

New Graph

[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]

$$\pi = [1, 1, 1, 1, 1, 1]$$

POSSIBLE RANKS

$$\begin{array}{l} 1 \times 6 \\ 2 \times 3 \end{array}$$

BASE DETERMINANT 91/512, .1777343750

NullSpace of Δ

{1, 2, 3, 4}, {5, 6}

Range of Δ : $[\lambda_1, \lambda_2, -\lambda_1 - \lambda_2 - \lambda_3, \lambda_3, \lambda_4, -\lambda_4]$

1 . Coloring, {}

R: [3, 3, 5, 5, 1, 1] **B**: [2, 4, 6, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A + \tau \Delta$:

[‘1‘ (‘1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘ , 1‘ (‘1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘ , 1‘ (‘1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘]‘

For $\tau=1/2$, [3, 1, 3, 1, 3, 1] . FixedPtCheck, [3, 1, 3, 1, 3, 1]

$\det(A + \tau \Delta) = 0$

Delta Range : $[y_4, -y_4 - y_1 - y_2, y_1, y_2, y_3, -y_3]$

$$[1, 1, 1, 1, 1, 1]$$

$$+ \quad \backslash; \quad - \quad \backslash; \quad \Delta$$

$$\begin{aligned} & \$ [[2, 0, 2, 0, 2, 0], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \\ & \$ [[0, 2, 0, 2, 0, 2], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \\ & \$ [[1, -1, 1, -1, 1, -1], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$ \end{aligned}$$

$$[-y_1, y_1, -y_1, y_1, -y_1, y_1]$$

$$p' = s^3 \quad p = s^2$$

$$S+ \quad \backslash; \quad S- \quad \backslash; \quad NM$$

$$\begin{aligned} & \$ [[0, 0, 0, 1, 1, 0], [0, 0, 0, 0, 1, 1], [1, 1, 0, 0, 0, 0], [1, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 1], [0, 0, 1, 1, 0, 0]] \\ & \$ [[0, 0, 1, 0, 0, 1], [0, 0, 0, 0, 1, 1], [1, 0, 0, 1, 0, 0], [1, 0, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0], [0, 1, 1, 0, 0, 0]] \$ \\ & \$ [[4, 3, 2, 2, 2, 3], [3, 4, 2, 2, 3, 2], [2, 2, 4, 4, 2, 2], [2, 2, 4, 4, 2, 2], [2, 3, 2, 2, 4, 3], [3, 2, 2, 2, 3, 4]] \$ \end{aligned}$$

CmmCk *true, true, true*

$$p' = s^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
1 vs 4	1 vs 4	1 vs 4	1 vs 3	1 vs 3

Omega Rank for R : cycles: $\{\{1, 3, 5\}\}$, net cycles: 1 . order: 3

$$\begin{aligned} & \$ [[2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$ \\ & [y_1, 0, y_1, 0, y_1, 0] \end{aligned}$$

$$[y_1, 0, y_1, 0, y_1, 0]$$

$$p = s - s^3 \quad p' = s - s^2$$

Omega Rank for B : cycles: $\{\{2, 4, 6\}\}$, net cycles: 1 . order: 3

$$\begin{aligned} & \$ [[0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2]] \$ \\ & [0, y_1, 0, y_1, 0, y_1] \end{aligned}$$

$$[0, y_1, 0, y_1, 0, y_1]$$

$$p = -s + s^2 \quad p = -s + s^3$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$$[0, x_3, x_1, x_2]$$

$$\text{For } A+2\Delta: [y_1, -3y_1 - 3y_2 - y_3 - 3y_4 - y_5, y_2, y_3, y_4, y_5]$$

$$\text{For } A-2\Delta: [-3y_1 - y_2 - 3y_3 - y_4 - 3y_5, y_1, y_2, y_3, y_4, y_5]$$

Range of $\{\Omega\Delta^i\}$: $[\mu_1, -\mu_1, \mu_1, -\mu_1, \mu_1, -\mu_1]$

rank of M is 6 , rank of N is 4

M N

\$ [[0, 0, 1, 0, 1, 0] , [0, 0, 0, 1, 0, 1] , [1, 0, 0, 0, 1, 0] , [0, 1, 0, 0, 0, 1] , [1, 0, 1, 0, 0, 0] , [0, 1, 0, 1, 0, 0]]
 \$ \$ [[0, 1, 2, 2, 2, 1] , [1, 0, 2, 2, 1, 2] , [2, 2, 0, 0, 2, 2] , [2, 2, 0, 0, 2, 2] , [2, 1, 2, 2, 0, 1] , [1, 2, 2, 2, 1, 0]] \$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [1, -1, 1, -1, 1, -1]$

ker M, [0, 0, 0, 0, 0, 0]

Range M, [$x_1, x_2, x_3, x_4, x_5, x_6$]

$\tau = 12$, $r' = 2/3$

Ranges

Action of R on ranges, [[1], [1]]

Action of B on ranges, [[2], [2]]

$\beta(\{1, 3, 5\}) = 1/2$

$\beta(\{2, 4, 6\}) = 1/2$

ker N, [$-\mu_2, \mu_2, -\mu_1, \mu_1, -\mu_2, \mu_2$]

Range of N

[$y_1 - y_3 + y_4, y_1, y_2, y_2, y_3, y_4$]

Partitions

Action of R on partitions, [[2], [2]]

Action of B on partitions, [[1], [1]]

$\alpha(\{\{3, 4\}, \{2, 5\}, \{1, 6\}\}) = 1/2$

$\alpha(\{\{5, 6\}, \{1, 2\}, \{3, 4\}\}) = 1/2$

$b1 = \{5, 6\}$, , $b2 = \{1, 2\}$, , $b3 = \{3, 4\}$, , $b4 = \{2, 5\}$, , $b5 = \{1, 6\}$

Action of R and B on the blocks of the partitions: \$ [[0, 0, 2, 0, 0] , [1, 0, 0, 0, 1] , [0, 1, 0, 1, 0] , [0, 0, 1, 0, 1] , [1, 0, 1, 0, 0]] \$ = \$ [[0, 0, 1, 0, 0] , [1, 0, 0, 0, 0] , [0, 1, 0, 0, 0] , [0, 0, 1, 0, 0] , [1, 0, 0, 0, 0]] \$ + \$ [[0, 0, 1, 0, 0] , [0, 0, 0, 0, 1] , [0, 0, 0, 1, 0] , [0, 0, 0, 0, 1] , [0, 0, 1, 0, 0]] \$
 ['3' , '1' , '2' , '3' , '1' , ['3' , '5' , '4' , '5' , '3'] with invariant measure [1, 1, 2, 1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-3 partition graph.

'

Sandwich	
Coloring	{ }
Rank	3
R,B	[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]
π_2	[0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
u_3	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

2. Coloring, {2}

R: [3, 4, 5, 5, 1, 1] **B:** [2, 3, 6, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘ [‘1‘ (‘ 1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘ , 1‘ (‘ 1 + τ^2 ‘)‘ , -1‘ (‘ 1 + τ ‘)‘ (‘ - 1 + τ ‘)‘ , 1‘ (‘ 1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘ ‘

For $\tau=1/2$, [6, 2, 5, 3, 6, 2] . FixedPtCheck, [6, 2, 5, 3, 6, 2]

$\det(A + \tau \Delta) = 0$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

[1, 1, 1, 1, 1, 1]

+ \ ; - \ ; Δ

\$ [[2, 0, 1, 1, 2, 0], [1, 1, 2, 0, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \$ [[0, 2, 1, 1, 0, 2], [1, 1, 0, 2, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \$ [[1, -1, 0, 0, 1, -1], [0, 0, 1, -1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$

[y₂, -y₂, y₁, -y₁, y₂, -y₂]

$$p' = s^3 \quad p = s^3$$

S+ \; S- \; NM

\$ [[1, 1, 0, 0, 1, 1], [2, 1, 0, 1, 0, 0], [0, 1, 1, 1, 0, 1], [0, 1, 1, 1, 0, 1], [1, 0, 1, 1, 1, 0], [0, 0, 1, 0, 2, 1]]
 \$ \$ [[1, 0, 0, 1, 1, 1], [2, 1, 0, 1, 0, 0], [0, 1, 2, 0, 1, 0], [0, 1, 2, 0, 1, 0], [1, 1, 0, 1, 0, 1], [0, 0, 0, 1, 1, 2]] \$
 \$ [[8, 6, 4, 4, 4, 6], [6, 8, 4, 4, 6, 4], [4, 4, 8, 8, 4, 4], [4, 4, 8, 8, 4, 4], [4, 6, 4, 4, 8, 6], [6, 4, 4, 4, 6, 8]] \$

CmmCk true, true, true

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 4	2 vs 5	2 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

\$ [[2, 0, 1, 1, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$

[$y_1, 0, y_2, y_1 - y_2, y_1, 0$]

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

\$ [[0, 2, 1, 1, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2]] \$

[$0, y_1, y_1 - y_2, y_2, 0, y_1$]

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

[$0, 0, x_1, x_2$]

For $A+2\Delta$: [$y_1, -3y_1 - 4y_2 - 3y_3 - y_4, y_2, y_2, y_3, y_4$]

For $A-2\Delta$: [$-3y_1 - 4y_2 - y_3 - 3y_4, y_1, y_2, y_2, y_3, y_4$]

Range of $\{\Omega\Delta^i\}$: [$\mu_2, -\mu_2, \mu_1, -\mu_1, \mu_2, -\mu_2$]

rank of M is 5 , rank of N is 4

M N

\$ [[0, 0, 1, 1, 2, 0], [0, 0, 1, 1, 0, 2], [1, 1, 0, 0, 1, 1], [1, 1, 0, 0, 1, 1], [2, 0, 1, 1, 0, 0], [0, 2, 1, 1, 0, 0]]
 \$ \$ [[0, 1, 2, 2, 2, 1], [1, 0, 2, 2, 1, 2], [2, 2, 0, 0, 2, 2], [2, 2, 0, 0, 2, 2], [2, 1, 2, 2, 0, 1], [1, 2, 2, 2, 1, 0]] \$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [1, -1, 0, 0, 1, -1]$

ker M, $[0, 0, -\lambda_1, \lambda_1, 0, 0]$

Range M, $[x_1, x_2, x_3, x_3, x_4, x_5]$

$\tau = 12$, $r' = 2/3$

Ranges

Action of R on ranges, $[[1], [1], [2], [2]]$

Action of B on ranges, $[[4], [4], [3], [3]]$

$\beta(\{1, 3, 5\}) = 1/4$

$\beta(\{1, 4, 5\}) = 1/4$

$\beta(\{2, 3, 6\}) = 1/4$

$\beta(\{2, 4, 6\}) = 1/4$

ker N, $[-\mu_2, \mu_2, -\mu_1, \mu_1, -\mu_2, \mu_2]$

Range of N

$[y_1 - y_3 + y_4, y_1, y_2, y_2, y_3, y_4]$

Partitions

Action of R on partitions, $[[1], [1]]$

Action of B on partitions, $[[2], [2]]$

$\alpha(\{\{5, 6\}, \{1, 2\}, \{3, 4\}\}) = 1/2$

$\alpha(\{\{3, 4\}, \{2, 5\}, \{1, 6\}\}) = 1/2$

$b_1 = \{5, 6\}$, $b_2 = \{1, 2\}$, $b_3 = \{3, 4\}$, $b_4 = \{2, 5\}$, $b_5 = \{1, 6\}$

Action of R and B on the blocks of the partitions: $\$ [[0, 0, 2, 0, 0], [1, 0, 0, 0, 1], [0, 1, 0, 1, 0], [0, 0, 1, 0, 1], [1, 0, 1, 0, 0]] \$ = \$ [[0, 0, 1, 0, 0], [1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [1, 0, 0, 0, 0]] \$ + \$ [[0, 0, 1, 0, 0], [0, 0, 0, 0, 1], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1], [0, 0, 1, 0, 0]] \$$
 $['3', '1', '2', '3', '1'], ['3', '5', '4', '5', '3']$ with invariant measure $[1, 1, 2, 1, 1]$

N by blocks, check: *true*. ' See partition graph.

' ' See level-3 partition graph.

