, 2] **B**: [2, 9, 4, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘27‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 + 3τ ‘)‘ , -54‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 3τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -9‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , 9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , -18‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 27‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 3τ ‘)‘‘]‘

For τ=1/2, [-338, -104, -102, -357, -136, -459, -238, -612, -26] . FixedPtCheck, [338, 104, 102, 357, 136, 459, 238, 612, 26]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	7 *vs* 8	4 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 0 . order: 4

$ [[3, 1, 2, 5, 1, 2, 0, 4, 0] , [0, 0, 1, 4, 2, 4, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0] , [0, 0, 1, 0, 2, 8, 0, 7, 0] , [0, 0, 2, 0, 1, 7, 0, 8, 0]] $

$$[3\,y_3, y_3, y_4, y_2, y_1, -y_2 + 3\,y_1 + 2\,y_4, 0, -4\,y_3 + 3\,y_4 + 2\,y_1, 0]$$

$p = -s^3 + s^5$ $p = -s^3 + s^7$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}}, net cycles: 1 . order: 6

$ [[3, 3, 0, 1, 3, 0, 6, 0, 2] , [2, 3, 0, 0, 6, 0, 4, 0, 3] , [3, 2, 0, 0, 4, 0, 6, 0, 3] , [3, 3, 0, 0, 6, 0, 4, 0, 2] , [2, 3, 0, 0, 4, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 4, 0, 3]] $

$$[4\,y_2, 4\,y_1, 0, 5\,y_2 + 5\,y_1 - 4\,y_4 - 4\,y_3 + 5\,y_5, 4\,y_4, 0, 4\,y_3, 0, 4\,y_5]$$

$p = s^2 + s^3 - s^5 - s^6$

Â» SYNC'D 5583/524288 , 0.01064872742

244 . Coloring, {3, 4, 5, 7, 8, 9}

R: [4, 4, 5, 8, 3, 7, 5, 6, 2] **B**: [2, 9, 4, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ 3 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘ , 9‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 3 , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 9‘ (‘ - 1 + τ ‘)‘ 4 ‘ (‘ 5 + 3τ + 3τ 2 + τ 3 ‘)‘‘]‘

For τ=1/2, [767, 236, 2295, 1326, 3060, 918, 1785, 1224, 59] . FixedPtCheck, [767, 236, 2295, 1326, 3060, 918, 1785, 1224, 59]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘ 2 ‘ (‘ τ ‘)‘ 2 ‘ (‘ 1 + τ ‘)‘ 3

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	7 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{3, 5}}, net cycles: 0 . order: 6

$$[0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

R = $ [[0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0,

1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[0, 1, -5, 22, -97, -1663/72, 7355/72] , [0, 1, -5, 22, -97, -1663/72, 7355/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 1, -5, 22, 371/72, -1663/72] , [0, 0, 0, 0, 0, 11/72, -7/72] , [0, 0, 0, 0, 1, 11/72, -79/72] , [0, 0, 0, 0, 0, -7/72, 11/72] , [0, 0, 0, 1, -5, -79/72, 371/72] , [1, -5, 22, -97, 428, 7355/72, -32479/72]] $ x $ [[0, 1, 2, 5, 4, 2, 1, 3, 0] , [0, 0, 4, 1, 3, 3, 2, 5, 0] , [0, 0, 3, 0, 6, 5, 3, 1, 0] , [0, 0, 6, 0, 6, 1, 5, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] , [0, 0, 7, 0, 11, 0, 0, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 1, 0, 0, 5, 1, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 0, y_4, 0, 0, y_3, y_4, y_5]$$

$p = - s^{3} + s^{6}$

Â» SYNC’D 63051/524288 , 0.1202602386

245 . Coloring, {3, 4, 6, 7, 8, 9}

R: [4, 4, 5, 8, 7, 8, 5, 6, 2] **B**: [2, 9, 4, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [-325, -100, -153, -357, -612, -459, -663, -612, -25] . FixedPtCheck, [325, 100, 153, 357, 612, 459, 663, 612, 25]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {5, 7}}, net cycles: 1 . order: 4

