New Graph

[3, 4, 4, 3, 2, 5], [6, 1, 2, 6, 1, 3]

π = [3, 4, 6, 5, 2, 4]

POSSIBLE RANKS

1 x 24
2 x 12
3 x 8
4 x 6

BASE DETERMINANT 55/256, .2148437500

NullSpace of Δ

{1, 2, 4}, {3, 5, 6}

Range of Δ: $[-\lambda_1 - \lambda_2, \lambda_1, -\lambda_3 - \lambda_4, \lambda_2, \lambda_3, \lambda_4]$

1 . Coloring, {}

R: [3, 4, 4, 3, 2, 5] **B**: [6, 1, 2, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+$\tau\Delta$:
‘[‘-12‘ (‘ 3 + 2τ + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 2 + τ + τ^2 ‘)‘ , -24‘ (‘ 3 + τ^2 ‘)‘ , 12‘ (‘ - 5 + τ - τ^2 + τ^3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-17, -44, -104, -111, -24, -32] . FixedPtCheck, [17, 44, 104, 111, 24, 32]

det(A + $\tau\,\Delta$) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 82/91, 0, 27/91, 3/91] , [0, 0, 0, 1, 0, 0] , [0, 0, 27/91, 0, 10/91, -9/91] , [0, 0, 3/91, 0, -9/91, 90/91]] $ =

$ [[0, 1/40, -1/4, -56/75, 76/75] , [0, -11/40, -3/4, 76/75, 4/75] , [-3/4, -47/40, 1/2, 22/75, 88/75] , [0, 1/40, -1/4, -56/75, 76/75] , [9/4, 77/40, 0, 58/75, -368/75] , [1/4, 41/40, 1/2, -26/75, -104/75]] $ x $ [[9/2, 5, 5, 5/2, 1, 6] , [9/2, 4, 25/4, 5/2, 3/2, 21/4] , [33/8, 81/16, 91/16, 41/16, 21/16, 21/4] , [153/32, 147/32, 359/64, 43/16, 21/16, 321/64] , [567/128, 1161/256, 1441/256, 653/256, 321/256, 717/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[0, y_1, y_2, y_3, y_4, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, -5/48, 7/48] , [0, 0, 7/48, -5/48] , [0, 0, 7/48, -5/48] , [0, 0, -5/48, 7/48] , [0, 1/4, -5/48, -5/48] , [1/4, -1/8, -5/48, 1/48]] $ x $ [[0, 2, 8, 10, 4, 0] , [0, 4, 10, 10, 0, 0] , [0, 0, 10, 14, 0, 0] , [0, 0, 14, 10, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 1 . order: 4

$$[y_4, y_3, y_2, 0, 0, y_1]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[19/96, 7/96, -5/96, -17/96] , [7/96, -5/96, -17/96, 19/96] , [-5/96, -17/96, 19/96, 7/96] , [19/96, 7/96, -5/96, -17/96] , [7/96, -5/96, -17/96, 19/96] , [-17/96, 19/96, 7/96, -5/96]] $ x $ [[6, 6, 4, 0, 0, 8] , [6, 4, 8, 0, 0, 6] , [4, 8, 6, 0, 0, 6] , [8, 6, 6, 0, 0, 4]] $

Â» SYNC'D 1/16 , 0.06250000000

2 . Coloring, {2}

R: [3, 1, 4, 3, 2, 5] **B**: [6, 4, 2, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘12‘ (‘ 3 + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ 2 + τ + τ ² ‘)‘‘ (‘ - 1 + τ ‘)‘ , -24‘ (‘ 3 + τ ² ‘)‘ , -12‘ (‘ 5 + 2τ ² + τ ⁴ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-39, -44, -104, -89, -24, -32] . FixedPtCheck, [39, 44, 104, 89, 24, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 82/91, 0, 27/91, 3/91] , [0, 0, 0, 1, 0, 0] , [0, 0, 27/91, 0, 10/91, -9/91] , [0, 0, 3/91, 0, -9/91, 90/91]] $ =

$ [[0, 1/40, -1/4, -56/75, 76/75] , [0, -11/40, -3/4, 76/75, 4/75] , [-3/4, -47/40, 1/2, 22/75, 88/75] , [0, 1/40, -1/4, -56/75, 76/75] , [9/4, 77/40, 0, 58/75, -368/75] , [1/4, 41/40, 1/2, -26/75, -104/75]] $ x $ [[5/2, 5, 5, 9/2, 1, 6] , [2, 4, 25/4, 5, 3/2, 21/4] , [17/8, 81/16, 91/16, 73/16, 21/16, 21/4] , [9/4, 147/32, 359/64, 167/32, 21/16, 321/64] , [273/128, 1161/256, 1441/256, 1241/256, 321/256, 717/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, y_5, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 0, 7/48, -5/48] , [0, 0, 1/4, -5/48, -5/48] , [0, 0, 0, -5/48, 7/48] , [0, 0, 0, 7/48, -5/48] , [0, 1/4, -1/8, -5/48, 1/48] , [1/4, -1/8, -3/16, 1/48, 1/12]] $ x $ [[4, 2, 8, 6, 4, 0] , [2, 4, 10, 8, 0, 0] , [4, 0, 10, 10, 0, 0] , [0, 0, 14, 10, 0, 0] , [0, 0, 10, 14, 0, 0]] $

Omega Rank for B : cycles: {{2, 3, 4, 6}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 7/96, -5/96, -17/96, 19/96] , [0, -5/96, -17/96, 19/96, 7/96] , [0, -17/96, 19/96, 7/96, -5/96] , [0, 7/96, -5/96, -17/96, 19/96] , [1/2, -5/96, -17/96, 19/96, -41/96] , [0, 19/96, 7/96, -5/96, -17/96]] $ x $ [[2, 6, 4, 4, 0, 8] , [0, 4, 8, 6, 0, 6] , [0, 8, 6, 4, 0, 6] , [0, 6, 6, 8, 0, 4] , [0, 6, 4, 6, 0, 8]] $

Â» SYNC'D 1/16 , 0.06250000000

3 . Coloring, {3}

R: [3, 4, 2, 3, 2, 5] **B**: [6, 1, 4, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘1‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 4‘ (‘ 1 + τ ‘)‘ , 2‘ (‘ 3 + τ ^{2} ‘)‘ , -1‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘ , -2‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -4‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [15, 48, 52, 49, 12, 16] . FixedPtCheck, [15, 48, 52, 49, 12, 16]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 82/91, 0, 27/91, 3/91] , [0, 0, 0, 1, 0, 0] , [0, 0, 27/91, 0, 10/91, -9/91] , [0, 0, 3/91, 0, -9/91, 90/91]] $ =
$ [[0, 101/192, -55/576, 5/36, -19/36] , [0, -355/192, -223/576, -19/36, 101/36] , [-3/4, -469/192, -409/576, 11/36, 131/36] , [0, 101/192, -55/576, 5/36, -19/36] , [9/4, 1451/192, 1319/576, 11/36, -445/36] , [1/4, 131/192, 287/576, -13/36, -37/36]] $ x $ [[9/2, 2, 5, 11/2, 1, 6] , [9/4, 3/2, 7, 17/4, 3/2, 15/2] , [9/4, 17/8, 29/4, 45/8, 15/8, 39/8] , [3, 73/32, 45/8, 191/32, 39/32, 189/32] , [21/8, 219/128, 427/64, 613/128, 189/128, 861/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[0, y_1, y_2, y_3, y_4, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, -5/72, 1/72, 7/72] , [0, 1/72, 7/72, -5/72] , [0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [0, 7/72, -5/72, 1/72] , [1/4, -5/72, 1/72, -11/72]] $ x $ [[0, 8, 8, 4, 4, 0] , [0, 12, 4, 8, 0, 0] , [0, 4, 8, 12, 0, 0] , [0, 8, 12, 4, 0, 0]] $

Omega Rank for B : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_3, 0, y_1, y_2, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 7/72, -5/72, 1/72] , [1/6, -5/72, 1/72, -5/72] , [0, -5/72, 1/72, 7/72] , [0, 7/72, -5/72,

1/72] , [1/6, -5/72, 1/72, -5/72] , [0, 1/72, 7/72, -5/72]] $ x $ [[6, 0, 4, 6, 0, 8] , [0, 0, 8, 4, 0, 12] , [0, 0, 12, 8, 0, 4] , [0, 0, 4, 12, 0, 8]] $

Â» SYNC'D 5/8 , 0.6250000000

4 . Coloring, {4}

R: [3, 4, 4, 6, 2, 5] **B**: [6, 1, 2, 3, 1, 3]

' See graph

' ' See pair graph

'

Ω for A+τΔ :
'['24' (' - 1 + τ ')'' (' 1 + τ 2 ')'' (' 3 + τ ')' , -24' (' 4 - τ + 3τ 2 + τ 3 + τ 4 ')' , 24' (' - 1 + τ ')'' (' 3 + τ ')'' (' 2 + τ + τ 2 ')' , 24' (' 1 + τ ')'' (' - 5 + τ - τ 2 + τ 3 ')' , 24' (' - 2 - τ - 2τ 2 + τ 3 ')'' (' 1 + τ ')' , 48' (' - 2 - τ - 2τ 2 + τ 3 ')''] '

For τ=1/2, [-35, -71, -77, -111, -69, -92] . FixedPtCheck, [35, 71, 77, 111, 69, 92]

det(A + τ Δ) = 1' (' - 1 + τ ')'' (' τ ')'' (' 1 + τ ')' 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =
$ [[-2641/16440, -3779/8220, -869/4110, 337/685, -992/2055, 592/685] , [-603/5480, -397/2740, 809/4110, -1411/2055, 64/685, 1424/2055] , [483/5480, 3263/4110, 148/2055, 101/2055, -24/685, -1904/2055] , [457/1096, 479/548, 935/822, 251/411, -320/411, -304/137] , [-1277/5480, -3529/1370, -5677/2055, -1043/685, 1688/2055, 12976/2055] , [-309/5480, -693/1370, -128/685, 497/2055, 1816/2055, -688/2055]] $ x $ [[9/2, 5, 15/2, 5/2, 1, 7/2] , [9/2, 47/8, 45/8, 25/8, 7/8, 4] , [81/16, 71/16, 207/32, 23/8, 1, 133/32] , [261/64, 653/128, 837/128, 349/128, 133/128, 289/64] , [1179/256, 661/128, 3303/512, 745/256, 289/256, 1915/512] , [4833/1024, 10487/2048, 12573/2048, 5947/2048, 1915/2048, 2141/512]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4, 5, 6}}, net cycles: 0 . order: 4

$$[0, y_3, y_4, y_5, y_1, y_2]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[1/3, -1/96, -5/96, 5/32, -37/96] , [0, -5/96, -1/96, -5/96, 5/32] , [0, -5/96, -1/96, -5/96, 5/32] , [0, 5/32, -5/96, -1/96, -5/96] , [0, -1/96, -5/96, 5/32, -5/96] , [0, -5/96, 5/32, -5/96, -1/96]] $ x $ [[0, 2, 3, 10, 4, 5] , [0, 4, 0, 5, 5, 10] , [0, 5, 0, 4, 10, 5] , [0, 10, 0, 5, 5, 4] , [0, 5, 0, 10, 4, 5]] $

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 1 . order: 4

$$[y_1, y_2, y_3, 0, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-11/96, -1/32, 5/96, 13/96] , [-1/32, 5/96, 13/96, -11/96] , [5/96, 13/96, -11/96, -1/32] , [13/96, -11/96, -1/32, 5/96] , [-1/32, 5/96, 13/96, -11/96] , [13/96, -11/96, -1/32, 5/96]] $ x $ [[6, 6, 9, 0, 0, 3] , [6, 9, 3, 0, 0, 6] , [9, 3, 6, 0, 0, 6] , [3, 6, 6, 0, 0, 9]] $

Â» SYNC'D 45/256 , 0.1757812500

5 . Coloring, {5}

R: [3, 4, 4, 3, 1, 5] **B**: [6, 1, 2, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘3‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 12‘ (‘ - 1 + τ ‘)‘ , -6‘ (‘ 3 + τ 2 ‘)‘ , -3‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ 2 ‘)‘ , 6‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 12‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-13, -16, -52, -51, -12, -16] . FixedPtCheck, [13, 16, 52, 51, 12, 16]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2, y_1, -y_3 - y_4, y_2, y_3, y_4]$

[3, 4, 6, 5, 2, 4]