'

Sandwich	
Coloring	{2}
Rank	3
R,B	[3, 4, 5, 5, 1, 1], [2, 3, 6, 6, 4, 2]
π_2	[0, 1, 1, 2, 0, 1, 1, 0, 2, 0, 1, 1, 1, 1, 0]
u_2	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]
u_3	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

3. Coloring, {3}

R: [3, 3, 6, 5, 1, 1] **B:** [2, 4, 5, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘ [‘3‘ (‘1 + τ ‘)‘ (‘4 - $\tau + \tau^3$ ‘)‘, -3‘ (‘-1 + τ ‘)‘ (‘4 + $\tau + 2\tau^2 + \tau^3$ ‘)‘, 12‘ (‘1 + τ ‘)‘, 6‘ (‘-1 + τ ‘)‘²‘ (‘2 + τ ‘)‘, 3‘ (‘-4 + $\tau + \tau^2$ ‘)‘ (‘-1 + τ ‘)‘ (‘1 + τ ‘)‘, -3‘ (‘-4 + $\tau - 5\tau^2 - \tau^3 + \tau^4$ ‘)‘]‘

For $\tau=1/2$, [87, 41, 96, 20, 39, 77] . FixedPtCheck, [87, 41, 96, 20, 39, 77]

$\det(A + \tau \Delta) = 0$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 5	5 vs 5	2 vs 4	4 vs 4

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 1/4, 3/4], [1/4, 0, 0, 3/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[11/20, 3/20, -9/20, 3/20, 0, 0], [3/20, 19/20, 3/20, -1/20, 0, 0], [-9/20, 3/20, 11/20, 3/20, 0, 0], [3/20, -1/20, 3/20, 19/20, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$
 $\$ [[11/60, -21/10, -209/180, 22/45, 124/45], [-7/60, 7/10, -227/180, -14/45, 52/45], [83/60, 7/10,$

223/180, -74/45, -68/45], [-19/60, -1/10, 241/180, -38/45, 4/45], [-13/60, 9/5, -23/180, 34/45, -92/45], [1/12, -1, -1/36, 14/9, -4/9]] \$ x \$ [[1/2, 3/2, 1/2, 3/2, 1, 1], [1/2, 9/8, 1/2, 15/8, 3/4, 5/4], [1/2, 21/16, 13/32, 45/32, 27/32, 49/32], [19/32, 195/128, 29/64, 207/128, 21/32, 37/32], [29/64, 21/16, 271/512, 837/512, 381/512, 679/512]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

\$ [[2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2]] \$

[y₁ + y₂, 0, y₁ + y₂, 0, y₁, y₂]

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

[0, y₁, 0, y₂, y₃, y₄]

B = \$ [[0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0, 0]] \$ x \$ [[0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, -5/18, 1/18, 7/18], [0, 7/18, -5/18, 1/18], [1, -5/18, 1/18, -11/18], [0, 1/18, 7/18, -5/18], [0, 7/18, -5/18, 1/18], [0, -5/18, 1/18, 7/18]] \$ x \$ [[0, 2, 0, 2, 1, 1], [0, 1, 0, 3, 0, 2], [0, 2, 0, 1, 0, 3], [0, 3, 0, 2, 0, 1]] \$

Â» SYNC'D 5/16 , 0.3125000000

4 . Coloring, {4}

R: [3, 3, 5, 6, 1, 1] **B:** [2, 4, 6, 5, 4, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$\begin{aligned} & ['3' (' - 4 - \tau + \tau^3 ') ' (' 1 + \tau ') ' , -3' (' - 1 + \tau ') ' (' - 4 + \tau + \tau^2 ') ' (' 1 + \tau ') ' , 6' (' - 2 + \tau ') ' (' 1 + \tau ') '^2 , 12' (' - 1 + \tau ') ' , 3' (' - 4 - \tau - 5\tau^2 + \tau^3 + \tau^4 ') ' , -3' (' - 4 - \tau + \tau^2 ') ' (' - 1 + \tau ') ' (' 1 + \tau ') '] ' \end{aligned}$$

For τ=1/2, [-105, -39, -108, -32, -89, -51] . FixedPtCheck, [105, 39, 108, 32, 89, 51]

$$\det(A + \tau \Delta) = 0$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[11/20, 3/20, -9/20, 3/20, 0, 0], [3/20, 19/20, 3/20, -1/20, 0, 0], [-9/20, 3/20, 11/20, 3/20, 0, 0], [3/20, -1/20, 3/20, 19/20, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$
 $\$ [[1, -5/6, -8/3, -40/9, 64/9], [-1, 19/6, 2/3, 8/9, -32/9], [1/3, -1/6, 40/9, 8/3, -64/9], [1/3, -13/6, 10/9, -8/3, 32/9], [-5/6, 2, -1/9, 40/9, -16/3], [7/6, -2, -31/9, -8/9, 16/3]] \$ \times \$ [[1/2, 3/2, 1/2, 3/2, 1, 1], [1/2, 9/8, 1/2, 15/8, 5/4, 3/4], [1/2, 15/16, 13/32, 57/32, 49/32, 27/32], [19/32, 129/128, 23/64, 237/128, 23/16, 3/4], [35/64, 129/128, 205/512, 939/512, 757/512, 375/512]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

$\$ [[2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$$

$[y_1 + y_2, 0, y_1 + y_2, 0, y_1, y_2]$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 4

$\$ [[0, 2, 0, 2, 1, 1], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$

$[0, y_1 - y_2 + y_3, 0, y_1, y_2, y_3]$

$$p = -s^3 + s^4$$

Â» SYNC'D 3/32 , 0.09375000000

5 . Coloring, {5}

R: [3, 3, 5, 5, 4, 1] **B**: [2, 4, 6, 6, 1, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$['-12' (' 1 + \tau ')'' (' - 1 + \tau ')', 6' (' - 1 + \tau ')'^2 (' 2 + \tau ')', 3' (' 1 + \tau ')'' (' - 1 + \tau ')'' (' - 4 - \tau + \tau^2 ')', -3' (' - 4 + \tau - 5\tau^2 - \tau^3 + \tau^4 ')', 12' (' 1 + \tau ')', -12' (' - 1 + \tau ')'']'$

For $\tau=1/2$, [48, 20, 51, 77, 96, 32] . FixedPtCheck, [48, 20, 51, 77, 96, 32]

$$\det(A + \tau \Delta) = 0$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 5	5 vs 5	3 vs 4	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], \\ & [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1/10, 3/10], \\ & [0, 0, 0, 0, 3/10, 9/10]] \$ = \\ & \$ [[0, 7/2, -2/3, 8/9, -32/9], [0, -3/2, 5/3, -16/9, 16/9], [0, 1/2, -1/3, 16/9, -16/9], [0, 1/2, -1/3, 16/9, -16/9], \\ & [3/2, -7/2, 5/6, -28/9, 40/9], [-1/2, 1/2, -7/6, 4/9, 8/9]] \$ \times \$ [[1, 3/2, 1/2, 1, 1/2, 3/2], [3/4, 15/8, 5/8, 5/4, 3/8, 9/8], \\ & [9/16, 45/32, 21/32, 3/2, 15/32, 45/32], [45/64, 189/128, 63/128, 75/64, 69/128, 207/128], [207/256, 891/512, 279/512, 159/128, 213/512, 639/512]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[1, 0, 2, 1, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[y_1 + y_2 - y_3, 0, y_1, y_2, y_3, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

$$[y_1, y_2, 0, y_3, 0, y_4]$$

$$\begin{aligned} \mathbf{B} = & \$ [[0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[0, 7/18, -5/18, 1/18], [0, 1/18, 7/18, -5/18], [0, -5/18, 1/18, 7/18], [0, -5/18, 1/18, 7/18], [1, -5/18, 1/18, -11/18], \\ & [0, 7/18, -5/18, 1/18]] \$ \times \$ [[1, 2, 0, 1, 0, 2], [0, 3, 0, 2, 0, 1], [0, 1, 0, 3, 0, 2], [0, 2, 0, 1, 0, 3]] \$ \end{aligned}$$

Â» SYNC'D 5/32 , 0.1562500000

6 . Coloring, {6}

R: [3, 3, 5, 5, 1, 2] **B**: [2, 4, 6, 6, 4, 1]

‘ See graph

' ' See pair graph

Ω for A+τΔ :

$$[-12(\tau^2 + \tau), 6(\tau - 1 + \tau^2), 3(\tau^2 + \tau - 4 + \tau - 2\tau^2 + \tau^3), -3(\tau^3 - 4 - \tau + \tau^3), -12(\tau^2 + \tau), 12(\tau - 1 + \tau^2)]$$

For τ=1/2, [-80, -44, -93, -35, -96, -32] . FixedPtCheck, [80, 44, 93, 35, 96, 32]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

bi =

$$\begin{aligned} & \$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [1/4, 0, 0, 3/4, 0, 0], \\ & [3/4, 1/4, 0, 0, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1/10, 3/10], \\ & [0, 0, 0, 0, 3/10, 9/10]] \times \$ [[0, 11/6, 1/9, 32/9, -16/3], [0, -25/6, 7/9, -16/9, 16/3], [0, 5/2, -7/3, 16/9, -16/9], [0, 5/2, -7/3, 16/9, -16/9], \\ & [3/2, -7/3, 17/9, -8/3, 16/9], [-1/2, -1/3, 17/9, -8/3, 16/9]] \times \$ [[1, 1, 1/2, 3/2, 1/2, 3/2], [5/4, 9/8, 1/2, 9/8, 1/2, 3/2], [5/4, 21/16, 19/32, 39/32, 13/32, 39/32], [65/64, 159/128, 41/64, 165/128, 29/64, 87/64], \\ & [145/128, 141/128, 289/512, 651/512, 247/512, 741/512]] \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

$$\begin{aligned} & \$ [[1, 1, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \times \\ & [-y_1 + y_2, y_1, y_2, 0, y_2, 0] \end{aligned}$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 6}}, net cycles: 1 . order: 4

$$\begin{aligned} & \$ [[1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1]] \times \\ & [y_1 - y_2 + y_3, y_1, 0, y_2, 0, y_3] \end{aligned}$$

$$p = -s + s^2 - s^3 + s^4$$

Â» SYNC'D 3/16 , 0.1875000000

7 . Coloring, {2, 3}

R: [3, 4, 6, 5, 1, 1] **B:** [2, 3, 5, 6, 4, 2]

' See graph

' ' See pair graph

'

Ω for $A+\tau\Delta$:

$$\left[3 \left(1 + \tau \right) \left(-4 + \tau - 2\tau^2 + \tau^3 \right), 3 \left(-1 + \tau \right) \left(4 + \tau + 2\tau^2 + \tau^3 \right), -12 \left(1 + \tau^2 \right), 6 \left(-1 + \tau \right) \left(2 + \tau + \tau^2 \right), 3 \left(4 + 3\tau + 4\tau^2 + \tau^3 \right) \left(-1 + \tau \right), -3 \left(4 - \tau + 3\tau^2 + \tau^3 + \tau^4 \right) \right]$$

For $\tau=1/2$, [-93, -41, -80, -44, -53, -71] . FixedPtCheck, [93, 41, 80, 44, 53, 71]

$$\det(A + \tau \Delta) = 1 \left(1 + \tau \right) \left(\tau \right)^2 \left(-1 + \tau \right)$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	3 vs 5	5 vs 5

bi =

$$\$ \left[\left[0, 3/4, 1/4, 0, 0, 0 \right], \left[0, 0, 3/4, 1/4, 0, 0 \right], \left[0, 0, 0, 0, 3/4, 1/4 \right], \left[0, 0, 0, 0, 1/4, 3/4 \right], \left[1/4, 0, 0, 3/4, 0, 0 \right], \left[1/4, 3/4, 0, 0, 0, 0 \right] \right] \times \$ \left[\left[1, 0, 0, 0, 0, 0 \right], \left[0, 1, 0, 0, 0, 0 \right], \left[0, 0, 1, 0, 0, 0 \right], \left[0, 0, 0, 1, 0, 0 \right], \left[0, 0, 0, 0, 1, 0 \right], \left[0, 0, 0, 0, 0, 1 \right] \right] \$ =$$

$$\$ \left[\left[461/246, 11/41, -83/123, -272/369, -80/369, -128/369 \right], \left[-31/246, 93/41, -83/123, -272/369, -80/369, -128/369 \right], \left[77/246, 15/41, 349/123, -356/369, -944/369, 64/369 \right], \left[-247/246, -79/41, 37/123, -104/369, 1648/369, -512/369 \right], \left[11/246, 49/41, 208/123, 1636/369, -416/369, -2240/369 \right], \left[-25/246, -89/41, -428/123, -632/369, -128/369, 2944/369 \right] \right] \times \$ \left[\left[1/2, 3/2, 1, 1, 1, 1 \right], \left[1/2, 9/8, 5/4, 9/8, 1, 1 \right], \left[1/2, 9/8, 31/32, 33/32, 39/32, 37/32 \right], \left[19/32, 159/128, 31/32, 153/128, 63/64, 65/64 \right], \left[1/2, 309/256, 553/512, 537/512, 525/512, 583/512 \right], \left[277/512, 2517/2048, 1055/1024, 2193/2048, 549/512, 541/512 \right] \right] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$$\$ \left[\left[2, 0, 1, 1, 1, 1 \right], \left[2, 0, 2, 0, 1, 1 \right], \left[2, 0, 2, 0, 0, 2 \right], \left[2, 0, 2, 0, 0, 2 \right], \left[2, 0, 2, 0, 0, 2 \right] \right] \$$$

$$\left[y_2 + y_3, 0, -y_1 + y_2 + y_3, y_1, y_2, y_3 \right]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$\left[0, y_5, y_4, y_3, y_2, y_1 \right]$$

$$B = \$ [[0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[5/6, -1/6, -1/6, -1/6, -1/6], [-1/6, 5/6, -1/6, -1/6, -1/6], [-1/6, -1/6, 5/6, -1/6, -1/6], [-1/6, -1/6, -1/6, 5/6], [-1/6, -1/6, -1/6, 5/6, -1/6], [5/6, -1/6, -1/6, -1/6, -1/6]] \$ x \$ [[0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 2, 1, 1], [0, 1, 1, 1, 2, 1]] \$$$

Â» SYNC'D 25/128 , 0.1953125000

8. Coloring, {2, 4}

R: [3, 4, 5, 6, 1, 1] **B:** [2, 3, 6, 5, 4, 2]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$['3' ('4 - 3\tau + \tau^2') ' ('1 + \tau') , 3' (' - 1 + \tau') ' (' - 4 + \tau + \tau^2') , 6' ('2 - \tau + \tau^2') , -12' (' - 1 + \tau') , 3' ('4 - 3\tau + 2\tau^2 + \tau^3') , -3' ('4 + \tau + \tau^2') ' (' - 1 + \tau') ']$$