$ [[0, 1, 0, 5, 4, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0]] $

$$[0, y_1, 0, 3\,y_1 - y_2 - 4\,y_3 + 3\,y_4, 2\,y_1 - 3\,y_3 + 2\,y_4, y_2, y_3, y_4, 0]$$

p' = - s ^{3} + s ^{5} p = - s ^{3} + s ^{5}

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 6

$$[y_1, y_2, y_3, y_4, 0, 0, y_5, 0, y_6]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 0, -4/27, 5/27, 1/54] , [0, 0, 0, 1/54, -4/27, 5/27] , [0, 1/2, -1/4, -4/27, -17/54, 29/108] , [0, 0, 1/2, 1/54, -4/27, -17/54] , [1/2, -1/4, -7/8, -17/54, 29/108, 157/216] , [0, 0, 1/2, 1/54, -4/27, -17/54] , [0, 0, 0, 5/27, 1/54, -4/27] , [0, 0, 0, 5/27, 1/54, -4/27] , [0, 0, 0, 5/27, 1/54, -4/27]] $ x $ [[6, 3, 2, 1, 0, 0, 4, 0, 2] , [6, 6, 0, 2, 0, 0, 1, 0, 3] , [4, 6, 0, 0, 0, 0, 2, 0, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8]] $

Â» SYNC'D 61/1024 , 0.05957031250

246 . Coloring, {3, 5, 6, 7, 8, 9}

R: [4, 4, 5, 7, 3, 8, 5, 6, 2] **B**: [2, 9, 4, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 4τ + τ ^{2} ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 + 4τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{4} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘ ^{4} ‘ (‘ 5 + 4τ + τ ^{2} ‘)‘‘]‘

For τ=1/2, [377, 116, 1377, 714, 1836, 306, 1071, 408, 29] . FixedPtCheck, [377, 116, 1377, 714, 1836, 306, 1071, 408, 29]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{3}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	5 *vs* 7	5 *vs* 6

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[0, 1, 2, 5, 4, 2, 3, 1, 0] , [0, 0, 4, 1, 5, 1, 5, 2, 0] , [0, 0, 5, 0, 9, 2, 1, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0]] $

$$[0, -y_1 + y_5 + 4\,y_3 - y_4, y_1, y_2, -y_2 + 4\,y_5 + y_3, y_5, y_4, y_3, 0]$$

p = $s^4 - s^6$ p' = $s^4 - s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: -1 . order: 3

$ [[6, 3, 0, 1, 0, 0, 3, 3, 2] , [8, 6, 0, 0, 0, 0, 0, 1, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] $

$$[y_1, y_2, 0, y_3, 0, 0, 3\,y_3, y_4, y_5]$$

p = $-s^3 + s^6$

Â» SYNC'D 6823/131072 , 0.05205535889

247 . Coloring, {4, 5, 6, 7, 8, 9}

R: [4, 4, 4, 8, 3, 8, 5, 6, 2] **B**: [2, 9, 5, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ^2 ‘)‘‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ^2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 1 + τ^2 ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{3} , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ^2 ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -18‘ (‘ 1 + τ^2 ‘)‘‘ (‘ 5 - 2τ + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , 9‘ (‘ - 1 + τ ‘)‘ ^{3} ‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘]‘

For τ=1/2, [-1118, -344, -918, -1785, -1224, -2295, -1326, -3060, -86] . FixedPtCheck, [1118, 344, 918, 1785, 1224, 2295, 1326, 3060, 86]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 vs 7	*7 vs 7*	*7 vs 7*	5 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}}, net cycles: -1 . order: 4

$ [[0, 1, 2, 6, 3, 2, 0, 4, 0] , [0, 0, 3, 3, 0, 4, 0, 8, 0] , [0, 0, 0, 3, 0, 8, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0]] $

$$[0, y_1, y_2, y_3, 3\,y_1, y_4, 0, y_5, 0]$$

$p = -s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 3

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 0, y_5]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 5/27, -4/27, 1/54] , [0, 0, 1/54, 5/27, -4/27] , [1, -6, 5/27, -31/27, 325/54] , [0, 1, 1/54, 5/27, -31/27] , [0, 1, 1/54, 5/27, -31/27] , [0, 1, 1/54, 5/27, -31/27] , [0, 0, -4/27, 1/54, 5/27] , [0, 0, -4/27, 1/54, 5/27] , [0, 0, -4/27, 1/54, 5/27]] $ x $ [[6, 3, 0, 0, 1, 0, 6, 0, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4]] $