+ \ ; - \ ; Δ

$ [[2, 0, 8, 10, 4, 0] , [6, 2, 10, 4, 0, 2] , [3, 3, 8, 6, 1, 3] , [6, 7, 14, 11, 3, 7]] $ $ [[4, 8, 4, 0, 0, 8] , [0, 6, 2, 6, 4, 6] , [3, 5, 4, 4, 3, 5] , [6, 9, 10, 9, 5, 9]] $ $ [[-1, -4, 2, 5, 2, -4] , [3, -2, 4, -1, -2, -2] , [0, -1, 2, 1, -1, -1] , [0, -1, 2, 1, -1, -1]] $

$$[-y_3 - y_1, y_3, -y_3 - y_2, y_1, y_2, y_3]$$

p = s 3 - 2s 4

S+ \ ; S- \ ; NM

$ [[2, 2, 3, 2, 1, 2] , [2, 3, 2, 3, 1, 1] , [1, 2, 3, 3, 1, 2] , [2, 2, 3, 2, 1, 2] , [1, 2, 3, 3, 1, 2] , [1, 1, 4, 2, 1, 3]] $ $ [[2, 1, 3, 3, 0, 3] , [2, 3, 2, 3, 1, 1] , [1, 3, 3, 2, 2, 1] , [2, 1, 3, 3, 0, 3] , [1, 3, 3, 2, 2, 1] , [1, 1, 4, 2, 1, 3]] $ $ [[9, 4, 0, 15, 0, 8] , [3, 12, 12, 5, 4, 0] , [0, 8, 18, 0, 6, 4] , [9, 4, 0, 15, 0, 8] , [0, 8, 18, 0, 6, 4] , [6, 0, 6, 10, 2, 12]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 4

$ [[2, 0, 8, 10, 4, 0] , [4, 0, 12, 8, 0, 0] , [0, 0, 12, 12, 0, 0] , [0, 0, 12, 12, 0, 0]] $

$$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3, 0]$$

p = s 3 - s 4

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 1 . order: 4

$ [[4, 8, 4, 0, 0, 8] , [8, 4, 8, 0, 0, 4] , [4, 8, 4, 0, 0, 8] , [8, 4, 8, 0, 0, 4]] $

$$[y_1, y_2, y_1, 0, 0, y_2]$$

p’ = s - s 3 p = s - s 3

Â« NOT SYNC’D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[0, 0, x_1, -2\,x_1]$
For A+2Δ : $[y_1, 9\,y_1 + 9\,y_2 - y_3, -4\,y_1 - 3\,y_2, y_1, y_2, y_3]$
For A-2Δ : $[y_1, y_3, -3\,y_3 - y_1 - 3\,y_2, y_1, -y_1 + 9\,y_3 + 9\,y_2, y_2]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_3 - \mu_1, \mu_3, -\mu_3 - \mu_2, \mu_1, \mu_2, \mu_3]$

rank of M is 6 , rank of N is 3

M N

$ [[0, 0, 3, 0, 0, 0] , [0, 0, 0, 0, 0, 4] , [3, 0, 0, 3, 0, 0] , [0, 0, 3, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 4, 0, 0, 0, 0]] $ $ [[0, 2, 3, 0, 3, 1] , [2, 0, 1, 2, 1, 3] , [3, 1, 0, 3, 0, 2] , [0, 2, 3, 0, 3, 1] , [3, 1, 0, 3, 0, 2] , [1, 3, 2, 1, 2, 0]] $

Check is ΩΔN zero? *true,* πΔ= [-1, -4, 2, 5, 2, -4]

ker M, [0, 0, 0, 0, 0, 0]
Range M, $[x_1, x_2, x_3, x_4, x_5, x_6]$

τ= 18 , r'= 1/2

Ranges

Action of R on ranges, [[3], [4], [3], [1]]
Action of B on ranges, [[2], [1], [2], [2]]
β({1, 3}) = 1/4
β({2, 6}) = 1/3
β({3, 4}) = 1/4
β({4, 5}) = 1/6

ker N, $[\mu_1 + \mu_3 - \mu_2, -\mu_1 - \mu_3, \mu_1, \mu_2, \mu_3, -\mu_1 - \mu_3]$
Range of N
$[y_1, y_2 + y_1 - y_3, y_2, y_1, y_2, y_3]$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 4, 6}, {2, 3, 5}]) = 2/3
α([{1, 2, 4}, {3, 5, 6}]) = 1/3
b1 = {1, 2, 4} ‘ , ‘ b2 = {1, 4, 6} ‘ , ‘ b3 = {2, 3, 5} ‘ , ‘ b4 = {3, 5, 6}

Action of R and B on the blocks of the partitions: $ [[0, 0, 2, 0] , [1, 0, 1, 0] , [0, 1, 0, 1] , [0, 2, 0, 0]] $ = $ [[0, 0, 1, 0] , [0, 0, 1, 0] , [0, 1, 0, 0] , [0, 1, 0, 0]] $ + $ [[0, 0, 1, 0] , [1, 0, 0, 0] , [0, 0, 0, 1] , [0, 1, 0, 0]] $
[‘3‘, ‘3‘, ‘2‘, ‘2‘], [‘3‘, ‘1‘, ‘4‘, ‘2‘] with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{5}
Rank	2
R,B	[3, 4, 4, 3, 1, 5], [6, 1, 2, 6, 2, 3]
π_2	[0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]
$\mathbf{u}_2$	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

6 . Coloring, {6}

R: [3, 4, 4, 3, 2, 3] **B**: [6, 1, 2, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-12‘ (‘ - 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , 24‘ (‘ - 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 12‘ (‘ 5 - 4τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 24‘ (‘ - 1 + τ ‘)‘ ^ 2 , -48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [11, 36, 120, 117, 8, 32] . FixedPtCheck, [11, 36, 120, 117, 8, 32]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 5	5 *vs* 5	3 *vs* 3	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =
$ [[0, 15/56, 31/28, -8/21, -20/21] , [0, 27/56, -37/84, -20/21, 20/21] , [1/4, -79/168, -107/126, 10/21, 40/63] , [0, 15/56, 31/28, -8/21, -20/21] , [-1/12, -113/504, -116/189, -10/21, 272/189] , [-1/12, -101/504, -73/378, 26/21, -136/189]] $ x $ [[9/2, 5, 3, 5/2, 3, 6] , [6, 3, 13/4, 2, 9/2, 21/4] , [45/8, 57/16, 53/16, 25/16, 63/16, 6] , [45/8, 111/32, 211/64, 55/32, 9/2, 345/64] , [765/128, 921/256, 815/256, 433/256, 1035/256, 705/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 2

$$[0, y_1, y_2, y_3, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, -5/48, 7/48] , [0, 7/48, -5/48] , [0, 7/48, -5/48] , [0, -5/48, 7/48] , [1/2, -5/48, -17/48] , [0, -5/48, 7/48]] $ x $ [[0, 2, 12, 10, 0, 0] , [0, 0, 10, 14, 0, 0] , [0, 0, 14, 10, 0, 0]] $

Omega Rank for B : cycles: {{1, 5, 6}}, net cycles: 0 . order: 3

$$[y_4, y_3, 0, 0, y_2, y_1]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, -11/72, 13/72, 1/72] , [0, 13/72, 1/72, -11/72] , [1/6, 1/72, -11/72, 1/72] , [0, -11/72, 13/72, 1/72] , [0, 13/72, 1/72, -11/72] , [0, 1/72, -11/72, 13/72]] $ x $ [[6, 6, 0, 0, 4, 8] , [10, 0, 0, 0, 8, 6] , [8, 0, 0, 0, 6, 10] , [6, 0, 0, 0, 10, 8]] $

Â» SYNC'D 5/32 , 0.1562500000

7 . Coloring, {2, 3}

R: [3, 1, 2, 3, 2, 5] **B**: [6, 4, 4, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘1‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 4‘ (‘ 1 + τ ‘)‘ , 2‘ (‘ 3 + τ ^{2} ‘)‘ , -1‘ (‘ 5 + 2τ + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -2‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -4‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [39, 48, 52, 25, 12, 16] . FixedPtCheck, [39, 48, 52, 25, 12, 16]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 82/91, 0, 27/91, 3/91] , [0, 0, 0, 1, 0, 0] , [0, 0, 27/91, 0, 10/91, -9/91] , [0, 0, 3/91, 0, -9/91, 90/91]] $ =

$ [[0, 101/192, -55/576, 5/36, -19/36] , [0, -355/192, -223/576, -19/36, 101/36] , [-3/4, -469/192, -409/576, 11/36, 131/36] , [0, 101/192, -55/576, 5/36, -19/36] , [9/4, 1451/192, 1319/576, 11/36, -445/36] , [1/4, 131/192, 287/576, -13/36, -37/36]] $ x $ [[5/2, 2, 5, 15/2, 1, 6] , [5/4, 3/2, 7, 21/4, 3/2, 15/2] , [3/2, 17/8, 29/4, 51/8, 15/8, 39/8] , [31/16, 73/32, 45/8, 225/32, 39/32, 189/32] , [95/64, 219/128, 427/64, 759/128, 189/128, 861/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_2, y_1, y_3, 0, y_4, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, -5/72, 1/72, 7/72] , [0, 1/72, 7/72, -5/72] , [0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [0, 7/72, -5/72, 1/72] , [1/4, -5/72, 1/72, -11/72]] $ x $ [[4, 8, 8, 0, 4, 0] , [8, 12, 4, 0, 0, 0] , [12, 4, 8, 0, 0, 0] , [4, 8, 12, 0, 0, 0]] $

Omega Rank for B : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, y_2, y_3, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [0, -5/72, 1/72, 7/72] , [0, 7/72, -5/72, 1/72] , [1/2, -5/72, 1/72, -29/72] , [0, 1/72, 7/72, -5/72]] $ x $ [[2, 0, 4, 10, 0, 8] , [0, 0, 8, 4, 0, 12] , [0, 0, 12, 8, 0, 4] , [0, 0, 4, 12, 0, 8]] $

Â» SYNC'D 5/8 , 0.6250000000

8 . Coloring, {2, 4}

R: [3, 1, 4, 6, 2, 5] **B**: [6, 4, 2, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ 3 - τ + τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ 4 - τ + 3τ ^ 2 + τ ^ 3 + τ ^ 4 ‘)‘ , 24‘ (‘ 6 - τ + τ ^ 2 + τ ^ 3 + τ ^ 4 ‘)‘ , 24‘ (‘ 5 + 2τ ^ 2 + τ ^ 4 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 2 - τ + τ ^ 2 ‘)‘ , 48‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ ^ 2 ‘)‘‘]‘

For τ=1/2, [69, 71, 95, 89, 63, 84] . FixedPtCheck, [69, 71, 95, 89, 63, 84]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ 1 + 3τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	6 *vs* 6	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-62393/181368, 90185/90684, 42937/45342, 50201/22671, -60208/22671, -25264/22671] , [-30269/181368, 11639/90684, 47827/45342, -10111/22671, -12448/22671, 464/22671] , [-161/181368, 30959/90684, -30049/22671, -15190/22671, -1696/22671, 40160/22671] , [20527/181368, -50299/90684, -6263/45342, 15557/22671, 6704/22671, -8176/22671] , [14945/16488, -2561/8244, -4220/2061, -6506/2061, -1712/2061, 11296/2061] , [14791/181368, -48757/90684, 32183/22671, 11582/22671, 61184/22671, -93664/22671]] $ x $ [[5/2, 5, 15/2, 9/2, 1, 7/2] , [2, 47/8, 53/8, 45/8, 7/8, 3] , [17/8, 83/16, 223/32, 97/16, 3/4, 93/32] , [119/64, 693/128, 929/128, 721/128, 93/128, 199/64] , [243/128, 45/8, 3595/512, 47/8, 199/256, 1435/512] , [2037/1024, 11183/2048, 14301/2048, 12235/2048, 1435/2048, 1481/512]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}}, net cycles: 1 . order: 6

$$[y_1, y_2, y_3, y_4, y_5, y_6]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-527/2736, 421/2736, -287/2736, -347/2736, 529/2736, 325/2736] , [421/2736, -287/2736, -347/2736, 529/2736, 325/2736, -527/2736] , [325/2736, -527/2736, 421/2736, -287/2736, -347/2736, 529/2736] , [529/2736, 325/2736, -527/2736, 421/2736, -287/2736, -347/2736] , [-287/2736, -347/2736, 529/2736, 325/2736, -527/2736, 421/2736] , [-347/2736, 529/2736, 325/2736, -527/2736, 421/2736, -287/2736]] $ x $ [[4, 2, 3, 6, 4, 5] , [2, 4, 4, 3, 5, 6] , [4, 5, 2, 4, 6, 3] , [5, 6, 4, 2, 3, 4] , [6, 3, 5, 4, 4, 2] , [3, 4, 6, 5, 2, 4]] $

