

# New Graph

[4, 3, 1, 2], [3, 4, 4, 3]

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$$\pi = [1, 1, 2, 2]$$

POSSIBLE RANKS

$$\begin{array}{l} 1 \times 6 \\ 2 \times 3 \end{array}$$

BASE DETERMINANT 3/16, .1875000000

*NullSpace* of  $\Delta$

$$\{1, 2, 3, 4\}$$

$$\text{Range of } \Delta: [-\lambda_1 - \lambda_2 - \lambda_3, \lambda_1, \lambda_2, \lambda_3]$$

1 . Coloring, { }

**R:** [4, 3, 1, 2]   **B:** [3, 4, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Omega$  for  $A + \tau \Delta$  :

$$[ '3' ('1 + \tau ' )', 3' ('1 + \tau ' )', 6, 6' ]'$$

For  $\tau=1/2$ , [3, 3, 4, 4] . FixedPtCheck, [3, 3, 4, 4]

$$\det(A + \tau \Delta) = 1' (' \tau ' )'' ('1 + \tau ' )' ^2$$

Delta Range : [-y<sub>1</sub> - y<sub>2</sub> - y<sub>3</sub>, y<sub>1</sub>, y<sub>2</sub>, y<sub>3</sub>]

$$[1, 1, 2, 2]$$

+            -             $\Delta$

$$\$ [ [2, 2, 1, 1], [1, 1, 5, 5], [5, 5, 7, 7] ] \$ \quad \$ [ [0, 0, 3, 3], [3, 3, 3, 3], [3, 3, 9, 9] ] \$ \quad \$ [ [1, 1, -1, -1], [-1, -1, 1, 1], [1, 1, -1, -1] ] \$$$

$$[-y_1, -y_1, y_1, y_1]$$

$$p' = s + 2s^2 \quad p = s - 4s^3$$

S+            S-            NM

$$\$ [ [1, 0, 1, 3], [0, 1, 3, 1], [2, 0, 2, 1], [0, 2, 1, 2] ] \$ \quad \$ [ [0, 1, 3, 1], [1, 0, 1, 3], [1, 1, 0, 3], [1, 1, 3, 0] ] \$ \quad \$ [ [3, 0, 2, 4], [0, 3, 4, 2], [1, 2, 6, 0], [2, 1, 0, 6] ] \$$$

CmmCk true, true, true

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
1 vs 3	2 vs 4	2 vs 4	2 vs 4	1 vs 2

Omega Rank for R : cycles: {{1, 2, 3, 4}}, net cycles: 1 . order: 4

$$\$ [ [2, 2, 1, 1], [1, 1, 2, 2], [2, 2, 1, 1], [1, 1, 2, 2] ] \$$$

$$[y_1, y_1, y_2, y_2]$$

$$p' = 1 - s^2 \quad p' = s - s^3$$

Omega Rank for B : cycles: {{3, 4}}, net cycles: 1 . order: 2

$$\$ [ [0, 0, 3, 3], [0, 0, 3, 3] ] \$$$

$$[0, 0, y_1, y_1]$$

$$p = -s + s^2$$

Â« NOT SYNC'D Â»

Nullspace of  $\{\Omega\Delta^i\}$  :

$$[x_2, x_1, -4x_2 + 2x_1]$$

$$\text{For } A+2\Delta: [-y_1, y_1, -y_2, y_2]$$

$$\text{For } A-2\Delta: [-y_2, y_2, -y_1, y_1]$$

Range of  $\{\Omega\Delta^i\}$ :  $[-\mu_1, -\mu_1, \mu_1, \mu_1]$

rank of M is 4 , rank of N is 3

M            N

$$\$ [ [0, 1, 0, 0], [1, 0, 0, 0], [0, 0, 0, 2], [0, 0, 2, 0] ] \$ \quad \$ [ [0, 3, 2, 1], [3, 0, 1, 2], [2, 1, 0, 3], [1, 2, 3, 0] ] \$$$

Check is  $\Omega\Delta N$  zero? *true*,  $\pi\Delta = [1, 1, -1, -1]$

ker M, [0, 0, 0, 0]

Range M,  $[x_1, x_2, x_3, x_4]$

$\tau = 8$ ,  $r' = 1/2$

Ranges

Action of R on ranges, [[2], [1]]

Action of B on ranges, [[2], [2]]