Â» SYNC’D 915/8192 , 0.1116943359

248 . Coloring, {2, 3, 4, 5, 6, 7, 8}

R: [4, 9, 5, 8, 3, 8, 5, 6, 1] **B**: [2, 4, 4, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘9‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ 3 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [247, 76, 387, 301, 516, 387, 301, 516, 57] . FixedPtCheck, [247, 76, 387, 301, 516, 387, 301, 516, 57]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2 - y_3 - y_4 - y_7, y_1, -y_5 - y_6, y_2, y_3, y_4, y_5, y_6, y_7]$

[3, 2, 1, 3, 2, 1, 3, 2, 1]

+ \; - \; Δ

$ [[1, 0, 2, 3, 4, 2, 0, 4, 2] , [8, 5, 4, 5, 2, 4, 3, 5, 0] , [12, 8, 2, 11, 7, 5, 13, 9, 5] , [23, 15, 7, 26, 15, 9, 25, 16, 8] , [47, 33, 15, 49, 32, 16, 46, 35, 15] , [94, 66, 32, 95, 61, 35, 95, 65, 33] , [193, 129, 61, 188, 127, 65, 193, 130, 66]] $ $ [[5, 4, 0, 3, 0, 0, 6, 0, 0] , [4, 3, 0, 7, 6, 0, 9, 3, 4] , [12, 8, 6, 13, 9, 3, 11, 7, 3] , [25, 17, 9, 22, 17, 7, 23, 16, 8] , [49, 31, 17, 47, 32, 16, 50, 29, 17] , [98, 62, 32, 97, 67, 29, 97, 63, 31] , [191, 127, 67, 196, 129, 63, 191, 126, 62]] $ $ [[-2, -2, 1, 0, 2, 1, -3, 2, 1] , [2, 1, 2, -1, -2, 2, -3, 1, -2] , [0, 0, -2, -1, -1, 1, 1, 1, 1] , [-1, -1, -1, 2, -1, 1, 1, 0, 0] , [-1, 1, -1, 1, 0, 0, -2, 3, -1] , [-2, 2, 0, -1, -3, 3, -1, 1, 1] , [1, 1, -3, -4, -1, 1, 1, 2, 2]] $

$[y_3, y_4, y_5, y_2, y_6, y_1, -2 y_4 - 3 y_6 - 3 y_1 - y_3 - y_2, 2 y_4 - y_5 + 3 y_6 + 3 y_1 + y_3 + y_2, -y_3 - y_4 - y_2 - y_6 - y_1]$

$p = - s^3 + s^4 + 8s^7$

S+ \; S- \; NM

$ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[14, 11, 4, 14, 8, 5, 12, 8, 4] , [14, 8, 2, 16, 10, 5, 10, 8, 7] , [15, 6, 3, 16, 15, 5, 9, 6, 5] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [10, 10, 9, 11, 6, 3, 19, 10, 2] , [10, 10, 6, 13, 6, 3, 17, 11, 4] , [16, 6, 3, 13, 13, 5, 11, 8, 5] , [16, 8, 3, 13, 10, 6, 11, 8, 5] , [15, 11, 4, 11, 6, 5, 14, 10, 4]] $ $ [[0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 7	4 *vs* 4

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[1, 0, 2, 3, 4, 2, 0, 4, 2] , [2, 0, 4, 1, 2, 4, 0, 5, 0] , [0, 0, 2, 2, 4, 5, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0]] $

$[y_1, 0, y_2, 3 y_1 - 4 y_2 - y_3 + 3 y_4 - y_5, 2 y_1 - 3 y_2 + 2 y_4, y_3, 0, y_4, y_5]$

$p' = - s^4 + s^6$ $p = - s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 1 . order: 4

$[y_4, y_3, 0, y_2, 0, 0, y_1, 0, 0]$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0]] $ = $ [[1/72, 1/72, 19/72, -17/72] , [-17/72, 1/72, 1/72, 19/72] , [-17/72, 1/72, 1/72, 19/72] , [19/72, -17/72, 1/72, 1/72] , [19/72, -17/72, 1/72, 1/72] , [19/72, -17/72, 1/72, 1/72] , [1/72, 19/72, -17/72, 1/72] , [1/72, 19/72, -17/72, 1/72] , [1/72, 1/72, 19/72, -17/72]] $ x $ [[5, 4, 0, 3, 0, 0, 6, 0, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0]] $