Omega Rank for B : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[y_4, y_3, y_2, y_1, 0, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 1/2, 25/72, -23/72, -35/72] , [0, 0, 25/72, -23/72, 1/72] , [0, 0, -23/72, 1/72, 25/72] , [0, 0, 1/72, 25/72, -23/72] , [1/2, -3/4, -23/72, -35/72, 79/72] , [0, 0, 1/72, 25/72, -23/72]] $ x $ [[2, 6, 9, 4, 0, 3] , [0, 9, 7, 6, 0, 2] , [0, 7, 8, 9, 0, 0] , [0, 8, 9, 7, 0, 0] , [0, 9, 7, 8, 0, 0]] $

Â» SYNC'D 33/256 , 0.1289062500

9 . Coloring, {2, 5}

R: [3, 1, 4, 3, 1, 5] **B**: [6, 4, 2, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘3‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 12‘ (‘ - 1 + τ ‘)‘ , -6‘ (‘ 3 + τ 2 ‘)‘ , -3‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , 6‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 12‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-21, -16, -52, -43, -12, -16] . FixedPtCheck, [21, 16, 52, 43, 12, 16]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2, y_1, -y_3 - y_4, y_2, y_3, y_4]$

[3, 4, 6, 5, 2, 4]

+ \ ; - \ ; Δ

$ [[6, 0, 8, 6, 4, 0] , [2, 2, 10, 8, 0, 2] , [1, 3, 8, 8, 1, 3] , [4, 7, 14, 13, 3, 7]] $ $ [[0, 8, 4, 4, 0, 8] , [4, 6, 2, 2, 4, 6] , [5, 5, 4, 2, 3, 5] , [8, 9, 10, 7, 5, 9]] $ $ [[3, -4, 2, 1, 2, -4] , [-1, -2, 4, 3, -2, -2] , [-2, -1, 2, 3, -1, -1] , [-2, -1, 2, 3, -1, -1]] $

$[-y_3 - y_1, y_3, -y_3 - y_2, y_1, y_2, y_3]$

p = s 3 - 2s 4
S+ \ ; S- \ ; NM
$ [[1, 2, 3, 3, 1, 2] , [2, 3, 2, 3, 1, 1] , [2, 2, 3, 2, 1, 2] , [1, 2, 3, 3, 1, 2] , [2, 2, 3, 2, 1, 2] , [1, 1, 4, 2, 1, 3]] $ $ [[3, 1, 3, 2, 0, 3] , [2, 3, 2, 3, 1, 1] , [0, 3, 3, 3, 2, 1] , [3, 1, 3, 2, 0, 3] , [0, 3, 3, 3, 2, 1] , [1, 1, 4, 2, 1, 3]] $ $ [[9, 4, 0, 15, 0, 8] , [3, 12, 12, 5, 4, 0] , [0, 8, 18, 0, 6, 4] , [9, 4, 0, 15, 0, 8] , [0, 8, 18, 0, 6, 4] , [6, 0, 6, 10, 2, 12]] $
CmmCk *true, true, true*

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$ [[6, 0, 8, 6, 4, 0] , [4, 0, 12, 8, 0, 0] , [0, 0, 12, 12, 0, 0] , [0, 0, 12, 12, 0, 0]] $$

$$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{2, 3, 4, 6}}, net cycles: 1 . order: 4

$$ [[0, 8, 4, 4, 0, 8] , [0, 4, 8, 8, 0, 4] , [0, 8, 4, 4, 0, 8] , [0, 4, 8, 8, 0, 4]] $$

$$[0, y_2, y_1, y_1, 0, y_2]$$

$p = - s + s^3$ p' = $- s + s^3$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[0, 0, x_1, -2\, x_1]$
For A+2Δ : $[y_3, y_2, -4\, y_3 - 3\, y_1, y_3, y_1, 9\, y_3 - y_2 + 9\, y_1]$
For A-2Δ : $[y_2, y_1, -3\, y_1 - y_2 - 3\, y_3, y_2, 9\, y_3 - y_2 + 9\, y_1, y_3]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_3 - \mu_1, \mu_3, -\mu_3 - \mu_2, \mu_1, \mu_2, \mu_3]$

rank of M is 6 , rank of N is 3

M N

$ [[0, 0, 1, 0, 2, 0] , [0, 0, 0, 0, 0, 4] , [1, 0, 0, 5, 0, 0] , [0, 0, 5, 0, 0, 0] , [2, 0, 0, 0, 0, 0] , [0, 4, 0, 0, 0, 0]]
$ $ [[0, 2, 3, 0, 3, 1] , [2, 0, 1, 2, 1, 3] , [3, 1, 0, 3, 0, 2] , [0, 2, 3, 0, 3, 1] , [3, 1, 0, 3, 0, 2] , [1, 3, 2, 1, 2, 0]] $

Check is ΩΔN zero? *true,* $\pi\Delta = [3, -4, 2, 1, 2, -4]$

ker M, [0, 0, 0, 0, 0, 0]
Range M, $[x_6, x_5, x_4, x_3, x_2, x_1]$

$\tau = 18$, r' = 1/2

Ranges

Action of R on ranges, [[4], [1], [2], [4]]
Action of B on ranges, [[3], [3], [4], [3]]
β({1, 3}) = 1/12
β({1, 5}) = 1/6
β({2, 6}) = 1/3
β({3, 4}) = 5/12

ker N, $[\mu_3, \mu_2, \mu_1, -\mu_3 - \mu_2, -\mu_2 - \mu_1, \mu_2]$
Range of N
$[y_1, y_2 + y_1 - y_3, y_2, y_1, y_2, y_3]$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 4, 6}, {2, 3, 5}]) = 2/3
α([{1, 2, 4}, {3, 5, 6}]) = 1/3
b1 = {1, 2, 4} ‘ , ‘ b2 = {1, 4, 6} ‘ , ‘ b3 = {2, 3, 5} ‘ , ‘ b4 = {3, 5, 6}

Action of R and B on the blocks of the partitions: $ [[0, 0, 2, 0] , [1, 0, 1, 0] , [0, 1, 0, 1] , [0, 2, 0, 0]] $ = $ [[0, 0, 1, 0] , [0, 0, 1, 0] , [0, 1, 0, 0] , [0, 1, 0, 0]] $ + $ [[0, 0, 1, 0] , [1, 0, 0, 0] , [0, 0, 0, 1] , [0, 1, 0, 0]] $
[‘3‘, ‘3‘, ‘2‘, ‘2‘], [‘3‘, ‘1‘, ‘4‘, ‘2‘] with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 5}
Rank	2
R,B	[3, 1, 4, 3, 1, 5], [6, 4, 2, 6, 2, 3]
π_2	[0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0, 0]
$\mathbf{u}_2$	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

10 . Coloring, {2, 6}

R: [3, 1, 4, 3, 2, 3] **B**: [6, 4, 2, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘12‘ (‘ - 3 - τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -12‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ ^ 2 + τ ^ 3 ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘ ^ 2 , -48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [29, 36, 120, 99, 8, 32] . FixedPtCheck, [29, 36, 120, 99, 8, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 5	5 *vs* 5	4 *vs* 4	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =
$ [[0, 15/56, 31/28, -8/21, -20/21] , [0, 27/56, -37/84, -20/21, 20/21] , [1/4, -79/168, -107/126, 10/21, 40/63] , [0, 15/56, 31/28, -8/21, -20/21] , [-1/12, -113/504, -116/189, -10/21, 272/189] , [-1/12, -101/504, -73/378, 26/21, -136/189]] $ x $ [[5/2, 5, 3, 9/2, 3, 6] , [7/2, 3, 13/4, 9/2, 9/2, 21/4] , [33/8, 57/16, 53/16, 49/16, 63/16, 6] , [123/32, 111/32, 211/64, 7/2, 9/2, 345/64] , [543/128, 921/256, 815/256, 877/256, 1035/256, 705/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 7/48, -5/48] , [0, 1/2, -5/48, -17/48] , [0, 0, -5/48, 7/48] , [0, 0, 7/48, -5/48] , [1/2, -1, -17/48, 43/48] , [0, 0, 7/48, -5/48]] $ x $ [[4, 2, 12, 6, 0, 0] , [2, 0, 10, 12, 0, 0] , [0, 0, 14, 10, 0, 0] , [0, 0, 10, 14, 0, 0]] $

Omega Rank for B : cycles: {{1, 5, 6}}, net cycles: 0 . order: 3

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 13/72, 1/72, -11/72] , [0, 1/6, 1/72, -11/72, 1/72] , [1/6, -1/9, -11/72, 1/72, 1/8] , [0, 0, 13/72, 1/72, -11/72] , [0, 0, 1/72, -11/72, 13/72] , [0, 0, -11/72, 13/72, 1/72]] $ x $ [[2, 6, 0, 4, 4, 8] , [4, 0, 0, 6, 8, 6] , [8, 0, 0, 0, 6, 10] , [6, 0, 0, 0, 10, 8] , [10, 0, 0, 0, 8, 6]] $

Â» SYNC'D 5/32 , 0.1562500000

11 . Coloring, {3, 4}

R: [3, 4, 2, 6, 2, 5] **B**: [6, 1, 4, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 4 - τ + τ ^{3} ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ‘)‘‘ (‘ 2 + τ + τ ^{2} ‘)‘ , 24‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘ , -24‘ (‘ 2 - τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} , -48‘ (‘ 2 - τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-6, -15, -11, -14, -9, -12] . FixedPtCheck, [6, 15, 11, 14, 9, 12]

det(A + τ Δ) = 1‘ (‘ - 1 + τ ‘)‘‘ (‘ τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-863/34104, 3511/8526, -529/4263, -271/1421, 304/4263, -144/1421] , [6527/11368, -675/1421, -1800/1421, -6733/4263, -2656/4263, 4848/1421] , [123/1624, -307/2436, -340/609, -148/609, -160/203, 1024/609] , [-2773/11368, 289/2842, 859/1421, 885/1421, 4880/4263, -9328/4263] , [-2087/11368, 2391/5684, 6203/4263, 12308/4263, 1104/1421, -7552/1421] , [-251/11368, 101/5684, 3044/4263, -192/1421, -96/1421, -1984/4263]] $ x $ [[9/2, 2, 15/2, 11/2, 1, 7/2] , [9/4, 17/8, 63/8, 49/8, 7/8, 19/4] , [9/4, 35/16, 279/32, 103/16, 19/16, 103/32] , [81/32, 317/128, 999/128, 907/128, 103/128, 211/64] , [315/128, 551/256, 4311/512, 1657/256, 211/256, 1879/512] , [1143/512, 4733/2048, 16839/2048,

14035/2048, 1879/2048, 3547/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4, 5, 6}}, net cycles: 0 . order: 4

$$[0, y_1, y_2, y_3, y_4, y_5]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[1/3, 5/96, -19/96, 7/32, -35/96] , [0, 7/32, -1/32, 5/96, -19/96] , [0, -1/32, 5/96, -19/96, 7/32] , [0, -19/96, 7/32, -1/32, 5/96] , [0, -1/32, 5/96, -19/96, 7/32] , [0, 5/96, -19/96, 7/32, -1/32]] $ x $ [[0, 8, 3, 4, 4, 5] , [0, 7, 0, 8, 5, 4] , [0, 5, 0, 7, 4, 8] , [0, 4, 0, 5, 8, 7] , [0, 8, 0, 4, 7, 5]] $

Omega Rank for B : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[y_4, 0, y_3, y_2, 0, y_1]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 1/6, -1/16, -1/16] , [1/6, -1/12, -1/16, 1/48] , [0, 0, -1/16, 5/48] , [0, 0, 5/48, -1/16] , [1/6, -1/12, -1/16, 1/48] , [0, 0, 5/48, -1/16]] $ x $ [[6, 0, 9, 6, 0, 3] , [0, 0, 9, 9, 0, 6] , [0, 0, 15, 9, 0, 0] , [0, 0, 9, 15, 0, 0]] $

Â» SYNC'D 45/256 , 0.1757812500

12 . Coloring, {3, 5}

R: [3, 4, 2, 3, 1, 5] **B**: [6, 1, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-12‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ^{2} ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ ^{2} ‘)‘ , 24‘ (‘ 3 + τ ^{2} ‘)‘ , 12‘ (‘ 5 + 2τ ^{2} + τ ^{4} ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , -48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [39, 84, 104, 89, 24, 32] . FixedPtCheck, [39, 84, 104, 89, 24, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	4 *vs* 5	5 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, y_4, y_5, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, 0, 13/72, 1/72, -11/72] , [0, 0, 1/72, -11/72, 13/72] , [0, 0, -11/72, 13/72, 1/72] , [0, 0, 13/72, 1/72, -11/72] , [0, 1/4, 1/72, -11/72, -5/72] , [1/4, -1/8, -11/72, -5/72, 5/36]] $ x $ [[2, 6, 8, 4, 4, 0] , [4, 8, 6, 6, 0, 0] , [0, 6, 10, 8, 0, 0] , [0, 10, 8, 6, 0, 0] , [0, 8, 6, 10, 0, 0]] $