$\beta(\{1, 2\}) = 1/3$

$\beta(\{3, 4\}) = 2/3$

ker N,  $[-\mu_1, -\mu_1, \mu_1, \mu_1]$

Range of N

$[y_3, -y_3 + y_1 + y_2, y_1, y_2]$

Partitions

Action of R on partitions, [[2], [1]]

Action of B on partitions, [[1], [1]]

$\alpha(\{1, 4\}, \{2, 3\}) = 2/3$

$\alpha(\{1, 3\}, \{2, 4\}) = 1/3$

$b_1 = \{1, 3\}$ ,  $b_2 = \{1, 4\}$ ,  $b_3 = \{2, 4\}$ ,  $b_4 = \{2, 3\}$

Action of R and B on the blocks of the partitions:  $\$ [ [0, 1, 0, 1], [1, 0, 0, 1], [0, 1, 0, 1], [0, 1, 1, 0] ] \$ =$   
 $\$ [ [0, 0, 0, 1], [1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0] ] \$ + \$ [ [0, 1, 0, 0], [0, 0, 0, 1], [0, 0, 0, 1], [0, 1, 0,$   
 $0] ] \$$

[‘4’, ‘1’, ‘2’, ‘3’], [‘2’, ‘4’, ‘4’, ‘2’] with invariant measure [1, 2, 1, 2]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
<b>Coloring</b>	{ }
<b>Rank</b>	2
<b>R,B</b>	[4, 3, 1, 2], [3, 4, 4, 3]
$\pi_2$	[1, 0, 0, 0, 0, 2]
$u_2$	[3, 2, 1, 1, 2, 3] (dim 1)
<b>wpp</b>	[2, 2, 2, 2]

2. Coloring, {2}

**R:** [4, 4, 1, 2]    **B:** [3, 3, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Omega$  for  $A+\tau\Delta$  :

‘ [ ‘3‘ (‘3 +  $\tau$  ‘)‘ (‘ - 1 +  $\tau$  ‘)‘ (‘ 1 +  $\tau$  ‘)‘ , -3‘ (‘3 +  $\tau^2$  ‘)‘ (‘ 1 +  $\tau$  ‘)‘ , 6‘ (‘3 +  $\tau$  ‘)‘ (‘ - 1 +  $\tau$  ‘)‘ , -6‘ (‘3 +  $\tau^2$  ‘)‘ ‘]‘

For  $\tau=1/2$ , [-21, -39, -28, -52] . FixedPtCheck, [21, 39, 28, 52]

$\det(A + \tau \Delta) = 0$

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	3 vs 3	3 vs 3	3 vs 3	2 vs 2

bi =

$\$ [ [0, 0, 3/4, 1/4], [0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4], [0, 1/4, 3/4, 0] ] \$ \times \$ [ [19/100, 27/100, -9/100, 27/100], [27/100, 91/100, 3/100, -9/100], [-9/100, 3/100, 99/100, 3/100], [27/100, -9/100, 3/100, 91/100] ] \$ =$   
 $\$ [ [1, -1/6, -2/3], [1, -1/6, -2/3], [0, 5/6, -2/3], [-1/2, -2/3, 4/3] ] \$ \times \$ [ [1/2, 1/2, 3, 2], [3/4, 1/2, 9/4, 5/2], [9/16, 5/8, 45/16, 2] ] \$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4}}, net cycles: 0 . order: 2

$$[y_1, y_3, 0, y_2]$$

$$R = \$ [ [0, 0, 0, 1], [0, 0, 0, 1], [1, 0, 0, 0], [0, 1, 0, 0] ] \$ \times \$ [ [1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 0], [0, 0, 0, 1] ] \$ = \$ [ [0, 1/3, -1/6], [0, 1/3, -1/6], [1/2, -1/6, -1/6], [0, -1/6, 1/3] ] \$ \times \$ [ [2, 2, 0, 2], [0, 2, 0, 4], [0, 4, 0, 2] ] \$$$

Omega Rank for B : cycles: {{3, 4}}, net cycles: 1 . order: 2

$$[0, 0, y_1, y_2]$$

$$B = \$ [ [0, 0, 1, 0], [0, 0, 1, 0], [0, 0, 0, 1], [0, 0, 1, 0] ] \$ \times \$ [ [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1] ] \$ = \$ [ [1/3, -1/6], [1/3, -1/6], [-1/6, 1/3], [1/3, -1/6] ] \$ \times \$ [ [0, 0, 4, 2], [0, 0, 2, 4] ] \$$$