Â» SYNC'D 615/32768 , 0.01876831055

249 . Coloring, {2, 3, 4, 5, 6, 7, 9}

R: [4, 9, 5, 8, 3, 8, 5, 1, 2] **B**: [2, 4, 4, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [301, 172, 225, 325, 300, 75, 175, 300, 129] . FixedPtCheck, [301, 172, 225, 325, 300, 75, 175, 300, 129]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 7	4 *vs* 5

Omega Rank for R : cycles: {{3, 5}, {2, 9}, {1, 4, 8}}, net cycles: 3 . order: 6

$ [[2, 1, 2, 3, 4, 0, 0, 4, 2] , [4, 2, 4, 2, 2, 0, 0, 3, 1] , [3, 1, 2, 4, 4, 0, 0, 2, 2] , [2, 2, 4, 3, 2, 0, 0, 4, 1] , [4, 1, 2, 2, 4, 0, 0, 3, 2] , [3, 2, 4, 4, 2, 0, 0, 2, 1] , [2, 1, 2, 3, 4, 0, 0, 4, 2]] $

$$[3\,y_1 - y_2 - y_3 + 3\,y_4, y_1, 2\,y_1, y_2, 2\,y_4, 0, 0, y_3, y_4]$$

p = s - s ^{3} - s ^{4} + s ^{6} p = - s + s ^{7} p = - s - s ^{2} + s ^{4} + s ^{5}

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$ [[4, 3, 0, 3, 0, 2, 6, 0, 0] , [6, 4, 0, 3, 0, 0, 5, 0, 0] , [5, 6, 0, 4, 0, 0, 3, 0, 0] , [3, 5, 0, 6, 0, 0, 4, 0, 0] , [4, 3, 0, 5, 0, 0, 6, 0, 0]] $

$$[y_1 - y_2 - y_3 + y_4, y_1, 0, y_2, 0, y_3, y_4, 0, 0]$$

$p = - s^2 + s^3 - s^4 + s^5$

Â» SYNC'D 5859/262144 , 0.02235031128

250 . Coloring, {2, 3, 4, 5, 6, 8, 9}

R: [4, 9, 5, 8, 3, 8, 1, 6, 2] **B**: [2, 4, 4, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘27‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 54‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ 3 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 27‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 5 + 3τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-546, -312, -150, -525, -200, -675, -350, -900, -234] . FixedPtCheck, [546, 312, 150, 525, 200, 675, 350, 900, 234]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 8	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}, {2, 9}}, net cycles: 2 . order: 4

$ [[3, 1, 2, 3, 1, 2, 0, 4, 2] , [0, 2, 1, 3, 2, 4, 0, 5, 1] , [0, 1, 2, 0, 1, 5, 0, 7, 2] , [0, 2, 1, 0, 2, 7, 0, 5, 1] , [0, 1, 2, 0, 1, 5, 0, 7, 2] , [0, 2, 1, 0, 2, 7, 0, 5, 1] , [0, 1, 2, 0, 1, 5, 0, 7, 2] , [0, 2, 1, 0, 2, 7, 0, 5, 1]] $

$$[y_1 + 3 y_4 - y_3, y_1, y_4, 3 y_1 + y_4 - y_2, y_1, y_2, 0, y_3, y_4]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$ $p = - s^3 + s^7$ $p' = - s^3 + s^7$

Omega Rank for B : cycles: {{5, 7}}, net cycles: 0 . order: 4

$ [[3, 3, 0, 3, 3, 0, 6, 0, 0] , [0, 3, 0, 3, 6, 0, 6, 0, 0] , [0, 0, 0, 3, 6, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0] , [0, 0, 0, 0, 9, 0, 9, 0, 0]] $

$$[y_1 - y_2 - y_3 + y_4, y_1, 0, y_2, y_3, 0, y_4, 0, 0]$$

p = - s ^{4} + s ^{5}

Â» SYNC'D 3861/1048576 , 0.003682136536

251 . Coloring, {2, 3, 4, 5, 7, 8, 9}

R: [4, 9, 5, 8, 3, 7, 5, 6, 2] **B**: [2, 4, 4, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 18‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -9‘ (‘ 1 + τ ^{2} ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^{2} ‘)‘ , 9‘ (‘ 5 + 3τ + 3τ ^{2} + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘]‘

