

New Graph

[4, 4, 4, 7, 7, 7, 1, 1, 1], [2, 9, 5, 8, 3, 8, 5, 6, 2]

$$\pi = [3, 2, 1, 3, 2, 1, 3, 2, 1]$$

POSSIBLE RANKS

1 x 18

2 x 9

3 x 6

BASE DETERMINANT 2151937075/68719476736, .3131480589e-1

NullSpace of Δ

{1, 2, 4, 5, 6, 9}, {3, 7, 8}

Nullspace of A

[[2, 5, 6, 9], {1, 4}] ‘ ‘ [{3, 8}, {7}]

1 . Coloring, { }

$$\Omega p(\Delta)=0: \quad p' = s^4 - 4s^6 \quad p' = s^3 - 8s^6 \quad p = s^2 - 32s^7 \quad p' = s^2 - 16s^6 \quad p' = s^5 - 2s^6$$

R: [4, 4, 4, 7, 7, 7, 1, 1, 1]

B: [2, 9, 5, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 7	2 vs 7	2 vs 7	1 vs 3	2 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$

$$[y_1, 0, 0, y_1, 0, 0, y_1, 0, 0]$$

$$p = -s + s^3 \quad p = -s + s^2$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{3, 5\}, \{6, 8\}\}$ order: 2

See Matrix

$$\$ [[0, 4, 2, 0, 4, 2, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 2, 4], [0, 4, 2, 0, 4, 2, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 2, 4], [0, 4, 2, 0, 4, 2, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 2, 4]] \$$$

$$[0, y_2, y_1, 0, y_2, y_1, 0, y_2, y_1]$$

$$p' = s - s^3 \quad p' = s^2 - s^4 \quad p' = -s^3 + s^5 \quad p = s - s^5$$

‘ See 3-level graph

‘

$$M \quad \setminus ; \quad N$$

$$\$ [[0, 0, 0, 3, 0, 0, 3, 0, 0], [0, 0, 0, 0, 2, 0, 0, 2, 0], [0, 0, 0, 0, 0, 1, 0, 0, 1], [3, 0, 0, 0, 0, 0, 3, 0, 0], [0, 2, 0, 0, 0, 0, 2, 0], [0, 0, 1, 0, 0, 0, 0, 0, 1], [3, 0, 0, 3, 0, 0, 0, 0, 0], [0, 2, 0, 0, 2, 0, 0, 0, 0], [0, 0, 1, 0, 0, 1, 0, 0, 0]] \$ \quad \$ [[0, 3, 3, 6, 4, 6, 6, 5, 3], [3, 0, 2, 4, 6, 4, 5, 6, 6], [3, 2, 0, 6, 6, 6, 3, 4, 6], [6, 4, 6, 0, 2, 0, 6, 6, 6], [4, 6, 6, 2, 0, 2, 6, 6, 4], [6, 4, 6, 0, 2, 0, 6, 6, 6], [6, 5, 3, 6, 6, 6, 0, 1, 3], [5, 6, 4, 6, 6, 6, 1, 0, 2], [3, 6, 6, 6, 4, 6, 3, 2, 0]] \$$$

$$\tau = 27, r' = 2/3$$

$$\mathbf{R}: [4, 4, 4, 7, 7, 7, 1, 1, 1]$$

$$\mathbf{B}: [2, 9, 5, 8, 3, 8, 5, 6, 2]$$

Ranges

Action of R on ranges, $[[1], [1], [1]]$

Action of B on ranges, $[[2], [3], [2]]$

Cycles: R , $\{\{1, 4, 7\}\}$, B , $\{\{2, 9\}, \{3, 5\}, \{6, 8\}\}$

$$\beta(\{1, 4, 7\}) = 1/2$$

$$\beta(\{2, 5, 8\}) = 1/3$$

$$\beta(\{3, 6, 9\}) = 1/6$$

Partitions

Action of R on partitions, $[[2], [2], [2]]$

Action of B on partitions, $[[3], [3], [1]]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 4	4 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 0, 4, 0, 0, 6, 0, 2], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0]] \$$$

$$[y_4, 0, 0, y_3, 0, 0, y_2, 0, y_1]$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4

See Matrix

$$\$ [[0, 4, 2, 2, 4, 2, 0, 4, 0], [0, 0, 4, 4, 2, 4, 0, 4, 0], [0, 0, 2, 0, 4, 4, 0, 8, 0], [0, 0, 4, 0, 2, 8, 0, 4, 0], [0, 0, 2, 0, 4, 4, 0, 8, 0], [0, 0, 4, 0, 2, 8, 0, 4, 0]] \$$$

$$[0, 2y_2 - y_4, y_1, 2y_1 - y_3, y_2, y_3, 0, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = s^3 - s^5$$

3. Coloring, {3}

R: [4, 4, 5, 7, 7, 7, 1, 1, 1]

B: [2, 9, 4, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	2 vs 4	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

\$ [[6, 0, 0, 5, 1, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$

$$[y_2, 0, 0, y_1, -y_1 + y_2, 0, y_2, 0, 0]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles: {{6, 8}, {2, 9}} order: 4

See Matrix

\$ [[0, 4, 2, 1, 3, 2, 0, 4, 2] , [0, 2, 3, 2, 0, 4, 0, 3, 4] , [0, 4, 0, 3, 0, 3, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 6, 4] , [0, 4, 0, 0, 0, 6, 0, 6, 2] , [0, 2, 0, 0, 0, 6, 0, 6, 4] , [0, 4, 0, 0, 0, 6, 0, 6, 2]] \$

$$[0, y_1 + y_2 + y_3 - y_5, -y_4 + y_1 + y_2 + y_3, y_1, y_2, y_3, 0, y_4, y_5]$$

$$p' = s^4 - s^6 \quad p = s^4 - s^6$$

4 . Coloring, {4}

R: [4, 4, 4, 8, 7, 7, 1, 1, 1]

B: [2, 9, 5, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 8	8 vs 8	2 vs 4	3 vs 7

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

\$ [[6, 0, 0, 6, 0, 0, 3, 3, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0] , [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$

$$[y_2 + y_1, 0, 0, y_2 + y_1, 0, 0, y_2, y_1, 0]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{6, 8}, {2, 9}, {3, 5}} order: 2

See Matrix

\$ [[0, 4, 2, 0, 4, 2, 3, 1, 2], [0, 2, 4, 0, 5, 1, 0, 2, 4], [0, 4, 5, 0, 4, 2, 0, 1, 2], [0, 2, 4, 0, 5, 1, 0, 2, 4], [0, 4, 5, 0, 4, 2, 0, 1, 2], [0, 2, 4, 0, 5, 1, 0, 2, 4], [0, 4, 5, 0, 4, 2, 0, 1, 2]] \$

$$[0, 2y_2, 2y_2 + y_1 - y_3, 0, y_2 + 2y_1, y_2, y_3, y_1, 2y_1]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

5. Coloring, {5}

$$\Omega p(\Delta)=0: \quad p = -s^3 + s^4 + 4s^5 - 8s^7$$

R: [4, 4, 4, 7, 3, 7, 1, 1, 1]

B: [2, 9, 5, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 4	2 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[6, 0, 2, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0]] \$

$$[y_3, 0, y_1, y_2, 0, 0, y_4, 0, 0]$$

Omega Rank for B : cycles: {{6, 8}, {5, 7}, {2, 9}} order: 2

See Matrix

\$ [[0, 4, 0, 0, 4, 2, 2, 4, 2], [0, 2, 0, 0, 2, 4, 4, 2, 4], [0, 4, 0, 0, 4, 2, 2, 4, 2], [0, 2, 0, 0, 2, 4, 4, 2, 4], [0, 4, 0, 0, 4, 2, 2, 4, 2], [0, 2, 0, 0, 2, 4, 4, 2, 4]] \$

$$[0, y_1, 0, 0, y_1, y_2, y_2, y_1, y_2]$$

$$p = -s + s^3 \quad p = -s + s^5 \quad p' = -s + s^5 \quad p' = -s + s^3$$

6. Coloring, {6}

R: [4, 4, 4, 7, 7, 8, 1, 1, 1]

B: [2, 9, 5, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 8	8 vs 8	2 vs 4	5 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[6, 0, 0, 6, 0, 0, 5, 1, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0] , [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$

[$y_2, 0, 0, y_2, 0, 0, y_2 - y_1, y_1, 0$]

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles: {{3, 5}, {2, 9}} order: 4

See Matrix

\$ [[0, 4, 2, 0, 4, 2, 1, 3, 2] , [0, 2, 4, 0, 3, 3, 2, 0, 4] , [0, 4, 3, 0, 6, 0, 3, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2] , [0, 2, 6, 0, 6, 0, 0, 0, 4] , [0, 4, 6, 0, 6, 0, 0, 0, 2]] \$

[$0, y_1 + y_2 - y_5, y_1 + y_2 - y_3 - y_4, 0, y_1, y_2, y_3, y_4, y_5$]

$$p = s^4 - s^6 \quad p' = -s^4 + s^6$$

7 . Coloring, {7}

$$\Omega p(\Delta)=0: \quad p = s^3 - 16s^5 + 8s^6 + 32s^7 \quad p' = s^3 - 4s^4 + 8s^6$$

R: [4, 4, 4, 7, 7, 7, 5, 1, 1]

B: [2, 9, 5, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 7	8 vs 8	8 vs 8	3 vs 4	3 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 0, 6, 3, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$$

$$[y_2 + y_3 - y_1, 0, 0, y_2, y_3, 0, y_1, 0, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{3, 5}, {6, 8}, {2, 9}} order: 2

See Matrix

$$\$ [[3, 4, 2, 0, 1, 2, 0, 4, 2], [0, 5, 1, 0, 2, 4, 0, 2, 4], [0, 4, 2, 0, 1, 2, 0, 4, 5], [0, 5, 1, 0, 2, 4, 0, 2, 4], [0, 4, 2, 0, 1, 2, 0, 4, 5], [0, 5, 1, 0, 2, 4, 0, 2, 4], [0, 4, 2, 0, 1, 2, 0, 4, 5]] \$$$

$$[2y_1 + y_2 - y_3, y_1 + 2y_2, y_1, 0, y_2, 2y_2, 0, 2y_1, y_3]$$

$$p' = -s^3 + s^5 \quad p' = -s^4 + s^6 \quad p = s^2 - s^4 \quad p' = s^2 - s^4$$

8 . Coloring, {8}

$$\Omega p(\Delta)=0: \quad p = s^2 - 6s^4 + 16s^7$$

R: [4, 4, 4, 7, 7, 7, 1, 6, 1]

B: [2, 9, 5, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 4	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[4, 0, 0, 6, 0, 2, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, 0, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{3, 5}, {2, 9}} order: 4

See Matrix

$$\$ [[2, 4, 2, 0, 4, 0, 0, 4, 2], [4, 4, 4, 0, 2, 0, 0, 0, 4], [0, 8, 2, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 2, 0, 0, 0, 8]] \$$$

$$[2y_1 - y_4, 2y_2 - y_3, y_1, 0, y_2, 0, 0, y_3, y_4]$$

$$p' = s^3 - s^5 \quad p = -s^3 + s^5$$

9. Coloring, {9}

R: [4, 4, 4, 7, 7, 7, 1, 1, 2]

B: [2, 9, 5, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	2 vs 4	4 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[5, 1, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$$

$$[-y_1 + y_2, y_1, 0, y_2, 0, 0, y_2, 0, 0]$$

$$p = s^2 - s^3 \quad p' = -s^2 + s^3$$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}, {6, 8}} order: 6

See Matrix

$$\$ [[1, 3, 2, 0, 4, 2, 0, 4, 2], [2, 1, 4, 0, 2, 4, 0, 2, 3], [3, 2, 2, 0, 4, 2, 0, 4, 1], [1, 3, 4, 0, 2, 4, 0, 2, 2], [2, 1, 2, 0, 4, 2, 0, 4, 3], [3, 2, 4, 0, 2, 4, 0, 2, 1], [1, 3, 2, 0, 4, 2, 0, 4, 2]] \$$$

$$[-y_1 + y_2 + y_3 - y_4, y_1, y_2, 0, y_3, y_2, 0, y_3, y_4]$$

$$p' = -s - s^2 + s^4 + s^5 \quad p = -s + s^7 \quad p = -s - s^2 + s^4 + s^5$$

10 . Coloring, {2, 3}

R: [4, 9, 5, 7, 7, 7, 1, 1, 1]

B: [2, 4, 4, 8, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[6, 0, 0, 3, 1, 0, 6, 0, 2], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0]] \$$$

$$[y_3, 0, 0, y_2, y_1, 0, y_4, 0, 2y_1]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{6, 8}} order: 4

See Matrix

$$\$ [[0, 4, 2, 3, 3, 2, 0, 4, 0], [0, 0, 3, 6, 0, 4, 0, 5, 0], [0, 0, 0, 3, 0, 5, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0]] \$$$

$$[0, 4y_3, 3y_1, 3y_2, 3y_3, 3y_4, 0, 3y_5, 0]$$

$$p = -s^4 + s^6$$

11 . Coloring, {2, 4}

R: [4, 9, 4, 8, 7, 7, 1, 1, 1]

B: [2, 4, 5, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$\$ [[6, 0, 0, 4, 0, 0, 3, 3, 2], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0]] \$$

$$[2y_1, 0, 0, 2y_2, 0, 0, 3y_3, 2y_4, 2y_3]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[0, 4, 2, 2, 4, 2, 3, 1, 0], [0, 0, 4, 4, 5, 1, 2, 2, 0], [0, 0, 5, 0, 6, 2, 4, 1, 0], [0, 0, 6, 0, 9, 1, 0, 2, 0], [0, 0, 9, 0, 6, 2, 0, 1, 0], [0, 0, 6, 0, 9, 1, 0, 2, 0], [0, 0, 9, 0, 6, 2, 0, 1, 0]] \$$

$$[0, -y_1 + 4y_5 - y_3 + y_4, y_1, -y_2 + y_5 + 4y_4, y_2, y_5, y_3, y_4, 0]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

12 . Coloring, $\{2, 5\}$

$$\Omega p(\Delta)=0: \quad p = s^2 - 2s^4 - 16s^7$$

R: [4, 9, 4, 7, 3, 7, 1, 1, 1]

B: [2, 4, 5, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	4 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[6, 0, 2, 4, 0, 0, 4, 0, 2], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0]] \$$

$[y_2, 0, y_3, y_1, 0, 0, y_4, 0, y_3]$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[0, 4, 0, 2, 4, 2, 2, 4, 0], [0, 0, 0, 4, 2, 4, 4, 4, 0], [0, 0, 0, 0, 4, 4, 2, 8, 0], [0, 0, 0, 0, 2, 8, 4, 4, 0], [0, 0, 0, 4, 4, 2, 8, 0], [0, 0, 0, 0, 2, 8, 4, 4, 0]] \$$

$[0, 2y_1 - y_4, 0, -y_2 + 2y_3, y_1, y_2, y_3, y_4, 0]$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

13 . Coloring, $\{2, 6\}$

R: $[4, 9, 4, 7, 7, 8, 1, 1, 1]$

B: $[2, 4, 5, 8, 3, 7, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	7 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[6, 0, 0, 4, 0, 0, 5, 1, 2], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0]] \$$

$[y_1, 0, 0, y_2, 0, 0, y_4, y_3, 2y_3]$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{3, 5}} order: 6

See Matrix

$$\$ [[0, 4, 2, 2, 4, 2, 1, 3, 0], [0, 0, 4, 4, 3, 3, 2, 2, 0], [0, 0, 3, 0, 6, 2, 3, 4, 0], [0, 0, 6, 0, 6, 4, 2, 0, 0], [0, 0, 6, 0, 8, 0, 4, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0]] \$$$

$$[0, y_2, y_1, y_4, y_3, y_7, y_6, y_5, 0]$$

14 . Coloring, {2, 7}

R: [4, 9, 4, 7, 7, 7, 5, 1, 1]

B: [2, 4, 5, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 0, 4, 3, 0, 6, 0, 2], [2, 0, 0, 3, 6, 0, 7, 0, 0], [0, 0, 0, 2, 7, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$$

$$[y_3, 0, 0, y_2, y_1, 0, -y_3 + y_2 + y_1 + y_4, 0, y_4]$$

$$p = s^4 - s^5$$

Omega Rank for B : cycles: {{6, 8}, {3, 5}} order: 4

See Matrix

$$\$ [[3, 4, 2, 2, 1, 2, 0, 4, 0], [0, 3, 1, 4, 2, 4, 0, 4, 0], [0, 0, 2, 3, 1, 4, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0]] \$$$

$$[2y_1 - y_5 + 3y_2 - y_3, 3y_1 + 2y_2 - y_4, y_1, y_5, y_2, y_3, 0, y_4, 0]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

15 . Coloring, {2, 8}

$$\Omega p(\Delta)=0: \quad p = -s^3 - s^4 + 4s^5 + 8s^7$$

R: [4, 9, 4, 7, 7, 7, 1, 6, 1]

B: [2, 4, 5, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	2 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[4, 0, 0, 4, 0, 2, 6, 0, 2], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0]] \$$$

$$[y_2, 0, 0, y_1, 0, y_4, y_3, 0, y_4]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}} order: 4

See Matrix

$$\$ [[2, 4, 2, 2, 4, 0, 0, 4, 0], [4, 2, 4, 4, 2, 0, 0, 2, 0], [2, 4, 2, 2, 4, 0, 0, 4, 0], [4, 2, 4, 4, 2, 0, 0, 2, 0], [2, 4, 2, 2, 4, 0, 0, 4, 0], [4, 2, 4, 4, 2, 0, 0, 2, 0]] \$$$

$$[y_2, y_1, y_2, y_2, y_1, 0, 0, y_1, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5 \quad p' = -s + s^3 \quad p = -s + s^3$$

16 . Coloring, {2, 9}

R: [4, 9, 4, 7, 7, 7, 1, 1, 2]

B: [2, 4, 5, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{2, 9\}\}$ order: 6

See Matrix

$\$ [[5, 1, 0, 4, 0, 0, 6, 0, 2], [6, 2, 0, 5, 0, 0, 4, 0, 1], [4, 1, 0, 6, 0, 0, 5, 0, 2], [5, 2, 0, 4, 0, 0, 6, 0, 1], [6, 1, 0, 5, 0, 0, 4, 0, 2]] \$$

$$[5y_1 - y_2 - y_3 + 5y_4, y_1, 0, y_2, 0, 0, y_3, 0, y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{3, 5\}\}$ order: 4

See Matrix

$\$ [[1, 3, 2, 2, 4, 2, 0, 4, 0], [0, 1, 4, 3, 2, 4, 0, 4, 0], [0, 0, 2, 1, 4, 4, 0, 7, 0], [0, 0, 4, 0, 2, 7, 0, 5, 0], [0, 0, 2, 0, 4, 5, 0, 7, 0], [0, 0, 4, 0, 2, 7, 0, 5, 0], [0, 0, 2, 0, 4, 5, 0, 7, 0]] \$$

$$[y_4, 3y_4 - 4y_3 + 3y_2 + 3y_1 - y_5, y_3, y_2, 2y_4 - 3y_3 + 2y_2 + 2y_1, y_1, 0, y_5, 0]$$

$$p' = -s^4 + s^6 \quad p = -s^4 + s^6$$

17 . Coloring, $\{3, 4\}$

R: [4, 4, 5, 8, 7, 7, 1, 1, 1]

B: [2, 9, 4, 7, 3, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	3 vs 5	4 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 0, 5, 1, 0, 3, 3, 0], [6, 0, 0, 6, 0, 0, 1, 5, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$$

$$[y_2 + y_3, 0, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{3, 4, 5, 7\}, \{2, 9\}\}$ order: 4

See Matrix

$$\$ [[0, 4, 2, 1, 3, 2, 3, 1, 2], [0, 2, 3, 2, 3, 1, 1, 2, 4], [0, 4, 3, 3, 1, 2, 2, 1, 2], [0, 2, 1, 3, 2, 1, 3, 2, 4], [0, 4, 2, 1, 3, 2, 3, 1, 2], [0, 2, 3, 2, 3, 1, 1, 2, 4], [0, 4, 3, 3, 1, 2, 2, 1, 2], [0, 2, 1, 3, 2, 1, 3, 2, 4]] \$$$

$$[0, 2y_2 + 2y_3 - 4y_4, y_1, y_2, y_3, y_2 + y_3 - 2y_4, -y_1 + 2y_2 + 2y_3 - 3y_4, y_4, 2y_4]$$

$$p = -s + s^5 \quad p' = -s + s^5 \quad p' = -s^2 + s^6 \quad p' = -s^3 + s^7$$

18. Coloring, $\{3, 5\}$

R: $[4, 4, 5, 7, 3, 7, 1, 1, 1]$

B: $[2, 9, 4, 8, 7, 8, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	3 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}\}$ order: 6