Â» SYNC'D 1/4 , 0.2500000000

3 . Coloring, {3}

**R:** [4, 3, 4, 2]    **B:** [3, 4, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

$$\begin{bmatrix} 3 & (-1 + \tau) & (3 + \tau^2) & 3 & (1 + \tau) & (-3 + \tau) & -6 & (3 + \tau^2) & 6 & (1 + \tau) & (-3 + \tau) \end{bmatrix}$$

For τ=1/2, [-13, -45, -52, -60] . FixedPtCheck, [13, 45, 52, 60]

$$\det(A + \tau \Delta) = 1 \cdot (\tau) \cdot (-1 + \tau) \cdot (1 + \tau)$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	4 vs 4	3 vs 4	3 vs 3	2 vs 3

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 1 . order: 3

$$[0, y_1, y_2, y_3]$$

$$R = \$ [ [0, 0, 0, 1], [0, 0, 1, 0], [0, 0, 0, 1], [0, 1, 0, 0] ] \$ \times \$ [ [0, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1] ] \$ = \$ [ [7/18, -5/18, 1/18], [-5/18, 1/18, 7/18], [7/18, -5/18, 1/18], [1/18, 7/18, -5/18] ] \$ \times \$ [ [0, 2, 1, 3], [0, 3, 2, 1], [0, 1, 3, 2] ] \$$$

Omega Rank for B : cycles: {{1, 3}}, net cycles: 0 . order: 2

$$\$ [ [2, 0, 3, 1], [3, 0, 3, 0], [3, 0, 3, 0] ] \$$$

$$[y_1 - y_2, 0, y_1, y_2]$$

$$p = -s^2 + s^3$$

Â» SYNC'D 1/4 , 0.2500000000

4 . Coloring, {4}

**R:** [4, 3, 1, 3] **B:** [3, 4, 4, 2]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$[ '3' ( '1 + \tau' )^2 ( '-3 + \tau' ) , 3' ( '-1 + \tau' ) ( '3 + \tau^2' ) , 6' ( '1 + \tau' ) ( '-3 + \tau' ) , -6' ( '3 + \tau^2' ) ]'$$

For τ=1/2, [-45, -13, -60, -52] . FixedPtCheck, [45, 13, 60, 52]

$$\det(A + \tau \Delta) = 1' ( ' \tau ' ) ( '-1 + \tau' ) ( '1 + \tau' )'$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	4 vs 4	3 vs 4	3 vs 3	2 vs 3

Omega Rank for R : cycles: {{1, 3, 4}}, net cycles: 1 . order: 3

$$[y_1, 0, y_2, y_3]$$

$$R = \$ [ [0, 0, 0, 1], [0, 0, 1, 0], [1, 0, 0, 0], [0, 0, 1, 0] ] \$ \times \$ [ [1, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1] ] \$ = \$ [ [-5/18, 1/18, 7/18], [7/18, -5/18, 1/18], [1/18, 7/18, -5/18], [7/18, -5/18, 1/18] ] \$ \times \$ [ [2, 0, 3, 1], [3, 0, 1, 2], [1, 0, 2, 3] ] \$$$

Omega Rank for B : cycles: {{2, 4}}, net cycles: 0 . order: 2

$$\$ [ [0, 2, 1, 3], [0, 3, 0, 3], [0, 3, 0, 3] ] \$$$

$$[0, -y_1 + y_2, y_1, y_2]$$

$$p = -s^2 + s^3$$

Â» SYNC'D 1/4 , 0.2500000000

5 . Coloring, {2, 3}

**R:** [4, 4, 4, 2]    **B:** [3, 3, 1, 3]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$\begin{bmatrix} (-3 + \tau)^2 & 3(1 + \tau)^2 & (-3 + \tau)^2 & 6(3 + \tau)^2 & 6(1 + \tau)^2 \\ (-3 + \tau)^2 & 3(1 + \tau)^2 & (-3 + \tau)^2 & 6(3 + \tau)^2 & 6(1 + \tau)^2 \\ (-3 + \tau)^2 & 3(1 + \tau)^2 & (-3 + \tau)^2 & 6(3 + \tau)^2 & 6(1 + \tau)^2 \\ (-3 + \tau)^2 & 3(1 + \tau)^2 & (-3 + \tau)^2 & 6(3 + \tau)^2 & 6(1 + \tau)^2 \\ (-3 + \tau)^2 & 3(1 + \tau)^2 & (-3 + \tau)^2 & 6(3 + \tau)^2 & 6(1 + \tau)^2 \end{bmatrix}$$