For τ=1/2, [33, 13, 28, 16, 25, 19] . FixedPtCheck, [33, 13, 28, 16, 25, 19]

$$\det(A + \tau \Delta) = 1' (' \tau ') ^ 2 ' (' 1 + \tau ') ' (' - 1 + \tau ')$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	6 vs 6	6 vs 6	3 vs 5	3 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[15/8, 7/24, -7/12, -3/4, -2/9, -4/9], [-1/8, 55/24, -7/12, -3/4, -2/9, -4/9], [11/24, 25/24, 31/12, -37/36, -10/3, 4/9], [-31/24, -77/24, 13/12, -7/36, 6, -20/9], [1/6, -4/3, -6, -14/9, -16/9, 32/3], [-1/12, 11/12, 7/2, 77/18, -4/9, -8]] \$ x \$ [[1/2, 3/2, 1, 1, 1, 1], [1/2, 9/8, 5/4, 9/8, 1, 1], [1/2, 9/8, 31/32, 33/32, 37/32, 39/32], [19/32, 165/128, 31/32, 147/128, 65/64, 63/64], [1/2, 303/256, 571/512, 555/512, 565/512, 519/512], [271/512, 2325/2048, 1037/1024, 2301/2048, 559/512, 567/512]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

$$\$ [[2, 0, 1, 1, 1, 1], [2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$$$

$$[y_2 + y_3, 0, -y_1 + y_2 + y_3, y_1, y_2, y_3]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^4$$

Omega Rank for B : cycles: {{4, 5}, {2, 3, 6}}, net cycles: 2 . order: 6

$$\$ [[0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1]] \$$$

$$[0, y_3, y_1, y_2, y_2, -y_3 - y_1 + 4y_2]$$

$$p = s - s^4 \quad p' = s - s^4$$

Â» SYNC'D 5/64 , 0.07812500000

9 . Coloring, {2, 5}

R: [3, 4, 5, 5, 4, 1] **B:** [2, 3, 6, 6, 1, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$['-12' (' 1 + \tau ')'' (' - 1 + \tau ')', 6' (' 2 + \tau ')'' (' - 1 + \tau ')'^2, -3' (' - 1 + \tau ')'' (' 4 + \tau + 2\tau^2 + \tau^3 ')', 3' (' 1 + \tau ')'' (' 4 - \tau + \tau^3 ')', 12' (' 1 + \tau ')', -12' (' - 1 + \tau ')'']'$$

For τ=1/2, [48, 20, 41, 87, 96, 32] . FixedPtCheck, [48, 20, 41, 87, 96, 32]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	3 vs 4	4 vs 4	3 vs 4	4 vs 4

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[91/100, 3/100, -9/100, 27/100, 0, 0], [3/100, 99/100, 3/100, -9/100, 0, 0], [-9/100, 3/100, 91/100, 27/100, 0, 0], [27/100, -9/100, 27/100, 19/100, 0, 0], [0, 0, 0, 0, 1/10, 3/10], [0, 0, 0, 0, 3/10, 9/10]] \$ =$$

$$\$ [[-1, 7/6, -8/9, 8/9], [1/2, -1/3, 16/9, -16/9], [-1/2, -2/3, -4/9, 16/9], [-1/2, -2/3, -4/9, 16/9], [11/4, -7/12, 10/9, -28/9], [-1/4, 13/12, -10/9, 4/9]] \$ \times \$ [[1, 3/2, 1, 1/2, 1/2, 3/2], [3/4, 15/8, 11/8, 1/2, 3/8, 9/8], [9/16, 45/32, 51/32, 9/16, 15/32, 45/32], [45/64, 189/128, 153/128, 15/32, 69/128, 207/128]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[1, 0, 1, 2, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[y_3 + y_1 - y_2, 0, y_3, y_1, y_2, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, 0, y_4]$$

$$\begin{aligned} B = \$ [[0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 7/18, -5/18, 1/18], [0, 1/18, 7/18, -5/18], [0, -5/18, 1/18, 7/18], [0, -5/18, 1/18, 7/18], [1, -5/18, 1/18, -11/18], [0, 7/18, -5/18, 1/18]] \$ \times \$ [[1, 2, 1, 0, 0, 2], [0, 3, 2, 0, 0, 1], [0, 1, 3, 0, 0, 2], [0, 2, 1, 0, 0, 3]] \$ \end{aligned}$$

Â» SYNC'D 5/32 , 0.1562500000

10 . Coloring, {2, 6}

R: [3, 4, 5, 5, 1, 2] **B:** [2, 3, 6, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

$$\text{‘ [‘ -12‘ (‘ 1 + } \tau^2 \text{ ‘)‘ , 6‘ (‘ 2 + } \tau + \tau^2 \text{ ‘)‘ ‘ (‘ - 1 + } \tau \text{ ‘)‘ , -3‘ (‘ 4 - } \tau + 3\tau^2 + \tau^3 + \tau^4 \text{ ‘)‘ , 3‘ (‘ 4 + } \tau + \tau^2 \text{ ‘)‘ ‘ (‘ 1 + } \tau \text{ ‘)‘ ‘ (‘ - 1 + } \tau \text{ ‘)‘ , -12‘ (‘ 1 + } \tau \text{ ‘)‘ , 12‘ (‘ - 1 + } \tau \text{ ‘)‘ ‘] ‘}$$

For τ=1/2, [-80, -44, -71, -57, -96, -32] . FixedPtCheck, [80, 44, 71, 57, 96, 32]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [1/4, 0, 0, 3/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1/10, 3/10], [0, 0, 0, 0, 3/10, 9/10]] \$ =$

$\$ [[0, 11/6, 1/9, 32/9, -16/3], [0, -25/6, 7/9, -16/9, 16/3], [0, 5/2, -7/3, 16/9, -16/9], [0, 5/2, -7/3, 16/9, -16/9], [3/2, -7/3, 17/9, -8/3, 16/9], [-1/2, -1/3, 17/9, -8/3, 16/9]] \$ \times \$ [[1, 1, 1, 1, 1/2, 3/2], [5/4, 9/8, 1, 5/8, 1/2, 3/2], [5/4, 21/16, 37/32, 21/32, 13/32, 39/32], [65/64, 159/128, 83/64, 81/128, 29/64, 87/64], [145/128, 141/128, 607/512, 333/512, 247/512, 741/512]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

$\$ [[1, 1, 1, 1, 2, 0], [2, 0, 1, 1, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$$

$[y_1, -y_1 + y_2, y_2 - y_3, y_3, y_2, 0]$

$$p = s^3 - s^5 \quad p' = s^3 - s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 0 . order: 4

$\$ [[1, 1, 1, 1, 0, 2], [2, 1, 1, 0, 0, 2], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2]] \$$

$[y_1 - y_2 - y_3 + y_4, y_1, y_2, y_3, 0, y_4]$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â» SYNC'D 3/16 , 0.1875000000

11 . Coloring, {3, 4}

R: [3, 3, 6, 6, 1, 1] **B:** [2, 4, 5, 5, 4, 2]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$['-6' (' 1 + \tau ')', 6' (' - 1 + \tau ')'' (' 1 + \tau ')', 3' (' - 2 + \tau ')'' (' 1 + \tau ')'^2, -3' (' 2 + \tau ')'' (' - 1 + \tau ')'^2, 6' (' - 1 + \tau ')', -6' (' 1 + \tau ')'']'$

For τ=1/2, [-24, -12, -27, -5, -8, -24] . FixedPtCheck, [24, 12, 27, 5, 8, 24]

det(A + τ Δ) = 0

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

$$[1, 1, 1, 1, 1, 1]$$

$$+ \quad \backslash; \quad - \quad \backslash; \quad \Delta$$

$$\begin{aligned} & \$ [[2, 0, 2, 0, 0, 2], [1, 0, 1, 2, 1, 1], [2, 2, 1, 3, 1, 3], [4, 3, 4, 5, 4, 4]] \$ \quad \$ [[0, 2, 0, 2, 2, 0], [1, 2, 1, 0, \\ & 1, 1], [2, 2, 3, 1, 3, 1], [4, 5, 4, 3, 4, 4]] \$ \quad \$ [[1, -1, 1, -1, -1, 1], [0, -1, 0, 1, 0, 0], [0, 0, -1, 1, -1, 1], \\ & [0, -1, 0, 1, 0, 0]] \$ \end{aligned}$$

$$[y_1, -3y_1 + y_3 - y_2, 2y_1 - y_3, y_2, -y_3, y_3]$$

$$p = s^2 - 4s^4$$

$$S+ \quad \backslash; \quad S- \quad \backslash; \quad NM$$

$$\begin{aligned} & \$ [[2, 2, 2, 2, 1, 1], [0, 2, 3, 1, 2, 2], [2, 1, 1, 2, 2, 2], [2, 1, 1, 2, 2, 2], [1, 2, 2, 1, 2, 2], [3, 2, 1, 2, 1, 1]] \\ & \$ \quad \$ [[2, 2, 2, 2, 1, 1], [0, 2, 3, 1, 2, 2], [2, 1, 1, 2, 2, 2], [2, 1, 1, 2, 2, 2], [1, 2, 2, 1, 2, 2], [3, 2, 1, 2, 1, \\ & 1]] \$ \quad \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, \\ & 0, 0, 0]] \$ \end{aligned}$$

CmmCk true, true, true

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 4	4 vs 4	1 vs 3	3 vs 3

bi =

$$\begin{aligned} & \$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, \\ & 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[11/20, 3/20, -9/20, 3/20, 0, 0], [3/20, 19/20, 3/20, -1/20, 0, 0], [-9/20, \\ & 3/20, 11/20, 3/20, 0, 0], [3/20, -1/20, 3/20, 19/20, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \$ \\ & = \end{aligned}$$

$$\begin{aligned} & \$ [[11/12, 5/36, -4/9, -4/9], [-1/12, 41/36, -4/9, -4/9], [-1/12, -7/36, 8/9, -4/9], [-1/12, -7/36, 8/9, \\ & -4/9], [-1/3, 1/18, -4/9, 8/9], [2/3, -17/18, -4/9, 8/9]] \$ \times \$ [[1/2, 3/2, 1/2, 3/2, 3/2, 1/2], [1/2, 3/4, 1/2, \\ & 9/4, 3/2, 1/2], [1/2, 3/4, 5/16, 27/16, 33/16, 11/16], [11/16, 57/64, 5/16, 135/64, 3/2, 1/2]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 1 . order: 3

$$\begin{aligned} & \$ [[2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2]] \$ \\ & [y_1, 0, y_1, 0, 0, y_1] \end{aligned}$$

$$p = -s + s^3 \quad p = -s + s^2$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 2

$$[0, y_1, 0, y_2, y_3, 0]$$

$$\begin{aligned} B = & \$ [[0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, \\ & 0]] \$ \times \$ [[0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \end{aligned}$$

$$0, 0, 0]] \$ = \$ [[1/2, -1/6, -1/6], [0, 1/3, -1/6], [0, -1/6, 1/3], [0, -1/6, 1/3], [0, 1/3, -1/6], [1/2, -1/6, -1/6]]] \$ \times \$ [[0, 2, 0, 2, 2, 0], [0, 0, 0, 4, 2, 0], [0, 0, 0, 2, 4, 0]]] \$$$

Â» SYNC'D 1/4 , 0.2500000000

12 . Coloring, {3, 5}

R: [3, 3, 6, 5, 4, 1] **B:** [2, 4, 5, 6, 1, 2]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$['-3' ('1 + \tau')'' ('4 - \tau + \tau^2')', 3' ('-1 + \tau')'' ('4 + 3\tau + \tau^2')', 3' ('1 + \tau')'' ('-4 - \tau + \tau^2')', 3' ('-4 - \tau + \tau^3')', 3' ('-4 + \tau + \tau^2')'' ('1 + \tau')', 3' ('-4 - 3\tau - 2\tau^2 + \tau^3')'']'$$

For τ=1/2, [-45, -23, -51, -35, -39, -47] . FixedPtCheck, [45, 23, 51, 35, 39, 47]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')'' ('\tau')'^2 ('1 + \tau')$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	6 vs 6	6 vs 6	3 vs 5	3 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[76/105, -487/630, -278/315, -482/105, 1184/315, 608/315], [-62/105, 809/630, -44/315, 34/105, -1408/315, 1184/315], [148/105, -451/630, -44/315, 34/105, -1408/315, 1184/315], [-74/105, -247/630, 1072/315, 298/105, 704/315, -2272/315], [-13/30, 124/45, 97/45, 58/15, -16/45, -352/45], [25/42, -136/63, -277/63, -58/21, 208/63, 352/63]] \$ \times \$ [[1, 3/2, 1/2, 1, 1, 1], [1, 3/2, 5/8, 11/8, 5/8, 7/8], [11/16, 45/32, 5/8, 41/32, 13/16, 19/16], [29/32, 45/32, 67/128, 161/128, 101/128, 143/128], [223/256, 777/512, 37/64, 641/512, 181/256, 275/256], [409/512, 747/512, 1223/2048, 2693/2048, 1529/2048, 2219/2048]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}, {1, 3, 6}}, net cycles: 2 . order: 6

$$\$ [[1, 0, 2, 1, 1, 1], [1, 0, 1, 1, 1, 2], [2, 0, 1, 1, 1, 1], [1, 0, 2, 1, 1, 1], [1, 0, 1, 1, 1, 2]] \$$$

$$[-y_1 + 4y_2 - y_3, 0, y_1, y_2, y_2, y_3]$$

$$p' = -s + s^4 \quad p = -s + s^4$$

Omega Rank for B : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

$$\$ [[1, 2, 0, 1, 1, 1], [1, 2, 0, 2, 0, 1], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2]] \$$$

$$[y_1, y_2, 0, y_3, y_2 - y_3, -y_1 + y_2]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Â» SYNC'D 29/256, 0.1132812500

