

New Graph

[2, 3, 2, 3], [4, 4, 1, 1]

$$\pi = [1, 1, 1, 1]$$

POSSIBLE RANKS

1 x 4
2 x 2

BASE DETERMINANT 117/512, .2285156250

NullSpace of Δ

{1, 2, 3, 4}

Range of Δ : $[-\lambda_1 - \lambda_2 - \lambda_3, \lambda_1, \lambda_2, \lambda_3]$

1 . Coloring, {}

R: [2, 3, 2, 3] **B**: [4, 4, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A + \tau \Delta$:

‘ [‘ -1‘ (‘ - 1 + τ ‘)‘ , 1‘ (‘ 1 + τ ‘)‘ , 1‘ (‘ 1 + τ ‘)‘ , -1‘ (‘ - 1 + τ ‘)‘]‘

For $\tau=1/2$, [1, 3, 3, 1] . FixedPtCheck, [1, 3, 3, 1]

$\det(A + \tau \Delta) = 0$

Delta Range : $[-y_1 - y_2 - y_3, y_1, y_2, y_3]$

[1, 1, 1, 1]

+ - Δ

$$\$ [[0, 2, 2, 0], [1, 1, 1, 1], [1, 1, 1, 1]] \$ \quad \$ [[2, 0, 0, 2], [1, 1, 1, 1], [1, 1, 1, 1]] \$ \quad \$ [[-1, 1, 1, -1], [0, 0, 0, 0], [0, 0, 0, 0]] \$$$

$$[y_1, -y_1, -y_1, y_1]$$

$$p = s^2$$

S+ S- NM

$$\$ [[1, 0, 1, 0], [1, 1, 0, 0], [0, 0, 1, 1], [0, 1, 0, 1]] \$ \quad \$ [[0, 0, 1, 1], [1, 0, 1, 0], [0, 1, 0, 1], [1, 1, 0, 0]] \$ \quad \$ [[2, 1, 1, 0], [1, 2, 0, 1], [1, 0, 2, 1], [0, 1, 1, 2]] \$$$

CmmCk *true, true, true*

$$p' = s^2$$

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
1 vs 3	1 vs 3	1 vs 3	1 vs 2	1 vs 2

Omega Rank for R : cycles: {{2, 3}}, net cycles: 1 . order: 2

$$\$ [[0, 2, 2, 0], [0, 2, 2, 0]] \$$$

$$[0, y_1, y_1, 0]$$

$$p = -s + s^2$$

Omega Rank for B : cycles: {{1, 4}}, net cycles: 1 . order: 2

$$\$ [[2, 0, 0, 2], [2, 0, 0, 2]] \$$$

$$[y_1, 0, 0, y_1]$$

$$p = -s + s^2$$

\hat{A} « NOT SYNC'D \hat{A} »

Nullspace of $\{\Omega\Delta^i\}$:

$$[0, x_1, x_2]$$

$$\text{For } A+2\Delta : [-3y_1 - 3y_2 - y_3, y_1, y_2, y_3]$$

$$\text{For } A-2\Delta : [y_1, -3y_1 - y_2 - 3y_3, y_2, y_3]$$

Range of $\{\Omega\Delta^i\}$: $[\mu_1, -\mu_1, -\mu_1, \mu_1]$

rank of M is 4 , rank of N is 3

M N

$\$ [[0, 0, 0, 1], [0, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0, 0]] \$ \quad \$ [[0, 1, 1, 2], [1, 0, 2, 1], [1, 2, 0, 1], [2, 1, 1, 0]] \$$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [-1, 1, 1, -1]$