For τ=1/2, [-7, -45, -28, -60] . FixedPtCheck, [7, 45, 28, 60]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	3 vs 3	3 vs 3	2 vs 2	2 vs 2

bi =

$$\begin{bmatrix} [0, 0, 3/4, 1/4], [0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4], [0, 1/4, 3/4, 0] \\ [3/20, 11/20, 3/20, -9/20], [-1/20, 3/20, 19/20, 3/20], [3/20, -9/20, 3/20, 11/20] \\ [-1/2, -2/3, 4/3], [-1/2, -2/3, 4/3], [0, 5/6, -2/3], [1, -1/6, -2/3] \\ [3/2, 1/2, 3, 1], [9/4, 1/4, 9/4, 5/4], [27/16, 5/16, 45/16, 19/16] \end{bmatrix} \times \begin{bmatrix} [19/20, 3/20, -1/20, 3/20], \\ [3/20, 11/20, 3/20, -9/20], [-1/20, 3/20, 19/20, 3/20], [3/20, -9/20, 3/20, 11/20] \\ [3/2, 1/2, 3, 1], [9/4, 1/4, 9/4, 5/4], [27/16, 5/16, 45/16, 19/16] \end{bmatrix} =$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4}}, net cycles: 1 . order: 2

$$[0, y_1, 0, y_2]$$

$$R = \begin{bmatrix} [0, 0, 0, 1], [0, 0, 0, 1], [0, 0, 0, 1], [0, 1, 0, 0] \\ [1/3, -1/6], [1/3, -1/6], [1/3, -1/6], [-1/6, 1/3] \end{bmatrix} \times \begin{bmatrix} [0, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 0], [0, 0, 0, 1] \\ [0, 2, 0, 4], [0, 4, 0, 2] \end{bmatrix}$$

Omega Rank for B : cycles: {{1, 3}}, net cycles: 1 . order: 2

$$[y_1, 0, y_2, 0]$$

$$B = \begin{bmatrix} [0, 0, 1, 0], [0, 0, 1, 0], [1, 0, 0, 0], [0, 0, 1, 0] \\ [1/3, -1/6], [1/3, -1/6], [-1/6, 1/3], [1/3, -1/6] \end{bmatrix} \times \begin{bmatrix} [1, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 0] \\ [2, 0, 4, 0], [4, 0, 2, 0] \end{bmatrix}$$

Â» SYNC'D 1/4 , 0.2500000000

6 . Coloring, {2, 4}

**R:** [4, 4, 1, 3]    **B:** [3, 3, 4, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

' [ '1' (' 1 + τ ' )' , -1' (' - 1 + τ ' )' , 2 , 2 ]'

For τ=1/2, [3, 1, 4, 4] . FixedPtCheck, [3, 1, 4, 4]

det(A + τ Δ) = 0

Delta Range : [-y<sub>1</sub> - y<sub>2</sub> - y<sub>3</sub>, y<sub>1</sub>, y<sub>2</sub>, y<sub>3</sub>]

[1, 1, 2, 2]

+            -            Δ

\$ [ [2, 0, 2, 2] , [1, 1, 2, 2] , [1, 1, 2, 2] ] \$    \$ [ [0, 2, 2, 2] , [1, 1, 2, 2] , [1, 1, 2, 2] ] \$    \$ [ [1, -1, 0, 0] , [0, 0, 0, 0] , [0, 0, 0, 0] ] \$

[y<sub>1</sub>, -y<sub>1</sub>, 0, 0]

p = s<sup>2</sup>

S+            S-            NM

\$ [ [0, 0, 1, 1] , [0, 0, 1, 1] , [0, 1, 0, 1] , [1, 0, 1, 0] ] \$    \$ [ [1, 1, 0, 0] , [1, 1, 0, 0] , [0, 0, 1, 0] , [0, 0, 0, 1] ] \$    \$ [ [2, 2, 2, 2] , [2, 2, 2, 2] , [1, 1, 4, 2] , [1, 1, 2, 4] ] \$