See Matrix

$$\$ [[6, 0, 2, 5, 1, 0, 4, 0, 0], [4, 0, 1, 6, 2, 0, 5, 0, 0], [5, 0, 2, 4, 1, 0, 6, 0, 0], [6, 0, 1, 5, 2, 0, 4, 0, 0], [4, 0, 2, 6, 1, 0, 5, 0, 0]] \$$$

$$[y_2, 0, y_1, -y_2 + 5y_1 + 5y_3 - y_4, y_3, 0, y_4, 0, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{5, 7\}, \{2, 9\}\}$ order: 2

See Matrix

$$\$ [[0, 4, 0, 1, 3, 2, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 3, 4], [0, 4, 0, 0, 3, 3, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 3, 4], [0, 4, 0, 0, 3, 3, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 3, 4], [0, 4, 0, 0, 3, 3, 2, 4, 2]] \$$$

$$[0, 8y_3 + 8y_1 - 10y_2, 0, y_3, 5y_3 + 5y_1 - 6y_2, y_1, y_2, 6y_3 + 6y_1 - 7y_2, -2y_3 - 2y_1 + 4y_2]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

19. Coloring, $\{3, 6\}$

R: [4, 4, 5, 7, 7, 8, 1, 1, 1]

B: [2, 9, 4, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	2 vs 5	4 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 0, 5, 1, 0, 5, 1, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$$

$$[y_1 + y_2, 0, 0, y_1, y_2, 0, y_1, y_2, 0]$$

$$p' = -s^2 + s^3 \quad p = s^2 - s^3 \quad p' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{3, 4, 5, 6, 7, 8\}, \{2, 9\}\}$ order: 6

See Matrix

$$\$ [[0, 4, 2, 1, 3, 2, 1, 3, 2], [0, 2, 3, 2, 1, 3, 2, 1, 4], [0, 4, 1, 3, 2, 1, 3, 2, 2], [0, 2, 2, 1, 3, 2, 1, 3, 4], [0, 4, 3, 2, 1, 3, 2, 1, 2], [0, 2, 1, 3, 2, 1, 3, 2, 4], [0, 4, 2, 1, 3, 2, 1, 3, 2], [0, 2, 3, 2, 1, 3, 2, 1, 4]] \$$$

$$[0, y_1 + y_2 + y_4 - y_3, y_1, y_2, y_4, y_1, y_2, y_4, y_3]$$

$$p = s - s^3 - s^4 + s^6 \quad p' = s + s^2 - s^4 - s^5 \quad p'' = -s^2 - s^3 + s^5 + s^6 \quad p''' = s^2 - s^4 - s^5 + s^7$$

20 . Coloring, {3, 7}

$$\Omega p(\Delta)=0: \quad p = -s^3 + s^4 - 4s^5 + 8s^7$$

R: [4, 4, 5, 7, 7, 7, 5, 1, 1]

B: [2, 9, 4, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	3 vs 4	4 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 0, 5, 4, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$$

$$[y_1 + y_2 - y_3, 0, 0, y_1, y_2, 0, y_3, 0, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{2, 9}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 4, 2, 1, 0, 2, 0, 4, 2], [0, 5, 0, 2, 0, 4, 0, 3, 4], [0, 4, 0, 0, 0, 3, 0, 6, 5], [0, 5, 0, 0, 0, 6, 0, 3, 4], [0, 4, 0, 0, 0, 3, 0, 6, 5], [0, 5, 0, 0, 0, 6, 0, 3, 4], [0, 4, 0, 0, 0, 3, 0, 6, 5]] \$$$

$$[9y_1 - 6y_2 - 6y_4 - 3y_3, 2y_1, 6y_1 - 4y_2 - 4y_4 - 2y_3, 2y_2, 0, 2y_4, 0, 2y_3, -5y_1 + 4y_2 + 4y_4 + 3y_3]$$

$$p = -s^3 + s^7 \quad p' = -s^3 + s^5 \quad p'' = s^3 - s^5$$

21 . Coloring, {3, 8}

R: [4, 4, 5, 7, 7, 7, 1, 6, 1]

B: [2, 9, 4, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	7 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[4, 0, 0, 5, 1, 2, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] \$

$[y_1, 0, 0, y_2, y_3, 2y_3, y_4, 0, 0]$

$$p = s^2 - s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

\$ [[2, 4, 2, 1, 3, 0, 0, 4, 2] , [4, 4, 3, 2, 0, 0, 0, 1, 4] , [1, 8, 0, 3, 0, 0, 0, 2, 4] , [2, 5, 0, 0, 0, 0, 0, 3, 8] , [3, 10, 0, 0, 0, 0, 0, 0, 5] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$

$[y_1, y_2, y_3, y_4, y_5, 0, 0, y_6, y_7]$

22 . Coloring, {3, 9}

R: [4, 4, 5, 7, 7, 7, 1, 1, 2]

B: [2, 9, 4, 8, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	2 vs 5	6 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$$\$ [[5, 1, 0, 5, 1, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$$

$$[y_1, y_2, 0, y_1, y_2, 0, y_1 + y_2, 0, 0]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{6, 8\}\}$ order: 6

See Matrix

$$\$ [[1, 3, 2, 1, 3, 2, 0, 4, 2], [2, 1, 3, 2, 0, 4, 0, 3, 3], [3, 2, 0, 3, 0, 3, 0, 6, 1], [1, 3, 0, 0, 0, 6, 0, 6, 2], [2, 1, 0, 0, 0, 6, 0, 6, 3], [3, 2, 0, 0, 0, 6, 0, 6, 1], [1, 3, 0, 0, 0, 6, 0, 6, 2], [2, 1, 0, 0, 0, 6, 0, 6, 3]] \$$$

$$[-y_1 + y_2 + y_3 + y_4 - y_6, y_1, -y_5 + y_2 + y_3 + y_4, y_2, y_3, y_4, 0, y_5, y_6]$$

$$p' = -s^4 + s^7 \quad p = -s^4 + s^7$$

23 . Coloring, $\{4, 5\}$

R: [4, 4, 4, 8, 3, 7, 1, 1, 1]

B: [2, 9, 5, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	2 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 2, 6, 0, 0, 1, 3, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0]] \$$$

$$[y_1, 0, 2y_4, y_2, 0, 0, y_4, y_3, 0]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{6, 8}, {2, 9}, {5, 7}} order: 2

See Matrix

$$\$ [[0, 4, 0, 0, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4], [0, 4, 0, 0, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4], [0, 4, 0, 0, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4]] \$$$

$$[0, 2y_2, 0, 0, y_2 + 2y_1, y_2, 2y_2 + y_1, y_1, 2y_1]$$

$$p = -s + s^3 \quad p' = -s + s^3 \quad p = -s + s^5 \quad p' = -s + s^5$$

24 . Coloring, {4, 6}

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^4 - 16s^7$$

R: [4, 4, 4, 8, 7, 8, 1, 1, 1]

B: [2, 9, 5, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	6 vs 7	7 vs 7	2 vs 4	4 vs 6

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

$$\$ [[6, 0, 0, 6, 0, 0, 2, 4, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$$

$$[y_2, 0, 0, y_2, 0, 0, y_1, y_2 - y_1, 0]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^3$$

Omega Rank for B : cycles: {{3, 5}, {2, 9}} order: 4

See Matrix

$$\$ [[0, 4, 2, 0, 4, 2, 4, 0, 2], [0, 2, 4, 0, 6, 0, 2, 0, 4], [0, 4, 6, 0, 6, 0, 0, 0, 2], [0, 2, 6, 0, 6, 0, 0, 0, 4], [0, 4, 6, 0, 6, 0, 0, 0, 2], [0, 2, 6, 0, 6, 0, 0, 0, 4]] \$$$

$$[0, y_4, y_3, 0, y_2, y_1, -y_3 + y_2 + y_1, 0, -y_4 + y_2 + y_1]$$

$$p' = s^3 - s^5 \quad p = s^3 - s^5$$

25 . Coloring, {4, 7}

$$\Omega p(\Delta)=0: \quad p = s - 4s^3 - 4s^4 + 4s^5 + 8s^6 \quad p' = s - 4s^3 - 4s^4 + 4s^5 + 8s^6$$

R: [4, 4, 4, 8, 7, 7, 5, 1, 1]

B: [2, 9, 5, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 7	9 vs 9	9 vs 9	3 vs 5	4 vs 8

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}} order: 6

See Matrix

$$\$ [[3, 0, 0, 6, 3, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 3, 6, 0], [6, 0, 0, 3, 3, 0, 3, 3, 0], [3, 0, 0, 6, 3, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 3, 6, 0]] \$$$

$$[-y_1 + 4y_3 - y_2, 0, 0, y_1, y_3, 0, y_3, y_2, 0]$$

$$p = s - s^4 \quad p' = -s + s^4$$

Omega Rank for B : cycles: {{3, 5}, {2, 9}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 4, 2, 0, 1, 2, 3, 1, 2], [3, 5, 1, 0, 2, 1, 0, 2, 4], [0, 7, 2, 0, 1, 2, 0, 1, 5], [0, 5, 1, 0, 2, 1, 0, 2, 7], [0, 7, 2, 0, 1, 2, 0, 1, 5], [0, 5, 1, 0, 2, 1, 0, 2, 7], [0, 7, 2, 0, 1, 2, 0, 1, 5], [0, 5, 1, 0, 2, 1, 0, 2, 7]] \$$$

$$[y_4 + 3y_2 - y_1, 3y_4 - y_3 + y_2, y_4, 0, y_2, y_4, y_3, y_2, y_1]$$

$$p' = s^5 - s^7 \quad p' = s^4 - s^6 \quad p' = s^3 - s^7 \quad p = s^3 - s^7$$

26 . Coloring, {4, 8}

R: [4, 4, 4, 8, 7, 7, 1, 6, 1]

B: [2, 9, 5, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 5	4 vs 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[4, 0, 0, 6, 0, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 2, 6, 0] , [2, 0, 0, 3, 0, 6, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 6, 3, 0] , [6, 0, 0, 3, 0, 3, 4, 2, 0]] \$

$[y_1, 0, 0, y_2, 0, y_3, y_4, y_5, 0]$

Omega Rank for B : cycles: {{3, 5}, {2, 9}} order: 4

See Matrix

\$ [[2, 4, 2, 0, 4, 0, 3, 1, 2] , [1, 4, 4, 0, 5, 0, 0, 0, 4] , [0, 5, 5, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 5, 0, 0, 0, 5] , [0, 5, 5, 0, 4, 0, 0, 0, 4] , [0, 4, 4, 0, 5, 0, 0, 0, 5] , [0, 5, 5, 0, 4, 0, 0, 0, 4]] \$

$[y_4 - y_2, y_3 + 2y_1, y_3, 0, y_4, 0, 3y_1, y_1, y_2]$

$p' = -s^4 + s^6$ $p' = -s^3 + s^5$ $p = s^3 - s^5$

27 . Coloring, {4, 9}

R: [4, 4, 4, 8, 7, 7, 1, 1, 2]

B: [2, 9, 5, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	2 vs 5	5 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$$\$ [[5, 1, 0, 6, 0, 0, 3, 3, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$$

$$[-y_2 + y_1, y_2, 0, y_1, 0, 0, 3y_2, -3y_2 + y_1, 0]$$

$$p = s^2 - s^3 \quad p' = -s^2 + s^3 \quad p'' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 9\}, \{6, 8\}\}$ order: 6

See Matrix

$$\$ [[1, 3, 2, 0, 4, 2, 3, 1, 2], [2, 1, 4, 0, 5, 1, 0, 2, 3], [3, 2, 5, 0, 4, 2, 0, 1, 1], [1, 3, 4, 0, 5, 1, 0, 2, 2], [2, 1, 5, 0, 4, 2, 0, 1, 3], [3, 2, 4, 0, 5, 1, 0, 2, 1], [1, 3, 5, 0, 4, 2, 0, 1, 2], [2, 1, 4, 0, 5, 1, 0, 2, 3]] \$$$

$$[y_1, -y_1 + 2y_5 + 2y_3 - y_2, 2y_5 + y_3 - y_4, 0, y_5 + 2y_3, y_5, y_4, y_3, y_2]$$

$$p = -s^2 + s^8 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 - s^3 + s^5 + s^6$$

28 . Coloring, $\{5, 6\}$

R: [4, 4, 4, 7, 3, 8, 1, 1, 1]

B: [2, 9, 5, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	4 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 2, 6, 0, 0, 3, 1, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0]] \$$$

$$[y_1, 0, 2y_4, y_2, 0, 0, y_3, y_4, 0]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{5, 7\}\}$ order: 4

See Matrix

$$\$ [[0, 4, 0, 0, 4, 2, 3, 3, 2], [0, 2, 0, 0, 3, 3, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4]] \$$$

$$[0, y_1 + y_3 - y_4, 0, 0, -y_2 + y_1 + y_3, y_2, y_1, y_3, y_4]$$

$$p' = s^3 - s^5 \quad p = -s^3 + s^5$$

29 . Coloring, $\{5, 7\}$

R: [4, 4, 4, 7, 3, 7, 5, 1, 1]

B: [2, 9, 5, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}\}$ order: 4

See Matrix

$$\$ [[3, 0, 2, 6, 3, 0, 4, 0, 0], [0, 0, 3, 5, 4, 0, 6, 0, 0], [0, 0, 4, 3, 6, 0, 5, 0, 0], [0, 0, 6, 4, 5, 0, 3, 0, 0], [0, 0, 5, 6, 3, 0, 4, 0, 0]] \$$$

$$[y_3, 0, y_4, y_2, y_3 + y_4 - y_2 + y_1, 0, y_1, 0, 0]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 4

See Matrix

$$\$ [[3, 4, 0, 0, 1, 2, 2, 4, 2], [2, 5, 0, 0, 0, 4, 1, 2, 4], [1, 6, 0, 0, 0, 2, 0, 4, 5], [0, 6, 0, 0, 0, 4, 0, 2, 6], [0, 6, 0, 0, 0, 2, 0, 4, 6], [0, 6, 0, 0, 0, 4, 0, 2, 6], [0, 6, 0, 0, 0, 2, 0, 4, 6]] \$$$

$$[-y_1 + y_2 + y_3 - y_5, y_2 + y_3 - y_4, 0, 0, y_1, y_2, y_4, y_3, y_5]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

30 . Coloring, {5, 8}

$$\Omega p(\Delta)=0: \quad p = s^3 - 3s^4 + 8s^7$$

R: [4, 4, 4, 7, 3, 7, 1, 6, 1]

B: [2, 9, 5, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[4, 0, 2, 6, 0, 2, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0]] \$$$

$$[y_1, 0, y_3, y_2, 0, y_3, y_4, 0, 0]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{2, 9}, {5, 7}} order: 4

See Matrix

$$\$ [[2, 4, 0, 0, 4, 0, 2, 4, 2], [4, 4, 0, 0, 2, 0, 4, 0, 4], [0, 8, 0, 0, 4, 0, 2, 0, 4], [0, 4, 0, 0, 2, 0, 4, 0, 8], [0, 8, 0, 0, 4, 0, 2, 0, 4], [0, 4, 0, 0, 2, 0, 4, 0, 8]] \$$$

$$[2y_1 - y_3, 2y_2 - y_4, 0, 0, y_2, 0, y_1, y_4, y_3]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

31 . Coloring, {5, 9}

R: [4, 4, 4, 7, 3, 7, 1, 1, 2]

B: [2, 9, 5, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	4 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[5, 1, 2, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0]] \$

$[y_3, y_4, 2y_4, y_1, 0, 0, y_2, 0, 0]$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}, {6, 8}} order: 6

See Matrix

\$ [[1, 3, 0, 0, 4, 2, 2, 4, 2], [2, 1, 0, 0, 2, 4, 4, 2, 3], [3, 2, 0, 0, 4, 2, 2, 4, 1], [1, 3, 0, 0, 2, 4, 4, 2, 2], [2, 1, 0, 0, 4, 2, 2, 4, 3], [3, 2, 0, 0, 2, 4, 4, 2, 1], [1, 3, 0, 0, 4, 2, 2, 4, 2]] \$

$[y_4, y_2, 0, 0, y_3, y_1, y_1, y_3, -y_4 - y_2 + y_3 + y_1]$

$$p = s - s^3 - s^4 + s^6 \quad p = -s + s^7 \quad p = s + s^2 - s^4 - s^5$$

32 . Coloring, {6, 7}

R: [4, 4, 4, 7, 7, 8, 5, 1, 1]

B: [2, 9, 5, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 5	6 vs 8

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[3, 0, 0, 6, 3, 0, 5, 1, 0], [1, 0, 0, 3, 5, 0, 9, 0, 0], [0, 0, 0, 1, 9, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$

$$[y_4, 0, 0, y_3, y_2, 0, y_1, y_5, 0]$$

Omega Rank for B : cycles: {{2, 9}, {3, 5}} order: 6

See Matrix

\$ [[3, 4, 2, 0, 1, 2, 1, 3, 2], [1, 5, 1, 0, 2, 3, 2, 0, 4], [2, 5, 2, 0, 1, 0, 3, 0, 5], [3, 7, 1, 0, 2, 0, 0, 0, 5], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8]] \$

$$[y_4, y_3, y_2, 0, y_1, -y_4 + 2y_2 + 3y_1 - y_6, -y_3 + 3y_2 + 2y_1 - y_5, y_5, y_6]$$

$$p' = s^5 - s^7 \quad p = s^5 - s^7$$

33 . Coloring, {6, 8}

R: [4, 4, 4, 7, 7, 8, 1, 6, 1]

B: [2, 9, 5, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	4 vs 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}} order: 6

See Matrix

\$ [[4, 0, 0, 6, 0, 2, 5, 1, 0], [5, 0, 0, 4, 0, 1, 6, 2, 0], [6, 0, 0, 5, 0, 2, 4, 1, 0], [4, 0, 0, 6, 0, 1, 5, 2, 0], [5, 0, 0, 4, 0, 2, 6, 1, 0]] \$

$$[y_1, 0, 0, -y_1 + 5y_2 - y_3 + 5y_4, 0, y_2, y_3, y_4, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: {{2, 9}, {3, 5}} order: 4

See Matrix

$$\$ [[2, 4, 2, 0, 4, 0, 1, 3, 2], [3, 4, 4, 0, 3, 0, 0, 0, 4], [0, 7, 3, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 3, 0, 0, 0, 7], [0, 7, 3, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 3, 0, 0, 0, 7], [0, 7, 3, 0, 4, 0, 0, 0, 4]] \$$$

$$[-16y_3 + 33y_2 - 48y_1 - 5y_4, 5y_3, -7y_3 + 16y_2 - 26y_1, 0, 5y_2, 0, 5y_1, 15y_1, 5y_4]$$

$$p = -s^3 + s^7 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^5$$

34 . Coloring, {6, 9}

R: [4, 4, 4, 7, 7, 8, 1, 1, 2]

B: [2, 9, 5, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	2 vs 5	6 vs 8

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[5, 1, 0, 6, 0, 0, 5, 1, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$$

$$[y_1, y_2, 0, y_2 + y_1, 0, 0, y_1, y_2, 0]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}} order: 6

See Matrix

$$\$ [[1, 3, 2, 0, 4, 2, 1, 3, 2], [2, 1, 4, 0, 3, 3, 2, 0, 3], [3, 2, 3, 0, 6, 0, 3, 0, 1], [1, 3, 6, 0, 6, 0, 0, 0, 2], [2, 1, 6, 0, 6, 0, 0, 0, 3], [3, 2, 6, 0, 6, 0, 0, 0, 1], [1, 3, 6, 0, 6, 0, 0, 0, 2], [2, 1, 6, 0, 6, 0, 0, 0, 3]] \$$$

$$[-y_1 + y_2 + y_6 - y_5, y_1, y_2 + y_6 - y_3 - y_4, 0, y_2, y_6, y_3, y_4, y_5]$$

$$p = -s^4 + s^7 \quad p' = -s^4 + s^7$$

35 . Coloring, {7, 8}

R: [4, 4, 4, 7, 7, 7, 5, 6, 1]