For τ=1/2, [413, 236, 1125, 650, 1500, 450, 875, 600, 177] . FixedPtCheck, [413, 236, 1125, 650, 1500, 450, 875, 600, 177]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘ ^{4} ‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	6 *vs* 8	4 *vs* 5

Omega Rank for R : cycles: {{3, 5}, {2, 9}}, net cycles: 1 . order: 6

$ [[0, 1, 2, 3, 4, 2, 1, 3, 2] , [0, 2, 4, 0, 3, 3, 2, 3, 1] , [0, 1, 3, 0, 6, 3, 3, 0, 2] , [0, 2, 6, 0, 6, 0, 3, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1]] $

$$[0, y_6, y_5, y_3, y_4, -15\, y_6 - y_3 - y_4 + 4\, y_5 + 4\, y_1 + 4\, y_2, y_1, y_2, -4\, y_6 + y_5 + y_1 + y_2]$$

p' = - s ^{5} + s ^{7} p = - s ^{5} + s ^{7}

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 3, 0, 0, 5, 1, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0] , [6, 3, 0, 3, 0, 0, 6, 0, 0]] $

$$[y_2, y_3, 0, y_4, 0, 0, y_2 - y_3 + y_4 - y_1, y_1, 0]$$

$p = - s^2 + s^3 - s^4 + s^5$

Â» SYNC'D 35637/1048576 , 0.03398609161

252 . Coloring, {2, 3, 4, 6, 7, 8, 9}

R: [4, 9, 5, 8, 7, 8, 5, 6, 2] **B**: [2, 4, 4, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ‘)‘ ^{2} , -9‘ (‘ 3 + τ ^{2} ‘)‘ , -18‘ (‘ 1 + τ ‘)‘ , 9‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-7, -4, -3, -7, -12, -9, -13, -12, -3] . FixedPtCheck, [7, 4, 3, 7, 12, 9, 13, 12, 3]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 7	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {5, 7}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 1, 0, 3, 4, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2]] $

$$[0, y_1, 0, -3\,y_1 - y_2 + 2\,y_3, -4\,y_1 + 2\,y_3, y_2, 2\,y_1, y_3, -2\,y_1 + y_3]$$

$p = - s^2 + s^4$ $p = - s^2 + s^6$ $p' = - s^2 + s^6$ $p' = - s^2 + s^4$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$ [[6, 3, 2, 3, 0, 0, 4, 0, 0] , [4, 6, 0, 5, 0, 0, 3, 0, 0] , [3, 4, 0, 6, 0, 0, 5, 0, 0] , [5, 3, 0, 4, 0, 0, 6, 0, 0] , [6, 5, 0, 3, 0, 0, 4, 0, 0]] $

$$[y_2, y_3, y_1, y_4, 0, 0, y_2 - y_3 - y_1 + y_4, 0, 0]$$

$p = -s^2 + s^3 - s^4 + s^5$

Â» SYNC'D 801/131072 , 0.006111145020

253 . Coloring, {2, 3, 5, 6, 7, 8, 9}

R: [4, 9, 5, 7, 3, 8, 5, 6, 2] **B**: [2, 4, 4, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 3 + τ ‘)‘ , 18‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 1 + τ ‘)‘ ^ 3 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 18‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -9‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , -18‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 5 + 4τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘]‘

For τ=1/2, [203, 116, 675, 350, 900, 150, 525, 200, 87] . FixedPtCheck, [203, 116, 675, 350, 900, 150, 525, 200, 87]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^ 4 ‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	9 *vs* 9	9 *vs* 9	4 *vs* 8	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}, {2, 9}}, net cycles: 2 . order: 4