Omega Rank for B : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, -11/72, 1/72, 13/72] , [0, 1/2, 1/72, 13/72, -47/72] , [0, 0, 1/72, 13/72, -11/72] , [0, 0, -11/72, 1/72, 13/72] , [1/2, -1, 13/72, -47/72, 73/72] , [0, 0, 13/72, -11/72, 1/72]] $ x $ [[4, 2, 4, 6, 0, 8] , [2, 0, 8, 4, 0, 10] , [0, 0, 10, 8, 0, 6] , [0, 0, 6, 10, 0, 8] , [0, 0, 8, 6, 0, 10]] $

Â» SYNC'D 3/16 , 0.1875000000

13 . Coloring, {3, 6}

R: [3, 4, 2, 3, 2, 3] **B**: [6, 1, 4, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-1‘ (‘ 3 + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 4‘ (‘ 1 + τ ‘)‘ , -2‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 1‘ (‘ 5 - 2τ + τ ^{2} ‘)‘‘ (‘ 1 + τ ‘)‘ , 2‘ (‘ - 1 + τ ‘)‘ ^{2} , -4‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [13, 48, 60, 51, 4, 16] . FixedPtCheck, [13, 48, 60, 51, 4, 16]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =

$ [[0, 101/192, -55/576, 5/36, -19/36] , [0, -355/192, -223/576, -19/36, 101/36] , [1/4, 57/64, 167/576, 11/36, -61/36] , [0, 101/192, -55/576, 5/36, -19/36] , [-1/12, -127/576, -25/576, 11/36, 1/12] , [-1/12, -247/576, 95/576, -13/36, 3/4]] $ x $ [[9/2, 2, 3, 11/2, 3, 6] , [15/4, 3/2, 4, 11/4, 9/2, 15/2] , [9/2, 17/8, 7/2, 27/8, 45/8, 39/8] , [93/16, 73/32, 51/16, 101/32, 117/32, 189/32] , [285/64, 219/128, 119/32, 379/128, 567/128, 861/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 1 . order: 3

$$[0, y_1, y_2, y_3, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[7/72, -5/72, 1/72] , [-5/72, 1/72, 7/72] , [1/72, 7/72, -5/72] , [7/72, -5/72, 1/72] , [1/72, 7/72, -5/72] , [7/72, -5/72, 1/72]] $ x $ [[0, 8, 12, 4, 0, 0] , [0, 12, 4, 8, 0, 0] , [0, 4, 8, 12, 0, 0]] $

Omega Rank for B : cycles: {{1, 5, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, 0, y_2, y_3, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [1/6, -5/72, 1/72, -5/72] , [0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [0, 1/72, 7/72, -5/72]] $ x $ [[6, 0, 0, 6, 4, 8] , [4, 0, 0, 0, 8, 12] , [8, 0, 0, 0, 12, 4] , [12, 0, 0, 0, 4, 8]] $

Â» SYNC’D 5/8 , 0.6250000000

14 . Coloring, {4, 5}

R: [3, 4, 4, 6, 1, 5] **B**: [6, 1, 2, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-24‘ (‘ 3 - τ + 5τ ^{2} + τ ^{3} ‘)‘ , 24‘ (‘ 4 + τ + 2τ ^{2} + τ ^{3} ‘)‘‘ (‘ - 1 + τ ‘)‘ , -24‘ (‘ 6 - τ + τ ^{2} + τ ^{3} + τ ^{4} ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 2 - τ - 2τ ^{2} + τ ^{3} ‘)‘ , 48‘ (‘ - 2 - τ - 2τ ^{2} + τ ^{3} ‘)‘‘]‘

For τ=1/2, [-62, -41, -95, -102, -69, -92] . FixedPtCheck, [62, 41, 95, 102, 69, 92]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-100397/741352, -256535/556014, -171433/278007, 132965/278007, -174832/278007, 390800/278007] , [-96265/741352, -46615/278007, 127199/278007, -153349/278007, 149648/278007, -29200/278007] , [66007/741352, 401203/556014, -137885/556014, 99308/278007, -165352/278007, -78784/278007] , [308491/741352, 482635/556014, 273089/278007, 66101/278007, -110800/278007, -573808/278007] , [-146309/741352, -677194/278007, -1331117/556014, -223366/278007, 75800/278007, 1556768/278007] , [-54569/741352, -242273/556014, 192415/556014, -66280/278007, 330104/278007, -206848/278007]] $ x $ [[7/2, 6, 15/2, 5/2, 1, 7/2] , [19/4, 51/8, 43/8, 27/8, 7/8, 13/4] , [5, 75/16, 197/32, 47/16, 13/16, 141/32] , [119/32, 669/128, 865/128, 347/128, 141/128, 287/64] , [537/128, 1509/256, 3239/512, 767/256, 287/256, 1775/512] , [2407/512, 11439/2048, 12075/2048, 6257/2048, 1775/2048, 3989/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-2309/47784, -1229/47784, 691/47784, -1205/47784, 6043/47784] , [6043/47784, -2309/47784, -1229/47784, 691/47784, -1205/47784] , [6043/47784, -2309/47784, -1229/47784, 691/47784, -1205/47784] , [-1205/47784, 6043/47784, -2309/47784, -1229/47784, 691/47784] , [-1229/47784, 691/47784, -1205/47784, 6043/47784, -2309/47784] , [691/47784, -1205/47784, 6043/47784, -2309/47784, -1229/47784]] $ x $ [[2, 0, 3, 10, 4, 5] , [4, 0, 2, 3, 5, 10] , [5, 0, 4, 2, 10, 3] , [10, 0, 5, 4, 3, 2] , [3, 0, 10, 5, 2, 4]] $

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 1 . order: 4

$$[y_1, y_3, y_2, 0, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-79/480, 41/480, -31/480, 89/480] , [41/480, -31/480, 89/480, -79/480] , [-31/480, 89/480, -79/480, 41/480] , [89/480, -79/480, 41/480, -31/480] , [-31/480, 89/480, -79/480, 41/480] , [89/480, -79/480, 41/480, -31/480]] $ x $ [[4, 8, 9, 0, 0, 3] , [8, 9, 3, 0, 0, 4] , [9, 3, 4, 0, 0, 8] , [3, 4, 8, 0,

0, 9]] $

Â» SYNC’D 189/512 , 0.3691406250

15 . Coloring, {4, 6}

R: [3, 4, 4, 6, 2, 3] **B**: [6, 1, 2, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘24‘ (‘ 3 + 2τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 , -24‘ (‘ 1 + τ ‘)‘‘ (‘ 4 - τ + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ 6 - 3τ + τ ^ 3 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘ ^ 2 ‘ (‘ 5 - 4τ + τ ^ 2 ‘)‘ , 24‘ (‘ - 2 - τ - 2τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -48‘ (‘ - 2 - τ - 2τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [17, 45, 111, 117, 23, 92] . FixedPtCheck, [17, 45, 111, 117, 23, 92]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 6	6 *vs* 6	4 *vs* 4	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =
$ [[-10781/326200, 7071/163100, -51137/244650, 95063/366975, -282736/366975, 275888/366975] , [-7127/46600, 1907/23300, 28871/34950, -21379/52425, 37088/52425, -53104/52425] , [-11141/326200, 91081/163100, -72566/122325, 169118/366975, -226096/366975, 97568/366975] , [5011/13048, -2981/6524, 47/9786, -5653/14679, -1168/14679, 8432/14679] , [37839/326200, -26149/163100, -67511/122325, -208822/366975, 113984/366975, 328928/366975] , [-19277/326200, -49043/163100, 59998/122325, 105746/366975, 271088/366975, -409504/366975]] $ x $ [[9/2, 5, 11/2, 5/2, 3, 7/2] , [6, 39/8, 31/8, 21/8, 21/8, 4] , [45/8, 57/16, 143/32, 35/16, 3, 165/32] , [315/64, 525/128, 555/128, 257/128, 495/128, 305/64] , [765/128, 135/32, 2011/512, 135/64, 915/256, 2147/512] , [5985/1024, 7863/2048, 8447/2048, 4171/2048, 6441/2048, 2565/512]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[0, y_1, y_2, y_3, 0, y_4]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, -65/504, 55/504, 31/504] , [0, 31/504, -65/504, 55/504] , [0, 31/504, -65/504, 55/504] , [0, 55/504, 31/504, -65/504] , [1/2, -65/504, 55/504, -221/504] , [0, -65/504, 55/504, 31/504]] $ x $ [[0, 2, 7, 10, 0, 5] , [0, 0, 5, 9, 0, 10] , [0, 0, 10, 5, 0, 9] , [0, 0, 9, 10, 0, 5]] $

Omega Rank for B : cycles: {{1, 5, 6}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 13/72, 1/72, -11/72] , [0, 0, 1/72, -11/72, 13/72] , [0, 1/5, -11/72, 13/72, -67/360] , [1/5, -6/25, 13/72, -67/360, 157/1800] , [0, 0, 1/72, -11/72, 13/72] , [0, 0, -11/72, 13/72, 1/72]] $ x $ [[6, 6, 5, 0, 4, 3] , [10, 5, 0, 0, 3, 6] , [8, 0, 0, 0, 6, 10] , [6, 0, 0, 0, 10, 8] , [10, 0, 0, 0, 8, 6]] $

Â» SYNC'D 343/1024 , 0.3349609375

16 . Coloring, {5, 6}

R: [3, 4, 4, 3, 1, 3] **B**: [6, 1, 2, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-3‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ 2 , 12‘ (‘ - 1 + τ ‘)‘ , 6‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 3‘ (‘ - 5 + τ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , -6‘ (‘ - 1 + τ ‘)‘ 2 , 12‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-7, -16, -60, -57, -4, -16] . FixedPtCheck, [7, 16, 60, 57, 4, 16]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2, y_1, -y_3 - y_4, y_2, y_3, y_4]$

[3, 4, 6, 5, 2, 4]

+ \ ; - \ ; Δ

$ [[2, 0, 12, 10, 0, 0] , [4, 2, 6, 6, 4, 2] , [5, 3, 6, 4, 3, 3] , [8, 7, 12, 9, 5, 7]] $ $ [[4, 8, 0, 0, 4, 8] , [2, 6, 6, 4, 0, 6] , [1, 5, 6, 6, 1, 5] , [4, 9, 12, 11, 3, 9]] $ $ [[-1, -4, 6, 5, -2, -4] , [1, -2, 0, 1, 2, -2] , [2, -1, 0, -1, 1, -1] , [2, -1, 0, -1, 1, -1]] $

$$[y_2, y_3, -y_3 - y_1, -y_2 - y_3, y_1, y_3]$$

$p = s^3 - 2s^4$

S+ \; S- \; NM

$ [[3, 2, 4, 1, 0, 2] , [2, 3, 2, 3, 1, 1] , [0, 1, 1, 2, 1, 1] , [3, 2, 4, 1, 0, 2] , [0, 1, 1, 2, 1, 1] , [1, 1, 4, 2, 1, 3]] $ $ [[1, 1, 2, 4, 1, 3] , [2, 3, 2, 3, 1, 1] , [2, 3, 4, 1, 1, 1] , [1, 1, 2, 4, 1, 3] , [2, 3, 4, 1, 1, 1] , [1, 1, 4, 2, 1, 3]] $ $ [[9, 4, 0, 15, 0, 8] , [3, 12, 12, 5, 4, 0] , [0, 8, 18, 0, 6, 4] , [9, 4, 0, 15, 0, 8] , [0, 8, 18, 0, 6, 4] , [6, 0, 6, 10, 2, 12]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	3 *vs* 5	3 *vs* 5	2 *vs* 3	2 *vs* 4

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 2

$ [[2, 0, 12, 10, 0, 0] , [0, 0, 12, 12, 0, 0] , [0, 0, 12, 12, 0, 0]] $

$$[y_1 - y_2, 0, y_1, y_2, 0, 0]$$

$p = -s^2 + s^3$

Omega Rank for B : cycles: {{1, 2, 5, 6}}, net cycles: 1 . order: 4

$ [[4, 8, 0, 0, 4, 8] , [8, 4, 0, 0, 8, 4] , [4, 8, 0, 0, 4, 8] , [8, 4, 0, 0, 8, 4]] $

$$[y_2, y_1, 0, 0, y_2, y_1]$$

$p' = -s + s^3$ $p = -s + s^3$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:
$[0, 0, x_1, -2\,x_1]$
For A+2Δ : $[y_1, -3\,y_2 - 3\,y_1 - y_3, y_2, y_1, -4\,y_1 - 3\,y_2, y_3]$
For A-2Δ : $[y_1, y_1 + y_3 - y_2, -4\,y_1 - 3\,y_3, y_1, y_3, y_2]$