ker M, $[0, 0, 0, 0]$

Range M, $[x_1, x_2, x_3, x_4]$

$\tau = 8$, $r' = 1/2$

Ranges

Action of R on ranges, $[[2], [2]]$

Action of B on ranges, $[[1], [1]]$

$\beta(\{1, 4\}) = 1/2$

$\beta(\{2, 3\}) = 1/2$

ker N, $[-\mu_1, \mu_1, \mu_1, -\mu_1]$

Range of N

$[y_3, y_2, y_1, -y_3 + y_2 + y_1]$

Partitions

Action of R on partitions, $[[1], [1]]$

Action of B on partitions, $[[2], [2]]$

$\alpha(\{\{1, 3\}, \{2, 4\}\}) = 1/2$

$\alpha(\{\{1, 2\}, \{3, 4\}\}) = 1/2$

$b_1 = \{1, 2\}$, $b_2 = \{1, 3\}$, $b_3 = \{3, 4\}$, $b_4 = \{2, 4\}$

Action of R and B on the blocks of the partitions: $\$ [[0, 1, 1, 0], [0, 0, 1, 1], [1, 0, 0, 1], [1, 1, 0, 0]] \$ =$
 $\$ [[0, 1, 0, 0], [0, 0, 0, 1], [0, 0, 0, 1], [0, 1, 0, 0]] \$ + \$ [[0, 0, 1, 0], [0, 0, 1, 0], [1, 0, 0, 0], [1, 0, 0, 0]] \$$

$['2', '4', '4', '2'], ['3', '3', '1', '1']$ with invariant measure $[1, 1, 1, 1]$

N by blocks, check: *true*. ' See partition graph.

' ' See level-2 partition graph.

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Sandwich	
Coloring	{ }
Rank	2
R,B	[2, 3, 2, 3], [4, 4, 1, 1]
π_2	[0, 0, 1, 1, 0, 0]
u_2	[1, 1, 2, 2, 1, 1] (dim 1)
wpp	[2, 2, 2, 2]

2. Coloring, {2}

R: [2, 4, 2, 3] **B:** [4, 3, 1, 1]

‘ See graph

‘ ‘ See pair graph

Ω for $A+\tau\Delta$:

‘ [‘ 2 ‘ (‘ - 1 + τ ‘) ‘ ‘ (‘ 5 + 2 τ + τ^2 ‘) ‘ , 2 ‘ (‘ 1 + τ ‘) ‘ ‘ (‘ - 5 + τ^2 ‘) ‘ , -2 ‘ (‘ 1 + τ ‘) ‘ ‘ (‘ 5 - 2 τ + τ^2 ‘) ‘ , 2 ‘ (‘ - 5 - τ - 3 τ^2 + τ^3 ‘) ‘] ‘

For $\tau=1/2$, [-25, -57, -51, -49] . FixedPtCheck, [25, 57, 51, 49]

$\det(A + \tau \Delta) = 1 \cdot (-1 + \tau) \cdot (\tau) \cdot (1 + \tau)$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 3	4 vs 4	4 vs 4	3 vs 3	2 vs 3

bi =

\$ [[0, 1/4, 0, 3/4], [0, 0, 3/4, 1/4], [3/4, 1/4, 0, 0], [3/4, 0, 1/4, 0]] \$ x \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ =
\$ [[-35/52, 20/13, 32/13, -40/13], [69/52, -6/13, 32/13, -40/13], [21/52, 14/13, -40/13, 24/13], [-3/52, -28/13, -24/13, 56/13]] \$ x \$ [[3/2, 1/2, 1, 1], [3/2, 5/8, 5/8, 5/4], [45/32, 17/32, 25/32, 41/32], [99/64, 35/64, 23/32, 19/16]] \$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 4}}, net cycles: 1 . order: 3

$$[0, y_3, y_2, y_1]$$

$$R = \$ [[0, 1, 0, 0], [0, 0, 0, 1], [0, 1, 0, 0], [0, 0, 1, 0]] \$ \times \$ [[0, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ = \$ [[3/4, -1/4, -1/4], [-1/4, 3/4, -1/4], [3/4, -1/4, -1/4], [-1/4, -1/4, 3/4]] \$ \times \$ [[0, 2, 1, 1], [0, 1, 1, 2], [0, 1, 2, 1]] \$$$

Omega Rank for B : cycles: {{1, 4}}, net cycles: 0 . order: 2

$$\$ [[2, 0, 1, 1], [2, 0, 0, 2], [2, 0, 0, 2]] \$$$

$$[y_2 + y_1, 0, y_2, y_1]$$

$$p = -s^2 + s^3$$

Â» SYNC'D 1/4 , 0.2500000000

3 . Coloring, {3}

R: [2, 3, 1, 3] **B:** [4, 4, 2, 1]

' See graph

' ' See pair graph

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Ω for A+τΔ :

$$['2' ('-5 - \tau - 3\tau^2 + \tau^3'), -2' ('1 + \tau')' ('5 - 2\tau + \tau^2'), 2' ('1 + \tau')' ('-5 + \tau^2'), 2' ('-1 + \tau')' ('5 + 2\tau + \tau^2')]'$$