B: [2, 9, 5, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	4 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[1, 0, 0, 6, 3, 2, 6, 0, 0], [0, 0, 0, 1, 6, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$$$

$$[y_1, 0, 0, y_3, y_2, 2y_1, y_4, 0, 0]$$

$$p = s^3 - s^5$$

Omega Rank for B : cycles: {{3, 5}, {2, 9}} order: 4

See Matrix

$$\$ [[5, 4, 2, 0, 1, 0, 0, 4, 2], [4, 7, 1, 0, 2, 0, 0, 0, 4], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8]] \$$$

$$[2y_1 + 3y_3 - y_4, 3y_1 + 2y_3 - y_2, y_1, 0, y_3, 0, 0, y_2, y_4]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

36 . Coloring, {7, 9}

R: [4, 4, 4, 7, 7, 7, 5, 1, 2]

B: [2, 9, 5, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	3 vs 5	4 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 6, 3, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[2y_1, y_1, 0, 3y_1 - y_3 + y_2, y_3, 0, y_2, 0, 0]$$

$$p' = -s^3 + s^4 \quad p = s^3 - s^4$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{6, 8\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 0, 1, 2, 0, 4, 2], [2, 4, 1, 0, 2, 4, 0, 2, 3], [3, 2, 2, 0, 1, 2, 0, 4, 4], [4, 3, 1, 0, 2, 4, 0, 2, 2], [2, 4, 2, 0, 1, 2, 0, 4, 3], [3, 2, 1, 0, 2, 4, 0, 2, 4], [4, 3, 2, 0, 1, 2, 0, 4, 2]] \$$

$$[y_1, -y_1 + 3y_2 + 3y_3 - y_4, y_2, 0, y_3, 2y_3, 0, 2y_2, y_4]$$

$$p = -s - s^2 + s^4 + s^5 \quad p = s - s^3 - s^4 + s^6 \quad p = -s + s^7$$

37 . Coloring, $\{8, 9\}$

R: [4, 4, 4, 7, 7, 7, 1, 6, 2]

B: [2, 9, 5, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[3, 1, 0, 6, 0, 2, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0]] \$$

$[y_1, y_2, 0, y_3, 0, 2y_2, y_4, 0, 0]$

$$p = s^2 - s^5$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[3, 3, 2, 0, 4, 0, 0, 4, 2], [6, 3, 4, 0, 2, 0, 0, 0, 3], [3, 6, 2, 0, 4, 0, 0, 0, 3], [3, 3, 4, 0, 2, 0, 0, 0, 6], [6, 3, 2, 0, 4, 0, 0, 0, 3], [3, 6, 4, 0, 2, 0, 0, 0, 3]] \$$

$[-y_1 + 2y_2 + 2y_3 - y_4 - y_5, y_1, y_2, 0, y_3, 0, 0, y_4, y_5]$

$$p = -s^2 - s^3 + s^5 + s^6$$

38 . Coloring, $\{2, 3, 4\}$

R: $[4, 9, 5, 8, 7, 7, 1, 1, 1]$

B: $[2, 4, 4, 7, 3, 8, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$\$ [[6, 0, 0, 3, 1, 0, 3, 3, 2], [8, 0, 0, 6, 0, 0, 1, 3, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0]] \$$

$[y_1, 0, 0, y_5, y_4, 0, y_3, y_2, 2y_4]$

$$p = -s^3 + s^6$$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[0, 4, 2, 3, 3, 2, 3, 1, 0], [0, 0, 3, 6, 3, 1, 3, 2, 0], [0, 0, 3, 3, 3, 2, 6, 1, 0], [0, 0, 3, 3, 6, 1, 3, 2, 0], [0, 0, 6, 3, 3, 2, 3, 1, 0], [0, 0, 3, 6, 3, 1, 3, 2, 0], [0, 0, 3, 3, 3, 2, 6, 1, 0]] \$$$

$$[0, -y_1 + 4y_3 - y_4 + y_5, y_1, -y_2 + y_3 + 4y_5, y_2, y_3, y_4, y_5, 0]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

39 . Coloring, {2, 3, 5}

R: [4, 9, 5, 7, 3, 7, 1, 1, 1]

B: [2, 4, 4, 8, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}} order: 6

See Matrix

$$\$ [[6, 0, 2, 3, 1, 0, 4, 0, 2], [6, 0, 1, 6, 2, 0, 3, 0, 0], [3, 0, 2, 6, 1, 0, 6, 0, 0], [6, 0, 1, 3, 2, 0, 6, 0, 0], [6, 0, 2, 6, 1, 0, 3, 0, 0], [3, 0, 1, 6, 2, 0, 6, 0, 0]] \$$$

$$[y_5, 0, y_4, y_3, y_2, 0, y_1, 0, -y_5 + 5y_4 - y_3 + 5y_2 - y_1]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[0, 4, 0, 3, 3, 2, 2, 4, 0], [0, 0, 0, 4, 2, 4, 3, 5, 0], [0, 0, 0, 0, 3, 5, 2, 8, 0], [0, 0, 0, 0, 2, 8, 3, 5, 0], [0, 0, 0, 0, 3, 5, 2, 8, 0], [0, 0, 0, 0, 2, 8, 3, 5, 0]] \$$$

$$[0, y_4, 0, y_3, -5 y_3 - 5 y_2 + 14 y_1, y_2, y_1, -y_4 - 14 y_3 - 14 y_2 + 39 y_1, 0]$$

$$p' = s^3 - s^5 \quad p = s^3 - s^5$$

40 . Coloring, {2, 3, 6}

R: [4, 9, 5, 7, 7, 8, 1, 1, 1]

B: [2, 4, 4, 8, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	7 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[6, 0, 0, 3, 1, 0, 5, 1, 2], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, y_4, 0, y_3, y_4, 2 y_4]$$

$$p' = -s^2 + s^5 \quad p = s^2 - s^5$$

Omega Rank for B : cycles: {{3, 4, 5, 6, 7, 8}} order: 6

See Matrix

$$\$ [[0, 4, 2, 3, 3, 2, 1, 3, 0], [0, 0, 3, 6, 1, 3, 2, 3, 0], [0, 0, 1, 3, 2, 3, 3, 6, 0], [0, 0, 2, 1, 3, 6, 3, 3, 0], [0, 0, 3, 2, 3, 3, 6, 1, 0], [0, 0, 3, 3, 6, 1, 3, 2, 0], [0, 0, 6, 3, 3, 2, 1, 3, 0]] \$$$

$$[0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

41 . Coloring, {2, 3, 7}

$$\Omega p(\Delta)=0: \quad p = s^2 + 6s^4 + 16s^7$$

R: [4, 9, 5, 7, 7, 7, 5, 1, 1]

B: [2, 4, 4, 8, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	5 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[3, 0, 0, 3, 4, 0, 6, 0, 2], [2, 0, 0, 3, 6, 0, 7, 0, 0], [0, 0, 0, 2, 7, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$

$$[y_1 + y_2 - y_3 + y_4, 0, 0, y_1, y_2, 0, y_3, 0, y_4]$$

$$p = -s^4 + s^5$$

Omega Rank for B : cycles: {{6, 8}} order: 4

See Matrix

\$ [[3, 4, 2, 3, 0, 2, 0, 4, 0], [0, 3, 0, 6, 0, 4, 0, 5, 0], [0, 0, 0, 3, 0, 5, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0]] \$

$$[3y_2, 2y_1, 2y_2, 2y_5, 0, 2y_4, 0, 2y_3, 0]$$

$$p = -s^4 + s^6$$

42 . Coloring, {2, 3, 8}

R: [4, 9, 5, 7, 7, 7, 1, 6, 1]

B: [2, 4, 4, 8, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3
See Matrix

$\$ [[4, 0, 0, 3, 1, 2, 6, 0, 2], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0]] \$$

$$[y_1, 0, 0, y_2, y_3, 2y_3, y_4, 0, 2y_3]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
See Matrix

$\$ [[2, 4, 2, 3, 3, 0, 0, 4, 0], [4, 2, 3, 6, 0, 0, 0, 3, 0], [3, 4, 0, 5, 0, 0, 0, 6, 0], [6, 3, 0, 4, 0, 0, 0, 5, 0], [5, 6, 0, 3, 0, 0, 0, 4, 0], [4, 5, 0, 6, 0, 0, 0, 3, 0]] \$$

$$[y_1, y_4, y_5, y_6, y_3, 0, 0, y_2, 0]$$

43 . Coloring, $\{2, 3, 9\}$

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^3 - 4s^5 - 8s^6 - 16s^7$$

R: $[4, 9, 5, 7, 7, 7, 1, 1, 2]$

B: $[2, 4, 4, 8, 3, 8, 5, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{2, 9\}\}$ order: 6
See Matrix

$\$ [[5, 1, 0, 3, 1, 0, 6, 0, 2], [6, 2, 0, 5, 0, 0, 4, 0, 1], [4, 1, 0, 6, 0, 0, 5, 0, 2], [5, 2, 0, 4, 0, 0, 6, 0, 1], [6, 1, 0, 5, 0, 0, 4, 0, 2], [4, 2, 0, 6, 0, 0, 5, 0, 1]] \$$

$$[5y_1 - y_4 - y_3 - y_2 + 5y_5, y_1, 0, y_4, y_3, 0, y_2, 0, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{6, 8}} order: 4

See Matrix

$$\$ [[1, 3, 2, 3, 3, 2, 0, 4, 0], [0, 1, 3, 5, 0, 4, 0, 5, 0], [0, 0, 0, 4, 0, 5, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0]] \$$$

$$[y_2, y_1, -7y_2 + 3y_1, -11y_2 + 4y_1 - y_4 + y_3, 3y_2, y_4, 0, y_3, 0]$$

$$p = s^4 - s^7 \quad p' = s^5 - s^6 \quad p'' = s^4 - s^6$$

44 . Coloring, {2, 4, 5}

R: [4, 9, 4, 8, 3, 7, 1, 1, 1]

B: [2, 4, 5, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

$$\$ [[6, 0, 2, 4, 0, 0, 1, 3, 2], [6, 0, 0, 8, 0, 0, 0, 4, 0], [4, 0, 0, 6, 0, 0, 0, 8, 0], [8, 0, 0, 4, 0, 0, 0, 6, 0], [6, 0, 0, 8, 0, 0, 0, 4, 0], [4, 0, 0, 6, 0, 0, 0, 8, 0]] \$$$

$$[y_1, 0, 2y_4, y_2, 0, 0, y_4, y_3, 2y_4]$$

$$p' = s^2 - s^5 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[0, 4, 0, 2, 4, 2, 5, 1, 0], [0, 0, 0, 4, 5, 1, 6, 2, 0], [0, 0, 0, 0, 6, 2, 9, 1, 0], [0, 0, 0, 0, 9, 1, 6, 2, 0], [0, 0, 0, 0, 6, 2, 9, 1, 0], [0, 0, 0, 0, 9, 1, 6, 2, 0]] \$$$

$$[0, 4y_3 + 4y_4 - 15y_1 - y_2, 0, y_3, y_4, y_3 + y_4 - 4y_1, y_2, y_1, 0]$$

$$p' = -s^3 + s^5 \quad p = -s^3 + s^5$$

45 . Coloring, {2, 4, 6}

$$\Omega p(\Delta)=0: \quad p = s^3 + s^4 + 4s^5 + 8s^7$$

R: [4, 9, 4, 8, 7, 8, 1, 1, 1]

B: [2, 4, 5, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	5 vs 6

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

$$\$ [[6, 0, 0, 4, 0, 0, 2, 4, 2], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0]] \$$$

$$[y_1, 0, 0, y_2, 0, 0, y_4, y_3, y_4]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: {{3, 5}} order: 4

See Matrix

$$\$ [[0, 4, 2, 2, 4, 2, 4, 0, 0], [0, 0, 4, 4, 6, 0, 4, 0, 0], [0, 0, 6, 0, 8, 0, 4, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0]] \$$$

$$[0, 2y_4, y_1, y_2, y_3, y_4, y_5, 0, 0]$$

$$p = -s^4 + s^6$$

46 . Coloring, {2, 4, 7}

R: [4, 9, 4, 8, 7, 7, 5, 1, 1]

B: [2, 4, 5, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	4 vs 8

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}} order: 6

See Matrix

$\$ [[3, 0, 0, 4, 3, 0, 3, 3, 2], [5, 0, 0, 3, 3, 0, 3, 4, 0], [4, 0, 0, 5, 3, 0, 3, 3, 0], [3, 0, 0, 4, 3, 0, 3, 5, 0], [5, 0, 0, 3, 3, 0, 3, 4, 0], [4, 0, 0, 5, 3, 0, 3, 3, 0]] \$$

$[-y_3 + 4y_2 - y_1 - y_4, 0, 0, y_3, y_2, 0, y_2, y_1, y_4]$

$$p' = s^2 - s^5 \quad p = s^2 - s^5$$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 7}, {6, 8}} order: 4

See Matrix

$\$ [[3, 4, 2, 2, 1, 2, 3, 1, 0], [3, 3, 1, 4, 2, 1, 2, 2, 0], [2, 3, 2, 3, 1, 2, 4, 1, 0], [4, 2, 1, 3, 2, 1, 3, 2, 0], [3, 4, 2, 2, 1, 2, 3, 1, 0], [3, 3, 1, 4, 2, 1, 2, 2, 0], [2, 3, 2, 3, 1, 2, 4, 1, 0], [4, 2, 1, 3, 2, 1, 3, 2, 0]] \$$

$[y_2 - y_1 + 3y_4, 3y_2 + y_4 - y_3, y_2, y_1, y_4, y_2, y_3, y_4, 0]$

$$p' = -s^3 + s^7 \quad p = -s + s^5 \quad p' = -s^2 + s^6 \quad p' = -s + s^5$$

47 . Coloring, {2, 4, 8}

R: [4, 9, 4, 8, 7, 7, 1, 6, 1]

B: [2, 4, 5, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 6	7 vs 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[4, 0, 0, 4, 0, 2, 3, 3, 2] , [5, 0, 0, 4, 0, 3, 2, 4, 0] , [2, 0, 0, 5, 0, 4, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 4, 5, 0] , [4, 0, 0, 3, 0, 5, 4, 2, 0] , [4, 0, 0, 4, 0, 2, 5, 3, 0]] \$

$[y_1, 0, 0, y_5, 0, y_2, y_3, y_4, y_6]$

Omega Rank for B : cycles: {{3, 5}} order: 6

See Matrix

\$ [[2, 4, 2, 2, 4, 0, 3, 1, 0] , [1, 2, 4, 4, 5, 0, 2, 0, 0] , [0, 1, 5, 2, 6, 0, 4, 0, 0] , [0, 0, 6, 1, 9, 0, 2, 0, 0] , [0, 0, 9, 0, 8, 0, 1, 0, 0] , [0, 0, 8, 0, 10, 0, 0, 0, 0] , [0, 0, 10, 0, 8, 0, 0, 0, 0]] \$

$[y_1, y_2, y_3, y_6, y_7, 0, y_4, y_5, 0]$

48 . Coloring, {2, 4, 9}

$\Omega p(\Delta)=0: p = s^2 - 2s^3 + 4s^5 + 8s^6 - 16s^7$

R: [4, 9, 4, 8, 7, 7, 1, 1, 2]

B: [2, 4, 5, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	9 vs 9	9 vs 9	5 vs 6	6 vs 8

Omega Rank for R : cycles: {{2, 9}, {1, 4, 8}} order: 6

See Matrix

\$ [[5, 1, 0, 4, 0, 0, 3, 3, 2] , [6, 2, 0, 5, 0, 0, 0, 4, 1] , [4, 1, 0, 6, 0, 0, 0, 5, 2] , [5, 2, 0, 4, 0, 0, 0, 6, 1] , [6, 1, 0, 5, 0, 0, 0, 4, 2] , [4, 2, 0, 6, 0, 0, 0, 5, 1]] \$

$$[5 y_1 - y_2 - y_4 - y_3 + 5 y_5, y_1, 0, y_2, 0, 0, y_4, y_3, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{3, 5}, {6, 8}} order: 6

See Matrix

$$\$ [[1, 3, 2, 2, 4, 2, 3, 1, 0], [0, 1, 4, 3, 5, 1, 2, 2, 0], [0, 0, 5, 1, 6, 2, 3, 1, 0], [0, 0, 6, 0, 8, 1, 1, 2, 0], [0, 0, 8, 0, 7, 2, 0, 1, 0], [0, 0, 7, 0, 8, 1, 0, 2, 0], [0, 0, 8, 0, 7, 2, 0, 1, 0], [0, 0, 7, 0, 8, 1, 0, 2, 0]] \$$$

$$[-y_3 - y_4 + 2 y_2 + 3 y_5, -y_1 + 3 y_2 - y_6 + 2 y_5, y_1, y_3, y_4, y_2, y_6, y_5, 0]$$

$$p' = -s^5 + s^7 \quad p = -s^5 + s^7$$

49 . Coloring, {2, 5, 6}

R: [4, 9, 4, 7, 3, 8, 1, 1, 1]

B: [2, 4, 5, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	6 vs 6

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

$$\$ [[6, 0, 2, 4, 0, 0, 3, 1, 2], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0]] \$$$

$$[y_1, 0, 2 y_4, y_3, 0, 0, y_2, y_4, 2 y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

$$\$ [[0, 4, 0, 2, 4, 2, 3, 3, 0], [0, 0, 0, 4, 3, 3, 6, 2, 0], [0, 0, 0, 0, 6, 2, 6, 4, 0], [0, 0, 0, 0, 6, 4, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$$$

$$[0, y_1, 0, y_4, y_2, y_3, y_6, y_5, 0]$$

50 . Coloring, {2, 5, 7}

R: [4, 9, 4, 7, 3, 7, 5, 1, 1]

B: [2, 4, 5, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	7 vs 7

Omega Rank for R : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 2, 4, 3, 0, 4, 0, 2], [2, 0, 3, 5, 4, 0, 4, 0, 0], [0, 0, 4, 5, 4, 0, 5, 0, 0], [0, 0, 4, 4, 5, 0, 5, 0, 0], [0, 0, 5, 4, 5, 0, 4, 0, 0], [0, 0, 5, 5, 4, 0, 4, 0, 0]] \$$$

$$[-y_1 + y_2 + y_3 - y_4 + y_5, 0, y_1, y_2, y_3, 0, y_4, 0, y_5]$$

$$p = -s^3 + s^4 - s^5 + s^6$$

Omega Rank for B : cycles: {{6, 8}} order: 6

See Matrix

$$\$ [[3, 4, 0, 2, 1, 2, 2, 4, 0], [2, 3, 0, 4, 0, 4, 1, 4, 0], [1, 2, 0, 3, 0, 4, 0, 8, 0], [0, 1, 0, 2, 0, 8, 0, 7, 0], [0, 0, 1, 0, 7, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0]] \$$$

$$[y_1, y_2, 0, y_3, y_4, y_5, y_6, y_7, 0]$$

51 . Coloring, {2, 5, 8}

$$\Omega p(\Delta)=0: \quad p' = s^5 - 2s^6 \quad p' = s^2 - 16s^6 \quad p' = s^4 - 4s^6 \quad p' = s^3 - 8s^6 \quad p = s^2 - 32s^7$$

R: [4, 9, 4, 7, 3, 7, 1, 6, 1]