13 . Coloring, {3, 6}

R: [3, 3, 6, 5, 1, 2] **B:** [2, 4, 5, 6, 4, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & ['3' ('4 + 3\tau + 4\tau^2 + \tau^3 ') ' (' - 1 + \tau ') ' , -3' ('4 - \tau + 3\tau^2 + \tau^3 + \tau^4 ') ' , 3' ('1 + \tau ') ' (' - \\ & 4 + \tau - 2\tau^2 + \tau^3 ') ' , 3' ('4 - \tau + \tau^3 ') ' (' - 1 + \tau ') ' , 3' ('1 + \tau ') ' ('4 - \tau + \tau^2 ') ' (' - 1 + \tau ') ' , \\ & 3' (' - 4 + \tau - 5\tau^2 - \tau^3 + \tau^4 ') '] ' \end{aligned}$$

For τ=1/2, [-53, -71, -93, -29, -45, -77] . FixedPtCheck, [53, 71, 93, 29, 45, 77]

$$\det(A + \tau \Delta) = 1' (' \tau ') ' ^2 (' 1 + \tau ') ' (' - 1 + \tau ') '$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 1/4, 3/4], [1/4, 0, 0, 3/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[2/3, 1/6, 8/3, 2/9, 64/9, -32/3], [-4/3, 1/6, -22/3, 14/9, -32/9, 32/3], [2, -11/6, 14/3, -14/3, 32/9, -32/9], [0, 1/6, 14/3, -14/3, 32/9, -32/9], [-1/6, 8/3, -13/3, 34/9, -16/3, 32/9], [-1/6, -4/3, -1/3, 34/9, -16/3, 32/9]] \$ \times \$ [[1, 1, 1/2, 3/2, 1, 1], [1, 1, 1/2, 3/2, 3/4, 5/4], [9/8, 17/16, 1/2, 21/16, 3/4, 5/4], [9/8, 37/32, 35/64, 87/64, 45/64, 71/64], [129/128, 287/256, 73/128, 357/256, 3/4, 37/32], [135/128, 535/512, 545/1024, 1437/1024, 795/1024, 1217/1024]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

$$\$ [[1, 1, 2, 0, 1, 1], [1, 1, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2]] \$$$

$$[-y_1 + y_2 + y_3, y_1, y_2 + y_3, 0, y_2, y_3]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{1, 2, 4, 6}}, net cycles: 0 . order: 4

$$\$ [[1, 1, 0, 2, 1, 1], [1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1]] \$$$

$$[y_3, y_4, 0, -y_3 + y_4 + y_1 + y_2, y_1, y_2]$$

$$p = s^2 - s^3 + s^4 - s^5$$

Â» SYNC'D 5/32 , 0.1562500000

14 . Coloring, {4, 5}

R: [3, 3, 5, 6, 4, 1] **B:** [2, 4, 6, 5, 1, 2]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & ['-3' (' 4 + \tau + 3\tau^2 - \tau^3 + \tau^4 ')', 3' (' - 1 + \tau ')'' (' 4 + \tau + 2\tau^2 + \tau^3 ')', 3' (' 1 + \tau ')'' (' - 4 \\ & + \tau - 2\tau^2 + \tau^3 ')', 3' (' - 4 + \tau - 5\tau^2 - \tau^3 + \tau^4 ')', 3' (' - 4 - \tau - 5\tau^2 + \tau^3 + \tau^4 ')', 3' (' 1 + \tau ')'' \\ &)'' (' - 4 + 3\tau - 4\tau^2 + \tau^3 ')', '' \end{aligned}$$

For τ=1/2, [-83, -41, -93, -77, -89, -81] . FixedPtCheck, [83, 41, 93, 77, 89, 81]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ')'' (' \tau ')'^2 (' - 1 + \tau ')'$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$\$ [[332/737, -3023/4422, 2066/2211, -6814/2211, 7520/2211, -1888/2211] , [-398/737, 4321/4422, -2308/2211, 1358/2211, -7168/2211, 7520/2211] , [1076/737, -4523/4422, -2308/2211, 1358/2211, -7168/2211, 7520/2211] , [-202/737, -2303/4422, 7880/2211, 230/2211, 6080/2211, -12064/2211] , [-683/1474, -1840/2211, -9817/2211, 4778/2211, -4432/2211, 12704/2211] , [541/1474, 4604/2211, 4487/2211, -910/2211, 5168/2211, -13792/2211]] \$ \times \$ [[1, 3/2, 1/2, 1, 1, 1] , [1, 3/2, 5/8, 11/8, 7/8, 5/8] , [13/16, 39/32, 5/8, 43/32, 19/16, 13/16] , [35/32, 39/32, 65/128, 155/128, 149/128, 103/128] , [275/256, 729/512, 37/64, 617/512, 265/256, 175/256] , [485/512, 675/512, 1279/2048, 2717/2048, 2147/2048, 1505/2048]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

$R = \$ [[0, 0, 1, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[5/6, -1/6, -1/6, -1/6, -1/6] , [5/6, -1/6, -1/6, -1/6, -1/6] , [-1/6, 5/6, -1/6, -1/6, -1/6] , [-1/6, -1/6, -1/6, 5/6, -1/6] , [-1/6, -1/6, 5/6, -1/6, -1/6] , [-1/6, -1/6, -1/6, -1/6, 5/6]] \$ \times \$ [[1, 0, 2, 1, 1, 1] , [1, 0, 1, 1, 2, 1] , [1, 0, 1, 2, 1, 1] , [1, 0, 1, 1, 1, 2] , [2, 0, 1, 1, 1, 1]] \$$

Omega Rank for B : cycles: {{1, 2, 4, 5}}, net cycles: 0 . order: 4

$\$ [[1, 2, 0, 1, 1, 1] , [1, 2, 0, 2, 1, 0] , [1, 1, 0, 2, 2, 0] , [2, 1, 0, 1, 2, 0] , [2, 2, 0, 1, 1, 0]] \$$

$$[y_3, y_4, 0, y_2, y_1, -y_3 + y_4 - y_2 + y_1]$$

$$p = s^2 - s^3 + s^4 - s^5$$

Â» SYNC'D 35/256 , 0.1367187500

15 . Coloring, {4, 6}

R: [3, 3, 5, 6, 1, 2] **B:** [2, 4, 6, 5, 4, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

' ['3' ('1 + τ')' ('4 - 3τ + 2τ² + τ³)' , -3' ('1 + τ')' ('4 + τ + τ²)' ('-1 + τ')' , 3' ('1 + τ')' ('4 - 3τ + τ²)' , 3' ('-4 - τ + τ³)' ('-1 + τ')' , 3' ('4 + τ + 3τ² - τ³ + τ⁴)' , 3' ('1 + τ')' ('-4 - τ + τ²)' ('-1 + τ')']'

For $\tau=1/2$, [75, 57, 99, 35, 83, 51] . FixedPtCheck, [75, 57, 99, 35, 83, 51]

$$\det(A + \tau \Delta) = 1' (' \tau ')^2 (' 1 + \tau ') (' - 1 + \tau ')'$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

bi =

$$\begin{aligned} & \$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[2/3, 1/6, 0, -38/9, -64/9, 32/3], [-4/3, 1/6, 6, 22/9, 32/9, -32/3], [2, -11/6, -10/3, 10/3, -32/9, 32/9], [0, 1/6, -10/3, 10/3, -32/9, 32/9], [-1/6, -4/3, 7/3, -22/9, 16/3, -32/9], [-1/6, 8/3, -5/3, -22/9, 16/3, -32/9]] \$ \times \$ [[1, 1, 1/2, 3/2, 1, 1], [1, 1, 1/2, 3/2, 5/4, 3/4], [7/8, 15/16, 1/2, 27/16, 5/4, 3/4], [7/8, 27/32, 29/64, 105/64, 89/64, 51/64], [121/128, 219/256, 55/128, 429/256, 43/32, 3/4], [115/128, 459/512, 461/1024, 1689/1024, 1397/1024, 759/1024]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 5}}, net cycles: 0 . order: 3

$$\begin{aligned} & \$ [[1, 1, 2, 0, 1, 1], [1, 1, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0]] \$ \\ & [-y_1 + y_2 + y_3, y_1, y_2 + y_3, 0, y_2, y_3] \end{aligned}$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\begin{aligned} & \$ [[1, 1, 0, 2, 1, 1], [1, 1, 0, 2, 2, 0], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$ \\ & [y_4 - y_3 + y_2 + y_1, y_4, 0, y_3, y_2, y_1] \end{aligned}$$

$$p = -s^4 + s^5$$

Â» SYNC'D 3/32 , 0.09375000000

16 . Coloring, {5, 6}

R: [3, 3, 5, 5, 4, 2] **B**: [2, 4, 6, 6, 1, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$[3^{(-1+\tau)}, 3^{(-1+\tau)}, 3^{(1+\tau)}(-1+\tau), -3^{(1+\tau^2)}, -3^{(1+\tau)}, 3^{(-1+\tau)}]$$

For τ=1/2, [-2, -2, -3, -5, -6, -2] . FixedPtCheck, [2, 2, 3, 5, 6, 2]

$$\det(A + \tau \Delta) = 0$$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

$$[1, 1, 1, 1, 1, 1]$$

$$+ \quad \backslash ; \quad - \quad \backslash ; \quad \Delta$$

$$[[0, 1, 2, 1, 2, 0], [2, 2, 1, 3, 3, 1], [4, 3, 4, 5, 4, 4], [8, 8, 7, 9, 9, 7]] \quad [[2, 1, 0, 1, 0, 2], [2, 2, 3, 1, 1, 3], [4, 5, 4, 3, 4, 4], [8, 8, 9, 7, 7, 9]] \quad [[-1, 0, 1, 0, 1, -1], [0, 0, -1, 1, 1, -1], [0, -1, 0, 1, 0, 0], [0, 0, -1, 1, 1, -1]]$$

$$[y_1 + y_3 + y_2, y_1, -2y_1 - y_3 - 2y_2, y_2, -y_3, y_3]$$

$$p = s^2 - 4s^4$$

$$S+ \quad \backslash ; \quad S- \quad \backslash ; \quad NM$$

$$[[2, 2, 2, 2, 1, 1], [0, 2, 3, 1, 2, 2], [2, 1, 1, 2, 2, 2], [2, 1, 1, 2, 2, 2], [1, 2, 2, 1, 2, 2], [3, 2, 1, 2, 1, 1]] \quad [[2, 2, 2, 2, 1, 1], [0, 2, 3, 1, 2, 2], [2, 1, 1, 2, 2, 2], [2, 1, 1, 2, 2, 2], [1, 2, 2, 1, 2, 2], [3, 2, 1, 2, 1, 1]] \quad [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]]$$

CmmCk true, true, true

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 4	4 vs 4	4 vs 4	3 vs 4	3 vs 4

bi =

$$[[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \quad [[99/100, 3/100, -9/100, 3/100, 0, 0], [3/100, 91/100, 27/100, -9/100, 0, 0], [-9/100, 27/100, 19/100, 27/100, 0, 0], [3/100, -9/100, 27/100, 91/100, 0, 0], [0, 0, 0, 0, 1/10, 3/10], [0, 0, 0, 0, 3/10, 9/10]] \quad [[-33/16, 25/16, -1, 5/3], [27/16, -35/16, 3, -7/3], [-21/16, 13/16, -7/3, 3], [-21/16, 13/16, -7/3, 3], [21/8, -9/8, 2, -10/3], [11/8, 1/8, 2/3, -2]] \quad [[3/2, 1, 1/2, 1, 1/2, 3/2], [3/2, 3/2, 5/8, 7/8, 3/8, 9/8], [9/8, 45/32, 3/4, 39/32, 3/8, 9/8], [9/8, 9/8, 81/128, 147/128, 63/128, 189/128]]$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[0, 1, 2, 1, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[0, y_3, y_2, y_1, -y_3 + y_2 + y_1, 0]$$

$$p = s^3 - s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 6}}, net cycles: 1 . order: 4

$$\$ [[2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1], [1, 1, 0, 2, 0, 2]] \$$$

$$[y_1 - y_2 + y_3, y_1, 0, y_2, 0, y_3]$$

$$p = -s + s^2 - s^3 + s^4$$

Â» SYNC'D 1/8 , 0.1250000000

17 . Coloring, {2, 3, 4}

R: [3, 4, 6, 6, 1, 1] **B:** [2, 3, 5, 5, 4, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$['-6(' 1 + \tau ')', 6(' - 1 + \tau ')', (' 1 + \tau ')', -3(' 2 - \tau + \tau^2 ')', (' 1 + \tau ')', 3(' - 1 + \tau ')', (' 2 + \tau + \tau^2 ')', 6(' - 1 + \tau ')', -6(' 1 + \tau ')']$$

For τ=1/2, [-24, -12, -21, -11, -8, -24] . FixedPtCheck, [24, 12, 21, 11, 8, 24]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	4 vs 4