Range of $\{\Omega\Delta^i\}$: $[\mu_2, \mu_3, -\mu_3 - \mu_1, -\mu_2 - \mu_3, \mu_1, \mu_3]$

rank of M is 6 , rank of N is 3

M N

$ [[0, 0, 1, 0, 2, 0] , [0, 0, 0, 0, 0, 4] , [1, 0, 0, 5, 0, 0] , [0, 0, 5, 0, 0, 0] , [2, 0, 0, 0, 0, 0] , [0, 4, 0, 0, 0, 0]] $ $ [[0, 2, 3, 0, 3, 1] , [2, 0, 1, 2, 1, 3] , [3, 1, 0, 3, 0, 2] , [0, 2, 3, 0, 3, 1] , [3, 1, 0, 3, 0, 2] , [1, 3, 2, 1, 2, 0]] $

Check is ΩΔN zero? *true,* πΔ= [-1, -4, 6, 5, -2, -4]

ker M, [0, 0, 0, 0, 0, 0]
Range M, $[x_1, x_2, x_3, x_4, x_5, x_6]$

τ= 18 , r'= 1/2

Ranges

Action of R on ranges, [[4], [1], [4], [4]]
Action of B on ranges, [[3], [3], [2], [3]]
β({1, 3}) = 1/12
β({1, 5}) = 1/6
β({2, 6}) = 1/3
β({3, 4}) = 5/12

ker N, $[-\mu_2 - \mu_3, \mu_3, \mu_1, \mu_2, -\mu_3 - \mu_1, \mu_3]$
Range of N
$[y_3, y_1, y_2, y_3, y_2, y_3 - y_1 + y_2]$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 4, 6}, {2, 3, 5}]) = 2/3
α([{1, 2, 4}, {3, 5, 6}]) = 1/3
b1 = {1, 2, 4} ‘ , ‘ b2 = {1, 4, 6} ‘ , ‘ b3 = {2, 3, 5} ‘ , ‘ b4 = {3, 5, 6}

Action of R and B on the blocks of the partitions: $ [[0, 0, 2, 0] , [1, 0, 1, 0] , [0, 1, 0, 1] , [0, 2, 0, 0]] $ = $ [[0, 0, 1, 0] , [0, 0, 1, 0] , [0, 1, 0, 0] , [0, 1, 0, 0]] $ + $ [[0, 0, 1, 0] , [1, 0, 0, 0] , [0, 0, 0, 1] , [0, 1, 0, 0]] $
[‘3‘, ‘3‘, ‘2‘, ‘2‘], [‘3‘, ‘1‘, ‘4‘, ‘2‘] with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{5, 6}
Rank	2
R,B	[3, 4, 4, 3, 1, 3], [6, 1, 2, 6, 2, 5]
π_2	[0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0, 0]
$\mathbf{u}_2$	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

17 . Coloring, {2, 3, 4}

R: [3, 1, 2, 6, 2, 5] **B**: [6, 4, 4, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ - 3 + τ ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 4 - τ + τ 3 ‘)‘ , -24‘ (‘ 6 - τ + τ 2 + τ 3 + τ 4 ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + 2τ + τ 2 ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 2 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2 , 48‘ (‘ - 1 + τ ‘)‘‘ (‘ 2 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [-18, -21, -19, -10, -9, -12] . FixedPtCheck, [18, 21, 19, 10, 9, 12]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-12871/76536, 1553/19134, 3212/3189, 38113/28701, 118576/28701, -181904/28701] , [29453/76536, -6686/9567, -1426/3189, -25469/28701, 46768/28701, 1744/28701] , [-17215/76536, -4874/9567, 10075/6378, 51808/28701, 92680/28701, -167552/28701] , [4145/76536, 9191/19134, -4018/3189, -28799/28701, -72176/28701, 122992/28701] , [-1843/76536, 10025/19134, 1375/6378,

80014/28701, -135992/28701, 36640/28701] , [20897/76536, 5164/9567, -7721/6378, -84836/28701, -116504/28701, 213952/28701]] $ x $ [[5/2, 2, 15/2, 15/2, 1, 7/2] , [5/4, 17/8, 71/8, 57/8, 7/8, 15/4] , [19/16, 39/16, 271/32, 33/4, 15/16, 87/32] , [21/16, 301/128, 1091/128, 1047/128, 87/128, 189/64] , [281/256, 589/256, 4443/512, 261/32, 189/256, 1551/512] , [289/256, 4821/2048, 17743/2048, 16863/2048, 1551/2048, 2931/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_5, y_4, y_3, 0, y_2, y_1]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/72, -23/72, 25/72] , [0, 0, -23/72, 25/72, 1/72] , [0, 0, 25/72, 1/72, -23/72] , [1/5, -4/25, -23/72, 53/360, 313/1800] , [0, 0, 25/72, 1/72, -23/72] , [0, 1/5, 1/72, -23/72, 53/360]] $ x $ [[4, 8, 3, 0, 4, 5] , [8, 7, 4, 0, 5, 0] , [7, 9, 8, 0, 0, 0] , [9, 8, 7, 0, 0, 0] , [8, 7, 9, 0, 0, 0]] $

Omega Rank for B : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[y_1, 0, y_2, y_3, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 1/2, 13/48, -35/48] , [0, 0, 13/48, -11/48] , [0, 0, 13/48, -11/48] , [0, 0, -11/48, 13/48] , [1/2, -3/4, -35/48, 49/48] , [0, 0, -11/48, 13/48]] $ x $ [[2, 0, 9, 10, 0, 3] , [0, 0, 13, 9, 0, 2] , [0, 0, 11, 13, 0, 0] , [0, 0, 13, 11, 0, 0]] $

Â» SYNC'D 81/1024 , 0.07910156250

18 . Coloring, {2, 3, 5}

R: [3, 1, 2, 3, 1, 5] **B**: [6, 4, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘12‘ (‘ 3 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , 24‘ (‘ 2 - τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ 3 + τ ^ 2 ‘)‘ , -12‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^ 2 + τ ^ 3 ‘)‘ , -24‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [81, 84, 104, 47, 24, 32] . FixedPtCheck, [81, 84, 104, 47, 24, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	4 *vs* 5	4 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, -11/72, 13/72, 1/72] , [0, 13/72, 1/72, -11/72] , [0, 1/72, -11/72, 13/72] , [0, -11/72, 13/72, 1/72] , [0, 13/72, 1/72, -11/72] , [1/4, 1/72, -11/72, -5/72]] $ x $ [[6, 6, 8, 0, 4, 0] , [10, 8, 6, 0, 0, 0] , [8, 6, 10, 0, 0, 0] , [6, 10, 8, 0, 0, 0]] $

Omega Rank for B : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[0, y_1, y_2, y_3, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 13/72, -11/72, 1/72] , [0, -11/72, 1/72, 13/72] , [0, -11/72, 1/72, 13/72] , [0, 13/72, -11/72, 1/72] , [1/2, 1/72, 13/72, -47/72] , [0, 1/72, 13/72, -11/72]] $ x $ [[0, 2, 4, 10, 0, 8] , [0, 0, 8, 6, 0, 10] , [0, 0, 10, 8, 0, 6] , [0, 0, 6, 10, 0, 8]] $

Â» SYNC'D 3/16 , 0.1875000000

19 . Coloring, {2, 3, 6}

R: [3, 1, 2, 3, 2, 3] **B**: [6, 4, 4, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-1‘ (‘ - 3 - τ - 5τ ^{2} + τ ^{3} ‘)‘ , 4‘ (‘ 1 + τ ‘)‘ , -2‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 1‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 2‘ (‘ - 1 + τ ‘)‘ ^{2} , -4‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [37, 48, 60, 27, 4, 16] . FixedPtCheck, [37, 48, 60, 27, 4, 16]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =

$ [[0, 101/192, -55/576, 5/36, -19/36] , [0, -355/192, -223/576, -19/36, 101/36] , [1/4, 57/64, 167/576, 11/36, -61/36] , [0, 101/192, -55/576, 5/36, -19/36] , [-1/12, -127/576, -25/576, 11/36, 1/12] , [-1/12, -247/576, 95/576, -13/36, 3/4]] $ x $ [[5/2, 2, 3, 15/2, 3, 6] , [11/4, 3/2, 4, 15/4, 9/2, 15/2] , [15/4, 17/8, 7/2, 33/8, 45/8, 39/8] , [19/4, 73/32, 51/16, 135/32, 117/32, 189/32] , [53/16, 219/128, 119/32, 525/128, 567/128, 861/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_1, y_2, y_3, 0, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[7/72, -5/72, 1/72] , [-5/72, 1/72, 7/72] , [1/72, 7/72, -5/72] , [7/72, -5/72, 1/72] , [1/72, 7/72, -5/72] , [7/72, -5/72, 1/72]] $ x $ [[4, 8, 12, 0, 0, 0] , [8, 12, 4, 0, 0, 0] , [12, 4, 8, 0, 0, 0]] $

Omega Rank for B : cycles: {{1, 5, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, 0, y_2, y_3, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 7/72, -5/72, 1/72] , [1/10, -5/72, 1/72, -1/360] , [1/10, -5/72, 1/72, -1/360] , [0, 7/72, -5/72, 1/72] , [0, -5/72, 1/72, 7/72] , [0, 1/72, 7/72, -5/72]] $ x $ [[2, 0, 0, 10, 4, 8] , [4, 0, 0, 0, 8, 12] , [8, 0, 0, 0, 12, 4] , [12, 0, 0, 0, 4, 8]] $

Â» SYNC'D 5/8 , 0.6250000000

20 . Coloring, {2, 4, 5}

R: [3, 1, 4, 6, 1, 5] **B**: [6, 4, 2, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-24‘ (‘ 1 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 24‘ (‘ 4 + τ + 2τ 2 + τ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ - 6 + τ - 3τ 2 - τ 3 + τ 4 ‘)‘ , -24‘ (‘ 5 - τ + 3τ 2 + τ 3 ‘)‘ , -24‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 2 - τ + τ 2 ‘)‘ , -48‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-78, -41, -101, -86, -63, -84] . FixedPtCheck, [78, 41, 101, 86, 63, 84]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ 2

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 3/4, 0, 1/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[1673/824, -1795/618, -5288/309, -605/309, -4304/309, 10480/309] , [-967/4120, 1127/1545, 1601/1545, 659/1545, -1024/1545, -1936/1545] , [3403/4120, -10483/6180, -11521/1545, -3022/1545, -1232/309, 22112/1545] , [-2627/4120, 413/618, 1918/309, 1691/1545, 7568/1545, -18832/1545] , [-1193/4120, 2951/6180, 12032/1545, -410/309, 11648/1545, -21856/1545] , [-5501/4120, 18041/6180, 17507/1545, 5054/1545, 2224/309, -36064/1545]] $ x $ [[3/2, 6, 15/2, 9/2, 1, 7/2] , [7/4, 51/8, 51/8, 51/8, 7/8, 9/4] , [29/16, 87/16, 221/32, 51/8, 9/16, 93/32] , [3/2, 717/128, 949/128, 743/128, 93/128, 189/64] , [405/256, 1563/256, 3555/512, 775/128, 189/256, 1319/512] , [219/128, 11799/2048, 14067/2048, 12933/2048, 1319/2048, 2765/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[y_1, 0, y_5, y_4, y_3, y_2]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-113/264, -89/264, 7/264, 127/264, 79/264] , [-89/264, 7/264, 127/264, 79/264, -113/264] , [79/264, -113/264, -89/264, 7/264, 127/264] , [127/264, 79/264, -113/264, -89/264, 7/264] , [-89/264, 7/264, 127/264, 79/264, -113/264] , [7/264, 127/264, 79/264, -113/264, -89/264]] $ x $ [[6, 0, 3, 6, 4, 5] , [4, 0, 6, 3, 5, 6] , [5, 0, 4, 6, 6, 3] , [6, 0, 5, 4, 3, 6] , [3, 0, 6, 5, 6, 4]] $

Omega Rank for B : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[0, y_1, y_2, y_3, 0, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0,

0, 0, 1]] $ = $ [[1/3, 1/72, 25/72, -47/72] , [0, 1/72, 25/72, -23/72] , [0, 25/72, -23/72, 1/72] , [0, -23/72, 1/72, 25/72] , [0, 25/72, -23/72, 1/72] , [0, -23/72, 1/72, 25/72]] $ x $ [[0, 8, 9, 4, 0, 3] , [0, 9, 7, 8, 0, 0] , [0, 7, 8, 9, 0, 0] , [0, 8, 9, 7, 0, 0]] $

Â» SYNC'D 85/1024 , 0.08300781250

21 . Coloring, {2, 4, 6}

R: [3, 1, 4, 6, 2, 3] **B**: [6, 4, 2, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ 3 + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ 4 - τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ - 2 + τ ‘)‘‘ (‘ 3 + τ 2 ‘)‘ , 24‘ (‘ - 5 + 3τ - 3τ 2 + τ 3 ‘)‘ , 24‘ (‘ 2 - τ + τ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -48‘ (‘ 2 - τ + τ 2 ‘)‘‘]‘