B: [2, 4, 5, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 7	2 vs 7	2 vs 7	2 vs 6	2 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[4, 0, 2, 4, 0, 2, 4, 0, 2], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$

$$[y_1, 0, y_2, y_1, 0, y_2, y_1, 0, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 4, 0, 2, 4, 0, 2, 4, 0], [4, 2, 0, 4, 2, 0, 4, 2, 0], [2, 4, 0, 2, 4, 0, 2, 4, 0], [4, 2, 0, 4, 2, 0, 4, 2, 0], [2, 4, 0, 2, 4, 0, 2, 4, 0], [4, 2, 0, 4, 2, 0, 4, 2, 0]] \$$

$$[y_1, y_2, 0, y_1, y_2, 0, y_1, y_2, 0]$$

$$p' = -s + s^3 \quad p = -s + s^5 \quad p' = -s + s^5 \quad p = -s + s^3$$

‘ See 3-level graph

‘

M \ ; **N**

$\$ [[0, 0, 0, 3, 0, 0, 3, 0, 0], [0, 0, 0, 0, 2, 0, 0, 2, 0], [0, 0, 0, 0, 0, 1, 0, 0, 1], [3, 0, 0, 0, 0, 0, 3, 0, 0], [0, 2, 0, 0, 0, 0, 2, 0], [0, 0, 1, 0, 0, 0, 0, 1], [3, 0, 0, 3, 0, 0, 0, 0], [0, 2, 0, 0, 2, 0, 0, 0], [0, 0, 1, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 16, 10, 20, 16, 20, 20, 8, 10], [16, 0, 17, 12, 20, 12, 20, 11], [10, 17, 0, 20, 15, 20, 10, 8, 20], [20, 12, 20, 0, 14, 0, 20, 14, 20], [16, 20, 15, 14, 0, 14, 10, 20, 11], [20, 12, 20, 0, 14, 0, 20, 14, 20], [20, 12, 10, 20, 10, 20, 0, 18, 10], [8, 20, 8, 14, 20, 14, 18, 0, 18], [10, 11, 20, 20, 11, 20, 10, 18, 0]] \$$

$$\tau = 27, r' = 2/3$$

R: [4, 9, 4, 7, 3, 7, 1, 6, 1]

B: [2, 4, 5, 8, 7, 8, 5, 1, 2]

Ranges

Action of R on ranges, [[1], [3], [1]]

Action of B on ranges, [[2], [1], [2]]

Cycles: R, {{1, 4, 7}}, B, {{1, 2, 4, 8}, {5, 7}}

$$\beta(\{1, 4, 7\}) = 1/2$$

$$\beta(\{2, 5, 8\}) = 1/3$$

$$\beta(\{3, 6, 9\}) = 1/6$$

Partitions

Action of R on partitions, [[2], [4], [2], [4], [2], [2], [2], [2]]

Action of B on partitions, [[3], [5], [1], [3], [6], [8], [5], [7]]

$$\alpha(\{\{1, 8, 9\}, \{2, 4, 6\}, \{3, 5, 7\}\}) = 1/12$$

$$\alpha(\{\{1, 3, 8\}, \{2, 7, 9\}, \{4, 5, 6\}\}) = 1/4$$

$$\alpha(\{\{1, 2, 9\}, \{3, 5, 7\}, \{4, 6, 8\}\}) = 1/6$$

$$\alpha(\{\{1, 3, 8\}, \{2, 4, 6\}, \{5, 7, 9\}\}) = 1/4$$

$$\alpha(\{\{1, 5, 9\}, \{2, 3, 7\}, \{4, 6, 8\}\}) = 2/15$$

$$\alpha(\{\{1, 5, 9\}, \{2, 4, 6\}, \{3, 7, 8\}\}) = 1/15$$

$$\alpha(\{\{1, 8, 9\}, \{2, 3, 7\}, \{4, 5, 6\}\}) = 1/60$$

$$\alpha(\{\{1, 2, 9\}, \{3, 7, 8\}, \{4, 5, 6\}\}) = 1/30$$

b1 = {1, 2, 9} ‘, ‘ b2 = {1, 3, 8} ‘, ‘ b3 = {1, 5, 9} ‘, ‘ b4 = {1, 8, 9} ‘, ‘ b5 = {2, 3, 7} ‘, ‘ b6 = {2, 4, 6} ‘, ‘ b7 = {2, 7, 9} ‘, ‘ b8 = {3, 5, 7} ‘, ‘ b9 = {3, 7, 8} ‘, ‘ b10 = {4, 5, 6} ‘, ‘ b11 = {4, 6, 8} ‘, ‘ b12 = {5, 7, 9}

Action of R and B on the blocks of the partitions: = [7, C, 7, 7, A, 2, 6, A, A, 2, 2, 6] [4, B, 9, B, 3, 1, 3, 8, A, 5, 6, 8]

with invariant measure [4, 10, 4, 2, 3, 8, 5, 5, 2, 6, 6, 5]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

$$[y_5, y_4, y_3, y_2, 0, 0, -y_5 + 5y_4 - y_3 - y_2 + 5y_1, 0, y_1]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{6, 8}, {5, 7}} order: 4

See Matrix

$$\$ [[1, 3, 0, 2, 4, 2, 2, 4, 0], [0, 1, 0, 3, 2, 4, 4, 4, 0], [0, 0, 0, 1, 4, 4, 2, 7, 0], [0, 0, 0, 0, 2, 7, 4, 5, 0], [0, 0, 0, 0, 4, 5, 2, 7, 0], [0, 0, 0, 0, 2, 7, 4, 5, 0], [0, 0, 0, 0, 4, 5, 2, 7, 0]] \$$$

$$[3y_1 - y_3 - 4y_4 - y_5 + 3y_2, y_1, 0, y_3, y_4, y_5, 2y_1 - 3y_4 + 2y_2, y_2, 0]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

53 . Coloring, {2, 6, 7}

R: [4, 9, 4, 7, 7, 8, 5, 1, 1]

B: [2, 4, 5, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	6 vs 8

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 0, 4, 3, 0, 5, 1, 2], [3, 0, 0, 3, 5, 0, 7, 0, 0], [0, 0, 0, 3, 7, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, 0, y_4, y_5, 2y_5]$$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}, {3, 5}} order: 6

See Matrix

$$\$ [[3, 4, 2, 2, 1, 2, 1, 3, 0], [1, 3, 1, 4, 2, 3, 2, 2, 0], [2, 1, 2, 3, 1, 2, 3, 4, 0], [3, 2, 1, 1, 2, 4, 2, 3, 0], [2, 3, 2, 2, 1, 3, 4, 1, 0], [4, 2, 1, 3, 2, 1, 3, 2, 0], [3, 4, 2, 2, 1, 2, 1, 3, 0], [1, 3, 1, 4, 2, 3, 2, 2, 0]] \$$$

$$[2y_1 - y_3 + 3y_2 - y_6, 3y_1 + 2y_2 - y_5 - y_4, y_1, y_3, y_2, y_6, y_5, y_4, 0]$$

$$p = -s + s^7 \quad p' = -s + s^7$$

54 . Coloring, {2, 6, 8}

R: [4, 9, 4, 7, 7, 8, 1, 6, 1]

B: [2, 4, 5, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}} order: 6

See Matrix

$$\$ [[4, 0, 0, 4, 0, 2, 5, 1, 2], [7, 0, 0, 4, 0, 1, 4, 2, 0], [4, 0, 0, 7, 0, 2, 4, 1, 0], [4, 0, 0, 4, 0, 1, 7, 2, 0], [7, 0, 0, 4, 0, 2, 4, 1, 0], [4, 0, 0, 7, 0, 1, 4, 2, 0]] \$$$

$$[-y_3 + 5y_1 - y_2 + 5y_4 - y_5, 0, 0, y_3, 0, y_1, y_2, y_4, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}} order: 4

See Matrix

$$\$ [[2, 4, 2, 2, 4, 0, 1, 3, 0], [3, 2, 4, 4, 3, 0, 0, 2, 0], [2, 3, 3, 2, 4, 0, 0, 4, 0], [4, 2, 4, 3, 3, 0, 0, 2, 0], [2, 4, 3, 2, 4, 0, 0, 3, 0], [3, 2, 4, 4, 3, 0, 0, 2, 0], [2, 3, 3, 2, 4, 0, 0, 4, 0]] \$$$

$$[-16y_1 - 5y_3 + 33y_2 - 16y_5, 5y_1, -7y_1 + 16y_2 - 5y_4 - 7y_5, 5y_3, 5y_2, 0, 5y_4, 5y_5, 0]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

55 . Coloring, {2, 6, 9}

R: [4, 9, 4, 7, 7, 8, 1, 1, 2]

B: [2, 4, 5, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	7 vs 8

Omega Rank for R : cycles: {{1, 4, 7}, {2, 9}} order: 6

See Matrix

\$ [[5, 1, 0, 4, 0, 0, 5, 1, 2], [6, 2, 0, 5, 0, 0, 4, 0, 1], [4, 1, 0, 6, 0, 0, 5, 0, 2], [5, 2, 0, 4, 0, 0, 6, 0, 1], [6, 1, 0, 5, 0, 0, 4, 0, 2], [4, 2, 0, 6, 0, 0, 5, 0, 1]] \$

$$[5y_1 - y_5 - y_4 - y_2 + 5y_3, y_1, 0, y_5, 0, 0, y_4, y_2, y_3]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: {{3, 5}} order: 8

See Matrix

\$ [[1, 3, 2, 2, 4, 2, 1, 3, 0], [0, 1, 4, 3, 3, 3, 2, 2, 0], [0, 0, 3, 1, 6, 2, 3, 3, 0], [0, 0, 6, 0, 6, 3, 2, 1, 0], [0, 0, 6, 0, 8, 1, 3, 0, 0], [0, 0, 8, 0, 9, 0, 1, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$

$$[y_1, y_1 - y_3 + y_4 + y_2 + y_5 - y_6 - y_7, y_3, y_4, y_2, y_5, y_6, y_7, 0]$$

$$p = -s^7 + s^8$$

56 . Coloring, {2, 7, 8}

R: [4, 9, 4, 7, 7, 7, 5, 6, 1]

B: [2, 4, 5, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[1, 0, 0, 4, 3, 2, 6, 0, 2], [2, 0, 0, 1, 6, 0, 9, 0, 0], [0, 0, 0, 2, 9, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$$

$$[y_1, 0, 0, y_2, y_3, y_5, y_4, 0, y_5]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{3, 5\}\}$ order: 4

See Matrix

$\$ [[5, 4, 2, 2, 1, 0, 0, 4, 0], [4, 5, 1, 4, 2, 0, 0, 2, 0], [2, 4, 2, 5, 1, 0, 0, 4, 0], [4, 2, 1, 4, 2, 0, 0, 5, 0], [5, 4, 2, 2, 1, 0, 0, 4, 0], [4, 5, 1, 4, 2, 0, 0, 2, 0]] \$$

$$[2y_1 - y_2 + 3y_3, 3y_1 + 2y_3 - y_4, y_1, y_2, y_3, 0, 0, y_4, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

57 . Coloring, $\{2, 7, 9\}$

R: $[4, 9, 4, 7, 7, 7, 5, 1, 2]$

B: $[2, 4, 5, 8, 3, 8, 1, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	4 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 4, 3, 0, 6, 0, 2], [0, 2, 0, 2, 6, 0, 7, 0, 1], [0, 1, 0, 0, 7, 0, 8, 0, 2], [0, 2, 0, 0, 8, 0, 7, 0, 1], [0, 1, 0, 0, 7, 0, 8, 0, 2], [0, 2, 0, 0, 8, 0, 7, 0, 1]] \$$

$$[2y_1 - y_3 + 3y_2, y_1, 0, y_4, 3y_1 - y_4 + 2y_2, 0, y_3, 0, y_2]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{3, 5}, {6, 8}} order: 4

See Matrix

$$\$ [[4, 3, 2, 2, 1, 2, 0, 4, 0], [0, 4, 1, 3, 2, 4, 0, 4, 0], [0, 0, 2, 4, 1, 4, 0, 7, 0], [0, 0, 1, 0, 2, 7, 0, 8, 0], [0, 0, 2, 0, 1, 8, 0, 7, 0], [0, 0, 1, 0, 2, 7, 0, 8, 0], [0, 0, 2, 0, 1, 8, 0, 7, 0]] \$$$

$$[y_2, 2y_1 + 3y_5 - y_4, y_1, -y_2 + 3y_1 + 2y_5 - y_3, y_5, y_3, 0, y_4, 0]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

58 . Coloring, {2, 8, 9}

R: [4, 9, 4, 7, 7, 7, 1, 6, 2]

B: [2, 4, 5, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 6

Omega Rank for R : cycles: {{2, 9}, {1, 4, 7}} order: 6

See Matrix

$$\$ [[3, 1, 0, 4, 0, 2, 6, 0, 2], [6, 2, 0, 3, 0, 0, 6, 0, 1], [6, 1, 0, 6, 0, 0, 3, 0, 2], [3, 2, 0, 6, 0, 0, 6, 0, 1], [6, 1, 0, 3, 0, 0, 6, 0, 2], [6, 2, 0, 6, 0, 0, 3, 0, 1]] \$$$

$$[5y_4 - y_3 - y_2 - y_1 + 5y_5, y_4, 0, y_3, 0, y_2, y_1, 0, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}} order: 4

See Matrix

$$\$ [[3, 3, 2, 2, 4, 0, 0, 4, 0], [4, 3, 4, 3, 2, 0, 0, 2, 0], [2, 4, 2, 3, 4, 0, 0, 3, 0], [3, 2, 4, 4, 2, 0, 0, 3, 0], [3, 3, 2, 2, 4, 0, 0, 4, 0], [4, 3, 4, 3, 2, 0, 0, 2, 0]] \$$$

$$[y_3, 3y_3 - 4y_1 + 3y_2 - y_4, y_1, y_2, 2y_3 - 3y_1 + 2y_2, 0, 0, y_4, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

59 . Coloring, {3, 4, 5}

R: [4, 4, 5, 8, 3, 7, 1, 1, 1]

B: [2, 9, 4, 7, 7, 8, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	3 vs 7

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}} order: 6

See Matrix

\$ [[6, 0, 2, 5, 1, 0, 1, 3, 0], [4, 0, 1, 6, 2, 0, 0, 5, 0], [5, 0, 2, 4, 1, 0, 0, 6, 0], [6, 0, 1, 5, 2, 0, 0, 4, 0], [4, 0, 2, 6, 1, 0, 0, 5, 0], [5, 0, 1, 4, 2, 0, 0, 6, 0]] \$

$$[5y_1 - y_5 + 5y_4 - y_3 - y_2, 0, y_1, y_5, y_4, 0, y_3, y_2, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{2, 9}, {5, 7}, {6, 8}} order: 2

See Matrix

\$ [[0, 4, 0, 1, 3, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4], [0, 4, 0, 0, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4], [0, 4, 0, 0, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 4, 2, 4], [0, 4, 0, 0, 4, 2, 5, 1, 2]] \$

$$[0, 2y_1, 0, -y_3 + y_1 + 2y_2, y_3, y_1, 2y_1 + y_2, y_2, 2y_2]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

60 . Coloring, {3, 4, 6}

R: [4, 4, 5, 8, 7, 8, 1, 1, 1]

B: [2, 9, 4, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$\$ [[6, 0, 0, 5, 1, 0, 2, 4, 0], [6, 0, 0, 6, 0, 0, 1, 5, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$

$$[y_2 + y_3, 0, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$$p = -s^3 + s^4 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{3, 4, 5, 7\}\}$ order: 4

See Matrix

$\$ [[0, 4, 2, 1, 3, 2, 4, 0, 2], [0, 2, 3, 2, 4, 0, 3, 0, 4], [0, 4, 4, 3, 3, 0, 2, 0, 2], [0, 2, 3, 4, 2, 0, 3, 0, 4], [0, 4, 2, 3, 3, 0, 4, 0, 2], [0, 2, 3, 2, 4, 0, 3, 0, 4], [0, 4, 4, 3, 3, 0, 2, 0, 2]] \$$

$$[0, y_5 + y_3 + y_2 - y_1, y_4, y_5, y_3, y_2, -y_4 + y_5 + y_3 + y_2, 0, y_1]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

61 . Coloring, $\{3, 4, 7\}$

R: [4, 4, 5, 8, 7, 7, 5, 1, 1]

B: [2, 9, 4, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	6 vs 8

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}\}$ order: 6
 See Matrix

$\$ [[3, 0, 0, 5, 4, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 4, 5, 0], [5, 0, 0, 3, 4, 0, 3, 3, 0], [3, 0, 0, 5, 3, 0, 4, 3, 0], [3, 0, 0, 3, 4, 0, 3, 5, 0]] \$$

$$[-7 y_1 + 11 y_2 + 11 y_3 - 7 y_4, 0, 0, 7 y_1, 7 y_2, 0, 7 y_3, 7 y_4, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 6
 See Matrix

$\$ [[3, 4, 2, 1, 0, 2, 3, 1, 2], [3, 5, 0, 2, 0, 1, 1, 2, 4], [1, 7, 0, 0, 0, 2, 2, 1, 5], [2, 6, 0, 0, 0, 1, 0, 2, 7], [0, 9, 0, 0, 0, 2, 0, 1, 6], [0, 6, 0, 0, 0, 1, 0, 2, 9], [0, 9, 0, 0, 0, 2, 0, 1, 6], [0, 6, 0, 0, 0, 1, 0, 2, 9]] \$$

$$[-y_2 + y_3 + 4 y_5 - y_6, -y_1 + 4 y_3 - y_4 + y_5, y_1, y_2, 0, y_3, y_4, y_5, y_6]$$

$$p = -s^5 + s^7 \quad p' = -s^5 + s^7$$

62 . Coloring, $\{3, 4, 8\}$

R: $[4, 4, 5, 8, 7, 7, 1, 6, 1]$

B: $[2, 9, 4, 7, 3, 8, 5, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	6 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 6, 7, 8\}\}$ order: 5
 See Matrix

$\$ [[4, 0, 0, 5, 1, 2, 3, 3, 0], [3, 0, 0, 4, 0, 3, 3, 5, 0], [3, 0, 0, 3, 0, 5, 3, 4, 0], [3, 0, 0, 3, 0, 4, 5, 3, 0], [5, 0, 0, 3, 0, 3, 4, 3, 0], [4, 0, 0, 5, 0, 3, 3, 3, 0]] \$$

$$[y_1, 0, 0, y_5, y_6, y_3, y_4, y_2, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{3, 4, 5, 7\}\}$ order: 4
 See Matrix

$\$ [[2, 4, 2, 1, 3, 0, 3, 1, 2], [1, 4, 3, 2, 3, 0, 1, 0, 4], [0, 5, 3, 3, 1, 0, 2, 0, 4], [0, 4, 1, 3, 2, 0, 3, 0, 5], [0, 5, 2, 1, 3, 0, 3, 0, 4], [0, 4, 3, 2, 3, 0, 1, 0, 5], [0, 5, 3, 3, 1, 0, 2, 0, 4], [0, 4, 1, 3, 2, 0, 3, 0, 5]] \$$

$$[y_6, y_5, y_4, y_3, y_2, 0, y_1, -y_5 + y_4 + y_1, -y_6 + y_3 + y_2]$$

$$p' = s^3 - s^7 \quad p = s^3 - s^7$$

63 . Coloring, $\{3, 4, 9\}$

R: [4, 4, 5, 8, 7, 7, 1, 1, 2]

B: [2, 9, 4, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 9	9 vs 9	3 vs 6	6 vs 9

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
 See Matrix

$\$ [[5, 1, 0, 5, 1, 0, 3, 3, 0], [6, 0, 0, 6, 0, 0, 1, 5, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$

$$[-y_1 + y_2 + y_3, y_1, 0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$$p' = -s^3 + s^5 \quad p = s^3 - s^4 \quad p' = -s^3 + s^4$$

Omega Rank for B : cycles: $\{\{3, 4, 5, 7\}, \{1, 2, 9\}, \{6, 8\}\}$
 See Matrix

$\$ [[1, 3, 2, 1, 3, 2, 3, 1, 2], [2, 1, 3, 2, 3, 1, 1, 2, 3], [3, 2, 3, 3, 1, 2, 2, 1, 1], [1, 3, 1, 3, 2, 1, 3, 2, 2], [2, 1, 2, 1, 3, 2, 3, 1, 3], [3, 2, 3, 2, 3, 1, 1, 2, 1], [1, 3, 3, 3, 1, 2, 2, 1, 2], [2, 1, 1, 3, 2, 1, 3, 2, 3], [3, 2, 2, 1, 3, 2, 3, 1, 1]] \$$

$$[-y_1 + 2y_3 + 2y_5 - y_6, y_1, 2y_3 + y_5 - y_4, -y_2 + y_3 + 2y_5, y_2, y_3, y_4, y_5, y_6]$$

$$p' = -1 - s - s^2 + s^4 + s^5 + s^6 \quad p' = 1 - s^3 - s^4 + s^7 \quad p' = s - s^4 - s^5 + s^8$$