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \$ =$$

$$\$ [[0, -13/12, -115/36, -4/9, 44/9], [0, 5/4, -7/36, -4/9, -4/9], [0, -1/12, 41/36, -4/9, -4/9], [0, -1/12, 41/36, -4/9, -4/9], [-1/2, -1/6, -17/18, 8/9, 8/9], [3/2, 1/6, 37/18, 8/9, -40/9]] \$ x \$ [[1/2, 3/2, 1, 1, 3/2, 1/2], [1/2, 3/4, 5/4, 3/2, 3/2, 1/2], [1/2, 3/4, 11/16, 21/16, 33/16, 11/16], [11/16, 57/64, 11/16, 111/64,$$

$3/2, 1/2], [1/2, 57/64, 215/256, 345/256, 465/256, 155/256]$] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

\$ [[2, 0, 1, 1, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2]] \$

$[y_2, 0, y_2 - y_1, y_1, 0, y_2]$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 4

$[0, y_1, y_2, y_3, y_4, 0]$

$B = \$ [[0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[1/2, -1/4, -1/6, 1/12], [0, 1/2, -1/6, -1/6], [0, 0, 1/3, -1/6], [0, 0, 1/3, -1/6], [0, 0, -1/6, 1/3], [1/2, -1/4, -1/6, 1/12]] \$ x \$ [[0, 2, 1, 1, 2, 0], [0, 0, 2, 2, 2, 0], [0, 0, 0, 2, 4, 0], [0, 0, 0, 4, 2, 0]] \$$

Â» SYNC'D 1/4 , 0.2500000000

18 . Coloring, {2, 3, 5}

R: [3, 4, 6, 5, 4, 1] **B:** [2, 3, 5, 6, 1, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

' ['-3' (' - 4 - 3τ - 2τ² + τ³ '), -3' (' - 1 + τ ')'' (' 4 + 3τ + τ² '), 3' (' 4 + τ + 2τ² + τ³ '), 3' (' 4 + τ + τ² ')'' (' 1 + τ ')', 3' (' 4 + 3τ + 4τ² + τ³ '), 3' (' 1 + τ ')'' (' 4 - τ + τ² ')'']'

For τ=1/2, [47, 23, 41, 57, 53, 45] . FixedPtCheck, [47, 23, 41, 57, 53, 45]

$$\det(A + \tau \Delta) = 0$$

Delta Range : $[y_4, -y_4 - y_1 - y_2, y_1, y_2, y_3, -y_3]$

[1, 1, 1, 1, 1, 1]

+ \; - \; \Delta

\$ [[1, 0, 1, 2, 1, 1], [2, 2, 3, 1, 3, 1], [2, 5, 4, 5, 2, 6], [12, 8, 5, 7, 9, 7]] \$ \$ [[1, 2, 1, 0, 1, 1], [2, 2, 1, 3, 1, 3], [6, 3, 4, 3, 6, 2], [4, 8, 11, 9, 7, 9]] \$ \$ [[0, -1, 0, 1, 0, 0], [0, 0, 1, -1, 1, -1], [-2, 1, 0, 1, -2, 2], [4, 0, -3, -1, 1, -1]] \$

$$[-y_1 - y_3, y_3 - y_2, y_1, y_2, -y_3, y_3]$$

$$p = s + 3s^2 + 4s^3 + 4s^4$$

S+ \; S- \; NM

\$ [[4, 0, 4, 0, 0, 4], [0, 4, 0, 4, 4, 0], [4, 0, 4, 0, 0, 4], [0, 4, 0, 4, 4, 0], [0, 4, 0, 4, 4, 0], [4, 0, 4, 0, 0, 4]] \$ \$ [[0, 4, 0, 4, 4, 0], [4, 0, 4, 0, 0, 4], [0, 4, 0, 4, 4, 0], [4, 0, 4, 0, 0, 4], [4, 0, 4, 0, 0, 4], [0, 4, 0, 4, 4, 0]] \$ \$ [[3, 0, 3, 0, 0, 3], [0, 3, 0, 3, 3, 0], [3, 0, 3, 0, 0, 3], [0, 3, 0, 3, 3, 0], [0, 3, 0, 3, 3, 0], [3, 0, 3, 0, 0, 3]] \$

CmmCk true, true, true

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 5	4 vs 5

Omega Rank for R : cycles: {{1, 3, 6}, {4, 5}}, net cycles: 2 . order: 6

\$ [[1, 0, 1, 2, 1, 1], [1, 0, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1], [1, 0, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1]] \$

$$[y_2, 0, y_2, 3y_2 - y_1, y_1, y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3 \quad p = -s + s^5$$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 0 . order: 4

\$ [[1, 2, 1, 0, 1, 1], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0], [2, 1, 1, 0, 2, 0], [2, 2, 1, 0, 1, 0]] \$

$$[y_1 - y_2 + y_3 - y_4, y_1, y_2, 0, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$$[x_1, 3x_1, 4x_1, 4x_1]$$

For $A+2\Delta$: $[-3y_2 + 2y_1, 9y_2 - 8y_1, -3y_2 + 2y_1, y_2, y_1, -y_1]$

For $A-2\Delta$: $[-3y_1 - 2y_2, y_1, -3y_1 - 2y_2, 9y_1 + 8y_2, -y_2, y_2]$

Range of $\{\Omega\Delta^i\}$: $[-\mu_1 - \mu_3, -\mu_2 + \mu_3, \mu_1, \mu_2, -\mu_3, \mu_3]$

rank of M is 2 , rank of N is 2

M N

\$ [[0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0]]
\$ \$ [[0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0]] \$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [0, -1, 0, 1, 0, 0]$

ker M, $[-\lambda_1 - \lambda_4, -\lambda_2 - \lambda_3, \lambda_1, \lambda_2, \lambda_3, \lambda_4]$

Range M, $[x_1, x_2, x_1, x_2, x_2, x_1]$

$\tau = 18$, $r' = 1/2$

Ranges

Action of R on ranges, $[[6], [7], [6], [8], [2], [9], [8], [3], [2]]$

Action of B on ranges, $[[4], [5], [1], [7], [4], [9], [3], [5], [1]]$

$\beta(\{1, 2\}) = 1/9$

$\beta(\{1, 4\}) = 1/9$

$\beta(\{1, 5\}) = 1/9$

$\beta(\{2, 3\}) = 1/9$

$\beta(\{2, 6\}) = 1/9$

$\beta(\{3, 4\}) = 1/9$

$\beta(\{3, 5\}) = 1/9$

$\beta(\{4, 6\}) = 1/9$

$\beta(\{5, 6\}) = 1/9$

ker N, $[\mu_2, -\mu_1 - \mu_3, -\mu_2 - \mu_4, \mu_1, \mu_3, \mu_4]$

Range of N

$[y_2, y_1, y_2, y_1, y_1, y_2]$

Partitions

$\alpha(\{\{2, 4, 5\}, \{1, 3, 6\}\}) = 1/1$

$b_1 = \{2, 4, 5\}$, $b_2 = \{1, 3, 6\}$

Action of R and B on the blocks of the partitions: $\$ [[1, 1], [1, 1]] \$ = \$ [[1, 0], [0, 1]] \$ + \$ [[0, 1], [1, 0]] \$$

$['1', '2'], ['2', '1']$ with invariant measure $[1, 1]$

N by blocks, check: *true*. ' See partition graph.

' ' See level-2 partition graph.

'

Right Group	
Coloring	{2, 3, 5}
Rank	2
R,B	[3, 4, 6, 5, 4, 1], [2, 3, 5, 6, 1, 2]
π_2	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]
u_2	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

19 . Coloring, {2, 3, 6}

R: [3, 4, 6, 5, 1, 2] **B:** [2, 3, 5, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

Ω for $A+\tau\Delta$:

‘ [‘1 , 1 , 1 , 1 , 1 , 1 ‘ ‘

For $\tau=1/2$, [1, 1, 1, 1, 1, 1] . FixedPtCheck, [1, 1, 1, 1, 1, 1]

$\det(A + \tau \Delta) = 1^6 (\tau^6)^2 (1 + \tau^2)^6$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

[1, 1, 1, 1, 1, 1]

+ \ ; - \ ; Δ

\$ [[1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \$ [[1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]] \$ \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$

[0, 0, 0, 0, 0, 0]

p = s

S+ \ ; S- \ ; NM

\$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0]] \$ \$ [[0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]] \$

0]] \$ [[5, 4, 4, 4, 4, 4] , [4, 5, 4, 4, 4, 4] , [4, 4, 5, 4, 4, 4] , [4, 4, 4, 5, 4, 4] , [4, 4, 4, 4, 5, 4] , [4, 4, 4, 4, 4, 5]] \$

CmmCk true, true, true

$$p' = s \quad p' = s^2 \quad p' = s^3$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
0 vs 4	1 vs 6	1 vs 6	1 vs 6	1 vs 6

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}}, net cycles: 1 . order: 6

\$ [[1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1]] \$

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5$$

Omega Rank for B : cycles: {{1, 2, 3, 4, 5, 6}}, net cycles: 1 . order: 6

\$ [[1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1]] \$

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$$p' = -1 + s^2 \quad p' = -1 + s \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$$[x_1, x_4, x_2, x_3]$$

$$\text{For } A+2\Delta : [-y_1 - y_2 - y_3 - y_4 - y_5, y_1, y_2, y_3, y_4, y_5]$$

$$\text{For } A-2\Delta : [-y_1 - y_2 - y_3 - y_4 - y_5, y_1, y_2, y_3, y_4, y_5]$$

Range of $\{\Omega\Delta^i\}$: [0, 0, 0, 0, 0, 0]

rank of M is 6 , rank of N is 6

M N

\$ [[0, 1, 1, 1, 1, 1] , [1, 0, 1, 1, 1, 1] , [1, 1, 0, 1, 1, 1] , [1, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 1] , [1, 1, 1, 1, 1, 0]] \$
 \$ [[0, 1, 1, 1, 1, 1] , [1, 0, 1, 1, 1, 1] , [1, 1, 0, 1, 1, 1] , [1, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 1] , [1, 1, 1, 1, 1, 0]] \$

Check is $\Omega\Delta N$ zero? true, $\pi\Delta = [0, 0, 0, 0, 0, 0]$

ker M, [0, 0, 0, 0, 0, 0]

Range M, [x₆, x₅, x₂, x₃, x₄, x₁]

$\tau = 6$, $r' = 5/6$

Ranges

Action of R on ranges, [[1]]

Action of B on ranges, [[1]]

$\beta(\{1, 2, 3, 4, 5, 6\}) = 1/1$

ker N, [0, 0, 0, 0, 0, 0]

Range of N

[y₁, y₂, y₆, y₃, y₄, y₅]

Partitions

$\alpha(\{\{1\}, \{2\}, \{3\}, \{4\}, \{6\}, \{5\}\}) = 1/1$

b1 = {1} ‘, ‘ b2 = {2} ‘, ‘ b3 = {3} ‘, ‘ b4 = {4} ‘, ‘ b5 = {6} ‘, ‘ b6 = {5}

Action of R and B on the blocks of the partitions: \$ [[0, 0, 0, 0, 1, 1], [1, 0, 0, 0, 1, 0], [1, 1, 0, 0, 0, 0], [0, 1, 0, 0, 0, 1], [0, 0, 1, 1, 0, 0], [0, 0, 1, 1, 0, 0]] \$ = \$ [[0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ + \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0]] \$
[‘6‘, ‘5‘, ‘1‘, ‘2‘, ‘3‘, ‘4‘], [‘5‘, ‘1‘, ‘2‘, ‘6‘, ‘4‘, ‘3‘] with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-6 partition graph.