For τ=1/2, [-49, -51, -57, -25] . FixedPtCheck, [49, 51, 57, 25]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')' ('1 + \tau')' ('\tau')$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	4 vs 4	4 vs 4	3 vs 3	2 vs 3

bi =

$$\$ [[0, 1/4, 0, 3/4], [0, 0, 1/4, 3/4], [1/4, 3/4, 0, 0], [3/4, 0, 1/4, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ = \$ [[-3/52, -28/13, -24/13, 56/13], [21/52, 14/13, -40/13, 24/13], [69/52, -6/13, 32/13, -40/13], [-35/52, 20/13, 32/13, -40/13]] \$ \times \$ [[1, 1, 1/2, 3/2], [5/4, 5/8, 5/8, 3/2], [41/32, 25/32, 17/32, 45/32],$$

[19/16, 23/32, 35/64, 99/64]] \$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_3, y_1, y_2, 0]$$

$$R = \$ [[0, 1, 0, 0], [0, 0, 1, 0], [1, 0, 0, 0], [0, 0, 1, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 0]] \$ = \$ [[-1/4, -1/4, 3/4], [3/4, -1/4, -1/4], [-1/4, 3/4, -1/4], [3/4, -1/4, -1/4]] \$ \times \$ [[1, 1, 2, 0], [2, 1, 1, 0], [1, 2, 1, 0]] \$$$

Omega Rank for B : cycles: {{1, 4}}, net cycles: 0 . order: 2

$$\$ [[1, 1, 0, 2], [2, 0, 0, 2], [2, 0, 0, 2]] \$$$

$$[-y_1 + y_2, y_1, 0, y_2]$$

$$p = -s^2 + s^3$$

Â» SYNC'D 1/4 , 0.2500000000

4 . Coloring, {4}

R: [2, 3, 2, 1] **B:** [4, 4, 1, 3]

' See graph

' ' See pair graph

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Ω for A+τΔ :

$$['-2' (' - 1 + \tau ')'' (' 5 + 2\tau + \tau^2 ')', 2' (' 5 - 2\tau + \tau^2 ')'' (' 1 + \tau ')', 2' (' 5 - \tau + 3\tau^2 + \tau^3 ')', 2' (' - 5 + \tau^2 ')'' (' - 1 + \tau ')'']'$$

For τ=1/2, [25, 51, 43, 19] . FixedPtCheck, [25, 51, 43, 19]

$$\det(A + \tau \Delta) = 1' (' \tau ')'' (' - 1 + \tau ')'' (' 1 + \tau ')'$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	4 vs 4	4 vs 4	2 vs 3	3 vs 3

bi =

$$\begin{aligned}
 & \$ [[0, 1/4, 0, 3/4], [0, 0, 1/4, 3/4], [3/4, 1/4, 0, 0], [1/4, 0, 3/4, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ = \\
 & \$ [[-19/20, -2, -8/15, 56/15], [37/20, 0, -16/15, -8/15], [1/4, 0, 8/3, -8/3], [-3/20, 2, -16/15, -8/15]] \\
 & \$ \times \$ [[1, 1/2, 1, 3/2], [9/8, 1/2, 5/4, 9/8], [39/32, 19/32, 31/32, 39/32], [33/32, 35/64, 17/16, 87/64]] \$
 \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3}}, net cycles: 0 . order: 2

$$\$ [[1, 2, 1, 0], [0, 2, 2, 0], [0, 2, 2, 0]] \$$$

$$[y_1 - y_2, y_1, y_2, 0]$$

$$p = -s^2 + s^3$$

Omega Rank for B : cycles: {{1, 3, 4}}, net cycles: 1 . order: 3

$$[y_1, 0, y_3, y_2]$$

$$\begin{aligned}
 B = & \$ [[0, 0, 0, 1], [0, 0, 0, 1], [1, 0, 0, 0], [0, 0, 1, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ = \\
 & \$ [[3/4, -1/4, -1/4], [3/4, -1/4, -1/4], [-1/4, -1/4, 3/4], [-1/4, 3/4, -1/4]] \$ \times \$ [[1, 0, 1, 2], [1, 0, 2, 1], [2, 0, 1, 1]] \$
 \end{aligned}$$