64 . Coloring, {3, 5, 6}

R: [4, 4, 5, 7, 3, 8, 1, 1, 1]

B: [2, 9, 4, 8, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}} order: 6

See Matrix

$$\$ [[6, 0, 2, 5, 1, 0, 3, 1, 0], [4, 0, 1, 6, 2, 0, 5, 0, 0], [5, 0, 2, 4, 1, 0, 6, 0, 0], [6, 0, 1, 5, 2, 0, 4, 0, 0], [4, 0, 2, 6, 1, 0, 5, 0, 0], [5, 0, 1, 4, 2, 0, 6, 0, 0]] \$$$

$$[5y_1 - y_2 + 5y_3 - y_4 - y_5, 0, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{2, 9}, {5, 7}} order: 4

See Matrix

$$\$ [[0, 4, 0, 1, 3, 2, 3, 3, 2], [0, 2, 0, 0, 3, 3, 5, 1, 4], [0, 4, 0, 0, 5, 1, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2]] \$$$

$$[0, y_3, 0, y_3 - y_1 - y_2 + y_5, y_1, y_2, y_3 - y_4 + y_5, y_4, y_5]$$

$$p' = -s^4 + s^6 \quad p = -s^4 + s^6$$

65 . Coloring, {3, 5, 7}

$$\Omega p(\Delta)=0: \quad p = s^2 - 2s^4 + 8s^5 - 16s^7$$

R: [4, 4, 5, 7, 3, 7, 5, 1, 1]

B: [2, 9, 4, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	4 vs 7

Omega Rank for R : cycles: {{3, 5}} order: 4

See Matrix

\$ [[3, 0, 2, 5, 4, 0, 4, 0, 0], [0, 0, 4, 3, 6, 0, 5, 0, 0], [0, 0, 6, 0, 9, 0, 3, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$

$$[-y_1 + y_2 + y_3 - y_4, 0, y_1, y_2, y_3, 0, y_4, 0, 0]$$

$$p = s^4 - s^5$$

Omega Rank for B : cycles: {{6, 8}, {2, 9}} order: 4

See Matrix

\$ [[3, 4, 0, 1, 0, 2, 2, 4, 2], [2, 5, 0, 0, 0, 4, 0, 3, 4], [0, 6, 0, 0, 0, 3, 0, 4, 5], [0, 5, 0, 0, 0, 4, 0, 3, 6], [0, 6, 0, 0, 0, 3, 0, 4, 5], [0, 5, 0, 0, 0, 4, 0, 3, 6], [0, 6, 0, 0, 0, 3, 0, 4, 5]] \$

$$[2y_1, 9y_1 - 15y_3 - 11y_2 + 9y_4, 0, 2y_3, 0, 2y_2, 4y_3, 7y_1 - 9y_3 - 9y_2 + 7y_4, 2y_4]$$

$$p = s^3 - s^5 \quad p' = -s^4 + s^6 \quad p'' = s^3 - s^5$$

66 . Coloring, {3, 5, 8}

R: [4, 4, 5, 7, 3, 7, 1, 6, 1]

B: [2, 9, 4, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[4, 0, 2, 5, 1, 2, 4, 0, 0], [4, 0, 1, 4, 2, 0, 7, 0, 0], [7, 0, 2, 4, 1, 0, 4, 0, 0], [4, 0, 1, 7, 2, 0, 4, 0, 0], [4, 0, 2, 4, 1, 0, 7, 0, 0], [7, 0, 1, 4, 2, 0, 4, 0, 0]] \$$

$$[5y_2 - y_3 + 5y_1 - y_4 - y_5, 0, y_2, y_3, y_1, y_4, y_5, 0, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 4

See Matrix

$\$ [[2, 4, 0, 1, 3, 0, 2, 4, 2], [4, 4, 0, 0, 2, 0, 3, 1, 4], [1, 8, 0, 0, 3, 0, 2, 0, 4], [0, 5, 0, 0, 2, 0, 3, 0, 8], [0, 8, 0, 0, 3, 0, 2, 0, 5], [0, 5, 0, 0, 2, 0, 3, 0, 8], [0, 8, 0, 0, 3, 0, 2, 0, 5]] \$$

$$[-14y_4 - y_2 + 39y_3 - 14y_1 - y_5, y_4, 0, y_2, y_3, 0, -5y_4 + 14y_3 - 5y_1, y_1, y_5]$$

$$p = -s^4 + s^6 \quad p' = s^4 - s^6$$

67 . Coloring, $\{3, 5, 9\}$

R: [4, 4, 5, 7, 3, 7, 1, 1, 2]

B: [2, 9, 4, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 5, 1, 0, 4, 0, 0], [4, 0, 1, 6, 2, 0, 5, 0, 0], [5, 0, 2, 4, 1, 0, 6, 0, 0], [6, 0, 1, 5, 2, 0, 4, 0, 0], [4, 0, 2, 6, 1, 0, 5, 0, 0], [5, 0, 1, 4, 2, 0, 6, 0, 0]] \$$

$$[y_4, y_5, y_3, y_1, y_2, 0, -y_4 - y_5 + 5y_3 - y_1 + 5y_2, 0, 0]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: {{6, 8}, {5, 7}, {1, 2, 9}} order: 6

See Matrix

$$\$ [[1, 3, 0, 1, 3, 2, 2, 4, 2], [2, 1, 0, 0, 2, 4, 3, 3, 3], [3, 2, 0, 0, 3, 3, 2, 4, 1], [1, 3, 0, 0, 2, 4, 3, 3, 2], [2, 1, 0, 0, 3, 3, 2, 4, 3], [3, 2, 0, 0, 2, 4, 3, 3, 1], [1, 3, 0, 0, 3, 3, 2, 4, 2], [2, 1, 0, 0, 2, 4, 3, 3, 3]] \$$$

$$[-y_1 + 6y_2 + 6y_3 - 6y_4 - y_5, y_1, 0, y_2, 5y_2 + 5y_3 - 6y_4, y_3, y_4, 6y_2 + 6y_3 - 7y_4, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = -s^2 + s^8 \quad p = s^2 - s^4 - s^5 + s^7$$

68 . Coloring, {3, 6, 7}

R: [4, 4, 5, 7, 7, 8, 5, 1, 1]

B: [2, 9, 4, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 5	8 vs 8

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[3, 0, 0, 5, 4, 0, 5, 1, 0], [1, 0, 0, 3, 5, 0, 9, 0, 0], [0, 0, 0, 1, 9, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, 0, y_4, y_5, 0]$$

Omega Rank for B : cycles: {{2, 9}} order: 8

See Matrix

$$\$ [[3, 4, 2, 1, 0, 2, 1, 3, 2], [1, 5, 0, 2, 0, 3, 2, 1, 4], [2, 5, 0, 0, 0, 1, 3, 2, 5], [3, 7, 0, 0, 0, 2, 1, 0, 5], [1, 8, 0, 0, 0, 0, 2, 0, 7], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7, y_8]$$

69 . Coloring, {3, 6, 8}

R: [4, 4, 5, 7, 7, 8, 1, 6, 1]

B: [2, 9, 4, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	8 vs 8

Omega Rank for R : cycles: {{1, 4, 7}, {6, 8}} order: 6

See Matrix

$$\$ [[4, 0, 0, 5, 1, 2, 5, 1, 0], [5, 0, 0, 4, 0, 1, 6, 2, 0], [6, 0, 0, 5, 0, 2, 4, 1, 0], [4, 0, 0, 6, 0, 1, 5, 2, 0], [5, 0, 0, 4, 0, 2, 6, 1, 0], [6, 0, 0, 5, 0, 1, 4, 2, 0]] \$$$

$$[-y_4 - y_2 + 5y_3 - y_1 + 5y_5, 0, 0, y_4, y_2, y_3, y_1, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{2, 9}} order: 8

See Matrix

$$\$ [[2, 4, 2, 1, 3, 0, 1, 3, 2], [3, 4, 3, 2, 1, 0, 0, 1, 4], [1, 7, 1, 3, 0, 0, 0, 2, 4], [2, 5, 0, 1, 0, 0, 0, 3, 7], [3, 9, 0, 0, 0, 0, 0, 1, 5], [1, 8, 0, 0, 0, 0, 0, 0, 9], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_2, y_1, y_5, y_4, y_3, 0, y_6, y_7, y_8]$$

70 . Coloring, {3, 6, 9}

$$\Omega p(\Delta)=0: p = s - 5s^5 - 12s^7 \quad p' = s + s^4 - 4s^6 \quad p'' = s^2 + 3s^4 + 4s^6 \quad p''' = -s^4 + s^5 - 2s^6$$

R: [4, 4, 5, 7, 7, 8, 1, 1, 2]

B: [2, 9, 4, 8, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 7	2 vs 9	3 vs 9	2 vs 6	3 vs 9

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[5, 1, 0, 5, 1, 0, 5, 1, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0], [6, 0, 0, 6, 0, 0, 6, 0, 0]] \$$

$[y_1, y_2, 0, y_1, y_2, 0, y_1, y_2, 0]$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 4, 5, 6, 7, 8\}\}$ order: 6

See Matrix

$\$ [[1, 3, 2, 1, 3, 2, 1, 3, 2], [2, 1, 3, 2, 1, 3, 2, 1, 3], [3, 2, 1, 3, 2, 1, 3, 2, 1], [1, 3, 2, 1, 3, 2, 1, 3, 2], [2, 1, 3, 2, 1, 3, 2, 1, 3], [3, 2, 1, 3, 2, 1, 3, 2, 1], [1, 3, 2, 1, 3, 2, 1, 3, 2], [2, 1, 3, 2, 1, 3, 2, 1, 3], [3, 2, 1, 3, 2, 1, 3, 2, 1]] \$$

$[y_1, y_2, y_3, y_1, y_2, y_3, y_1, y_2, y_3]$

$$p' = -s^2 + s^5 \quad p' = -1 + s^6 \quad p' = -s^2 + s^8 \quad p' = -s + s^7 \quad p' = -1 + s^3 \quad p' = -s + s^4$$

‘ See 3-level graph

‘

M \; N

$\$ [[0, 0, 0, 3, 0, 0, 3, 0, 0], [0, 0, 0, 0, 2, 0, 0, 2, 0], [0, 0, 0, 0, 0, 1, 0, 0, 1], [3, 0, 0, 0, 0, 0, 3, 0, 0], [0, 2, 0, 0, 0, 0, 2, 0], [0, 0, 1, 0, 0, 0, 0, 0, 1], [3, 0, 0, 3, 0, 0, 0, 0, 0], [0, 2, 0, 0, 2, 0, 0, 0, 0], [0, 0, 1, 0, 0, 1, 0, 0, 0]] \$$
 $\$ [[0, 186, 280, 535, 414, 480, 535, 470, 310], [186, 0, 293, 434, 535, 405, 450, 535, 372], [280, 293, 0, 520, 525, 535, 270, 252, 535], [535, 434, 520, 0, 126, 90, 535, 510, 460], [414, 535, 525, 126, 0, 180, 530, 535, 365], [480, 405, 535, 90, 180, 0, 500, 485, 535], [535, 450, 270, 535, 530, 500, 0, 90, 300], [470, 535, 252, 510, 535, 485, 90, 0, 333], [310, 372, 535, 460, 365, 535, 300, 333, 0]] \$$

71 . Coloring, {3, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s^3 + s^4 - 8s^7$$

R: [4, 4, 5, 7, 7, 7, 5, 6, 1]

B: [2, 9, 4, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	6 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[1, 0, 0, 5, 4, 2, 6, 0, 0], [0, 0, 0, 1, 6, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, 2y_1, y_4, 0, 0]$$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

$$\$ [[5, 4, 2, 1, 0, 0, 0, 4, 2], [4, 7, 0, 2, 0, 0, 0, 1, 4], [1, 8, 0, 0, 0, 0, 0, 2, 7], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5, y_6]$$

72 . Coloring, {3, 7, 9}

R: [4, 4, 5, 7, 7, 7, 5, 1, 2]

B: [2, 9, 4, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 5	6 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[2, 1, 0, 5, 4, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$

$$[2y_3, y_3, 0, 3y_3 - y_1 + y_2, y_1, 0, y_2, 0, 0]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}} order: 6

See Matrix

\$ [[4, 3, 2, 1, 0, 2, 0, 4, 2], [2, 4, 0, 2, 0, 4, 0, 3, 3], [3, 2, 0, 0, 0, 3, 0, 6, 4], [4, 3, 0, 0, 0, 6, 0, 3, 2], [2, 4, 0, 0, 0, 3, 0, 6, 3], [3, 2, 0, 0, 0, 6, 0, 3, 4], [4, 3, 0, 0, 0, 3, 0, 6, 2]] \$

$$[y_3, y_4, y_3 + y_4 - y_1 - y_2 - y_6 + y_5, y_1, 0, y_2, 0, y_6, y_5]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

73 . Coloring, {3, 8, 9}

R: [4, 4, 5, 7, 7, 7, 1, 6, 2]

B: [2, 9, 4, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	7 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3
 See Matrix

$\$ [[3, 1, 0, 5, 1, 2, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0], [4, 0, 0, 8, 0, 0, 6, 0, 0], [6, 0, 0, 4, 0, 0, 8, 0, 0], [8, 0, 0, 6, 0, 0, 4, 0, 0]] \$$

$$[y_1, y_3, 0, y_2, y_3, 2y_3, y_4, 0, 0]$$

$$p' = s^2 - s^5 \quad p = s^2 - s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 6
 See Matrix

$\$ [[3, 3, 2, 1, 3, 0, 0, 4, 2], [6, 3, 3, 2, 0, 0, 0, 1, 3], [4, 6, 0, 3, 0, 0, 0, 2, 3], [5, 4, 0, 0, 0, 0, 0, 3, 6], [9, 5, 0, 0, 0, 0, 0, 0, 4], [4, 9, 0, 0, 0, 0, 0, 0, 5], [5, 4, 0, 0, 0, 0, 0, 0, 9]] \$$

$$[y_2, y_1, y_7, y_6, y_5, 0, 0, y_4, y_3]$$

74 . Coloring, $\{4, 5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^3 + s^4 - 8s^7$$

R: $[4, 4, 4, 8, 3, 8, 1, 1, 1]$

B: $[2, 9, 5, 7, 7, 7, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 4	3 vs 5

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
 See Matrix

$\$ [[6, 0, 2, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0]] \$$

$$[y_2, 0, y_1, y_4, 0, 0, 0, y_3, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[0, 4, 0, 0, 4, 2, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2]] \$$

$$[0, y_2 - y_3, 0, 0, y_2 - y_1, y_1, y_2, 0, y_3]$$

$$p = s^2 - s^4 \quad p' = -s^2 + s^4$$

75 . Coloring, $\{4, 5, 7\}$

R: $[4, 4, 4, 8, 3, 7, 5, 1, 1]$

B: $[2, 9, 5, 7, 7, 8, 1, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[3, 0, 2, 6, 3, 0, 1, 3, 0], [3, 0, 3, 5, 1, 0, 0, 6, 0], [6, 0, 1, 6, 0, 0, 0, 5, 0], [5, 0, 0, 7, 0, 0, 0, 6, 0], [6, 0, 0, 5, 0, 0, 0, 7, 0], [7, 0, 0, 6, 0, 0, 0, 5, 0]] \$$

$$[y_6, 0, y_5, y_3, y_4, 0, y_1, y_2, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[3, 4, 0, 0, 1, 2, 5, 1, 2], [5, 5, 0, 0, 0, 1, 1, 2, 4], [1, 9, 0, 0, 0, 2, 0, 1, 5], [0, 6, 0, 0, 0, 1, 0, 2, 9], [0, 9, 0, 0, 0, 2, 0, 1, 6], [0, 6, 0, 0, 0, 1, 0, 2, 9], [0, 9, 0, 0, 0, 2, 0, 1, 6]] \$$

$$[y_2 + 4y_4 - y_1 - y_5, 4y_2 - y_3 + y_4, 0, 0, y_1, y_2, y_3, y_4, y_5]$$

$$p = s^4 - s^6 \quad p' = -s^4 + s^6$$

76 . Coloring, {4, 5, 8}

R: [4, 4, 4, 8, 3, 7, 1, 6, 1]

B: [2, 9, 5, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 6	4 vs 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[4, 0, 2, 6, 0, 2, 1, 3, 0] , [1, 0, 0, 6, 0, 3, 2, 6, 0] , [2, 0, 0, 1, 0, 6, 3, 6, 0] , [3, 0, 0, 2, 0, 6, 6, 1, 0] , [6, 0, 0, 3, 0, 1, 6, 2, 0] , [6, 0, 0, 6, 0, 2, 1, 3, 0]] \$

$$[y_3, 0, y_2, y_1, 0, y_5, y_6, y_4, 0]$$

Omega Rank for B : cycles: {{5, 7}, {2, 9}} order: 4

See Matrix

\$ [[2, 4, 0, 0, 4, 0, 5, 1, 2] , [1, 4, 0, 0, 5, 0, 4, 0, 4] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5] , [0, 5, 0, 0, 4, 0, 5, 0, 4] , [0, 4, 0, 0, 5, 0, 4, 0, 5]] \$

$$[y_1 - y_4, y_2 - y_3, 0, 0, y_1, 0, y_2, y_3, y_4]$$

$$p' = s^3 - s^5 \quad p = s^3 - s^5$$

77 . Coloring, {4, 5, 9}

R: [4, 4, 4, 8, 3, 7, 1, 1, 2]

B: [2, 9, 5, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	4 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
See Matrix

$\$ [[5, 1, 2, 6, 0, 0, 1, 3, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0]] \$$

$$[y_1, y_2, 2y_2, y_3, 0, 0, y_2, y_4, 0]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}, \{6, 8\}\}$ order: 6
See Matrix

$\$ [[1, 3, 0, 0, 4, 2, 5, 1, 2], [2, 1, 0, 0, 5, 1, 4, 2, 3], [3, 2, 0, 0, 4, 2, 5, 1, 1], [1, 3, 0, 0, 5, 1, 4, 2, 2], [2, 1, 0, 0, 4, 2, 5, 1, 3], [3, 2, 0, 0, 5, 1, 4, 2, 1], [1, 3, 0, 0, 4, 2, 5, 1, 2]] \$$

$$[y_1, -y_1 + 2y_2 - 2y_3 - y_4, 0, 0, y_2, y_2 - 2y_3, 2y_2 - 3y_3, y_3, y_4]$$

$$p = -s + s^7 \quad p' = s + s^2 - s^4 - s^5 \quad p = -s - s^2 + s^4 + s^5$$

78 . Coloring, $\{4, 6, 7\}$

R: $[4, 4, 4, 8, 7, 8, 5, 1, 1]$

B: $[2, 9, 5, 7, 3, 7, 1, 6, 2]$

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}\}$ order: 6
See Matrix

\$ [[3, 0, 0, 6, 3, 0, 2, 4, 0] , [4, 0, 0, 3, 2, 0, 3, 6, 0] , [6, 0, 0, 4, 3, 0, 2, 3, 0] , [3, 0, 0, 6, 2, 0, 3, 4, 0] , [4, 0, 0, 3, 3, 0, 2, 6, 0]] \$

$$[-5 y_1 + 13 y_2 + 13 y_3 - 5 y_4, 0, 0, 5 y_1, 5 y_2, 0, 5 y_3, 5 y_4, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: {{2, 9}, {3, 5}} order: 4
See Matrix

\$ [[3, 4, 2, 0, 1, 2, 4, 0, 2] , [4, 5, 1, 0, 2, 0, 2, 0, 4] , [2, 8, 2, 0, 1, 0, 0, 0, 5] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7]] \$

$$[2 y_5 + 3 y_3 - y_2 - y_1, y_4, y_5, 0, y_3, y_2, -y_4 + 3 y_5 + 2 y_3, 0, y_1]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

79 . Coloring, {4, 6, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 - 2s^4 + 8s^5 - 16s^7$$

R: [4, 4, 4, 8, 7, 8, 1, 6, 1]

B: [2, 9, 5, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 5	3 vs 6

Omega Rank for R : cycles: {{6, 8}} order: 4
See Matrix

\$ [[4, 0, 0, 6, 0, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 0, 8, 0] , [0, 0, 0, 2, 0, 8, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0]] \$

$$[y_1, 0, 0, y_2, 0, y_5, y_4, y_3, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{3, 5\}\}$ order: 2