$ [[0, 1, 2, 3, 4, 2, 3, 1, 2] , [0, 2, 4, 0, 5, 1, 3, 2, 1] , [0, 1, 5, 0, 7, 2, 0, 1, 2] , [0, 2, 7, 0, 5, 1, 0, 2, 1] , [0, 1, 5, 0, 7, 2, 0, 1, 2] , [0, 2, 7, 0, 5, 1, 0, 2, 1] , [0, 1, 5, 0, 7, 2, 0, 1, 2] , [0, 2, 7, 0, 5, 1, 0, 2, 1]] $

$$[0, y_3 + y_4 - 3\,y_1, y_2, y_3, y_4, y_1, 3\,y_3 + 3\,y_4 - 8\,y_1 - y_2, y_3 + y_4 - 3\,y_1, y_1]$$

$p = -s^3 + s^5$ $p = -s^3 + s^7$ $p' = -s^3 + s^7$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{1, 2, 4, 8}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 3, 0, 0, 3, 3, 0] , [6, 6, 0, 3, 0, 0, 0, 3, 0] , [3, 6, 0, 6, 0, 0, 0, 3, 0] , [3, 3, 0, 6, 0, 0, 0, 6, 0] , [6, 3, 0, 3, 0, 0, 0, 6, 0]] $

$$[y_1 - y_4 + y_3 + y_2, y_1, 0, y_4, 0, 0, y_3, y_2, 0]$$

$p = -s^2 + s^3 - s^4 + s^5$

Â» SYNC'D 675/131072 , 0.005149841309

254 . Coloring, {2, 4, 5, 6, 7, 8, 9}

R: [4, 9, 4, 8, 3, 8, 5, 6, 2] **B**: [2, 4, 5, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -9‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ ^ 2 ‘)‘ , 9‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - τ + 3τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-602, -344, -450, -875, -600, -1125, -650, -1500, -258] . FixedPtCheck, [602, 344, 450, 875, 600, 1125, 650, 1500, 258]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	7 *vs* 7	7 *vs* 7	5 *vs* 7	4 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {2, 9}}, net cycles: 1 . order: 4

$ [[0, 1, 2, 4, 3, 2, 0, 4, 2] , [0, 2, 3, 2, 0, 4, 0, 6, 1] , [0, 1, 0, 3, 0, 6, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 9, 1] , [0, 1, 0, 0, 0, 9, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 9, 1] , [0, 1, 0, 0, 0, 9, 0, 6, 2]] $

$$[0, y_5, y_4, y_3, y_2, -15\,y_5 + 4\,y_4 - y_3 - y_2 + 4\,y_1, 0, y_1, -4\,y_5 + y_4 + y_1]$$

p' = $-s^4 + s^6$ p = $-s^4 + s^6$

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 0 . order: 4

$ [[6, 3, 0, 2, 1, 0, 6, 0, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0] , [6, 3, 0, 3, 0, 0, 6, 0, 0]] $

$$[y_1 - y_2 - y_3 + y_4, y_1, 0, y_2, y_3, 0, y_4, 0, 0]$$

$p = -s^2 + s^3 - s^4 + s^5$

Â» SYNC'D 485/16384 , 0.02960205078

255 . Coloring, {3, 4, 5, 6, 7, 8, 9}

R: [4, 4, 5, 8, 3, 8, 5, 6, 2] **B**: [2, 9, 4, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -18‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 3 , -9‘ (‘ 3 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 18‘ (‘ 5 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 9‘ (‘ - 5 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 3 ‘]‘

For τ=1/2, [247, 76, 459, 357, 612, 459, 357, 612, 19] . FixedPtCheck, [247, 76, 459, 357, 612, 459, 357, 612, 19]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	4 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{6, 8}, {3, 5}}, net cycles: 1 . order: 4

$ [[0, 1, 2, 5, 4, 2, 0, 4, 0] , [0, 0, 4, 1, 2, 4, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0] , [0, 0, 2, 0, 4, 7, 0, 5, 0] , [0, 0, 4, 0, 2, 5, 0, 7, 0]] $

$$[0, 3\,y_1 - 4\,y_2 + 3\,y_3 - y_4, 2\,y_1 - 3\,y_2 + 2\,y_3, y_1, y_2, y_3, 0, y_4, 0]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{1, 2, 9}}, net cycles: 0 . order: 3

$$[y_5, y_4, 0, y_3, 0, 0, y_2, 0, y_1]$$

B = $ [[0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 5/27, -4/27, 1/54] , [0, 0, 1/54, 5/27, -4/27] , [1, -6, 5/27, -31/27, 325/54] , [0, 1, 1/54, 5/27, -31/27] , [0, 1, 1/54, 5/27, -31/27] , [0, 1, 1/54, 5/27, -31/27] , [0, 0,