Â» SYNC'D 1/4 , 0.2500000000

5 . Coloring, {2, 3}

R: [2, 4, 1, 3] **B:** [4, 3, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for A+τΔ :

$$['1, 1, 1, 1']$$

For τ=1/2, [1, 1, 1, 1] . FixedPtCheck, [1, 1, 1, 1]

$$\det(A + \tau \Delta) = 1 (' \tau ') (' 1 + \tau^2 ')$$

Delta Range : [-y₁ - y₂ - y₃, y₁, y₂, y₃]

$$[1, 1, 1, 1]$$

$$+ \quad - \quad \Delta$$

$$\$ [[1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1]] \$ \quad \$ [[1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1]] \$ \quad \$ [[0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0]] \$$$

$$[0, 0, 0, 0]$$

$$p = s$$

S+ S- NM

$$\$ [[0, 1, 0, 0], [0, 0, 0, 1], [1, 0, 0, 0], [0, 0, 1, 0]] \$ \quad \$ [[0, 0, 0, 1], [0, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0, 0]] \$ \quad \$ [[3, 2, 2, 2], [2, 3, 2, 2], [2, 2, 3, 2], [2, 2, 2, 3]] \$$$

CmmCk *true, true, true*

$$p' = s \quad p' = s^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
0 vs 3	1 vs 4	1 vs 4	1 vs 4	1 vs 4

Omega Rank for R : cycles: $\{\{1, 2, 3, 4\}\}$, net cycles: 1 . order: 4

$$\$ [[1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1]] \$$$

$$[y_1, y_1, y_1, y_1]$$

$$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3$$

Omega Rank for B : cycles: $\{\{1, 4\}, \{2, 3\}\}$, net cycles: 2 . order: 2

$$\$ [[1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1], [1, 1, 1, 1]] \$$$

$$[y_1, y_1, y_1, y_1]$$

$$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$$[x_1, x_2, x_3]$$

$$\text{For } A+2\Delta : [-y_1 - y_2 - y_3, y_1, y_2, y_3]$$

$$\text{For } A-2\Delta : [-y_1 - y_2 - y_3, y_1, y_2, y_3]$$

Range of $\{\Omega\Delta^i\}$: $[0, 0, 0, 0]$

rank of M is 4 , rank of N is 4

M N

$$\$ [[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1], [1, 1, 1, 0]] \$ \quad \$ [[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1], [1, 1, 1, 0]] \$$$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [0, 0, 0, 0]$

ker M, $[0, 0, 0, 0]$

Range M, $[x_2, x_1, x_3, x_4]$

$\tau = 4$, $r' = 3/4$

Ranges

Action of R on ranges, $[[1]]$

Action of B on ranges, $[[1]]$

$\beta(\{1, 2, 3, 4\}) = 1/1$

ker N, $[0, 0, 0, 0]$

Range of N

$[y_1, y_2, y_3, y_4]$

Partitions

$\alpha(\{\{1\}, \{2\}, \{3\}, \{4\}\}) = 1/1$

$b1 = \{1\}$, $b2 = \{2\}$, $b3 = \{3\}$, $b4 = \{4\}$

Action of R and B on the blocks of the partitions: $\$ [[0, 0, 1, 1], [1, 0, 1, 0], [0, 1, 0, 1], [1, 1, 0, 0]] \$ =$
 $\$ [[0, 0, 1, 0], [1, 0, 0, 0], [0, 0, 0, 1], [0, 1, 0, 0]] \$ + \$ [[0, 0, 0, 1], [0, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0,$
 $0]] \$$

$['3', '1', '4', '2'], ['4', '3', '2', '1']$ with invariant measure $[1, 1, 1, 1]$

N by blocks, check: true. ' See partition graph.

' ' See level-4 partition graph.

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Right Group	
Coloring	$\{2, 3\}$
Rank	4
R,B	$[2, 4, 1, 3], [4, 3, 2, 1]$
π_2	$[1, 1, 1, 1, 1, 1]$
u_2	$[1, 1, 1, 1, 1, 1]$ (dim 2)
wpp	$[1, 1, 1, 1]$
π_4	$[1]$
u_4	$[1]$

6. Coloring, {2, 4}

R: [2, 4, 2, 1] **B:** [4, 3, 1, 3]

' See graph

' ' See pair graph

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Ω for $A + \tau \Delta$:

' ['1' ('5 - τ + $3\tau^2 + \tau^3$ ') , 1' ('1 + τ ')'' ('5 - $2\tau + \tau^2$ ') , -1' ('-1 + τ ')'' ('5 + $2\tau + \tau^2$ ' , -1' ('-5 - $\tau - 3\tau^2 + \tau^3$ ')'']'

For $\tau=1/2$, [43, 51, 25, 49] . FixedPtCheck, [43, 51, 25, 49]

$\det(A + \tau \Delta) = 0$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 3	3 vs 3	3 vs 3	3 vs 3	3 vs 3

bi =

\$ [[0, 1/4, 0, 3/4], [0, 0, 3/4, 1/4], [3/4, 1/4, 0, 0], [1/4, 0, 3/4, 0]] \$ x \$ [[91/100, 27/100, 3/100, -9/100], [27/100, 19/100, -9/100, 27/100], [3/100, -9/100, 99/100, 3/100], [-9/100, 27/100, 3/100, 91/100]] \$ =

\$ [[-3/4, -1, 2], [5/4, -3/5, -2/5], [-3/4, 7/5, -2/5], [5/4, 1/5, -6/5]] \$ x \$ [[1, 1/2, 3/2, 1], [11/8, 5/8, 9/8, 7/8], [17/16, 5/8, 9/8, 19/16]] \$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 4}}, net cycles: 1 . order: 3

$$[y_1, y_2, 0, y_3]$$

$R = \$ [[0, 1, 0, 0], [0, 0, 0, 1], [0, 1, 0, 0], [1, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 0], [0, 0, 0, 1]] \$ = \$ [[3/4, -1/4, -1/4], [-1/4, 3/4, -1/4], [3/4, -1/4, -1/4], [-1/4, -1/4, 3/4]] \$ x \$ [[1, 2, 0, 1], [1, 1, 0, 2], [2, 1, 0, 1]] \$$

Omega Rank for B : cycles: {{1, 3, 4}}, net cycles: 1 . order: 3

$$[y_1, 0, y_3, y_2]$$

$$B = \$ [[0, 0, 0, 1], [0, 0, 1, 0], [1, 0, 0, 0], [0, 0, 1, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 0, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]] \$ = \$ [[-1/4, -1/4, 3/4], [3/4, -1/4, -1/4], [-1/4, 3/4, -1/4], [3/4, -1/4, -1/4]] \$ \times \$ [[1, 0, 2, 1], [2, 0, 1, 1], [1, 0, 1, 2]] \$$$

Â» SYNC'D 3/8 , 0.3750000000

7 . Coloring, {3, 4}

R: [2, 3, 1, 1] **B:** [4, 4, 2, 3]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$[' 1' (' 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') , -1' (' -5 - \tau - 3\tau^2 + \tau^3 ') , 1' (' 5 - \tau + 3\tau^2 + \tau^3 ') , -1' (' 5 + 2\tau + \tau^2 ') ' (' -1 + \tau ') ']$$

For τ=1/2, [51, 49, 43, 25] . FixedPtCheck, [51, 49, 43, 25]

$$\det(A + \tau \Delta) = 0$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 vs 3	3 vs 3	3 vs 3	3 vs 3	3 vs 3

bi =

$$\$ [[0, 1/4, 0, 3/4], [0, 0, 1/4, 3/4], [1/4, 3/4, 0, 0], [1/4, 0, 3/4, 0]] \$ \times \$ [[19/100, 27/100, 27/100, -9/100], [27/100, 91/100, -9/100, 3/100], [27/100, -9/100, 91/100, 3/100], [-9/100, 3/100, 3/100, 99/100]] \$ =$$

$$\$ [[5/4, -3/5, -2/5], [5/4, 1/5, -6/5], [-3/4, -1, 2], [-3/4, 7/5, -2/5]] \$ \times \$ [[1/2, 1, 1, 3/2], [5/8, 7/8, 11/8, 9/8], [5/8, 19/16, 17/16, 9/8]] \$$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_1, y_2, y_3, 0]$$

$$R = \$ [[0, 1, 0, 0], [0, 0, 1, 0], [1, 0, 0, 0], [1, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 0]] \$ = \$ [[-1/4, 3/4, -1/4], [-1/4, -1/4, 3/4], [3/4, -1/4, -1/4], [3/4, -1/4, -1/4]] \$ \times \$ [[2, 1, 1, 0], [1, 2, 1, 0], [1, 1, 2, 0]] \$$$