See Matrix

$\$ [[2, 4, 2, 0, 4, 0, 4, 0, 2], [0, 4, 4, 0, 6, 0, 0, 0, 4], [0, 4, 6, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 6, 0, 0, 0, 4], [0, 4, 6, 0, 4, 0, 0, 0, 4], [0, 4, 4, 0, 6, 0, 0, 0, 4]] \$$

$$[-5y_3 + 2y_1 + 2y_2, -4y_3 + 2y_1 + 2y_2, y_1, 0, y_2, 0, -10y_3 + 4y_1 + 4y_2, 0, y_3]$$

$$p = s^2 - s^4 \quad p' = -s^2 + s^4 \quad p'' = -s^3 + s^5$$

80 . Coloring, $\{4, 6, 9\}$

R: $[4, 4, 4, 8, 7, 8, 1, 1, 2]$

B: $[2, 9, 5, 7, 3, 7, 5, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	2 vs 5	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$\$ [[5, 1, 0, 6, 0, 0, 2, 4, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$

$$[y_1, y_1 - y_2, 0, 2y_1 - y_2, 0, 0, 2y_1 - 2y_2, y_2, 0]$$

$$p = s^2 - s^3 \quad p' = -s^2 + s^4 \quad p'' = -s^2 + s^3$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[1, 3, 2, 0, 4, 2, 4, 0, 2], [2, 1, 4, 0, 6, 0, 2, 0, 3], [3, 2, 6, 0, 6, 0, 0, 0, 1], [1, 3, 6, 0, 6, 0, 0, 0, 2], [2, 1, 6, 0, 6, 0, 0, 0, 3], [3, 2, 6, 0, 6, 0, 0, 0, 1], [1, 3, 6, 0, 6, 0, 0, 0, 2]] \$$

$$[y_5, y_4, y_3, 0, y_2, y_1, -y_3 + y_2 + y_1, 0, -y_5 - y_4 + y_2 + y_1]$$

$$p = s^3 - s^6 \quad p' = s^3 - s^6$$

81 . Coloring, {4, 7, 8}

R: [4, 4, 4, 8, 7, 7, 5, 6, 1]

B: [2, 9, 5, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	4 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

\$ [[1, 0, 0, 6, 3, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 5, 6, 0] , [0, 0, 0, 0, 5, 6, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$

$$[y_4, 0, 0, y_5, y_1, y_6, y_2, y_3, 0]$$

Omega Rank for B : cycles: {{2, 9}, {3, 5}} order: 4

See Matrix

\$ [[5, 4, 2, 0, 1, 0, 3, 1, 2] , [4, 7, 1, 0, 2, 0, 0, 0, 4] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7] , [0, 7, 1, 0, 2, 0, 0, 0, 8] , [0, 8, 2, 0, 1, 0, 0, 0, 7]] \$

$$[2y_1 + 3y_2 - y_4, 3y_1 + 2y_2 - 4y_3, y_1, 0, y_2, 0, 3y_3, y_3, y_4]$$

$$p' = -s^4 + s^6 \quad p' = s^3 - s^5 \quad p = s^3 - s^5$$

82 . Coloring, {4, 7, 9}

R: [4, 4, 4, 8, 7, 7, 5, 1, 2]

B: [2, 9, 5, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 8	8 vs 8	4 vs 6	5 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 6, 3, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 3, 6, 0], [6, 0, 0, 3, 3, 0, 3, 3, 0], [3, 0, 0, 6, 3, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 3, 6, 0], [6, 0, 0, 3, 3, 0, 3, 3, 0]] \$$

$$[-y_2 - y_1 + 4y_3 - y_4, y_2, 0, y_1, y_3, 0, y_3, y_4, 0]$$

$$p' = -s^2 + s^5 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 0, 1, 2, 3, 1, 2], [5, 4, 1, 0, 2, 1, 0, 2, 3], [3, 5, 2, 0, 1, 2, 0, 1, 4], [4, 3, 1, 0, 2, 1, 0, 2, 5], [5, 4, 2, 0, 1, 2, 0, 1, 3], [3, 5, 1, 0, 2, 1, 0, 2, 4], [4, 3, 2, 0, 1, 2, 0, 1, 5], [5, 4, 1, 0, 2, 1, 0, 2, 3]] \$$

$$[-y_1 + 4y_5 - y_4 + 4y_3 - y_2, y_1, y_5, 0, y_3, y_5, y_4, y_3, y_2]$$

$$p = -s^2 + s^8 \quad p = -s^2 - s^3 + s^5 + s^6 \quad p' = s^2 + s^3 - s^5 - s^6$$

83 . Coloring, $\{4, 8, 9\}$

R: $[4, 4, 4, 8, 7, 7, 1, 6, 2]$

B: $[2, 9, 5, 7, 3, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	3 vs 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[3, 1, 0, 6, 0, 2, 3, 3, 0] , [3, 0, 0, 4, 0, 3, 2, 6, 0] , [2, 0, 0, 3, 0, 6, 3, 4, 0] , [3, 0, 0, 2, 0, 4, 6, 3, 0] , [6, 0, 0, 3, 0, 3, 4, 2, 0] , [4, 0, 0, 6, 0, 2, 3, 3, 0]] \$

$$[y_1, y_2, 0, y_3, 0, y_4, y_5, y_6, 0]$$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}} order: 6

See Matrix

\$ [[3, 3, 2, 0, 4, 0, 3, 1, 2] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 5, 0, 0, 0, 3] , [3, 3, 5, 0, 4, 0, 0, 0, 3]] \$

$$[y_2 + y_3, y_2 + y_3, 3y_3 - y_1, 0, y_1, 0, 3y_2, y_2, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

84 . Coloring, {5, 6, 7}

R: [4, 4, 4, 7, 3, 8, 5, 1, 1]

B: [2, 9, 5, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	6 vs 7

Omega Rank for R : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

\$ [[3, 0, 2, 6, 3, 0, 3, 1, 0] , [1, 0, 3, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0]] \$

$$[y_2, 0, y_3, y_1, y_6, 0, y_4, y_5, 0]$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

\$ [[3, 4, 0, 0, 1, 2, 3, 3, 2], [3, 5, 0, 0, 0, 3, 3, 0, 4], [3, 7, 0, 0, 0, 0, 3, 0, 5], [3, 8, 0, 0, 0, 0, 0, 0, 7], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$

$$[y_2, y_1, 0, 0, y_4, y_3, y_5, 3y_4, y_6]$$

$$p = s^5 - s^7$$

85 . Coloring, {5, 6, 8}

R: [4, 4, 4, 7, 3, 8, 1, 6, 1]

B: [2, 9, 5, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}, {6, 8}} order: 6

See Matrix

\$ [[4, 0, 2, 6, 0, 2, 3, 1, 0], [3, 0, 0, 6, 0, 1, 6, 2, 0], [6, 0, 0, 3, 0, 2, 6, 1, 0], [6, 0, 0, 6, 0, 1, 3, 2, 0], [3, 0, 0, 6, 0, 2, 6, 1, 0], [6, 0, 0, 3, 0, 1, 6, 2, 0]] \$

$$[y_1, 0, y_3, -y_1 - y_3 + 5y_2 - y_5 + 5y_4, 0, y_2, y_5, y_4, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{5, 7}, {2, 9}} order: 4

See Matrix

\$ [[2, 4, 0, 0, 4, 0, 3, 3, 2], [3, 4, 0, 0, 3, 0, 4, 0, 4], [0, 7, 0, 0, 4, 0, 3, 0, 4], [0, 4, 0, 0, 3, 0, 4, 0, 7], [0, 7, 0, 0, 4, 0, 3, 0, 4], [0, 4, 0, 0, 3, 0, 4, 0, 7]] \$

$$[5y_1, -16y_1 + 33y_3 - 5y_4 - 16y_2, 0, 0, -7y_1 + 16y_3 - 7y_2, 0, 5y_3, 5y_4, 5y_2]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

86 . Coloring, {5, 6, 9}

R: [4, 4, 4, 7, 3, 8, 1, 1, 2]

B: [2, 9, 5, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	5 vs 7

Omega Rank for R : cycles: {{1, 4, 7}} order: 3

See Matrix

\$ [[5, 1, 2, 6, 0, 0, 3, 1, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0] , [8, 0, 0, 6, 0, 0, 4, 0, 0] , [4, 0, 0, 8, 0, 0, 6, 0, 0] , [6, 0, 0, 4, 0, 0, 8, 0, 0]] \$

$$[y_1, y_3, 2y_3, y_4, 0, 0, y_2, y_3, 0]$$

$$p' = -s^2 + s^5 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}} order: 6

See Matrix

\$ [[1, 3, 0, 0, 4, 2, 3, 3, 2] , [2, 1, 0, 0, 3, 3, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2] , [2, 1, 0, 0, 6, 0, 6, 0, 3] , [3, 2, 0, 0, 6, 0, 6, 0, 1] , [1, 3, 0, 0, 6, 0, 6, 0, 2]] \$

$$[y_1, -y_1 + y_5 + y_4 - y_3, 0, 0, -y_2 + y_5 + y_4, y_2, y_5, y_4, y_3]$$

$$p = -s^3 + s^6 \quad p' = -s^3 + s^6$$

87 . Coloring, {5, 7, 8}

R: [4, 4, 4, 7, 3, 7, 5, 6, 1]

B: [2, 9, 5, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}\}$ order: 4
See Matrix

$\$ [[1, 0, 2, 6, 3, 2, 4, 0, 0], [0, 0, 3, 3, 4, 0, 8, 0, 0], [0, 0, 4, 3, 8, 0, 3, 0, 0], [0, 0, 8, 4, 3, 0, 3, 0, 0], [0, 0, 3, 8, 3, 0, 4, 0, 0], [0, 0, 3, 3, 4, 0, 8, 0, 0]] \$$

$[y_4, 0, y_3, y_1, y_2, 2y_4, y_5, 0, 0]$

$$p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 4
See Matrix

$\$ [[5, 4, 0, 0, 1, 0, 2, 4, 2], [6, 7, 0, 0, 0, 0, 1, 0, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$

$[y_1, y_2, 0, 0, y_3, 0, y_4, 4y_3, y_5]$

$$p = -s^4 + s^6$$

88 . Coloring, $\{5, 7, 9\}$

R: [4, 4, 4, 7, 3, 7, 5, 1, 2]

B: [2, 9, 5, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	4 vs 6	4 vs 7

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 2, 6, 3, 0, 4, 0, 0], [0, 0, 3, 5, 4, 0, 6, 0, 0], [0, 0, 4, 3, 6, 0, 5, 0, 0], [0, 0, 6, 4, 5, 0, 3, 0, 0], [0, 0, 5, 6, 3, 0, 4, 0, 0], [0, 0, 3, 5, 4, 0, 6, 0, 0]] \$$

$$[2y_1, y_1, y_2, 3y_1 + y_2 - y_3 + y_4, y_3, 0, y_4, 0, 0]$$

$$p' = -s^2 + s^3 - s^4 + s^5 \quad p = s^2 - s^6$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[4, 3, 0, 0, 1, 2, 2, 4, 2], [4, 4, 0, 0, 0, 4, 1, 2, 3], [4, 4, 0, 0, 0, 2, 0, 4, 4], [4, 4, 0, 0, 0, 4, 0, 2, 4], [4, 4, 0, 0, 0, 2, 0, 4, 4], [4, 4, 0, 0, 0, 4, 0, 2, 4], [4, 4, 0, 0, 0, 2, 0, 4, 4]] \$$

$$[2y_3 + 2y_2, 2y_3 + 2y_2 - 2y_1, 0, 0, 2y_1, 2y_4, 2y_3, 3y_3 + 3y_2 - 2y_4, 2y_2]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^7$$

89 . Coloring, $\{5, 8, 9\}$

R: $[4, 4, 4, 7, 3, 7, 1, 6, 2]$

B: $[2, 9, 5, 8, 7, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}\}$ order: 3

See Matrix

$\$ [[3, 1, 2, 6, 0, 2, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0], [6, 0, 0, 8, 0, 0, 4, 0, 0], [4, 0, 0, 6, 0, 0, 8, 0, 0], [8, 0, 0, 4, 0, 0, 6, 0, 0]] \$$

$$[y_1, y_2, 2y_2, y_4, 0, 2y_2, y_3, 0, 0]$$

$$p' = -s^2 + s^5 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}\}$ order: 6
See Matrix

$$\$ [[3, 3, 0, 0, 4, 0, 2, 4, 2], [6, 3, 0, 0, 2, 0, 4, 0, 3], [3, 6, 0, 0, 4, 0, 2, 0, 3], [3, 3, 0, 0, 2, 0, 4, 0, 6], [6, 3, 0, 0, 4, 0, 2, 0, 3], [3, 6, 0, 0, 2, 0, 4, 0, 3]] \$$$

$$[-y_1 + 2y_2 + 2y_3 - y_4 - y_5, y_1, 0, 0, y_2, 0, y_3, y_4, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

90 . Coloring, $\{6, 7, 8\}$

R: [4, 4, 4, 7, 7, 8, 5, 6, 1]

B: [2, 9, 5, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	4 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4
See Matrix

$$\$ [[1, 0, 0, 6, 3, 2, 5, 1, 0], [0, 0, 0, 1, 5, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0]] \$$$

$$[-15y_2 - y_1 + 4y_4 + 4y_3, 0, 0, y_4, y_3, y_2, y_1, y_4 + y_3 - 4y_2, 0]$$

$$p' = -s^3 + s^5 \quad p = s^3 - s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}, \{3, 5\}\}$ order: 4
See Matrix

$$\$ [[5, 4, 2, 0, 1, 0, 1, 3, 2], [4, 7, 1, 0, 2, 0, 0, 0, 4], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 1, 0, 0, 0, 7]] \$$$

$$[2 y_1 + 3 y_2 - y_3, 3 y_1 + 2 y_2 - 4 y_4, y_1, 0, y_2, 0, y_4, 3 y_4, y_3]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^7$$

91 . Coloring, {6, 7, 9}

R: [4, 4, 4, 7, 7, 8, 5, 1, 2]

B: [2, 9, 5, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 8	8 vs 8	5 vs 6	7 vs 8

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[2, 1, 0, 6, 3, 0, 5, 1, 0], [1, 0, 0, 3, 5, 0, 9, 0, 0], [0, 0, 0, 1, 9, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$$

$$[y_1, y_4, 0, y_3, y_2, 0, y_5, y_4, 0]$$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 9}} order: 6

See Matrix

$$\$ [[4, 3, 2, 0, 1, 2, 1, 3, 2], [3, 4, 1, 0, 2, 3, 2, 0, 3], [5, 3, 2, 0, 1, 0, 3, 0, 4], [7, 5, 1, 0, 2, 0, 0, 0, 3], [3, 7, 2, 0, 1, 0, 0, 0, 5], [5, 3, 1, 0, 2, 0, 0, 0, 7], [7, 5, 2, 0, 1, 0, 0, 0, 3], [3, 7, 1, 0, 2, 0, 0, 0, 5]] \$$$

$$[y_3, -y_3 + 5 y_2 + 5 y_1 - y_7 - y_5 - y_6 - y_4, y_2, 0, y_1, y_7, y_5, y_6, y_4]$$

$$p = -s^4 - s^5 + s^7 + s^8$$

92 . Coloring, {6, 8, 9}

R: [4, 4, 4, 7, 7, 8, 1, 6, 2]

B: [2, 9, 5, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[3, 1, 0, 6, 0, 2, 5, 1, 0], [5, 0, 0, 4, 0, 1, 6, 2, 0], [6, 0, 0, 5, 0, 2, 4, 1, 0], [4, 0, 0, 6, 0, 1, 5, 2, 0], [5, 0, 0, 4, 0, 2, 6, 1, 0], [6, 0, 0, 5, 0, 1, 4, 2, 0]] \$$

$$[y_3, -y_3 - y_1 + 5y_2 - y_5 + 5y_4, 0, y_1, 0, y_2, y_5, y_4, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[3, 3, 2, 0, 4, 0, 1, 3, 2], [5, 3, 4, 0, 3, 0, 0, 0, 3], [3, 5, 3, 0, 4, 0, 0, 0, 3], [3, 3, 4, 0, 3, 0, 0, 0, 5], [5, 3, 3, 0, 4, 0, 0, 0, 3], [3, 5, 4, 0, 3, 0, 0, 0, 3], [3, 3, 3, 0, 4, 0, 0, 0, 5]] \$$

$$[-7y_1 + 11y_2 + 11y_3 - 10y_4 - 7y_5, 7y_1, 7y_2, 0, 7y_3, 0, 7y_4, 21y_4, 7y_5]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = -s^2 - s^3 + s^5 + s^6$$

93 . Coloring, $\{7, 8, 9\}$

R: [4, 4, 4, 7, 7, 7, 5, 6, 2]

B: [2, 9, 5, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	4 vs 5	5 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$$\$ [[0, 1, 0, 6, 3, 2, 6, 0, 0], [0, 0, 0, 1, 6, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$$$

$$[0, y_4, 0, y_2, y_3, 2y_4, y_1, 0, 0]$$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$$\$ [[6, 3, 2, 0, 1, 0, 0, 4, 2], [6, 6, 1, 0, 2, 0, 0, 0, 3], [3, 6, 2, 0, 1, 0, 0, 0, 6], [6, 3, 1, 0, 2, 0, 0, 0, 6], [6, 6, 2, 0, 1, 0, 0, 0, 3], [3, 6, 1, 0, 2, 0, 0, 0, 6]] \$$$

$$[-y_1 + 5y_2 + 5y_3 - y_5 - y_4, y_1, y_2, 0, y_3, 0, 0, y_5, y_4]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

94 . Coloring, $\{2, 3, 4, 5\}$

R: $[4, 9, 5, 8, 3, 7, 1, 1, 1]$

B: $[2, 4, 4, 7, 7, 8, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	4 vs 6

Omega Rank for R : cycles: $\{\{3, 5\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$$\$ [[6, 0, 2, 3, 1, 0, 1, 3, 2], [6, 0, 1, 6, 2, 0, 0, 3, 0], [3, 0, 2, 6, 1, 0, 0, 6, 0], [6, 0, 1, 3, 2, 0, 0, 6, 0], [6, 0, 2, 6, 1, 0, 0, 3, 0], [3, 0, 1, 6, 2, 0, 0, 6, 0], [6, 0, 2, 3, 1, 0, 0, 6, 0]] \$$$

$$[y_3, 0, y_2, -y_3 + 5y_2 + 5y_1 - 3y_4 - y_5, y_1, 0, y_4, y_5, 2y_4]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{5, 7}, {6, 8}} order: 4
See Matrix

$$\$ [[0, 4, 0, 3, 3, 2, 5, 1, 0], [0, 0, 0, 4, 5, 1, 6, 2, 0], [0, 0, 0, 0, 6, 2, 9, 1, 0], [0, 0, 0, 0, 9, 1, 6, 2, 0], [0, 0, 0, 0, 6, 2, 9, 1, 0], [0, 0, 0, 0, 9, 1, 6, 2, 0]] \$$$

$$[0, y_4, 0, y_3, -y_3 - 15y_1 + 4y_4 + 4y_2, y_1, y_2, y_4 - 4y_1 + y_2, 0]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

95 . Coloring, {2, 3, 4, 6}

R: [4, 9, 5, 8, 7, 8, 1, 1, 1]

B: [2, 4, 4, 7, 3, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{1, 4, 8}} order: 3
See Matrix

$$\$ [[6, 0, 0, 3, 1, 0, 2, 4, 2], [8, 0, 0, 6, 0, 0, 1, 3, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0]] \$$$

$$[y_1, 0, 0, y_5, y_4, 0, y_2, y_3, 2y_4]$$

$$p = s^3 - s^6$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4
See Matrix

$$\$ [[0, 4, 2, 3, 3, 2, 4, 0, 0], [0, 0, 3, 6, 4, 0, 5, 0, 0], [0, 0, 4, 3, 5, 0, 6, 0, 0], [0, 0, 5, 4, 6, 0, 3, 0, 0], [0, 0, 6, 5, 3, 0, 4, 0, 0], [0, 0, 3, 6, 4, 0, 5, 0, 0]] \$$$

$$[0, 2 y_5, y_2, y_1, y_4, y_5, y_3, 0, 0]$$

$$p = s^2 - s^6$$

96 . Coloring, {2, 3, 4, 7}

R: [4, 9, 5, 8, 7, 7, 5, 1, 1]