-4/27, 1/54, 5/27] , [0, 0, -4/27, 1/54, 5/27] , [0, 0, -4/27, 1/54, 5/27]] $ x $ [[6, 3, 0, 1, 0, 0, 6, 0, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4]] $

Â» SYNC'D 215/4096 , 0.05249023438

256 . Coloring, {2, 3, 4, 5, 6, 7, 8, 9}

R: [4, 9, 5, 8, 3, 8, 5, 6, 2] **B**: [2, 4, 4, 7, 7, 7, 1, 1, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

'['9' (' 3 + τ ')'' (' - 5 + τ 2 ')'' (' - 1 + τ ')' , 18' (' - 5 + τ 2 ')'' (' - 1 + τ ')' , 9' (' 1 + τ ')' 2 ' (' 5 + 2τ + τ 2 ')' , -9' (' 3 + τ ')'' (' - 1 + τ ')'' (' 5 + 2τ + τ 2 ')' , 18' (' 1 + τ ')'' (' 5 + 2τ + τ 2 ')' , 9' (' 1 + τ ')' 2 ' (' 5 + 2τ + τ 2 ')' , -9' (' 3 + τ ')'' (' - 1 + τ ')'' (' 5 + 2τ + τ 2 ')' , 18' (' 1 + τ ')'' (' 5 + 2τ + τ 2 ')' , 9' (' - 5 + τ 2 ')'' (' 1 + τ ')'' (' - 1 + τ ')'']'

For τ=1/2, [133, 76, 225, 175, 300, 225, 175, 300, 57] . FixedPtCheck, [133, 76, 225, 175, 300, 225, 175, 300, 57]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
7 *vs* 7	8 *vs* 8	8 *vs* 8	3 *vs* 7	3 *vs* 4

Omega Rank for R : cycles: {{6, 8}, {3, 5}, {2, 9}}, net cycles: 2 . order: 2

$ [[0, 1, 2, 3, 4, 2, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 5, 1] , [0, 1, 2, 0, 4, 5, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 5, 1] , [0, 1, 2, 0, 4, 5, 0, 4, 2] , [0, 2, 4, 0, 2, 4, 0, 5, 1] , [0, 1, 2, 0, 4, 5, 0, 4, 2]] $

$$[0, y_1 + y_2 - 2\,y_3, 2\,y_1 + 2\,y_2 - 4\,y_3, y_1, 2\,y_3, y_2, 0, 2\,y_1 + 2\,y_2 - 3\,y_3, y_3]$$

p' = - s 2 + s 4 p = - s 2 + s 6 p' = - s 2 + s 6 p = - s 2 + s 4

Omega Rank for B : cycles: {{1, 2, 4, 7}}, net cycles: 1 . order: 4

$ [[6, 3, 0, 3, 0, 0, 6, 0, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0]] $

$$[y_1 - y_2 + y_3, y_1, 0, y_2, 0, 0, y_3, 0, 0]$$

p = - s + s 2 - s 3 + s 4

Â» SYNC’D 405/65536 , 0.006179809570

SUMMARY		
Graph Type		CC
ν(A)		2
ν(Δ)		2
π		[3, 2, 1, 3, 2, 1, 3, 2, 1]
Dbly Stoch		false

SANDWICH		Total 4
No .	**Coloring**	**Rank**
1	{}	3
2	{2, 5, 8}	3
3	{2, 3, 5, 6, 8, 9}	6
4	{3, 6, 9}	3

RT GROUPS		Total 1	
No .	**Coloring**	**Rank**	**Solv**
1	{2, 4, 7, 9}	2	Not Solvable

CC Colorings		Total 1
No .	**Coloring**	**Sandwich,Rank**
1	{}	true, 3

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	**2^{n-1}**
213	0	243 , 247	28 , 44	5	256	256