‘

Right Group	
Coloring	{2, 3, 6}
Rank	6
R,B	[3, 4, 6, 5, 1, 2], [2, 3, 5, 6, 4, 1]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 1)
wpp	[1, 1, 1, 1, 1, 1]
π_6	[1]
u_6	[1]

20 . Coloring, {2, 4, 5}

R: [3, 4, 5, 6, 4, 1] **B:** [2, 3, 6, 5, 1, 2]

' See graph

' ' See pair graph

,

Ω for $A+\tau\Delta$:

' ['-3' (' 1 + \tau ')'' (' - 4 + 3\tau - 4\tau ^ 2 + \tau ^ 3 ')' , -3' (' 4 + \tau + 2\tau ^ 2 + \tau ^ 3 ')'' (' - 1 + \tau ')' , 3' (' 4 - \tau + 3\tau ^ 2 + \tau ^ 3 + \tau ^ 4 ')' , 3' (' 4 - \tau + \tau ^ 3 ')'' (' 1 + \tau ')' , 3' (' 4 - 3\tau + 2\tau ^ 2 + \tau ^ 3 ')'' (' 1 + \tau ')' , 3' (' 4 + \tau + 3\tau ^ 2 - \tau ^ 3 + \tau ^ 4 ')']'

For $\tau=1/2$, [81, 41, 71, 87, 75, 83] . FixedPtCheck, [81, 41, 71, 87, 75, 83]

$\det(A + \tau \Delta) = 0$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	5 vs 5	3 vs 5

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0] , [0, 0, 3/4, 1/4, 0, 0] , [0, 0, 0, 0, 1/4, 3/4] , [0, 0, 0, 0, 3/4, 1/4] , [3/4, 0, 0, 1/4, 0, 0] , [1/4, 3/4, 0, 0, 0, 0]] \$ \times \$ [[91/100, 3/100, -9/100, 27/100, 0, 0] , [3/100, 99/100, 3/100, -9/100, 0, 0] , [-9/100, 3/100, 91/100, 27/100, 0, 0] , [27/100, -9/100, 27/100, 19/100, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[-23/84, -65/84, -37/42, -44/21, 88/21] , [-29/84, 169/84, 89/42, -20/21, -8/3] , [-23/84, -137/84, -1/42, 52/21, -8/21] , [139/84, 1/84, 89/42, -20/21, -8/3] , [25/84, 115/84, -55/42, 52/21, -8/3] , [-5/84, -83/84, -85/42, -20/21, 88/21]] \$ \times \$ [[1, 3/2, 1, 1/2, 1, 1] , [1, 3/2, 11/8, 5/8, 5/8, 7/8] , [11/16, 45/32, 11/8, 17/32, 13/16, 19/16] , [29/32, 45/32, 157/128, 71/128, 95/128, 149/128] , [217/256, 795/512, 41/32, 275/512, 185/256, 271/256]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$[y_1, 0, y_2, y_3, y_4, y_5]$

$R = \$ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[-1/6, -1/6, -1/6, 5/6, -1/6] , [5/6, -1/6, -1/6, -1/6, -1/6] , [-1/6, -1/6, -1/6, -1/6, 5/6] , [-1/6, 5/6, -1/6, -1/6, -1/6] , [5/6, -1/6, -1/6, -1/6, -1/6] , [-1/6, -1/6, 5/6, -1/6, -1/6]] \$ \times \$ [[1, 0, 1, 2, 1, 1] , [1, 0, 1, 1, 1, 2] , [2, 0, 1, 1, 1, 1] , [1, 0, 2, 1, 1, 1] , [1, 0, 1, 1, 2, 1]] \$$

Omega Rank for B : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

\$ [[1, 2, 1, 0, 1, 1] , [1, 2, 2, 0, 0, 1] , [0, 2, 2, 0, 0, 2] , [0, 2, 2, 0, 0, 2] , [0, 2, 2, 0, 0, 2]] \$

[y₁ + y₂ - y₃, y₁ + y₂, y₁, 0, y₂, y₃]

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Â» SYNC'D 9/64 , 0.1406250000

21 . Coloring, {2, 4, 6}

R: [3, 4, 5, 6, 1, 2] **B**: [2, 3, 6, 5, 4, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

' ['1 , 1 , 1 , 1 , 1 , 1 ']'

For τ=1/2, [1, 1, 1, 1, 1, 1] . FixedPtCheck, [1, 1, 1, 1, 1, 1]

$$\det(A + \tau \Delta) = 1' (' \tau ')' ^2 (' 1 + \tau ^2 ')'$$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

[1, 1, 1, 1, 1, 1]

+ \ ; - \ ; Δ

\$ [[1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1]] \$ \$ [[1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1] , [1, 1, 1, 1, 1, 1]] \$ \$ [[0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] \$

[0, 0, 0, 0, 0, 0]

$$p' = s^2 \quad p' = s^3 \quad p = s$$

S+ \ ; S- \ ; NM

\$ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0]] \$ \$ [[0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0]] \$ \$ [[5, 4, 4, 4, 4, 4] , [4, 5, 4, 4, 4, 4] , [4, 4, 5, 4, 4, 4] , [4, 4, 4, 5, 4, 4] , [4, 4, 4, 4, 5, 4] , [4, 4, 4, 4, 4, 5]] \$

CmmCk true, true, true

$$p' = s$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
0 vs 4	1 vs 6	1 vs 6	1 vs 6	1 vs 6

Omega Rank for R : cycles: $\{\{2, 4, 6\}, \{1, 3, 5\}\}$, net cycles: 2 . order: 3

\$ [[1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]]
\$

[$y_1, y_1, y_1, y_1, y_1, y_1$]

$$p' = -1 + s \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5 \quad p' = -1 + s^2$$

Omega Rank for B : cycles: $\{\{4, 5\}, \{1, 2, 3, 6\}\}$, net cycles: 2 . order: 4

\$ [[1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1]]
\$

[$y_1, y_1, y_1, y_1, y_1, y_1$]

$$p' = -1 + s^2 \quad p' = -1 + s \quad p' = -1 + s^3 \quad p' = -1 + s^5 \quad p' = -1 + s^4$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

[x_1, x_2, x_4, x_3]

For $A+2\Delta$: [$y_3, y_2, -y_3 - y_2 - y_1 - y_4 - y_5, y_1, y_4, y_5$]

For $A-2\Delta$: [$y_3, y_1, y_2, -y_3 - y_2 - y_1 - y_4 - y_5, y_4, y_5$]

Range of $\{\Omega\Delta^i\}$: [0, 0, 0, 0, 0, 0]

rank of M is 6 , rank of N is 6

M N

\$ [[0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0]]
\$ \$ [[0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0]] \$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [0, 0, 0, 0, 0, 0]$

ker M, [0, 0, 0, 0, 0, 0]

Range M, [$x_1, x_2, x_3, x_4, x_5, x_6$]

$\tau = 6$, $r' = 5/6$

Ranges

Action of R on ranges, [[1]]

Action of B on ranges, [[1]]

$$\beta(\{1, 2, 3, 4, 5, 6\}) = 1/1$$

ker N, [0, 0, 0, 0, 0, 0]

Range of N

$$[y_6, y_5, y_4, y_3, y_2, y_1]$$

Partitions

$$\alpha(\{\{1\}, \{2\}, \{3\}, \{4\}, \{6\}, \{5\}\}) = 1/1$$

$$b_1 = \{1\} \text{ , ' } b_2 = \{2\} \text{ , ' } b_3 = \{3\} \text{ , ' } b_4 = \{4\} \text{ , ' } b_5 = \{6\} \text{ , ' } b_6 = \{5\}$$

Action of R and B on the blocks of the partitions: \$ [[0, 0, 0, 0, 1, 1] , [1, 0, 0, 0, 1, 0] , [1, 1, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 1] , [0, 0, 1, 1, 0, 0] , [0, 0, 1, 1, 0, 0]] \$ = \$ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0]] \$ + \$ [[0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0]] \$
 ['6' , '5' , '1' , '2' , '4' , '3' , ['5' , '1' , '2' , '6' , '3' , '4'] with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-6 partition graph.

'

Right Group	
Coloring	{2, 4, 6}
Rank	6
R,B	[3, 4, 5, 6, 1, 2], [2, 3, 6, 5, 4, 1]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 2)
wpp	[1, 1, 1, 1, 1, 1]
π_6	[1]
u_6	[1]

22 . Coloring, {2, 5, 6}

R: [3, 4, 5, 5, 4, 2] **B:** [2, 3, 6, 6, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘ [‘ $3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘ , $3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘ , $3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘ , $-3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘ , $-3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘ , $3^{\tau} (-1 + \tau^{\tau})^{\tau}$ ‘] ‘ ‘

For $\tau=1/2$, [-1, -1, -1, -3, -3, -1] . FixedPtCheck, [1, 1, 1, 3, 3, 1]

$\det(A + \tau \Delta) = 0$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1/10, 3/10], [0, 0, 0, 0, 3/10, 9/10]] \times \$ [[0, 47/16, -37/48, 5/3, -11/3], [0, -53/16, 45/16, -7/3, 3], [0, 27/16, -35/16, 3, -7/3], [0, 27/16, -35/16, 3, -7/3], [3/2, -15/8, 15/8, -10/3, 2], [-1/2, -9/8, 11/24, -2, 10/3]] \times \$ [[3/2, 1, 1, 1/2, 1/2, 3/2], [3/2, 3/2, 9/8, 3/8, 3/8, 9/8], [9/8, 45/32, 3/2, 15/32, 3/8, 9/8], [9/8, 9/8, 171/128, 57/128, 63/128, 189/128], [189/128, 621/512, 9/8, 207/512, 57/128, 171/128]] \times \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}}, net cycles: -1 . order: 2

$\$ [[0, 1, 1, 2, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \times \$$

$[0, y_2, y_2, y_1, y_1, 0]$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 1 . order: 4

$\$ [[2, 1, 1, 0, 0, 2], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2]] \times \$$

$[y_1 - y_2 + y_3, y_1, y_2, 0, 0, y_3]$

$$p = -s + s^2 - s^3 + s^4$$

$\hat{A} \gg \text{SYNC'D } 1/8, 0.1250000000$

23 . Coloring, {3, 4, 5}

R: [3, 3, 6, 6, 4, 1] **B:** [2, 4, 5, 5, 1, 2]

' See graph

' ' See pair graph

,

Ω for $A+\tau\Delta$:

' [-12' (' 1 + τ^2 ')', 6' (' - 1 + τ ')', (' 2 + $\tau + \tau^2$ ')', 3' (' - 4 + $\tau - 2\tau^2 + \tau^3$ ')', (' 1 + τ ')', -3' (' - 4 - $\tau + \tau^3$ ')', (' - 1 + τ ')', 12' (' - 1 + τ ')', -12' (' 1 + τ ')']'

For $\tau=1/2$, [-80, -44, -93, -35, -32, -96] . FixedPtCheck, [80, 44, 93, 35, 32, 96]

$\det(A + \tau \Delta) = 0$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	2 vs 4

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4, 0, 0], [1/4, 3/4, 0, 0, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \times =$

$\$ [[0, 1/6, -16/9, -16/9, 32/9], [0, 5/2, 11/9, -16/9, -16/9], [0, -5/6, 17/9, 8/9, -16/9], [0, -5/6, 17/9, 8/9, -16/9], [-1/2, -7/6, -23/18, 20/9, 8/9], [3/2, 1/6, -35/18, -4/9, 8/9]] \times \$ [[1, 3/2, 1/2, 1, 3/2, 1/2], [5/4, 9/8, 5/8, 3/2, 9/8, 3/8], [15/16, 39/32, 19/32, 9/8, 51/32, 17/32], [85/64, 141/128, 69/128, 21/16, 165/128, 55/128], [275/256, 675/512, 311/512, 147/128, 711/512, 237/512]] \times$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$[y_4, 0, y_3, y_2, 0, y_1]$

$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \times \$ [[0, -5/18, 1/18, 7/18], [0, -5/18, 1/18, 7/18], [0, 7/18, -5/18, 1/18], [0, 7/18, -5/18, 1/18], [1, -5/18, 1/18, -11/18], [0, 1/18, 7/18, -5/18]] \times \$ [[1, 0, 2, 1, 0, 2], [2, 0, 1, 0, 0, 3], [3, 0, 2, 0, 0, 1], [1, 0, 3, 0, 0, 2]] \times$

Omega Rank for B : cycles: {{1, 2, 4, 5}}, net cycles: 1 . order: 4

$$\$ [[1, 2, 0, 1, 2, 0], [2, 1, 0, 2, 1, 0], [1, 2, 0, 1, 2, 0], [2, 1, 0, 2, 1, 0]] \$$$

$$[y_1, y_2, 0, y_1, y_2, 0]$$

$$p' = -s + s^3 \quad p = -s + s^3$$

Â» SYNC'D 15/32 , 0.4687500000

24 . Coloring, {3, 4, 6}

R: [3, 3, 6, 6, 1, 2] **B:** [2, 4, 5, 5, 4, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{bmatrix} -12(\tau - 1 + \tau^2) & 6(2 - \tau + \tau^2) & 3(4 - 3\tau + \tau^2) & 2 \\ -3(4 - \tau + \tau^3) & -12(\tau - 1 + \tau^2) & 12(\tau - 1 + \tau^2) & 2 \end{bmatrix}$$

For τ=1/2, [48, 84, 99, 29, 32, 96] . FixedPtCheck, [48, 84, 99, 29, 32, 96]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \$ =$$

$$\$ [[0, 1/2, -19/9, -32/9, 16/3], [0, 5/2, 11/9, 16/9, -16/3], [0, -3/2, 5/3, -16/9, 16/9], [0, -3/2, 5/3, -16/9, 16/9], [-1/2, 1, -11/9, 8/3, -16/9], [3/2, -1, -11/9, 8/3, -16/9]] \$ \times \$ [[1, 1, 1/2, 3/2, 3/2, 1/2], [3/4, 7/8, 1/2, 15/8, 3/2, 1/2], [3/4, 11/16, 13/32, 57/32, 57/32, 19/32], [57/64, 91/128, 23/64, 237/128, 105/64, 35/64], [105/128, 103/128, 205/512, 903/512, 849/512, 283/512]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

$$\$ [[1, 1, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2]] \$$$

$$[y_1, y_2, y_1 + y_2, 0, 0, y_1 + y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[1, 1, 0, 2, 2, 0], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[y_2, y_3, 0, -y_2 + y_3 + y_1, y_1, 0]$$

$$p = -s^3 + s^4$$

Â» SYNC'D 1/16 , 0.06250000000

25 . Coloring, {3, 5, 6}

R: [3, 3, 6, 5, 4, 2] **B:** [2, 4, 5, 6, 1, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$['-3' (' 4 + 3\tau + \tau^2 ')'' (' - 1 + \tau ')', 3' (' 4 + \tau + 2\tau^2 + \tau^3 ')', 12' (' 1 + \tau ')', 6' (' 2 + \tau + \tau^2 ')', 3' (' 4 - \tau + \tau^2 ')'' (' 1 + \tau ')', -3' (' - 4 - 3\tau - 2\tau^2 + \tau^3 ')'']'$$