B: [2, 4, 4, 7, 3, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}} order: 6

See Matrix

\$ [[3, 0, 0, 3, 4, 0, 3, 3, 2], [5, 0, 0, 3, 3, 0, 4, 3, 0], [3, 0, 0, 5, 4, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 4, 5, 0], [5, 0, 0, 3, 4, 0, 3, 3, 0], [3, 0, 0, 5, 3, 0, 4, 3, 0]] \$

$$[-7 y_3 + 11 y_4 + 11 y_1 - 7 y_5 - 7 y_2, 0, 0, 7 y_3, 7 y_4, 0, 7 y_1, 7 y_5, 7 y_2]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}} order: 4

See Matrix

\$ [[3, 4, 2, 3, 0, 2, 3, 1, 0], [3, 3, 0, 6, 0, 1, 3, 2, 0], [3, 3, 0, 3, 0, 2, 6, 1, 0], [6, 3, 0, 3, 0, 1, 3, 2, 0], [3, 6, 0, 3, 0, 2, 3, 1, 0], [3, 3, 0, 6, 0, 1, 3, 2, 0], [3, 3, 0, 3, 0, 2, 6, 1, 0]] \$

$$[-y_5 + y_2 + 4 y_1, y_3, y_4, y_5, 0, y_2, -y_3 - y_4 + 4 y_2 + y_1, y_1, 0]$$

$$p = s^2 - s^6 \quad p' = s^2 - s^6$$

97 . Coloring, {2, 3, 4, 8}

R: [4, 9, 5, 8, 7, 7, 1, 6, 1]

B: [2, 4, 4, 7, 3, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	7 vs 7

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[4, 0, 0, 3, 1, 2, 3, 3, 2] , [5, 0, 0, 4, 0, 3, 3, 3, 0] , [3, 0, 0, 5, 0, 3, 3, 4, 0] , [3, 0, 0, 3, 0, 4, 3, 5, 0] , [3, 0, 0, 3, 0, 5, 4, 3, 0] , [4, 0, 0, 3, 0, 3, 5, 3, 0] , [5, 0, 0, 4, 0, 3, 3, 3, 0]] \$

$$[y_1, 0, 0, y_6, y_5, y_3, y_4, y_2, 2y_5]$$

$$p = -s^2 + s^7$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

\$ [[2, 4, 2, 3, 3, 0, 3, 1, 0] , [1, 2, 3, 6, 3, 0, 3, 0, 0] , [0, 1, 3, 5, 3, 0, 6, 0, 0] , [0, 0, 3, 4, 6, 0, 5, 0, 0] , [0, 0, 6, 3, 5, 0, 4, 0, 0] , [0, 0, 5, 6, 4, 0, 3, 0, 0] , [0, 0, 4, 5, 3, 0, 6, 0, 0]] \$

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, y_7, 0]$$

98 . Coloring, {2, 3, 4, 9}

R: [4, 9, 5, 8, 7, 7, 1, 1, 2]

B: [2, 4, 4, 7, 3, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	6 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{2, 9\}\}$ order: 6

See Matrix

$\$ [[5, 1, 0, 3, 1, 0, 3, 3, 2], [6, 2, 0, 5, 0, 0, 1, 3, 1], [4, 1, 0, 6, 0, 0, 0, 5, 2], [5, 2, 0, 4, 0, 0, 0, 6, 1], [6, 1, 0, 5, 0, 0, 0, 4, 2], [4, 2, 0, 6, 0, 0, 0, 5, 1], [5, 1, 0, 4, 0, 0, 0, 6, 2]] \$$

$$[5 y_4 - y_2 - y_3 - y_5 - y_1 + 5 y_6, y_4, 0, y_2, y_3, 0, y_5, y_1, y_6]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

Omega Rank for B : cycles: $\{\{3, 4, 5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[1, 3, 2, 3, 3, 2, 3, 1, 0], [0, 1, 3, 5, 3, 1, 3, 2, 0], [0, 0, 3, 4, 3, 2, 5, 1, 0], [0, 0, 3, 3, 5, 1, 4, 2, 0], [0, 0, 5, 3, 4, 2, 3, 1, 0], [0, 0, 4, 5, 3, 1, 3, 2, 0], [0, 0, 3, 4, 3, 2, 5, 1, 0], [0, 0, 3, 3, 5, 1, 4, 2, 0]] \$$

$$[y_6, y_5, y_3, y_4, -y_6 - y_4 + 2 y_2 + 3 y_1, y_2, -y_5 - y_3 + 3 y_2 + 2 y_1, y_1, 0]$$

$$p' = -s^3 + s^7 \quad p = -s^3 + s^7$$

99 . Coloring, $\{2, 3, 5, 6\}$

R: $[4, 9, 5, 7, 3, 8, 1, 1, 1]$

B: $[2, 4, 4, 8, 7, 7, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	6 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[6, 0, 2, 3, 1, 0, 3, 1, 2], [6, 0, 1, 6, 2, 0, 3, 0, 0], [3, 0, 2, 6, 1, 0, 6, 0, 0], [6, 0, 1, 3, 2, 0, 6, 0, 0], [6, 0, 2, 6, 1, 0, 3, 0, 0], [3, 0, 1, 6, 2, 0, 6, 0, 0], [6, 0, 2, 3, 1, 0, 6, 0, 0]] \$$

$$[5 y_1 - y_2 + 5 y_3 - y_4 - 3 y_5, 0, y_1, y_2, y_3, 0, y_4, y_5, 2 y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: {{5, 7}} order: 6
See Matrix

$$\$ [[0, 4, 0, 3, 3, 2, 3, 3, 0], [0, 0, 0, 4, 3, 3, 5, 3, 0], [0, 0, 0, 0, 5, 3, 6, 4, 0], [0, 0, 0, 0, 6, 4, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$$$

$$[0, y_6, 0, y_5, y_4, y_3, y_2, y_1, 0]$$

100 . Coloring, {2, 3, 5, 7}

$$\Omega p(\Delta)=0: \quad p = s^3 + 3s^4 + 8s^7$$

R: [4, 9, 5, 7, 3, 7, 5, 1, 1]

B: [2, 4, 4, 8, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 6
See Matrix

$$\$ [[3, 0, 2, 3, 4, 0, 4, 0, 2], [2, 0, 4, 3, 6, 0, 3, 0, 0], [0, 0, 6, 2, 7, 0, 3, 0, 0], [0, 0, 7, 0, 9, 0, 2, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$$$

$$[y_4, 0, y_1, y_2, y_3, 0, -y_4 - y_1 + y_2 + y_3 + y_5, 0, y_5]$$

$$p = -s^5 + s^6$$

Omega Rank for B : cycles: {{6, 8}} order: 6
See Matrix

$$\$ [[3, 4, 0, 3, 0, 2, 2, 4, 0], [2, 3, 0, 4, 0, 4, 0, 5, 0], [0, 2, 0, 3, 0, 5, 0, 8, 0], [0, 0, 0, 2, 0, 8, 0, 8, 0], [0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 10, 0, 8, 0]] \$$$

$$[y_1, y_2, 0, y_4, 0, y_3, y_5, y_6, 0]$$

101 . Coloring, {2, 3, 5, 8}

R: [4, 9, 5, 7, 3, 7, 1, 6, 1]

B: [2, 4, 4, 8, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}} order: 6

See Matrix

$$\$ [[4, 0, 2, 3, 1, 2, 4, 0, 2], [6, 0, 1, 4, 2, 0, 5, 0, 0], [5, 0, 2, 6, 1, 0, 4, 0, 0], [4, 0, 1, 5, 2, 0, 6, 0, 0], [6, 0, 2, 4, 1, 0, 5, 0, 0], [5, 0, 1, 6, 2, 0, 4, 0, 0], [4, 0, 2, 5, 1, 0, 6, 0, 0]] \$$$

$$[5y_1 - y_2 + 5y_3 - y_4 - 2y_5, 0, y_1, y_2, y_3, y_5, y_4, 0, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}} order: 4

See Matrix

$$\$ [[2, 4, 0, 3, 3, 0, 2, 4, 0], [4, 2, 0, 4, 2, 0, 3, 3, 0], [3, 4, 0, 2, 3, 0, 2, 4, 0], [4, 3, 0, 4, 2, 0, 3, 2, 0], [2, 4, 0, 3, 3, 0, 2, 4, 0], [4, 2, 0, 4, 2, 0, 3, 3, 0]] \$$$

$$[-14y_1 - y_3 + 39y_4 - 14y_2, y_1, 0, y_3, y_4, 0, -5y_1 + 14y_4 - 5y_2, y_2, 0]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

102 . Coloring, {2, 3, 5, 9}

$$\Omega p(\Delta)=0: \quad p = s^2 - 8s^4 - 12s^5 + 8s^6 + 16s^7$$

R: [4, 9, 5, 7, 3, 7, 1, 1, 2]

B: [2, 4, 4, 8, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	8 vs 8	8 vs 8	4 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}, \{2, 9\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 3, 1, 0, 4, 0, 2], [4, 2, 1, 5, 2, 0, 3, 0, 1], [3, 1, 2, 4, 1, 0, 5, 0, 2], [5, 2, 1, 3, 2, 0, 4, 0, 1], [4, 1, 2, 5, 1, 0, 3, 0, 2], [3, 2, 1, 4, 2, 0, 5, 0, 1], [5, 1, 2, 3, 1, 0, 4, 0, 2]] \$$

$$[y_1, y_2, y_3, -y_1 + 4y_3 + 4y_2 - y_4, y_2, 0, y_4, 0, y_3]$$

$$p = -s - s^2 + s^4 + s^5 \quad p = -s + s^7 \quad p = s - s^3 - s^4 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[1, 3, 0, 3, 3, 2, 2, 4, 0], [0, 1, 0, 3, 2, 4, 3, 5, 0], [0, 0, 0, 1, 3, 5, 2, 7, 0], [0, 0, 0, 0, 2, 7, 3, 6, 0], [0, 0, 0, 0, 3, 6, 2, 7, 0], [0, 0, 0, 0, 2, 7, 3, 6, 0], [0, 0, 0, 0, 3, 6, 2, 7, 0]] \$$

$$[9y_1 - 4y_2 - 13y_4 - 4y_5 + 9y_3, 4y_1, 0, 4y_2, 4y_4, 4y_5, 5y_1 - 9y_4 + 5y_3, 4y_3, 0]$$

$$p' = -s^4 + s^6 \quad p = -s^4 + s^6$$

103 . Coloring, $\{2, 3, 6, 7\}$

R: [4, 9, 5, 7, 7, 8, 5, 1, 1]

B: [2, 4, 4, 8, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	7 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[3, 0, 0, 3, 4, 0, 5, 1, 2] , [3, 0, 0, 3, 5, 0, 7, 0, 0] , [0, 0, 0, 3, 7, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$

$$[y_3, 0, 0, y_1, y_2, 0, y_4, y_5, 2y_5]$$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}} order: 6

See Matrix

\$ [[3, 4, 2, 3, 0, 2, 1, 3, 0] , [1, 3, 0, 6, 0, 3, 2, 3, 0] , [2, 1, 0, 3, 0, 3, 3, 6, 0] , [3, 2, 0, 1, 0, 6, 3, 3, 0] , [3, 3, 0, 2, 0, 3, 6, 1, 0] , [6, 3, 0, 3, 0, 1, 3, 2, 0] , [3, 6, 0, 3, 0, 2, 1, 3, 0]] \$

$$[y_4, y_3, y_2, y_1, 0, y_7, y_6, y_5, 0]$$

104 . Coloring, {2, 3, 6, 8}

R: [4, 9, 5, 7, 7, 8, 1, 6, 1]

B: [2, 4, 4, 8, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	7 vs 7

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}} order: 6

See Matrix

\$ [[4, 0, 0, 3, 1, 2, 5, 1, 2] , [7, 0, 0, 4, 0, 1, 4, 2, 0] , [4, 0, 0, 7, 0, 2, 4, 1, 0] , [4, 0, 0, 4, 0, 1, 7, 2, 0] , [7, 0, 0, 4, 0, 2, 4, 1, 0] , [4, 0, 0, 7, 0, 1, 4, 2, 0] , [4, 0, 0, 4, 0, 2, 7, 1, 0]] \$

$$[y_5, 0, 0, y_2, y_3, y_4, -y_5 - y_2 - 3y_3 + 5y_4 + 5y_1, y_1, 2y_3]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
See Matrix

$\$ [[2, 4, 2, 3, 3, 0, 1, 3, 0], [3, 2, 3, 6, 1, 0, 0, 3, 0], [3, 3, 1, 5, 0, 0, 0, 6, 0], [6, 3, 0, 4, 0, 0, 0, 5, 0], [5, 6, 0, 3, 0, 0, 0, 4, 0], [4, 5, 0, 6, 0, 0, 0, 3, 0], [3, 4, 0, 5, 0, 0, 0, 6, 0]] \$$

$$[y_4, y_1, y_2, y_3, y_6, 0, y_7, y_5, 0]$$

105 . Coloring, $\{2, 3, 6, 9\}$

R: $[4, 9, 5, 7, 7, 8, 1, 1, 2]$

B: $[2, 4, 4, 8, 3, 7, 5, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 9	9 vs 9	5 vs 7	7 vs 8

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 7\}\}$ order: 6
See Matrix

$\$ [[5, 1, 0, 3, 1, 0, 5, 1, 2], [6, 2, 0, 5, 0, 0, 4, 0, 1], [4, 1, 0, 6, 0, 0, 5, 0, 2], [5, 2, 0, 4, 0, 0, 6, 0, 1], [6, 1, 0, 5, 0, 0, 4, 0, 2], [4, 2, 0, 6, 0, 0, 5, 0, 1], [5, 1, 0, 4, 0, 0, 6, 0, 2]] \$$

$$[y_5, y_4, 0, y_3, y_2, 0, -y_5 + 5y_4 - y_3 - 2y_2 + 5y_1, y_2, y_1]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{3, 4, 5, 6, 7, 8\}\}$ order: 6
See Matrix

$\$ [[1, 3, 2, 3, 3, 2, 1, 3, 0], [0, 1, 3, 5, 1, 3, 2, 3, 0], [0, 0, 1, 4, 2, 3, 3, 5, 0], [0, 0, 2, 1, 3, 5, 3, 4, 0], [0, 0, 3, 2, 3, 4, 5, 1, 0], [0, 0, 3, 3, 5, 1, 4, 2, 0], [0, 0, 5, 3, 4, 2, 1, 3, 0], [0, 0, 4, 5, 1, 3, 2, 3, 0]] \$$

$$[y_1 + y_2 - y_3 - y_4 - y_7 + y_6 + y_5, y_1, y_2, y_3, y_4, y_7, y_6, y_5, 0]$$

$$p = s^3 - s^4 + s^5 - s^6 + s^7 - s^8$$

106 . Coloring, {2, 3, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^4 + 8s^5 + 16s^7$$

R: [4, 9, 5, 7, 7, 7, 5, 6, 1]

B: [2, 4, 4, 8, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[1, 0, 0, 3, 4, 2, 6, 0, 2], [2, 0, 0, 1, 6, 0, 9, 0, 0], [0, 0, 0, 2, 9, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$$$

$$[y_1, 0, 0, y_5, y_4, y_3, y_2, 0, y_3]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

$$\$ [[5, 4, 2, 3, 0, 0, 0, 4, 0], [4, 5, 0, 6, 0, 0, 0, 3, 0], [3, 4, 0, 5, 0, 0, 0, 6, 0], [6, 3, 0, 4, 0, 0, 0, 5, 0], [5, 6, 0, 3, 0, 0, 0, 4, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5, 0]$$

107 . Coloring, {2, 3, 7, 9}

R: [4, 9, 5, 7, 7, 7, 5, 1, 2]

B: [2, 4, 4, 8, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{5, 7}, {2, 9}} order: 4

See Matrix

\$ [[2, 1, 0, 3, 4, 0, 6, 0, 2], [0, 2, 0, 2, 6, 0, 7, 0, 1], [0, 1, 0, 0, 7, 0, 8, 0, 2], [0, 2, 0, 0, 8, 0, 7, 0, 1], [0, 1, 0, 0, 7, 0, 8, 0, 2], [0, 2, 0, 0, 8, 0, 7, 0, 1]] \$

$$[2y_1 - y_2 + 3y_3, y_1, 0, 3y_1 - y_4 + 2y_3, y_4, 0, y_2, 0, y_3]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{6, 8}} order: 4

See Matrix

\$ [[4, 3, 2, 3, 0, 2, 0, 4, 0], [0, 4, 0, 5, 0, 4, 0, 5, 0], [0, 0, 0, 4, 0, 5, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0]] \$

$$[2y_1, y_1 + y_2 + y_3 - y_4, y_1, y_2, 0, y_3, 0, y_4, 0]$$

$$p = -s^4 + s^5 \quad p = -s^4 + s^6$$

108 . Coloring, {2, 3, 8, 9}

R: [4, 9, 5, 7, 7, 7, 1, 6, 2]

B: [2, 4, 4, 8, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{2, 9\}\}$ order: 6

See Matrix

$$\$ [[3, 1, 0, 3, 1, 2, 6, 0, 2], [6, 2, 0, 3, 0, 0, 6, 0, 1], [6, 1, 0, 6, 0, 0, 3, 0, 2], [3, 2, 0, 6, 0, 0, 6, 0, 1], [6, 1, 0, 3, 0, 0, 6, 0, 2], [6, 2, 0, 6, 0, 0, 3, 0, 1], [3, 1, 0, 6, 0, 0, 6, 0, 2]] \$$$

$$[5y_1 - y_2 - 3y_3 - y_5 + 5y_4, y_1, 0, y_2, y_3, 2y_3, y_5, 0, y_4]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4

See Matrix

$$\$ [[3, 3, 2, 3, 3, 0, 0, 4, 0], [4, 3, 3, 5, 0, 0, 0, 3, 0], [3, 4, 0, 6, 0, 0, 0, 5, 0], [5, 3, 0, 4, 0, 0, 0, 6, 0], [6, 5, 0, 3, 0, 0, 0, 4, 0], [4, 6, 0, 5, 0, 0, 0, 3, 0]] \$$$

$$[y_4, y_2, y_3, -y_4 + y_2 + y_3 - y_1 + y_5, y_1, 0, 0, y_5, 0]$$

$$p = -s^3 + s^4 - s^5 + s^6$$

109 . Coloring, $\{2, 4, 5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^4 + 8s^5 + 16s^7$$

R: [4, 9, 4, 8, 3, 8, 1, 1, 1]

B: [2, 4, 5, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	4 vs 5

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 2, 4, 0, 0, 0, 4, 2], [6, 0, 0, 8, 0, 0, 0, 4, 0], [4, 0, 0, 6, 0, 0, 0, 8, 0], [8, 0, 0, 4, 0, 0, 0, 6, 0], [6, 0, 0, 8, 0, 0, 0, 4, 0]] \$$$

$$[y_1, 0, y_4, y_2, 0, 0, 0, y_3, y_4]$$

$$p = s^2 - s^5$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[0, 4, 0, 2, 4, 2, 6, 0, 0], [0, 0, 0, 4, 6, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$$

$$[0, 2 y_2, 0, y_3, y_4, y_2, y_1, 0, 0]$$

$$p = s^3 - s^5$$

110 . Coloring, {2, 4, 5, 7}

R: [4, 9, 4, 8, 3, 7, 5, 1, 1]

B: [2, 4, 5, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	5 vs 7

Omega Rank for R : cycles: {{1, 4, 8}} order: 6

See Matrix

$$\$ [[3, 0, 2, 4, 3, 0, 1, 3, 2], [5, 0, 3, 5, 1, 0, 0, 4, 0], [4, 0, 1, 8, 0, 0, 0, 5, 0], [5, 0, 0, 5, 0, 0, 0, 8, 0], [8, 0, 0, 5, 0, 0, 0, 5, 0], [5, 0, 0, 8, 0, 0, 0, 5, 0], [5, 0, 0, 5, 0, 0, 0, 8, 0]] \$$$

$$[y_2, 0, y_1, y_6, y_5, 0, y_4, y_3, 2 y_4]$$

$$p = -s^4 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 4, 0, 2, 1, 2, 5, 1, 0], [5, 3, 0, 4, 0, 1, 3, 2, 0], [3, 5, 0, 3, 0, 2, 4, 1, 0], [4, 3, 0, 5, 0, 1, 3, 2, 0], [3, 4, 0, 3, 0, 2, 5, 1, 0], [5, 3, 0, 4, 0, 1, 3, 2, 0], [3, 5, 0, 3, 0, 2, 4, 1, 0]] \$$$

$$[y_3 + 4y_5 - y_1 - y_2, 4y_3 - y_4 + y_5, 0, y_1, y_2, y_3, y_4, y_5, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