CmmCk true, true, true

p' = s<sup>2</sup>

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 vs 3	1 vs 3	1 vs 3	1 vs 3	1 vs 3

Omega Rank for R : cycles: {{1, 3, 4}}, net cycles: 1 . order: 3

\$ [ [2, 0, 2, 2] , [2, 0, 2, 2] , [2, 0, 2, 2] ] \$

[y<sub>1</sub>, 0, y<sub>1</sub>, y<sub>1</sub>]



$$p = -s + s^2 \quad p' = -s + s^3$$

Omega Rank for B : cycles:  $\{\{2, 3, 4\}\}$ , net cycles: 1 . order: 3

$$\$ [ [0, 2, 2, 2], [0, 2, 2, 2], [0, 2, 2, 2] ] \$$$

$$[0, y_1, y_1, y_1]$$

$$p = s - s^3 \quad p' = s - s^2$$

Â« NOT SYNC'D Â»

Nullspace of  $\{\Omega\Delta^i\}$  :

$$[0, x_1, x_2]$$

$$\text{For } A+2\Delta : [y_1, -3y_1 - 4y_2 - 4y_3, y_2, y_3]$$

$$\text{For } A-2\Delta : [-3y_1 - 4y_2 - 4y_3, y_1, y_2, y_3]$$

$$\text{Range of } \{\Omega\Delta^i\} : [-\mu_1, \mu_1, 0, 0]$$

rank of M is 3 , rank of N is 3

M            N

$$\$ [ [0, 0, 1, 1], [0, 0, 1, 1], [1, 1, 0, 2], [1, 1, 2, 0] ] \$ \quad \$ [ [0, 0, 1, 1], [0, 0, 1, 1], [1, 1, 0, 1], [1, 1, 1, 0] ] \$$$

Check is  $\Omega\Delta N$  zero? *true*,  $\pi\Delta = [1, -1, 0, 0]$

$$\ker M, [-\lambda_1, \lambda_1, 0, 0]$$

$$\text{Range } M, [x_1, x_1, x_2, x_3]$$

$$\tau = 6, r' = 2/3$$

Ranges

$$\text{Action of } R \text{ on ranges, } [[1], [1]]$$

$$\text{Action of } B \text{ on ranges, } [[2], [2]]$$

$$\beta(\{1, 3, 4\}) = 1/2$$

$$\beta(\{2, 3, 4\}) = 1/2$$

$$\ker N, [-\mu_1, \mu_1, 0, 0]$$

Range of N

$$[y_3, y_3, y_2, y_1]$$

Partitions

$$\alpha(\{1, 2\}, \{3\}, \{4\}) = 1/1$$

$$b1 = \{1, 2\}, \quad b2 = \{3\}, \quad b3 = \{4\}$$

Action of R and B on the blocks of the partitions:  $\$ [ [0, 1, 1], [1, 0, 1], [1, 1, 0] ] \$ = \$ [ [0, 1, 0], [0, 0, 1], [1, 0, 0] ] \$ + \$ [ [0, 0, 1], [1, 0, 0], [0, 1, 0] ] \$$   
 $[ '2', '3', '1' ], [ '3', '1', '2' ]$  with invariant measure  $[1, 1, 1]$

N by blocks, check: true . ' See partition graph.

' ' See level-3 partition graph.

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<b>Right Group</b>	
<b>Coloring</b>	{2, 4}
<b>Rank</b>	3
<b>R,B</b>	[4, 4, 1, 3], [3, 3, 4, 2]
$\pi_2$	[0, 1, 1, 1, 1, 2]
$u_2$	[0, 1, 1, 1, 1, 1] (dim 1)
<b>wpp</b>	[2, 2, 1, 1]
$\pi_3$	[0, 0, 1, 1]
$u_3$	[0, 0, 1, 1]