For τ=1/2, [23, 41, 48, 44, 45, 47] . FixedPtCheck, [23, 41, 48, 44, 45, 47]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ x \$ [[99/100, 3/100, -9/100, 3/100, 0, 0], [3/100, 91/100, 27/100, -9/100, 0, 0], [-9/100, 27/100, 19/100, 27/100, 0, 0], [3/100, -9/100, 27/100, 91/100, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[1/9, -43/18, 10/3, -40/9, 32/9], [-5/9, 77/18, -8/3, 56/9, -64/9], [19/9, -79/18, 10/3, -40/9, 32/9], [-1, -83/18, 4/3, -8/3, 64/9], [1/18, 14/3, -11/3, 40/9, -16/3], [5/18, 22/9, -5/3, 8/9, -16/9]] \$ x \$ [[3/2, 1, 1/2, 1, 1, 1], [3/2, 11/8, 5/8, 1, 5/8, 7/8], [9/8, 43/32, 23/32, 19/16, 23/32, 29/32], [39/32, 137/128, 79/128, 19/16, 107/128, 137/128], [183/128, 605/512, 293/512, 259/256, 389/512, 535/512]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}, {2, 3, 6}}, net cycles: 2 . order: 6

$$\begin{aligned} & \$ [[0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2]] \$ \\ & [0, -y_1 + 4y_2 - y_3, y_1, y_2, y_2, y_3] \end{aligned}$$

$$p' = -s + s^4 \quad p = -s + s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 6}}, net cycles: 0 . order: 4

$$\begin{aligned} & \$ [[2, 1, 0, 1, 1, 1], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1], [1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2]] \$ \\ & [y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4] \end{aligned}$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â» SYNC'D 25/256 , 0.09765625000

26 . Coloring, {4, 5, 6}

R: [3, 3, 5, 6, 4, 2] **B:** [2, 4, 6, 5, 1, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & \text{' ['-3' (' 4 + \tau + 2\tau^2 + \tau^3 ')', (' - 1 + \tau ')', 3' (' 4 - \tau + 3\tau^2 + \tau^3 + \tau^4 ')', 6' (' 1 + \tau ')', (' 2 - } \\ & \tau + \tau^2 ')', 12' (' 1 + \tau^2 ')', 3' (' 4 + \tau + 3\tau^2 - \tau^3 + \tau^4 ')', -3' (' 1 + \tau ')', (' - 4 + 3\tau - 4\tau^2 + \tau^3 } \\ & \text{')', ']' } \end{aligned}$$

For τ=1/2, [41, 71, 84, 80, 83, 81] . FixedPtCheck, [41, 71, 84, 80, 83, 81]

$$\det(A + \tau \Delta) = 0$$

Delta Range : [y4, -y4 - y1 - y2, y1, y2, y3, -y3]

$$[1, 1, 1, 1, 1, 1]$$

$$+ \quad \backslash ; \quad - \quad \backslash ; \quad \Delta$$

$$\begin{aligned} & \$ [[0, 1, 2, 1, 1, 1], [2, 3, 1, 2, 3, 1], [4, 3, 5, 4, 3, 5], [8, 9, 7, 8, 9, 7]] \$ \quad \$ [[2, 1, 0, 1, 1, 1], [2, 1, 3, 2, \\ & 1, 3], [4, 5, 3, 4, 5, 3], [8, 7, 9, 8, 7, 9]] \$ \quad \$ [[-1, 0, 1, 0, 0, 0], [0, 1, -1, 0, 1, -1], [0, -1, 1, 0, -1, 1], [0, \\ & 1, -1, 0, 1, -1]] \$ \end{aligned}$$

$$[y_2, -y_1, -y_2 + y_1, 0, -y_1, y_1]$$

$$p = s^2 - 4s^4$$

S+ \ ; S- \ ; NM

\$ [[2, 1, 2, 3, 2, 0], [0, 1, 2, 0, 2, 0], [2, 0, 1, 3, 3, 1], [2, 2, 1, 1, 1, 3], [1, 2, 1, 2, 0, 4], [3, 3, 1, 1, 0, 2]]
 \$ \$ [[2, 3, 2, 1, 0, 2], [0, 1, 1, 1, 0, 2], [2, 2, 1, 1, 1, 3], [2, 0, 1, 3, 3, 1], [1, 2, 3, 0, 4, 0], [3, 1, 1, 3, 2, 0]] \$ \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$

CmmCk true, true, true

$$p' = s^2 + 2s^3$$

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
2 vs 4	5 vs 5	5 vs 5	5 vs 5	4 vs 5

bi =

\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 1/4, 3/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ x \$ [[99/100, 3/100, -9/100, 3/100, 0, 0], [3/100, 91/100, 27/100, -9/100, 0, 0], [-9/100, 27/100, 19/100, 27/100, 0, 0], [3/100, -9/100, 27/100, 91/100, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =

\$ [[-1/6, 47/6, -25/6, 22/3, -32/3], [-1/6, -61/6, 47/6, -32/3, 40/3], [11/6, 35/6, -25/6, 22/3, -32/3], [-7/6, 47/6, -43/6, 34/3, -32/3], [1/3, -26/3, 35/6, -32/3, 40/3], [1/3, -8/3, 11/6, -14/3, 16/3]] \$ x \$ [[3/2, 1, 1/2, 1, 1, 1], [3/2, 11/8, 5/8, 1, 7/8, 5/8], [9/8, 41/32, 23/32, 5/4, 29/32, 23/32], [39/32, 131/128, 77/128, 19/16, 143/128, 109/128], [189/128, 577/512, 287/512, 67/64, 533/512, 383/512]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[0, y_1, y_2, y_3, y_4, y_5]$$

R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0]] \$ x \$ [[0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[5/6, -1/6, -1/6, -1/6, -1/6], [5/6, -1/6, -1/6, -1/6, -1/6], [-1/6, 5/6, -1/6, -1/6, -1/6], [-1/6, -1/6, -1/6, 5/6, -1/6], [-1/6, -1/6, 5/6, -1/6, -1/6], [-1/6, -1/6, -1/6, -1/6, 5/6]] \$ x \$ [[0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 2, 1], [0, 1, 1, 2, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1]] \$

Omega Rank for B : cycles: {{1, 2, 4, 5}}, net cycles: 0 . order: 4

\$ [[2, 1, 0, 1, 1, 1], [2, 2, 0, 1, 1, 0], [1, 2, 0, 2, 1, 0], [1, 1, 0, 2, 2, 0], [2, 1, 0, 1, 2, 0]] \$

$$[y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â» SYNC'D 55/256 , 0.2148437500

27 . Coloring, {2, 3, 4, 5}

R: [3, 4, 6, 6, 4, 1] **B:** [2, 3, 5, 5, 1, 2]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

' [-12' (' 1 + τ ² ')' , 6' (' 2 + τ + τ ² ')' (' - 1 + τ ')' , -3' (' 4 - τ + 3τ ² + τ ³ + τ ⁴ ')' , 3' (' 4 + τ + τ ² ')' (' 1 + τ ')' (' - 1 + τ ')' , 12' (' - 1 + τ ')' , -12' (' 1 + τ ')']'

For τ=1/2, [-80, -44, -71, -57, -32, -96] . FixedPtCheck, [80, 44, 71, 57, 32, 96]

det(A + τ Δ) = 0

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	4 vs 4	4 vs 4	4 vs 4	2 vs 4

bi =

\$ [[0, 3/4, 1/4, 0, 0, 0] , [0, 0, 3/4, 1/4, 0, 0] , [0, 0, 0, 0, 3/4, 1/4] , [0, 0, 0, 0, 3/4, 1/4] , [3/4, 0, 0, 1/4, 0, 0] , [1/4, 3/4, 0, 0, 0, 0]] \$ x \$ [[91/100, 3/100, -9/100, 27/100, 0, 0] , [3/100, 99/100, 3/100, -9/100, 0, 0] , [-9/100, 3/100, 91/100, 27/100, 0, 0] , [27/100, -9/100, 27/100, 19/100, 0, 0] , [0, 0, 0, 0, 9/10, 3/10] , [0, 0, 0, 0, 3/10, 1/10]] \$ =

\$ [[5/3, 25/18, -10/9, -16/9] , [-5/6, 17/9, 8/9, -16/9] , [-5/6, -13/9, 14/9, 8/9] , [-5/6, -13/9, 14/9, 8/9] , [-1/12, -31/36, -10/9, 20/9] , [23/12, 17/36, -16/9, -4/9]] \$ x \$ [[1, 3/2, 1, 1/2, 3/2, 1/2] , [5/4, 9/8, 11/8, 3/4, 9/8, 3/8] , [15/16, 39/32, 37/32, 9/16, 51/32, 17/32] , [85/64, 141/128, 147/128, 45/64, 165/128, 55/128]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

[y₁, 0, y₂, y₃, 0, y₄]

R = \$ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [1, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, -5/18, 1/18, 7/18] , [1/2, -5/18, 1/18, -1/9] , [0, 7/18, -5/18, 1/18] , [0, 7/18, -5/18, 1/18] , [1/2, -5/18, 1/18, -1/9] , [0, 1/18, 7/18, -5/18]] \$ x \$ [[1, 0, 1, 2, 0, 2] , [2, 0, 1, 0, 0, 3] , [3, 0, 2, 0, 0, 1] , [1, 0, 3, 0, 0, 2]] \$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 1 . order: 4

$$\$ [[1, 2, 1, 0, 2, 0], [2, 1, 2, 0, 1, 0], [1, 2, 1, 0, 2, 0], [2, 1, 2, 0, 1, 0]] \$$$

$$[y_1, y_2, y_1, 0, y_2, 0]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

Â» SYNC'D 15/32 , 0.4687500000

28 . Coloring, {2, 3, 4, 6}

R: [3, 4, 6, 6, 1, 2] **B:** [2, 3, 5, 5, 4, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & ['-12' ('1 + \tau')'' ('-1 + \tau')', 6' ('1 + \tau')'' ('2 - \tau + \tau^2')', -3' ('4 + \tau + \tau^2')'' ('1 + \tau')' \\ &)'' ('-1 + \tau')', 3' ('4 - \tau + 3\tau^2 + \tau^3 + \tau^4')', -12' ('-1 + \tau')', 12' ('1 + \tau')'']' \end{aligned}$$

For τ=1/2, [48, 84, 57, 71, 32, 96] . FixedPtCheck, [48, 84, 57, 71, 32, 96]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [1/4, 0, 0, 3/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \$ =$$

$$\$ [[0, 1/2, -19/9, -32/9, 16/3], [0, 5/2, 11/9, 16/9, -16/3], [0, -3/2, 5/3, -16/9, 16/9], [0, -3/2, 5/3, -16/9, 16/9], [-1/2, 1, -11/9, 8/3, -16/9], [3/2, -1, -11/9, 8/3, -16/9]] \$ \times \$ [[1, 1, 1, 1, 3/2, 1/2], [3/4, 7/8, 1, 11/8, 3/2, 1/2], [3/4, 11/16, 27/32, 43/32, 57/32, 19/32], [57/64, 91/128, 45/64, 193/128, 105/64, 35/64], [105/128, 103/128, 387/512, 721/512, 849/512, 283/512]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

$$\$ [[1, 1, 1, 1, 0, 2], [0, 2, 1, 1, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2]] \$$$

$$[y_3, y_2, y_1, y_3 + y_2 - y_1, 0, y_3 + y_2]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[1, 1, 1, 1, 2, 0], [0, 1, 1, 2, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[y_1 - y_2 - y_3 + y_4, y_1, y_2, y_3, y_4, 0]$$

$$p = -s^4 + s^5$$

Â» SYNC'D 1/16 , 0.06250000000

29 . Coloring, {2, 3, 5, 6}

R: [3, 4, 6, 5, 4, 2] **B:** [2, 3, 5, 6, 1, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & ['3' ('4 - \tau + \tau^3') ' (' - 1 + \tau ') ' , 3' ('4 + \tau + 2\tau^2 + \tau^3') ' (' - 1 + \tau ') ' , 12' (' - 1 + \tau ') ' , \\ & -6' ('2 - \tau + \tau^2') ' ('1 + \tau ') ' , -3' ('4 - \tau + 3\tau^2 + \tau^3 + \tau^4') ' , 3' ('4 - \tau + \tau^2') ' (' - 1 + \tau ') ' , \\ & ('1 + \tau ') '] \end{aligned}$$

For τ=1/2, [-29, -41, -32, -84, -71, -45] . FixedPtCheck, [29, 41, 32, 84, 71, 45]

$$\det(A + \tau \Delta) = 1' (' \tau ') ' ^2 (' - 1 + \tau ') ' ('1 + \tau ') '$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 vs 4	6 vs 6	6 vs 6	4 vs 5	4 vs 5

bi =

$$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 1/4, 3/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[-69/58, 17/29, -299/29, 220/29, -1040/87, 448/29], [3/2, -1, 17, -12, 16, -64/3], [-93/58, 33/29, -403/29, 352/29, -400/29, 1408/87], [47/58, -41/29, -299/29, 220/29, -1040/87, 448/29], [45/58, -59/29, 340/29, -320/29, 448/29, -1280/87], [41/58, 79/29, 168/29, -124/29, 544/87, -320/29]] \$ \times \$ [[3/2, 1, 1, 1/2, 1, 1], [3/2, 11/8, 9/8, 1/2, 7/8, 5/8], [9/8, 41/32, 45/32, 9/16, 31/32, 21/32], [39/32, 129/128, 159/128, 9/16, 153/128, 99/128], [189/128, 567/512, 543/512, 141/256, 549/512, 375/512], [693/512, 2643/2048, 2457/2048, 279/512, 1911/2048, 1389/2048]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 5}}, net cycles: 0 . order: 4

$$\$ [[0, 1, 1, 2, 1, 1], [0, 1, 0, 2, 2, 1], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0]] \$$$

$$[0, y_2, y_3, y_4, y_1, y_2 + y_3 - y_4 + y_1]$$

$$p = -s^4 + s^5$$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 0 . order: 4

$$\$ [[2, 1, 1, 0, 1, 1], [2, 2, 1, 0, 1, 0], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0], [2, 1, 1, 0, 2, 0]] \$$$

$$[y_2, y_2 + y_1 - y_3 - y_4, y_1, 0, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â» SYNC'D 27/256 , 0.1054687500