For τ=1/2, [-13, -15, -39, -33, -7, -28] . FixedPtCheck, [13, 15, 39, 33, 7, 28]

det(A + τ Δ) = 1‘ (‘ 1 + 3τ 2 ‘)‘‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 6	6 *vs* 6	5 *vs* 5	3 *vs* 6

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[2707/7992, -3805/3996, -5573/1998, -2173/2997, -2080/2997, 14576/2997] , [-5741/7992, 9503/3996, -791/1998, 4097/2997, -9760/2997, 2000/2997] , [14917/7992, 242/999, -2176/999, -7723/2997, -2968/2997, 11024/2997] , [-2069/7992, -2809/3996, 8443/1998, 1559/2997, 6656/2997, -17872/2997] , [-1211/7992, -2125/999, 209/999, -3745/2997, 11432/2997, -1360/2997] , [-13475/7992, -85/999, 368/999, 9041/2997, 1736/2997, -6448/2997]] $ x $ [[5/2, 5, 11/2, 9/2, 3, 7/2] , [7/2, 39/8, 39/8, 41/8, 21/8, 3] , [51/16, 69/16, 175/32, 39/8, 9/4, 125/32] , [177/64, 597/128, 695/128, 589/128, 375/128, 231/64] , [861/256, 615/128, 2583/512, 1243/256, 693/256, 1651/512] , [3309/1024, 9135/2048, 10831/2048, 9963/2048, 4953/2048, 1913/512]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_2, y_1, y_4, y_3, 0, y_5]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/72, -23/72, 25/72] , [0, 1/2, -23/72, 25/72, -35/72] , [0, 0, 25/72, 1/72, -23/72] , [0, 0, -23/72, 25/72, 1/72] , [1/2, -1, 25/72, -35/72, 49/72] , [0, 0, 1/72, -23/72, 25/72]] $ x $ [[4, 2, 7, 6, 0, 5] , [2, 0, 9, 7, 0, 6] , [0, 0, 8, 9, 0, 7] , [0, 0, 7, 8, 0, 9] , [0, 0, 9, 7, 0, 8]] $

Omega Rank for B : cycles: {{1, 5, 6}, {2, 3, 4}}, net cycles: 2 . order: 3

$ [[2, 6, 5, 4, 4, 3] , [4, 5, 4, 6, 3, 2] , [3, 4, 6, 5, 2, 4] , [2, 6, 5, 4, 4, 3] , [4, 5, 4, 6, 3, 2] , [3, 4, 6, 5, 2, 4]] $

$$[9\, y_1 - 11\, y_2 - 2\, y_3, 2\, y_1, 2\, y_1 - 2\, y_2 + 2\, y_3, 11\, y_1 - 13\, y_2 - 2\, y_3, 2\, y_2, 2\, y_3]$$

p’ = - 1 + s ^{3} p’ = - s + s ^{4} p’ = - s ^{2} + s ^{5}

Â» SYNC’D 3/128 , 0.02343750000

22 . Coloring, {2, 5, 6}

R: [3, 1, 4, 3, 1, 3] **B**: [6, 4, 2, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-3‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 12‘ (‘ - 1 + τ ‘)‘ , 6‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 3‘ (‘ - 5 - τ - 3τ ^{2} + τ ^{3} ‘)‘ , -6‘ (‘ - 1 + τ ‘)‘ ^{2} , 12‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-15, -16, -60, -49, -4, -16] . FixedPtCheck, [15, 16, 60, 49, 4, 16]

det(A + τ Δ) = 0

Delta Range : $[-y_1 - y_2, y_1, -y_3 - y_4, y_2, y_3, y_4]$

[3, 4, 6, 5, 2, 4]

+ \ ; - \ ; Δ

$ [[6, 0, 12, 6, 0, 0] , [0, 2, 6, 10, 4, 2] , [3, 3, 6, 6, 3, 3] , [6, 7, 12, 11, 5, 7]] $ $ [[0, 8, 0, 4, 4, 8] , [6, 6, 6, 0, 0, 6] , [3, 5, 6, 4, 1, 5] , [6, 9, 12, 9, 3, 9]] $ $ [[3, -4, 6, 1, -2, -4] , [-3, -2, 0, 5, 2, -2] , [0, -1, 0, 1, 1, -1] , [0, -1, 0, 1, 1, -1]] $

$$[-y_2 - y_3, y_2, -y_1 - y_2, y_3, y_1, y_2]$$

$p = s^3 - 2s^4$

S+ \ ; S- \ ; NM

$ [[1, 1, 2, 1, 0, 1] , [2, 3, 2, 3, 1, 1] , [1, 2, 2, 3, 2, 2] , [1, 1, 2, 1, 0, 1] , [1, 2, 2, 3, 2, 2] , [1, 1, 4, 2, 1, 3]] $ $ [[2, 1, 2, 3, 1, 3] , [2, 3, 2, 3, 1, 1] , [1, 3, 4, 2, 1, 1] , [2, 1, 2, 3, 1, 3] , [1, 3, 4, 2, 1, 1] , [1, 1, 4, 2, 1, 3]] $ $ [[9, 4, 0, 15, 0, 8] , [3, 12, 12, 5, 4, 0] , [0, 8, 18, 0, 6, 4] , [9, 4, 0, 15, 0, 8] , [0, 8, 18, 0, 6, 4] , [6, 0, 6, 10, 2, 12]] $

CmmCk *true, true, true*

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	3 *vs* 5	3 *vs* 5	2 *vs* 3	2 *vs* 4

Omega Rank for R : cycles: {{3, 4}}, net cycles: 0 . order: 2

$ [[6, 0, 12, 6, 0, 0] , [0, 0, 12, 12, 0, 0] , [0, 0, 12, 12, 0, 0]] $

$[y_1 - y_2, 0, y_1, y_2, 0, 0]$

$p = - s^2 + s^3$

Omega Rank for B : cycles: {{2, 4, 5, 6}}, net cycles: 1 . order: 4

$ [[0, 8, 0, 4, 4, 8] , [0, 4, 0, 8, 8, 4] , [0, 8, 0, 4, 4, 8] , [0, 4, 0, 8, 8, 4]] $

$[0, y_2, 0, y_1, y_1, y_2]$

$p = - s + s^3$ p’ = - s + s^3

Â« NOT SYNC’D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$[0, 0, x_1, -2\, x_1]$

For A+2Δ : $[y_2, y_3, y_1, y_2, -4\, y_2 - 3\, y_1, -y_3 - 3\, y_1 - 3\, y_2]$

For A-2Δ : $[y_3, y_2, -4\, y_3 - 3\, y_1, y_3, y_1, -y_2 + y_3 + y_1]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_3 - \mu_1, \mu_3, -\mu_3 - \mu_2, \mu_1, \mu_2, \mu_3]$

rank of M is 6 , rank of N is 3

M N

$ [[0, 0, 3, 0, 0, 0] , [0, 0, 0, 0, 0, 4] , [3, 0, 0, 3, 0, 0] , [0, 0, 3, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 4, 0, 0, 0, 0]] $ $ [[0, 2, 3, 0, 3, 1] , [2, 0, 1, 2, 1, 3] , [3, 1, 0, 3, 0, 2] , [0, 2, 3, 0, 3, 1] , [3, 1, 0, 3, 0, 2] , [1, 3, 2, 1, 2, 0]] $

Check is ΩΔN zero? *true*, πΔ= [3, -4, 6, 1, -2, -4]

ker M, [0, 0, 0, 0, 0, 0]
Range M, $[x_1, x_2, x_3, x_4, x_5, x_6]$

τ= 18 , r'= 1/2

Ranges

Action of R on ranges, [[3], [1], [3], [1]]
Action of B on ranges, [[2], [4], [2], [2]]
β({1, 3}) = 1/4
β({2, 6}) = 1/3
β({3, 4}) = 1/4
β({4, 5}) = 1/6

ker N, $[\mu_3, \mu_1, \mu_2, -\mu_3 - \mu_1, -\mu_2 - \mu_1, \mu_1]$
Range of N
$[y_1, y_1 + y_2 - y_3, y_2, y_1, y_2, y_3]$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 4, 6}, {2, 3, 5}]) = 2/3
α([{1, 2, 4}, {3, 5, 6}]) = 1/3
b1 = {1, 2, 4} ‘ , ‘ b2 = {1, 4, 6} ‘ , ‘ b3 = {2, 3, 5} ‘ , ‘ b4 = {3, 5, 6}

Action of R and B on the blocks of the partitions: $ [[0, 0, 2, 0] , [1, 0, 1, 0] , [0, 1, 0, 1] , [0, 2, 0, 0]] $ = $ [[0, 0, 1, 0] , [0, 0, 1, 0] , [0, 1, 0, 0] , [0, 1, 0, 0]] $ + $ [[0, 0, 1, 0] , [1, 0, 0, 0] , [0, 0, 0, 1] , [0, 1, 0, 0]] $
[‘3‘, ‘3‘, ‘2‘, ‘2‘], [‘3‘, ‘1‘, ‘4‘, ‘2‘] with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 5, 6}
Rank	2
R,B	[3, 1, 4, 3, 1, 3], [6, 4, 2, 6, 2, 5]
π_2	[0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]
$\mathbf{u}_2$	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

23 . Coloring, {3, 4, 5}

R: [3, 4, 2, 6, 1, 5] **B**: [6, 1, 4, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘24‘ (‘ 1 + τ ‘)‘‘ (‘ 3 - τ + τ 2 + τ 3 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ 4 - τ + τ 3 ‘)‘ , 24‘ (‘ 6 - τ + τ 2 + τ 3 + τ 4 ‘)‘ , 24‘ (‘ 5 + 2τ 2 + τ 4 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘ 2 ‘ (‘ 2 - τ + τ 2 ‘)‘ , 48‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ 2 ‘)‘‘]‘

For τ=1/2, [69, 87, 95, 89, 63, 84] . FixedPtCheck, [69, 87, 95, 89, 63, 84]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ 1 + 3τ 2 ‘)‘‘ (‘ τ ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	5 *vs* 6	6 *vs* 6	5 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}}, net cycles: 1 . order: 6

$$[y_1, y_2, y_3, y_4, y_5, y_6]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-17/144, -11/144, -5/144, 19/144, -11/144, 31/144] , [-11/144, 31/144, -17/144, -11/144, -5/144, 19/144] , [31/144, -17/144, -11/144, -5/144, 19/144, -11/144] , [19/144, -11/144, 31/144, -17/144, -11/144, -5/144] , [-11/144, -5/144, 19/144, -11/144, 31/144, -17/144] , [-5/144, 19/144, -11/144, 31/144,

-17/144, -11/144]] $ x $ [[2, 6, 3, 4, 4, 5] , [4, 3, 2, 6, 5, 4] , [5, 2, 4, 3, 4, 6] , [4, 4, 5, 2, 6, 3] , [6, 5, 4, 4, 3, 2] , [3, 4, 6, 5, 2, 4]] $

Omega Rank for B : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 1/2, 13/48, -35/48] , [0, 1/2, -1, -35/48, 61/48] , [0, 0, 0, 13/48, -11/48] , [0, 0, 0, -11/48, 13/48] , [1/2, -1, 5/4, 61/48, -95/48] , [0, 0, 0, -11/48, 13/48]] $ x $ [[4, 2, 9, 6, 0, 3] , [2, 0, 9, 9, 0, 4] , [0, 0, 13, 9, 0, 2] , [0, 0, 11, 13, 0, 0] , [0, 0, 13, 11, 0, 0]] $

Â» SYNC'D 9/128 , 0.07031250000

24 . Coloring, {3, 4, 6}

R: [3, 4, 2, 6, 2, 3] **B**: [6, 1, 4, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ 4 - 3τ + τ ^{2} ‘)‘ , 24‘ (‘ 6 - 3τ + τ ^{3} ‘)‘ , 24‘ (‘ 5 - 2τ + τ ^{2} ‘)‘ , -24‘ (‘ 2 - τ + τ ^{2} ‘)‘‘ (‘ - 1 + τ ‘)‘ , 48‘ (‘ 2 - τ + τ ^{2} ‘)‘‘]‘

For τ=1/2, [10, 33, 37, 34, 7, 28] . FixedPtCheck, [10, 33, 37, 34, 7, 28]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2}