7 . Coloring, {3, 4}

**R:** [4, 3, 4, 3]    **B:** [3, 4, 1, 2]

' See graph

' ' See pair graph

,

$\Omega$  for  $A+\tau\Delta$  :

$$[ '3' ( ' - 1 + \tau ' ) , 3' ( ' - 1 + \tau ' ) , -6 , -6' ]'$$

For  $\tau=1/2$ , [-1, -1, -4, -4] . FixedPtCheck, [1, 1, 4, 4]

$$\det(A + \tau \Delta) = 1' ( ' - 1 + \tau ' ) ^ 2 ( ' \tau ' )'$$

Delta Range :  $[-y_1 - y_2 - y_3, y_1, y_2, y_3]$

$$[1, 1, 2, 2]$$

$$+ \quad - \quad \Delta$$

$$\$ [ [0, 0, 3, 3], [1, 1, 5, 5], [3, 3, 9, 9] ] \$ \quad \$ [ [2, 2, 1, 1], [3, 3, 3, 3], [5, 5, 7, 7] ] \$ \quad \$ [ [-1, -1, 1, 1], [-1, -1, 1, 1], [-1, -1, 1, 1] ] \$$$

$$[-y_1, -y_1, y_1, y_1]$$

$$p = s - 4s^3$$

$$S+ \quad S- \quad NM$$

$$\$ [ [1, 0, 3, 1], [0, 1, 1, 3], [2, 0, 2, 1], [0, 2, 1, 2] ] \$ \quad \$ [ [0, 1, 1, 3], [1, 0, 3, 1], [1, 1, 0, 3], [1, 1, 3, 0] ] \$ \quad \$ [ [1, 0, 2, 0], [0, 1, 0, 2], [1, 0, 2, 0], [0, 1, 0, 2] ] \$$$

CmmCk true, true, true

$$p' = s - 2s^2$$

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
1 vs 3	2 vs 4	2 vs 4	1 vs 2	2 vs 4

Omega Rank for R : cycles: {{3, 4}}, net cycles: 1 . order: 2

$$\$ [ [0, 0, 3, 3], [0, 0, 3, 3] ] \$$$

$$[0, 0, y_1, y_1]$$

$$p = -s + s^2$$

Omega Rank for B : cycles: {{1, 3}, {2, 4}}, net cycles: 2 . order: 2

$$\$ [ [2, 2, 1, 1], [1, 1, 2, 2], [2, 2, 1, 1], [1, 1, 2, 2] ] \$$$

$$[y_1, y_1, y_2, y_2]$$

$$p' = -s + s^3 \quad p' = -1 + s^2$$

$\hat{A} \ll \text{NOT SYNC'D } \hat{A} \gg$

Nullspace of  $\{\Omega\Delta^i\}$  :

$$[x_1, x_2, -4x_1 - 2x_2]$$

$$\text{For } A+2\Delta : [y_1, -y_1, -y_2, y_2]$$

$$\text{For } A-2\Delta : [-y_1, y_1, -y_2, y_2]$$

Range of  $\{\Omega\Delta^i\}$ :  $[\mu_1, \mu_1, -\mu_1, -\mu_1]$

rank of M is 4 , rank of N is 2

$$\begin{array}{cc} & \text{M} & \text{N} \\ \$ & [ [0, 1, 0, 0], [1, 0, 0, 0], [0, 0, 0, 2], [0, 0, 2, 0] ] \$ & [ [0, 1, 0, 1], [1, 0, 1, 0], [0, 1, 0, 1], [1, 0, 1, 0] ] \$ \end{array}$$

Check is  $\Omega\Delta N$  zero? *true*,  $\pi\Delta = [-1, -1, 1, 1]$

ker M,  $[0, 0, 0, 0]$

Range M,  $[x_1, x_2, x_3, x_4]$

$\tau = 8$ ,  $r' = 1/2$

Ranges

Action of R on ranges,  $[[2], [2]]$

Action of B on ranges,  $[[2], [1]]$

$\beta(\{1, 2\}) = 1/3$

$\beta(\{3, 4\}) = 2/3$

ker N,  $[\mu_1, \mu_2, -\mu_1, -\mu_2]$

Range of N

$[y_1, y_2, y_1, y_2]$

Partitions

$\alpha(\{\{1, 3\}, \{2, 4\}\}) = 1/1$

$b_1 = \{1, 3\}$ ,  $b_2 = \{2, 4\}$

Action of R and B on the blocks of the partitions:  $\$ [ [1, 1], [1, 1] ] \$ = \$ [ [0, 1], [1, 0] ] \$ + \$ [ [1, 0], [0, 1] ] \$$

$['2', '1'], ['1', '2']$  with invariant measure  $[1, 1]$

N by blocks, check: *true*. ' See partition graph.