30 . Coloring, {2, 4, 5, 6}

R: [3, 4, 5, 6, 4, 2] **B:** [2, 3, 6, 5, 1, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$['3' ('-1 + \tau')' ('-4 - \tau + \tau^3')', 3' ('4 - \tau + 3\tau^2 + \tau^3 + \tau^4')', -6' ('-1 + \tau')' ('2 + \tau + \tau^2')', 12' ('1 + \tau')', -3' ('4 + \tau + \tau^2')' ('-1 + \tau')' ('1 + \tau')', 3' ('4 + \tau + 3\tau^2 - \tau^3 + \tau^4')']'$$

For τ=1/2, [35, 71, 44, 96, 57, 83] . FixedPtCheck, [35, 71, 44, 96, 57, 83]

$$\det(A + \tau \Delta) = 1' (' \tau ')'^2 ' ('-1 + \tau')' ('1 + \tau')$$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

$$[1, 1, 1, 1, 1, 1]$$

$$+ \quad \backslash ; \quad - \quad \backslash ; \quad \Delta$$

$$\$ [[0, 1, 1, 2, 1, 1], [2, 3, 1, 2, 1, 3], [4, 5, 3, 4, 3, 5], [8, 9, 7, 8, 7, 9]] \$ \quad \$ [[2, 1, 1, 0, 1, 1], [2, 1, 3, 2, 3, 1], [4, 3, 5, 4, 5, 3], [8, 7, 9, 8, 9, 7]] \$ \quad \$ [[-1, 0, 0, 1, 0, 0], [0, 1, -1, 0, -1, 1], [0, 1, -1, 0, -1, 1], [0, 1, -1, 0, -1, 1]] \$$$

$$[-y_1, y_2, -y_2, y_1, -y_2, y_2]$$

$$p = s^2 - 4s^4$$

S+ \ ; S- \ ; NM

\$ [[2, 3, 2, 1, 2, 0], [0, 1, 2, 0, 0, 2], [2, 0, 1, 3, 1, 3], [2, 2, 1, 1, 3, 1], [1, 2, 2, 1, 2, 2], [3, 1, 0, 4, 2, 0]]
 \$ \$ [[2, 1, 2, 3, 0, 2], [0, 1, 1, 1, 2, 0], [2, 2, 1, 1, 3, 1], [2, 0, 1, 3, 1, 3], [1, 2, 2, 1, 2, 2], [3, 3, 2, 0, 0, 2]] \$ \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$

CmmCk true, true, true

$$p' = s^2 - 2s^3$$

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
2 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

bi =

\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 1/4, 3/4], [0, 0, 0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =

\$ [[-2/3, 1/2, 14/3, -31/9, 40/9, -16/3], [2/3, -5/6, -14/3, 37/9, -8, 80/9], [-1, 5/6, 4, -1, 56/9, -80/9], [4/3, -3/2, 14/3, -31/9, 40/9, -16/3], [1/6, 8/3, -14/3, 34/9, -16/3, 32/9], [1/2, -5/3, -4, 0, -16/9, 64/9]] \$ x \$ [[3/2, 1, 1, 1/2, 1, 1], [3/2, 11/8, 9/8, 1/2, 5/8, 7/8], [9/8, 43/32, 45/32, 1/2, 21/32, 31/32], [39/32, 139/128, 165/128, 1/2, 93/128, 151/128], [183/128, 619/512, 573/512, 29/64, 357/512, 559/512], [687/512, 2755/2048, 2589/2048, 61/128, 1269/2048, 1951/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

\$ [[0, 1, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2]] \$

$$[0, y_1 - y_2 + y_3, y_1, y_1 + y_3, y_2, y_3]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 6}}, net cycles: 0 . order: 4

\$ [[2, 1, 1, 0, 1, 1], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2], [2, 1, 1, 0, 0, 2]] \$

$$[y_1 - y_2 + y_3 + y_4, y_1, y_2, 0, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Â» SYNC'D 47/256 , 0.1835937500

31 . Coloring, {3, 4, 5, 6}

R: [3, 3, 6, 6, 4, 2] **B:** [2, 4, 5, 5, 1, 1]

' See graph

' ' See pair graph

,

Ω for $A+\tau\Delta$:

' [-6' (' - 1 + τ ')', 6' (' 1 + τ ^ 2 ')', 3' (' 2 - τ + τ ^ 2 ')'' (' 1 + τ ')', -3' (' - 1 + τ ')'' (' 2 + τ + τ ^ 2 ')', -6' (' - 1 + τ ')', 6' (' 1 + τ ')''']'

For $\tau=1/2$, [8, 20, 21, 11, 8, 24] . FixedPtCheck, [8, 20, 21, 11, 8, 24]

$\det(A + \tau \Delta) = 0$

Delta Range : [y₄, -y₄ - y₁ - y₂, y₁, y₂, y₃, -y₃]

[1, 1, 1, 1, 1, 1]

+ \; - \; \Delta

\$ [[0, 1, 2, 1, 0, 2] , [2, 4, 1, 1, 1, 3] , [4, 5, 6, 1, 6, 2] , [8, 6, 9, 9, 9, 7]] \$ \$ [[2, 1, 0, 1, 2, 0] , [2, 0, 3, 3, 3, 1] , [4, 3, 2, 7, 2, 6] , [8, 10, 7, 7, 7, 9]] \$ \$ [[-1, 0, 1, 0, -1, 1] , [0, 2, -1, -1, -1, 1] , [0, 1, 2, -3, 2, -2] , [0, -2, 1, 1, 1, -1]] \$

[y₁ + y₂ - y₃, y₁, -2 y₁ - 2 y₂ + y₃, y₂, -y₃, y₃]

p = s² + 4s⁴

S+ \; S- \; NM

\$ [[2, 2, 2, 2, 1, 1] , [0, 2, 3, 1, 2, 2] , [2, 1, 1, 2, 2, 2] , [2, 1, 1, 2, 2, 2] , [1, 2, 2, 1, 2, 2] , [3, 2, 1, 2, 1, 1]]

\$ \$ [[2, 2, 2, 2, 1, 1] , [0, 2, 3, 1, 2, 2] , [2, 1, 1, 2, 2, 2] , [2, 1, 1, 2, 2, 2] , [1, 2, 2, 1, 2, 2] , [3, 2, 1, 2, 1, 1]] \$

\$ \$ [[0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0]] \$

\$

CmmCk true, true, true

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 4	4 vs 4	4 vs 4	3 vs 4

bi =

\$ [[0, 3/4, 1/4, 0, 0, 0] , [0, 0, 1/4, 3/4, 0, 0] , [0, 0, 0, 0, 3/4, 1/4] , [0, 0, 0, 0, 3/4, 1/4] , [3/4, 0, 0, 1/4, 0, 0] , [3/4, 1/4, 0, 0, 0, 0]] \$ x \$ [[99/100, 3/100, -9/100, 3/100, 0, 0] , [3/100, 91/100, 27/100, -9/100, 0, 0] , [-9/100, 27/100, 19/100, 27/100, 0, 0] , [3/100, -9/100, 27/100, 91/100, 0, 0] , [0, 0, 0, 0, 9/10, 3/10] , [0,

$0, 0, 0, 3/10, 1/10]] \$ =$
 $\$ [[7/12, 19/108, 52/27, -68/27] , [-29/12, 55/108, -20/27, 76/27] , [19/12, -185/108, 28/27, -20/27]$
 $, [19/12, -185/108, 28/27, -20/27] , [-2/3, 77/54, -56/27, 40/27] , [1/3, 71/54, -32/27, -8/27]] \$ \times \$ [[3/2,$
 $1, 1/2, 1, 3/2, 1/2] , [3/2, 5/4, 5/8, 9/8, 9/8, 3/8] , [9/8, 39/32, 11/16, 39/32, 21/16, 7/16] , [21/16, 61/64,$
 $75/128, 159/128, 183/128, 61/128]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 6}}, net cycles: 0 . order: 3

$$[0, y_1, y_2, y_3, 0, y_4]$$

$R = \$ [[0, 0, 1, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 1, 0, 0] , [0, 1, 0, 0, 0,$
 $0]] \$ \times \$ [[0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0,$
 $0, 0, 1]] \$ = \$ [[0, -5/18, 1/18, 7/18] , [0, -5/18, 1/18, 7/18] , [0, 7/18, -5/18, 1/18] , [0, 7/18, -5/18, 1/18] ,$
 $[1, -5/18, 1/18, -11/18] , [0, 1/18, 7/18, -5/18]] \$ \times \$ [[0, 1, 2, 1, 0, 2] , [0, 2, 1, 0, 0, 3] , [0, 3, 2, 0, 0, 1] ,$
 $[0, 1, 3, 0, 0, 2]] \$$

Omega Rank for B : cycles: {{1, 2, 4, 5}}, net cycles: 1 . order: 4

$$\$ [[2, 1, 0, 1, 2, 0] , [2, 2, 0, 1, 1, 0] , [1, 2, 0, 2, 1, 0] , [1, 1, 0, 2, 2, 0]] \$$$

$$[y_3, y_2, 0, y_1, y_3 - y_2 + y_1, 0]$$

$$p = -s + s^2 - s^3 + s^4$$

Â» SYNC'D 3/8 , 0.3750000000

32 . Coloring, {2, 3, 4, 5, 6}

R: [3, 4, 6, 6, 4, 2] **B:** [2, 3, 5, 5, 1, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$['-6' (' - 1 + \tau ')', 6' (' 1 + \tau ^2 ')', -3' (' - 1 + \tau ')'' (' 2 + \tau + \tau ^2 ')', 3' (' 1 + \tau ')'' (' 2 - \tau + \tau ^2 ')', -6' (' - 1 + \tau ')', 6' (' 1 + \tau ')'']'$$

For τ=1/2, [8, 20, 11, 21, 8, 24] . FixedPtCheck, [8, 20, 11, 21, 8, 24]

$$\det(A + \tau \Delta) = 0$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 4

bi =

$\$ [[0, 3/4, 1/4, 0, 0, 0], [0, 0, 3/4, 1/4, 0, 0], [0, 0, 0, 0, 3/4, 1/4], [0, 0, 0, 0, 3/4, 1/4], [3/4, 0, 0, 1/4, 0, 0], [3/4, 1/4, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 9/10, 3/10], [0, 0, 0, 0, 3/10, 1/10]] \$ =$

$\$ [[0, -1/12, 187/108, -68/27, 28/27], [0, 19/12, 7/108, 76/27, -116/27], [0, -29/12, 55/108, -20/27, 76/27], [0, -29/12, 55/108, -20/27, 76/27], [-1/2, 3/2, -109/54, 40/27, -8/27], [3/2, 11/6, -43/54, -8/27, -56/27]] \$ \times \$ [[3/2, 1, 1, 1/2, 3/2, 1/2], [3/2, 5/4, 9/8, 5/8, 9/8, 3/8], [9/8, 39/32, 21/16, 19/32, 21/16, 7/16], [21/16, 61/64, 153/128, 81/128, 183/128, 61/128], [183/128, 565/512, 267/256, 305/512, 351/256, 117/256]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 4, 6}}, net cycles: 0 . order: 3

$[0, y_1, y_2, y_3, 0, y_4]$

$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0]] \$ \times \$ [[0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[1, -5/18, 1/18, -11/18], [0, -5/18, 1/18, 7/18], [0, 7/18, -5/18, 1/18], [0, 7/18, -5/18, 1/18], [0, -5/18, 1/18, 7/18], [0, 1/18, 7/18, -5/18]] \$ \times \$ [[0, 1, 1, 2, 0, 2], [0, 2, 0, 1, 0, 3], [0, 3, 0, 2, 0, 1], [0, 1, 0, 3, 0, 2]] \$$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 1 . order: 4

$\$ [[2, 1, 1, 0, 2, 0], [2, 2, 1, 0, 1, 0], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0]] \$$

$[y_1, y_2, y_3, 0, y_1 - y_2 + y_3, 0]$

$p = -s + s^2 - s^3 + s^4$

$\hat{A} \gg \text{SYNC'D } 3/8, 0.3750000000$

SUMMARY	
Graph Type	CC
$v(A)$	2
$v(\Delta)$	2
π	[1, 1, 1, 1, 1, 1]
Dbly Stoch	true

SANDWICH		Total 2
No .	Coloring	Rank
1	{}	3
2	{2}	3

RT GROUPS		Total 3	
No .	Coloring	Rank	Solv
1	{2, 3, 6}	6	["group", Not Solvable]
2	{2, 3, 5}	2	Not Solvable
3	{2, 4, 6}	6	["group", Not Solvable]

CC Colorings		Total 1
No .	Coloring	Sandwich,Rank
1	{}	true, 3

Δ -RANK'D	SC'D !RK'D	τ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
22	0	24 , 27	7 , 6	5	32	32