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 4	4 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-261/2152, -67/1614, -496/807, -619/807, 240/269, 560/807] , [801/2152, -639/1076, -559/538, -47/269, 536/807, 656/807] , [-39/1076, 333/2152, 803/538, 907/1614, 512/807, -744/269] , [291/2152, 499/538, -19/807, -7/269, -416/269, 464/807] , [-50/269, -1541/2152, -162/269, 1375/1614, 584/807, -8/269] , [-57/1076, -879/2152, -659/1614, -785/1614, -192/269, 568/269]] $ x $ [[9/2, 2, 11/2, 11/2, 3, 7/2] , [15/4, 17/8, 49/8, 37/8, 21/8, 19/4] , [57/16, 35/16, 179/32, 41/8, 57/16, 127/32] , [69/16, 293/128, 733/128, 607/128, 381/128, 253/64] , [1011/256, 557/256, 2879/512, 623/128, 759/256, 2263/512] ,

[987/256, 4397/2048, 11761/2048, 9751/2048, 6789/2048, 4279/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4, 6}}, net cycles: 1 . order: 4

$$[0, y_1, y_2, y_3, 0, y_4]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-1/32, 5/96, -19/96, 7/32] , [-19/96, 7/32, -1/32, 5/96] , [7/32, -1/32, 5/96, -19/96] , [5/96, -19/96, 7/32, -1/32] , [7/32, -1/32, 5/96, -19/96] , [-1/32, 5/96, -19/96, 7/32]] $ x $ [[0, 8, 7, 4, 0, 5] , [0, 7, 5, 8, 0, 4] , [0, 5, 4, 7, 0, 8] , [0, 4, 8, 5, 0, 7]] $

Omega Rank for B : cycles: {{3, 4}, {1, 5, 6}}, net cycles: 2 . order: 6

$ [[6, 0, 5, 6, 4, 3] , [4, 0, 6, 5, 3, 6] , [3, 0, 5, 6, 6, 4] , [6, 0, 6, 5, 4, 3] , [4, 0, 5, 6, 3, 6]] $

$$[11 y_4, 0, 11 y_3, 11 y_1, 11 y_2, -11 y_4 + 13 y_3 + 13 y_1 - 11 y_2]$$

$p = s + s^2 - s^4 - s^5$

Â» SYNC'D 37/1024 , 0.03613281250

25 . Coloring, {3, 5, 6}

R: [3, 4, 2, 3, 1, 3] **B**: [6, 1, 4, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘-12‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 - τ - τ 2 + τ 3 ‘)‘ , 24‘ (‘ - 2 - τ - 2τ 2 + τ 3 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 12‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + 3τ - 3τ 2 + τ 3 ‘)‘ , -24‘ (‘ - 1 + τ ‘)‘ 2 , 48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-29, -92, -120, -99, -8, -32] . FixedPtCheck, [29, 92, 120, 99, 8, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =

$ [[0, 35/104, -19/78, 2/13, -8/39] , [0, -51/104, 1/52, -8/39, 28/39] , [1/4, -7/78, 35/312, 1/39, -10/39] , [0, 35/104, -19/78, 2/13, -8/39] , [-1/12, 47/468, -11/312, 19/39, -50/117] , [-1/12, -23/234, 33/104, -5/13, 34/117]] $ x $ [[7/2, 3, 3, 11/2, 3, 6] , [3, 3, 15/4, 3, 9/2, 27/4] , [27/8, 69/16, 51/16, 57/16, 81/16, 9/2] , [9/2, 147/32, 183/64, 111/32, 27/8, 333/64] , [549/128, 831/256, 843/256, 843/256, 999/256, 765/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[y_4, y_3, y_1, y_2, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[0, -1/24, -1/24, 1/8] , [0, -1/24, 1/8, -1/24] , [0, 1/8, -1/24, -1/24] , [0, -1/24, -1/24, 1/8] , [1/2, -1/24, 1/8, -13/24] , [0, -1/24, -1/24, 1/8]] $ x $ [[2, 6, 12, 4, 0, 0] , [0, 12, 6, 6, 0, 0] , [0, 6, 6, 12, 0, 0] , [0, 6, 12, 6, 0, 0]] $

Omega Rank for B : cycles: {{1, 2, 5, 6}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 11/96, -3/32, 1/32, -1/96] , [0, -3/32, 1/32, -1/96, 11/96] , [1/6, -3/32, 1/32, -1/96, -5/96] , [0, 11/96, -3/32, 1/32, -1/96] , [0, 1/32, -1/96, 11/96, -3/32] , [0, -1/96, 11/96, -3/32, 1/32]] $ x $ [[4, 2, 0, 6, 4, 8] , [2, 4, 0, 0, 8, 10] , [4, 8, 0, 0, 10, 2] , [8, 10, 0, 0, 2, 4] , [10, 2, 0, 0, 4, 8]] $

Â» SYNC’D 15/32 , 0.4687500000

26 . Coloring, {4, 5, 6}

R: [3, 4, 4, 6, 1, 3] **B**: [6, 1, 2, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-24‘ (‘ 3 + τ ^ 2 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ - 4 - τ + τ ^ 3 ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ - 2 + τ ‘)‘‘ (‘ 3 + τ ^ 2 ‘)‘ , -24‘ (‘ 1 + τ ‘)‘‘ (‘ - 5 + τ ^ 2 ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ - 2 - τ - 2τ ^ 2 + τ ^ 3 ‘)‘ , -48‘ (‘ - 2 - τ - 2τ ^ 2 + τ ^ 3 ‘)‘‘]‘

For τ=1/2, [26, 35, 117, 114, 23, 92] . FixedPtCheck, [26, 35, 117, 114, 23, 92]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 4	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[679/102344, -3473/76758, -25526/38379, 84509/115137, -127984/115137, 129296/115137] , [-18081/102344, 12295/76758, 39419/38379, -66073/115137, 101408/115137, -146896/115137] , [2377/102344, 78725/153516, -62113/76758, 63761/115137, -105736/115137, 78224/115137] , [34687/102344, -34643/76758, 9358/38379, -37567/115137, 9488/115137, 17744/115137] , [15601/102344, -41179/153516, -81443/76758, 3887/115137, -77848/115137, 214256/115137] , [-11567/102344, -30271/153516, 69947/76758, -47935/115137, 180248/115137, -196720/115137]] $ x $ [[7/2, 6, 11/2, 5/2, 3, 7/2] , [21/4, 51/8, 29/8, 23/8, 21/8, 13/4] , [87/16, 75/16, 137/32, 5/2, 39/16, 149/32] , [33/8, 645/128, 563/128, 287/128, 447/128, 301/64] , [1191/256, 1515/256, 1991/512, 151/64, 903/256, 1871/512] , [681/128, 11391/2048, 7877/2048, 5021/2048, 5613/2048, 4177/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, y_4, y_2, 0, y_3]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, -7/72, 17/72, -7/72] , [0, -7/72, -7/72, 17/72] , [0, -7/72, -7/72, 17/72] , [0, 17/72, -7/72, -7/72] , [1/2, 17/72, -7/72, -43/72] , [0, -7/72, 17/72, -7/72]] $ x $ [[2, 0, 7, 10, 0, 5] , [0, 0, 7, 7, 0, 10] , [0, 0, 10, 7, 0, 7] , [0, 0, 7, 10, 0, 7]] $

Omega Rank for B : cycles: {{1, 2, 5, 6}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 41/480, -31/480, 89/480, -79/480] , [0, -31/480, 89/480, -79/480, 41/480] , [0, 89/480, -79/480, 41/480, -31/480] , [1/5, -79/480, 41/480, -31/480, -7/480] , [0, 89/480, -79/480, 41/480, -31/480]

, [0, -79/480, 41/480, -31/480, 89/480]] $ x $ [[4, 8, 5, 0, 4, 3] , [8, 9, 0, 0, 3, 4] , [9, 3, 0, 0, 4, 8] , [3, 4, 0, 0, 8, 9] , [4, 8, 0, 0, 9, 3]] $

Â» SYNC'D 105/512 , 0.2050781250

27 . Coloring, {2, 3, 4, 5}

R: [3, 1, 2, 6, 1, 5] **B**: [6, 4, 4, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ - 3 + τ ^{2} ‘)‘ , -24‘ (‘ 4 - τ + τ ^{3} ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ - 6 + τ - 3τ ^{2} - τ ^{3} + τ ^{4} ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 5 + τ + τ ^{2} + τ ^{3} ‘)‘ , 24‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ , 48‘ (‘ 1 + τ ‘)‘‘ (‘ 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-99, -87, -101, -47, -45, -60] . FixedPtCheck, [99, 87, 101, 47, 45, 60]

det(A + τ Δ) = 1‘ (‘ 1 + τ ‘)‘ ^{2} ‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	5 *vs* 6	5 *vs* 5	3 *vs* 4

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_3, y_4, y_5, 0, y_1, y_2]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 0, 13/72, 1/72, -11/72] , [0, 0, 1/72, -11/72, 13/72] , [0, 0, -11/72, 13/72, 1/72] , [1/5, -4/25, 13/72, -67/360, 13/1800] , [0, 0, 1/72, -11/72, 13/72] , [0, 1/5, -11/72, 13/72, -67/360]] $ x $ [[6, 6, 3, 0, 4, 5] , [10, 3, 6, 0, 5, 0] , [8, 6, 10, 0, 0, 0] , [6, 10, 8, 0, 0, 0] , [10, 8, 6, 0, 0, 0]] $

Omega Rank for B : cycles: {{3, 4}}, net cycles: -1 . order: 2

$ [[0, 2, 9, 10, 0, 3] , [0, 0, 13, 11, 0, 0] , [0, 0, 11, 13, 0, 0] , [0, 0, 13, 11, 0, 0]] $

$$[0, 2\,y_1, 2\,y_2, 2\,y_3, 0, 3\,y_1]$$

p = - s ^{2} + s ^{4}

Â» SYNC’D 65/1024 , 0.06347656250

28 . Coloring, {2, 3, 4, 6}

R: [3, 1, 2, 6, 2, 3] **B**: [6, 4, 4, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘24‘ (‘ 3 + τ ² ‘)‘ , 24‘ (‘ 4 - 3τ + τ ² ‘)‘‘ (‘ 1 + τ ‘)‘ , -24‘ (‘ - 2 + τ ‘)‘‘ (‘ 3 + τ ² ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ ‘)‘ , 24‘ (‘ 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ² , -48‘ (‘ 2 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [26, 33, 39, 18, 5, 20] . FixedPtCheck, [26, 33, 39, 18, 5, 20]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ² ‘ (‘ 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 4	4 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [3/4, 1/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =
$ [[59/168, 47/42, -31/63, 271/189, 688/189, -1136/189] , [11/168, -55/42, 2/63, 445/189, -128/189, -80/189] , [-55/168, -107/84, -47/126, 19/27, 136/189, 16/27] , [-13/168, -1/6, -103/63, -149/189, 304/189, 208/189] , [-55/168, 1/84, 523/126, 907/189, -872/189, -752/189] , [113/168, 31/12, 109/126, -1115/189, -536/189, 880/189]] $ x $ [[5/2, 2, 11/2, 15/2, 3, 7/2] , [11/4, 17/8, 57/8, 45/8, 21/8, 15/4] , [5/2, 39/16, 187/32, 111/16, 45/16, 111/32] , [87/32, 277/128, 857/128, 795/128, 333/128, 231/64] , [319/128, 595/256, 3195/512, 1701/256, 693/256, 1839/512] , [1337/512, 4581/2048, 13321/2048, 13155/2048, 5517/2048, 3615/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_4, y_3, y_2, 0, 0, y_1]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 25/72, 1/72, -23/72] , [0, 1/72, -23/72, 25/72] , [0, -23/72, 25/72, 1/72] , [1/5, 1/72, -23/72, 53/360] , [0, -23/72, 25/72, 1/72] , [0, 25/72, 1/72, -23/72]] $ x $ [[4, 8, 7, 0, 0, 5] , [8, 7, 9, 0, 0, 0] , [7, 9, 8, 0, 0, 0] , [9, 8, 7, 0, 0, 0]] $

Omega Rank for B : cycles: {{1, 5, 6}, {3, 4}}, net cycles: 2 . order: 6

$ [[2, 0, 5, 10, 4, 3] , [4, 0, 10, 5, 3, 2] , [3, 0, 5, 10, 2, 4] , [2, 0, 10, 5, 4, 3] , [4, 0, 5, 10, 3, 2]] $

$$[3\,y_3,\ 0,\ 5\,y_3 - 3\,y_4 + 5\,y_1 + 5\,y_2,\ 3\,y_4,\ 3\,y_1,\ 3\,y_2]$$

$p = -s - s^2 + s^4 + s^5$

Â» SYNC'D 21/128 , 0.1640625000

29 . Coloring, {2, 3, 5, 6}

R: [3, 1, 2, 3, 1, 3] **B**: [6, 4, 4, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :
‘[‘12‘ (‘ 1 + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 24‘ (‘ - 2 - τ - 2τ ^ 2 + τ ^ 3 ‘)‘ , 24‘ (‘ 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , -12‘ (‘ - 5 - 3τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , -24‘ (‘ - 1 + τ ‘)‘ ^ 2 , 48‘ (‘ - 1 + τ ‘)‘‘]‘