Omega Rank for B : cycles: {{2, 3, 4}}, net cycles: 1 . order: 3

$$[0, y_3, y_1, y_2]$$

$$B = \begin{bmatrix} [0, 0, 0, 1] \\ [0, 0, 0, 1] \\ [0, 1, 0, 0] \\ [0, 0, 1, 0] \end{bmatrix} \times \begin{bmatrix} [0, 0, 0, 0] \\ [0, 1, 0, 0] \\ [0, 0, 1, 0] \\ [0, 0, 0, 1] \end{bmatrix} = \begin{bmatrix} [3/4, -1/4, -1/4] \\ [3/4, -1/4, -1/4] \\ [-1/4, -1/4, 3/4] \\ [-1/4, 3/4, -1/4] \end{bmatrix} \times \begin{bmatrix} [0, 1, 1, 2] \\ [0, 1, 2, 1] \\ [0, 2, 1, 1] \end{bmatrix}$$

$\hat{A} \gg \text{SYNC'D } 3/8, 0.3750000000$

8. Coloring, {2, 3, 4}

R: [2, 4, 1, 1] **B:** [4, 3, 2, 3]

' See graph

' ' See pair graph

Ω for $A + \tau \Delta$:

$$\begin{bmatrix} 2(1 + \tau) & (-5 + \tau^2) & 2(-5 - \tau - 3\tau^2 + \tau^3) & 2(-1 + \tau) & (5 + 2\tau + \tau^2) \\ -2(1 + \tau) & (5 - 2\tau + \tau^2) & & & \end{bmatrix}$$

For $\tau=1/2$, [-57, -49, -25, -51] . FixedPtCheck, [57, 49, 25, 51]

$$\det(A + \tau \Delta) = 1(1 + \tau)(\tau)(-1 + \tau)$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 3	4 vs 4	4 vs 4	3 vs 3	2 vs 3

bi =

$$\begin{bmatrix} [0, 1/4, 0, 3/4] \\ [0, 0, 3/4, 1/4] \\ [1/4, 3/4, 0, 0] \\ [1/4, 0, 3/4, 0] \end{bmatrix} \times \begin{bmatrix} [1, 0, 0, 0] \\ [0, 1, 0, 0] \\ [0, 0, 1, 0] \\ [0, 0, 0, 1] \end{bmatrix} = \begin{bmatrix} [69/52, -6/13, 32/13, -40/13] \\ [-3/52, -28/13, -24/13, 56/13] \\ [-35/52, 20/13, 32/13, -40/13] \\ [21/52, 14/13, -40/13, 24/13] \end{bmatrix} \times \begin{bmatrix} [1/2, 1, 3/2, 1] \\ [5/8, 5/4, 3/2, 5/8] \\ [17/32, 41/32, 45/32, 25/32] \\ [35/64, 19/16, 99/64, 23/32] \end{bmatrix}$$

Check x AllOnes: [1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 4}}, net cycles: 1 . order: 3

$$[y_1, y_2, 0, y_3]$$

$$R = \$ [[0, 1, 0, 0], [0, 0, 0, 1], [1, 0, 0, 0], [1, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 0], [0, 0, 0, 1]] \$ = \$ [[-1/4, 3/4, -1/4], [-1/4, -1/4, 3/4], [3/4, -1/4, -1/4], [3/4, -1/4, -1/4]] \$ \times \$ [[2, 1, 0, 1], [1, 2, 0, 1], [1, 1, 0, 2]] \$$$

Omega Rank for B : cycles: {{2, 3}}, net cycles: 0 . order: 2

$$\$ [[0, 1, 2, 1], [0, 2, 2, 0], [0, 2, 2, 0]] \$$$

$$[0, y_1, y_2, -y_1 + y_2]$$

$$p = -s^2 + s^3$$

Â» SYNC'D 1/4 , 0.2500000000

SUMMARY	
Graph Type	CC
v(A)	1
v(Δ)	1
π	[1, 1, 1, 1]
Dbly Stoch	true

SANDWICH		Total 1
No .	Coloring	Rank
1	{}	2

RT GROUPS		Total 1	
No .	Coloring	Rank	Solv
1	{2, 3}	4	["group", Solvable]

CC Colorings		Total 1
No .	Coloring	Sandwich,Rank
1	{}	true, 2

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
6	0	6, 6	5, 3	2	8	8