' ' See level-2 partition graph.

'

Right Group	
<b>Coloring</b>	{3, 4}
<b>Rank</b>	2
<b>R,B</b>	[4, 3, 4, 3], [3, 4, 1, 2]
$\pi_2$	[1, 0, 0, 0, 0, 2]
$u_2$	[1, 0, 1, 1, 0, 1] (dim 1)
<b>wpp</b>	[2, 2, 2, 2]

8. Coloring, {2, 3, 4}

**R:** [4, 4, 4, 3]    **B:** [3, 3, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Omega$  for  $A+\tau\Delta$  :

‘ [ ‘3‘ (‘ - 1 +  $\tau$  ‘)‘ (‘ 3 +  $\tau^2$  ‘)‘ , -3‘ (‘ - 1 +  $\tau$  ‘)‘ (‘ 1 +  $\tau$  ‘)‘ (‘ - 3 +  $\tau$  ‘)‘ , -6‘ (‘ 3 +  $\tau^2$  ‘)‘ , 6‘ (‘ 1 +  $\tau$  ‘)‘ (‘ - 3 +  $\tau$  ‘)‘ ]‘

For  $\tau=1/2$ , [-13, -15, -52, -60] . FixedPtCheck, [13, 15, 52, 60]

$\det(A + \tau \Delta) = 0$

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	3 vs 3	3 vs 3	2 vs 2	3 vs 3

bi =

$\$ [ [0, 0, 3/4, 1/4], [0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4], [0, 3/4, 1/4, 0] ] \$ \times \$ [ [91/100, 3/100, -9/100, 27/100], [3/100, 99/100, 3/100, -9/100], [-9/100, 3/100, 91/100, 27/100], [27/100, -9/100, 27/100, 19/100] ] \$ =$

$\$ [ [0, 5/6, -2/3], [0, 5/6, -2/3], [-1/2, -2/3, 4/3], [1, -1/6, -2/3] ] \$ \times \$ [ [3/2, 3/2, 2, 1], [3/2, 3/4, 5/2, 5/4], [15/8, 15/16, 2, 19/16] ] \$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4}}, net cycles: 1 . order: 2

$$[0, 0, y_2, y_1]$$

$$R = \$ [ [0, 0, 0, 1], [0, 0, 0, 1], [0, 0, 0, 1], [0, 0, 1, 0] ] \$ \times \$ [ [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1] ] \$ = \$ [ [1/3, -1/6], [1/3, -1/6], [1/3, -1/6], [-1/6, 1/3] ] \$ \times \$ [ [0, 0, 2, 4], [0, 0, 4, 2] ] \$$$

Omega Rank for B : cycles: {{1, 3}}, net cycles: 0 . order: 2

$$[y_1, y_2, y_3, 0]$$

$$B = \$ [ [0, 0, 1, 0], [0, 0, 1, 0], [1, 0, 0, 0], [0, 1, 0, 0] ] \$ \times \$ [ [1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 0] ] \$ = \$ [ [0, 1/3, -1/6], [0, 1/3, -1/6], [0, -1/6, 1/3], [1/2, -1/6, -1/6] ] \$ \times \$ [ [2, 2, 2, 0], [2, 0, 4, 0], [4, 0, 2, 0] ] \$$$

Â» SYNC'D 1/4 , 0.2500000000

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SUMMARY	
<b>Graph Type</b>	CC
v(A)	1
v(Δ)	1
π	[1, 1, 2, 2]
<b>Dbly Stoch</b>	false

SANDWICH		Total 1
No .	Coloring	Rank
<b>1</b>	{}	2

<b>RT GROUPS</b>		Total 2	
<b>No .</b>	<b>Coloring</b>	<b>Rank</b>	<b>Solv</b>
<b>1</b>	{2, 4}	3	Solvable
<b>2</b>	{3, 4}	2	Solvable

<b><math>\Delta</math>-RANK'D</b>	<b>SC'D !RK'D</b>	<b><math>\tau</math>-RANK'D</b>	<b>R/B RANK'D</b>	<b>NOT SYNC'D</b>	<b>Total Runs</b>	<b><math>2^{n-1}</math></b>
5	0	5, 3	5, 3	3	8	8

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