For τ=1/2, [-75, -92, -120, -53, -8, -32] . FixedPtCheck, [75, 92, 120, 53, 8, 32]

det(A + τ Δ) = 0

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	4 *vs* 4

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 1/4, 0, 0, 3/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 2/11, 0, 3/11, 3/11] , [0, 0, 0, 1, 0, 0] , [0, 0, 3/11, 0, 10/11, -1/11] , [0, 0, 3/11, 0, -1/11, 10/11]] $ =
$ [[0, 35/104, -19/78, 2/13, -8/39] , [0, -51/104, 1/52, -8/39, 28/39] , [1/4, -7/78, 35/312, 1/39, -10/39] , [0, 35/104, -19/78, 2/13, -8/39] , [-1/12, 47/468, -11/312, 19/39, -50/117] , [-1/12, -23/234, 33/104, -5/13, 34/117]] $ x $ [[3/2, 3, 3, 15/2, 3, 6] , [3/2, 3, 15/4, 9/2, 9/2, 27/4] , [15/8, 69/16, 51/16, 81/16, 81/16, 9/2] , [75/32, 147/32, 183/64, 45/8, 27/8, 333/64] , [255/128, 831/256, 843/256, 1431/256, 999/256, 765/128]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_2, y_3, y_1, 0, 0, 0]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] $ = $ [[1/8, -1/24, -1/24] , [-1/24, -1/24, 1/8] , [-1/24, 1/8, -1/24] , [1/8, -1/24, -1/24] , [-1/24, -1/24, 1/8] , [1/8, -1/24, -1/24]] $ x $ [[6, 6, 12, 0, 0, 0] , [6, 12, 6, 0, 0, 0] , [12, 6, 6, 0, 0, 0]] $

Omega Rank for B : cycles: {{2, 4, 5, 6}}, net cycles: 1 . order: 4

$$[0, y_3, 0, y_1, y_2, y_4]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[-1/96, 11/96, -3/32, 1/32] , [11/96, -3/32, 1/32, -1/96] , [11/96, -3/32, 1/32, -1/96] , [-1/96, 11/96, -3/32, 1/32] , [-3/32, 1/32, -1/96, 11/96] , [1/32, -1/96, 11/96, -3/32]] $ x $ [[0, 2, 0, 10, 4, 8] , [0, 4, 0, 2, 8, 10] , [0, 8, 0, 4, 10, 2] , [0, 10, 0, 8, 2, 4]] $

Â» SYNC'D 15/32 , 0.4687500000

30 . Coloring, {2, 4, 5, 6}

R: [3, 1, 4, 6, 1, 3] **B**: [6, 4, 2, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘-24‘ (‘ 3 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ - 4 - τ + τ^3 ‘)‘ , 24‘ (‘ 6 + 3τ + τ^2 - 3τ^3 + τ^4 ‘)‘ , -24‘ (‘ - 5 - τ - 3τ^2 + τ^3 ‘)‘ , -24‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ^2 ‘)‘ , 48‘ (‘ 1 + τ ‘)‘‘ (‘ 2 - τ + τ^2 ‘)‘‘]‘

For τ=1/2, [6, 5, 17, 14, 3, 12] . FixedPtCheck, [6, 5, 17, 14, 3, 12]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^{2} ‘ (‘ 1 + τ ‘)‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 4	5 *vs* 5

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [1/4, 0, 0, 3/4, 0, 0] , [0, 3/4, 0, 1/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[1583/2216, -257/831, -13/277, -1301/2493, -3056/2493, 3568/2493] , [-235/2216, 361/1108,

-386/831, 874/2493, -3200/2493, 3040/2493] , [623/2216, -2633/3324, -3605/3324, -5557/4986, 916/2493, 5944/2493] , [-781/2216, -127/1662, 754/831, 1423/2493, 2944/2493, -5456/2493] , [297/2216, 10/277, 3367/3324, -5011/4986, 3172/2493, -3512/2493] , [-505/2216, 3895/3324, 1615/3324, 7487/4986, -1148/2493, -6056/2493]] $ x $ [[3/2, 6, 11/2, 9/2, 3, 7/2] , [9/4, 51/8, 37/8, 47/8, 21/8, 9/4] , [9/4, 87/16, 177/32, 95/16, 27/16, 101/32] , [57/32, 693/128, 743/128, 699/128, 303/128, 203/64] , [249/128, 1569/256, 2731/512, 1411/256, 609/256, 1383/512] , [1089/512, 11847/2048, 10845/2048, 12145/2048, 4149/2048, 2905/1024]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{3, 4, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, y_2, y_3, 0, y_4]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 79/504, -41/504, -17/504] , [1/6, -41/504, -17/504, -5/504] , [0, -17/504, 79/504, -41/504] , [0, -41/504, -17/504, 79/504] , [1/6, -41/504, -17/504, -5/504] , [0, 79/504, -41/504, -17/504]] $ x $ [[6, 0, 7, 6, 0, 5] , [0, 0, 11, 7, 0, 6] , [0, 0, 6, 11, 0, 7] , [0, 0, 7, 6, 0, 11]] $

Omega Rank for B : cycles: {{2, 3, 4}}, net cycles: 0 . order: 3

$$[0, y_3, y_1, y_2, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[1/3, -4/9, 25/72, -47/72, 11/24] , [0, 0, 25/72, -23/72, 1/72] , [0, 0, -23/72, 1/72, 25/72] , [0, 0, 1/72, 25/72, -23/72] , [0, 0, -23/72, 1/72, 25/72] , [0, 1/3, 1/72, 25/72, -47/72]] $ x $ [[0, 8, 5, 4, 4, 3] , [0, 9, 4, 8, 3, 0] , [0, 7, 8, 9, 0, 0] , [0, 8, 9, 7, 0, 0] , [0, 9, 7, 8, 0, 0]] $

Â» SYNC’D 289/1024 , 0.2822265625

31 . Coloring, {3, 4, 5, 6}

R: [3, 4, 2, 6, 1, 3] **B**: [6, 1, 4, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ - 1 + τ ‘)‘‘ (‘ 3 + τ ² ‘)‘ , 24‘ (‘ - 4 + τ - 2τ ² + τ ³ ‘)‘ , 24‘ (‘ - 2 + τ ‘)‘‘ (‘ 3 + τ ² ‘)‘ , 24‘ (‘ - 5 + 3τ - 3τ ² + τ ³ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘‘ (‘ 2 - τ + τ ² ‘)‘ , -48‘ (‘ 2 - τ + τ ² ‘)‘‘]‘

For τ=1/2, [-13, -31, -39, -33, -7, -28] . FixedPtCheck, [13, 31, 39, 33, 7, 28]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ 1 + 3τ 2 ‘)‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 6

bi =

$ [[0, 0, 1/4, 0, 0, 3/4] , [3/4, 0, 0, 1/4, 0, 0] , [0, 1/4, 0, 3/4, 0, 0] , [0, 0, 3/4, 0, 0, 1/4] , [1/4, 3/4, 0, 0, 0, 0] , [0, 0, 1/4, 0, 3/4, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ =

$ [[-395/824, 37/103, -649/309, -157/309, 1160/309, -304/309] , [253/824, -371/206, -335/618, 602/309, 8/309, 32/309] , [-343/412, 1597/824, 2453/1236, 422/309, 716/309, -2080/309] , [1129/824, 25/206, -229/309, -163/309, -1000/309, 944/309] , [-539/412, -551/824, 1883/1236, 413/309, -52/309, -208/309] , [203/412, -983/824, -859/1236, -1120/309, -676/309, 2240/309]] $ x $ [[7/2, 3, 11/2, 11/2, 3, 7/2] , [3, 29/8, 47/8, 39/8, 21/8, 4] , [27/8, 55/16, 173/32, 85/16, 3, 111/32] , [213/64, 461/128, 729/128, 629/128, 333/128, 247/64] , [429/128, 27/8, 2807/512, 331/64, 741/256, 1907/512] , [3333/1024, 7253/2048, 11567/2048, 10149/2048, 5721/2048, 1949/512]] $

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4, 6}}, net cycles: 0 . order: 4

$$[y_2, y_3, y_4, y_5, 0, y_1]$$

R = $ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 89/480, -127/480, 41/480, 17/480] , [0, 41/480, 17/480, 89/480, -127/480] , [0, 17/480, 89/480, -127/480, 41/480] , [0, -127/480, 41/480, 17/480, 89/480] , [1/2, -127/480, 41/480, 17/480, -151/480] , [0, 89/480, -127/480, 41/480, 17/480]] $ x $ [[2, 6, 7, 4, 0, 5] , [0, 7, 7, 6, 0, 4] , [0, 7, 4, 7, 0, 6] , [0, 4, 6, 7, 0, 7] , [0, 6, 7, 4, 0, 7]] $

Omega Rank for B : cycles: {{3, 4}, {1, 2, 5, 6}}, net cycles: 2 . order: 4

$ [[4, 2, 5, 6, 4, 3] , [2, 4, 6, 5, 3, 4] , [4, 3, 5, 6, 4, 2] , [3, 4, 6, 5, 2, 4] , [4, 2, 5, 6, 4, 3] , [2, 4, 6, 5, 3, 4]] $

$$[10\,y_1, -23\,y_1 + 39\,y_2 - 23\,y_3 - 10\,y_4, -11\,y_1 + 23\,y_2 - 11\,y_3, 10\,y_2, 10\,y_3, 10\,y_4]$$

p’ = - 1 + s 4 p’ = - s + s 5

Â» SYNC’D 3/256 , 0.01171875000

32 . Coloring, {2, 3, 4, 5, 6}

R: [3, 1, 2, 6, 1, 3] **B**: [6, 4, 4, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

‘[‘24‘ (‘ 3 - 2τ + τ ^ 2 ‘)‘‘ (‘ 1 + τ ‘)‘ ^ 2 , -24‘ (‘ - 4 + τ - 2τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ 1 + τ ‘)‘ , 24‘ (‘ 6 + 3τ + τ ^ 2 - 3τ ^ 3 + τ ^ 4 ‘)‘ , 24‘ (‘ - 5 - 3τ - τ ^ 2 + τ ^ 3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 24‘ (‘ - 1 + τ ‘)‘ ^ 2 ‘ (‘ 2 + τ ‘)‘‘ (‘ 1 + τ ‘)‘ , -48‘ (‘ - 1 + τ ‘)‘‘ (‘ 2 + τ ‘)‘‘ (‘ 1 + τ ‘)‘‘]‘

For τ=1/2, [81, 93, 119, 53, 15, 60] . FixedPtCheck, [81, 93, 119, 53, 15, 60]

det(A + τ Δ) = 1‘ (‘ τ ‘)‘‘ (‘ 1 + τ ‘)‘‘ (‘ - 1 + τ ‘)‘ ^ 2

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	5 *vs* 6	4 *vs* 4	5 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, 0, y_4]$$

R = $ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0]] $ x $ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[0, 79/504, -41/504, -17/504] , [0, -41/504, -17/504, 79/504] , [0, -17/504, 79/504, -41/504] , [1/5, -41/504, -17/504, -109/2520] , [0, -41/504, -17/504, 79/504] , [0, 79/504, -41/504, -17/504]] $ x $ [[6, 6, 7, 0, 0, 5] , [6, 7, 11, 0, 0, 0] , [7, 11, 6, 0, 0, 0] , [11, 6, 7, 0, 0, 0]] $

Omega Rank for B : cycles: {{3, 4}}, net cycles: 0 . order: 4

$$[0, y_2, y_1, y_3, y_4, y_5]$$

B = $ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0]] $ x $ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] $ = $ [[1/3, -4/9, 10/27, 85/144, -349/432] , [0, 0, 0, -5/48, 7/48] , [0, 0, 0, -5/48, 7/48] , [0, 0, 0, 7/48, -5/48] , [0, 0, 1/3, 7/48, -7/16] , [0, 1/3, -4/9, -7/16, 85/144]] $ x $ [[0, 2, 5, 10, 4, 3] , [0, 4, 10, 7, 3, 0] , [0, 3, 7, 14, 0, 0] , [0, 0, 14, 10, 0, 0] , [0, 0, 10, 14, 0, 0]] $

Â» SYNC’D 13/64 , 0.2031250000

SUMMARY		
Graph Type		NOT CC
ν(A)		1
ν(Δ)		2
π		[3, 4, 6, 5, 2, 4]
Dbly Stoch		false

SANDWICH		Total 4
No .	**Coloring**	**Rank**
1	{2, 5, 6}	2
2	{5}	2
3	{5, 6}	2
4	{2, 5}	2

RT GROUPS		Total 0	
No .	**Coloring**	**Rank**	**Solv**

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	**2^{n-1}**
28	0	24 , 23	28 , 23	4	32	32