111 . Coloring, {2, 4, 5, 8}

R: [4, 9, 4, 8, 3, 7, 1, 6, 1]

B: [2, 4, 5, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 7	6 vs 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[4, 0, 2, 4, 0, 2, 1, 3, 2] , [3, 0, 0, 6, 0, 3, 2, 4, 0] , [2, 0, 0, 3, 0, 4, 3, 6, 0] , [3, 0, 0, 2, 0, 6, 4, 3, 0] , [4, 0, 0, 3, 0, 3, 6, 2, 0] , [6, 0, 0, 4, 0, 2, 3, 3, 0] , [3, 0, 0, 6, 0, 3, 2, 4, 0]] \$

$$[y_1, 0, y_5, y_2, 0, y_3, y_6, y_4, y_5]$$

$$p = -s^2 + s^7$$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

\$ [[2, 4, 0, 2, 4, 0, 5, 1, 0] , [1, 2, 0, 4, 5, 0, 6, 0, 0] , [0, 1, 0, 2, 6, 0, 9, 0, 0] , [0, 0, 0, 1, 9, 0, 8, 0, 0] , [0, 0, 0, 0, 8, 0, 10, 0, 0] , [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$

$$[y_2, y_1, 0, y_4, y_3, 0, y_5, y_6, 0]$$

112 . Coloring, {2, 4, 5, 9}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^5 - 8s^6 + 16s^7$$

R: [4, 9, 4, 8, 3, 7, 1, 1, 2]

B: [2, 4, 5, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 4, 0, 0, 1, 3, 2], [4, 2, 0, 7, 0, 0, 0, 4, 1], [4, 1, 0, 4, 0, 0, 0, 7, 2], [7, 2, 0, 4, 0, 0, 0, 4, 1], [4, 1, 0, 7, 0, 0, 0, 4, 2], [4, 2, 0, 4, 0, 0, 0, 7, 1], [7, 1, 0, 4, 0, 0, 0, 4, 2]] \$$

$$[5y_1 - y_2 - 3y_3 - y_4 + 5y_5, y_1, 2y_3, y_2, 0, 0, y_3, y_4, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[1, 3, 0, 2, 4, 2, 5, 1, 0], [0, 1, 0, 3, 5, 1, 6, 2, 0], [0, 0, 0, 1, 6, 2, 8, 1, 0], [0, 0, 0, 0, 8, 1, 7, 2, 0], [0, 0, 0, 0, 7, 2, 8, 1, 0], [0, 0, 0, 0, 8, 1, 7, 2, 0], [0, 0, 0, 0, 7, 2, 8, 1, 0]] \$$

$$[-y_1 - y_2 + 2y_5 + 3y_4, 3y_5 - y_3 + 2y_4, 0, y_1, y_2, y_5, y_3, y_4, 0]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

113 . Coloring, $\{2, 4, 6, 7\}$

R: [4, 9, 4, 8, 7, 8, 5, 1, 1]

B: [2, 4, 5, 7, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[3, 0, 0, 4, 3, 0, 2, 4, 2], [6, 0, 0, 3, 2, 0, 3, 4, 0], [4, 0, 0, 6, 3, 0, 2, 3, 0], [3, 0, 0, 4, 2, 0, 3, 6, 0], [6, 0, 0, 3, 3, 0, 2, 4, 0], [4, 0, 0, 6, 2, 0, 3, 3, 0]] \$$

$$[-5y_1 + 13y_2 + 13y_3 - 5y_4 - 5y_5, 0, 0, 5y_1, 5y_2, 0, 5y_3, 5y_4, 5y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{3, 5\}\}$ order: 4

See Matrix

$\$ [[3, 4, 2, 2, 1, 2, 4, 0, 0], [4, 3, 1, 4, 2, 0, 4, 0, 0], [4, 4, 2, 3, 1, 0, 4, 0, 0], [4, 4, 1, 4, 2, 0, 3, 0, 0], [3, 4, 2, 4, 1, 0, 4, 0, 0], [4, 3, 1, 4, 2, 0, 4, 0, 0], [4, 4, 2, 3, 1, 0, 4, 0, 0]] \$$

$$[y_5, y_4, y_3, y_2, y_1, -y_5 + 2y_3 - y_2 + 3y_1, -y_4 + 3y_3 + 2y_1, 0, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

114 . Coloring, $\{2, 4, 6, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^3 + 3s^4 + 8s^7$$

R: $[4, 9, 4, 8, 7, 8, 1, 6, 1]$

B: $[2, 4, 5, 7, 3, 7, 5, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{6, 8\}\}$ order: 4

See Matrix

$\$ [[4, 0, 0, 4, 0, 2, 2, 4, 2], [4, 0, 0, 4, 0, 4, 0, 6, 0], [0, 0, 0, 4, 0, 6, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0]] \$$

$$[y_1, 0, 0, y_2, 0, y_3, y_5, y_4, y_5]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: {{3, 5}} order: 6

See Matrix

$$\$ [[2, 4, 2, 2, 4, 0, 4, 0, 0], [0, 2, 4, 4, 6, 0, 2, 0, 0], [0, 0, 6, 2, 6, 0, 4, 0, 0], [0, 0, 6, 0, 10, 0, 2, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, y_5, 0, y_6, 0, 0]$$

115 . Coloring, {2, 4, 6, 9}

R: [4, 9, 4, 8, 7, 8, 1, 1, 2]

B: [2, 4, 5, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{1, 4, 8}, {2, 9}} order: 6

See Matrix

$$\$ [[5, 1, 0, 4, 0, 0, 2, 4, 2], [6, 2, 0, 5, 0, 0, 0, 4, 1], [4, 1, 0, 6, 0, 0, 0, 5, 2], [5, 2, 0, 4, 0, 0, 0, 6, 1], [6, 1, 0, 5, 0, 0, 0, 4, 2], [4, 2, 0, 6, 0, 0, 0, 5, 1]] \$$$

$$[5y_1 - y_2 - y_3 - y_5 + 5y_4, y_1, 0, y_2, 0, 0, y_3, y_5, y_4]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: {{3, 5}} order: 6

See Matrix

$$\$ [[1, 3, 2, 2, 4, 2, 4, 0, 0], [0, 1, 4, 3, 6, 0, 4, 0, 0], [0, 0, 6, 1, 8, 0, 3, 0, 0], [0, 0, 8, 0, 9, 0, 1, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$$$

$$[y_3, y_2, y_1, -3y_3 + y_2 + y_1 - y_5 + y_4, y_5, 2y_3, y_4, 0, 0]$$

$$p' = s^5 - s^6 \quad p = s^5 - s^7$$

116 . Coloring, {2, 4, 7, 8}

R: [4, 9, 4, 8, 7, 7, 5, 6, 1]

B: [2, 4, 5, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 7	5 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

$$\$ [[1, 0, 0, 4, 3, 2, 3, 3, 2], [2, 0, 0, 1, 3, 3, 5, 4, 0], [0, 0, 0, 2, 5, 4, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$$$

$$[y_3, 0, 0, y_1, y_2, y_6, y_7, y_4, y_5]$$

Omega Rank for B : cycles: {{3, 5}, {1, 2, 4, 7}} order: 4

See Matrix

$$\$ [[5, 4, 2, 2, 1, 0, 3, 1, 0], [4, 5, 1, 4, 2, 0, 2, 0, 0], [2, 4, 2, 5, 1, 0, 4, 0, 0], [4, 2, 1, 4, 2, 0, 5, 0, 0], [5, 4, 2, 2, 1, 0, 4, 0, 0], [4, 5, 1, 4, 2, 0, 2, 0, 0], [2, 4, 2, 5, 1, 0, 4, 0, 0]] \$$$

$$[2y_1 - y_4 + 3y_3, y_5, y_1, y_4, y_3, 0, y_2, -y_5 + 3y_1 + 2y_3 - y_2, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

117 . Coloring, {2, 4, 7, 9}

$$\Omega p(\Delta)=0: \quad p = s - 8s^3 - 12s^4 + 32s^6 + 32s^7$$

R: [4, 9, 4, 8, 7, 7, 5, 1, 2]

B: [2, 4, 5, 7, 3, 8, 1, 6, 1]

' See graph

' ' See pair graph

'

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 8	7 vs 8	4 vs 7	4 vs 8

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 4, 3, 0, 3, 3, 2], [3, 2, 0, 2, 3, 0, 3, 4, 1], [4, 1, 0, 3, 3, 0, 3, 2, 2], [2, 2, 0, 4, 3, 0, 3, 3, 1], [3, 1, 0, 2, 3, 0, 3, 4, 2], [4, 2, 0, 3, 3, 0, 3, 2, 1], [2, 1, 0, 4, 3, 0, 3, 3, 2]] \$$

$$[y_1, y_2 - y_3, 0, -y_1 + 3y_2 - y_4, y_2, 0, y_2, y_4, y_3]$$

$$p = -s - s^2 + s^4 + s^5 \quad p = -s + s^7 \quad p = s - s^3 - s^4 + s^6$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{6, 8\}, \{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[4, 3, 2, 2, 1, 2, 3, 1, 0], [3, 4, 1, 3, 2, 1, 2, 2, 0], [2, 3, 2, 4, 1, 2, 3, 1, 0], [3, 2, 1, 3, 2, 1, 4, 2, 0], [4, 3, 2, 2, 1, 2, 3, 1, 0], [3, 4, 1, 3, 2, 1, 2, 2, 0], [2, 3, 2, 4, 1, 2, 3, 1, 0], [3, 2, 1, 3, 2, 1, 4, 2, 0]] \$$

$$[y_2, 2y_1 - y_3 + 2y_4, y_1, -y_2 + 2y_1 + 2y_4, y_4, y_1, y_3, y_4, 0]$$

$$p' = -s^2 + s^6 \quad p' = -s^3 + s^7 \quad p = -s + s^5 \quad p' = -s + s^5$$

M \ ; N

$\$ [[0, 142, 0, 0, 104, 87, 191, 0, 46], [142, 0, 52, 94, 0, 0, 0, 92, 0], [0, 52, 0, 0, 0, 57, 81, 0, 0], [0, 94, 0, 0, 162, 46, 171, 0, 97], [104, 0, 0, 162, 0, 0, 0, 114, 0], [87, 0, 57, 46, 0, 0, 0, 0, 0], [191, 0, 81, 171, 0, 0, 0, 127, 0], [0, 92, 0, 0, 114, 0, 127, 0, 47], [46, 0, 0, 97, 0, 0, 0, 47, 0]] \$ \quad \$ [[0, 1, 0, 0, 1, 1, 1, 0, 1], [1, 0, 1, 1, 0, 0, 0, 1, 0], [0, 1, 0, 0, 1, 1, 1, 0, 1], [0, 1, 0, 0, 1, 1, 1, 0, 1], [1, 0, 1, 1, 0, 0, 0, 1, 0], [1, 0, 1, 1, 0, 0, 0, 1, 0], [1, 0, 1, 1, 0, 0, 0, 1, 0], [0, 1, 0, 0, 1, 1, 1, 0, 1], [1, 0, 1, 1, 0, 0, 0, 1, 0]] \$$

$$\tau = 41, r' = 1/2$$

R: [4, 9, 4, 8, 7, 7, 5, 1, 2]

B: [2, 4, 5, 7, 3, 8, 1, 6, 1]

Ranges

Action of R on ranges, [[14], [13], [13], [11], [7], [14], [17], [5], [13], [11], [16], [16], [15], [8], [4], [2], [1]]

Action of B on ranges, [[7], [6], [8], [1], [1], [11], [13], [12], [15], [2], [10], [16], [4], [4], [9], [3], [3]]

Cycles: R , {{5, 7}, {2, 9}, {1, 4, 8}}, B , {{3, 5}, {6, 8}, {1, 2, 4, 7}}

$$\beta(\{1, 2\}) = 71/855$$

$$\beta(\{1, 5\}) = 52/855$$

$$\beta(\{1, 6\}) = 29/570$$

$$\beta(\{1, 7\}) = 191/1710$$

$$\beta(\{1, 9\}) = 23/855$$

$$\beta(\{2, 3\}) = 26/855$$

$$\beta(\{2, 4\}) = 47/855$$

$$\beta(\{2, 8\}) = 46/855$$

$$\beta(\{3, 6\}) = 1/30$$

$$\beta(\{3, 7\}) = 9/190$$

$$\beta(\{4, 5\}) = 9/95$$

$$\beta(\{4, 6\}) = 23/855$$

$$\beta(\{4, 7\}) = 1/10$$

$$\beta(\{4, 9\}) = 97/1710$$

$$\beta(\{5, 8\}) = 1/15$$

$$\beta(\{7, 8\}) = 127/1710$$

$$\beta(\{8, 9\}) = 47/1710$$

Partitions

$$\alpha(\{\{1, 3, 4, 8\}, \{2, 5, 6, 7, 9\}\}) = 1/1$$

$$b_1 = \{1, 3, 4, 8\} \text{ , ' , ' } b_2 = \{2, 5, 6, 7, 9\}$$

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

,

Right Group	
Coloring	{2, 4, 7, 9}
Rank	2
R,B	[4, 9, 4, 8, 7, 7, 5, 1, 2], [2, 4, 5, 7, 3, 8, 1, 6, 1]
π_2	[142, 0, 0, 104, 87, 191, 0, 46, 52, 94, 0, 0, 0, 92, 0, 0, 0, 57, 81, 0, 0, 162, 46, 171, 0, 97, 0, 0, 114, 0, 0, 0, 0, 127, 0, 47]
u_2	[1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1] (dim 1)
wpp	[4, 5, 4, 4, 5, 5, 5, 4, 5]

118 . Coloring, {2, 4, 8, 9}

R: [4, 9, 4, 8, 7, 7, 1, 6, 2]

B: [2, 4, 5, 7, 3, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	6 vs 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 6, 7, 8}}

See Matrix

\$ [[3, 1, 0, 4, 0, 2, 3, 3, 2], [3, 2, 0, 3, 0, 3, 2, 4, 1], [2, 1, 0, 3, 0, 4, 3, 3, 2], [3, 2, 0, 2, 0, 3, 4, 3, 1], [4, 1, 0, 3, 0, 3, 3, 2, 2], [3, 2, 0, 4, 0, 2, 3, 3, 1], [3, 1, 0, 3, 0, 3, 2, 4, 2]] \$

$$[y_6, y_5, 0, y_4, 0, y_3, y_2, -y_6 + 5y_5 - y_4 - y_3 - y_2 + 5y_1, y_1]$$

$$p = -s - s^2 + s^6 + s^7$$

Omega Rank for B : cycles: {{3, 5}} order: 6

See Matrix

\$ [[3, 3, 2, 2, 4, 0, 3, 1, 0] , [1, 3, 4, 3, 5, 0, 2, 0, 0] , [0, 1, 5, 3, 6, 0, 3, 0, 0] , [0, 0, 6, 1, 8, 0, 3, 0, 0] , [0, 0, 8, 0, 9, 0, 1, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$

$$[y_2, y_1, y_2 - y_1 + y_5 + y_4 - y_3 - y_6, y_5, y_4, 0, y_3, y_6, 0]$$

$$p = s^6 - s^7$$

119 . Coloring, {2, 5, 6, 7}

R: [4, 9, 4, 7, 3, 8, 5, 1, 1]

B: [2, 4, 5, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	7 vs 7

Omega Rank for R : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

\$ [[3, 0, 2, 4, 3, 0, 3, 1, 2] , [3, 0, 3, 5, 3, 0, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0] , [0, 0, 4, 3, 5, 0, 6, 0, 0] , [0, 0, 5, 4, 6, 0, 3, 0, 0] , [0, 0, 6, 5, 3, 0, 4, 0, 0] , [0, 0, 3, 6, 4, 0, 5, 0, 0]] \$

$$[y_5, 0, y_4, y_3, y_2, 0, y_1, y_6, 2y_6]$$

$$p = s^3 - s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}} order: 6

See Matrix

\$ [[3, 4, 0, 2, 1, 2, 3, 3, 0] , [3, 3, 0, 4, 0, 3, 3, 2, 0] , [3, 3, 0, 3, 0, 2, 3, 4, 0] , [3, 3, 0, 3, 0, 4, 2, 3, 0] , [2, 3, 0, 3, 0, 3, 4, 3, 0] , [4, 2, 0, 3, 0, 3, 3, 3, 0] , [3, 4, 0, 2, 0, 3, 3, 3, 0]] \$

$$[y_1, y_2, 0, y_3, y_7, y_4, y_5, y_6, 0]$$

120 . Coloring, {2, 5, 6, 8}

R: [4, 9, 4, 7, 3, 8, 1, 6, 1]

B: [2, 4, 5, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}, {6, 8}} order: 6

See Matrix

\$ [[4, 0, 2, 4, 0, 2, 3, 1, 2], [5, 0, 0, 6, 0, 1, 4, 2, 0], [4, 0, 0, 5, 0, 2, 6, 1, 0], [6, 0, 0, 4, 0, 1, 5, 2, 0], [5, 0, 0, 6, 0, 2, 4, 1, 0], [4, 0, 0, 5, 0, 1, 6, 2, 0], [6, 0, 0, 4, 0, 2, 5, 1, 0]] \$

$$[-2y_3 - y_1 + 5y_5 - y_4 + 5y_2, 0, y_3, y_1, 0, y_5, y_4, y_2, y_3]$$

$$p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}} order: 4

See Matrix

\$ [[2, 4, 0, 2, 4, 0, 3, 3, 0], [3, 2, 0, 4, 3, 0, 4, 2, 0], [2, 3, 0, 2, 4, 0, 3, 4, 0], [4, 2, 0, 3, 3, 0, 4, 2, 0], [2, 4, 0, 2, 4, 0, 3, 3, 0], [3, 2, 0, 4, 3, 0, 4, 2, 0]] \$

$$[-16y_4 - 5y_3 + 33y_2 - 16y_1, 5y_4, 0, 5y_3, 5y_2, 0, -7y_4 + 16y_2 - 7y_1, 5y_1, 0]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

121 . Coloring, {2, 5, 6, 9}

R: [4, 9, 4, 7, 3, 8, 1, 1, 2]

B: [2, 4, 5, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	6 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 7\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 4, 0, 0, 3, 1, 2], [4, 2, 0, 7, 0, 0, 4, 0, 1], [4, 1, 0, 4, 0, 0, 7, 0, 2], [7, 2, 0, 4, 0, 0, 4, 0, 1], [4, 1, 0, 7, 0, 0, 4, 0, 2], [4, 2, 0, 4, 0, 0, 7, 0, 1], [7, 1, 0, 4, 0, 0, 4, 0, 2]] \$$

$$[5y_1 - y_2 - y_3 - 3y_4 + 5y_5, y_1, 2y_4, y_2, 0, 0, y_3, y_4, y_5]$$

$$p = -s^2 + s^4 + s^5 - s^7 \quad p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 3, 0, 2, 4, 2, 3, 3, 0], [0, 1, 0, 3, 3, 3, 6, 2, 0], [0, 0, 0, 1, 6, 2, 6, 3, 0], [0, 0, 0, 0, 6, 3, 8, 1, 0], [0, 0, 0, 0, 8, 1, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[y_6, y_5, 0, y_4, y_3, y_2, y_1, y_6 - y_5 + y_4 + y_3 + y_2 - y_1, 0]$$

$$p = s^6 - s^7$$

122 . Coloring, $\{2, 5, 7, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^3 - 16s^5 + 8s^6 - 32s^7 \quad p' = s^3 + 4s^4 + 8s^6$$

R: [4, 9, 4, 7, 3, 7, 5, 6, 1]

B: [2, 4, 5, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 7	8 vs 8	8 vs 8	6 vs 7	6 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}\}$ order: 4

See Matrix

\$ [[1, 0, 2, 4, 3, 2, 4, 0, 2], [2, 0, 3, 3, 4, 0, 6, 0, 0], [0, 0, 4, 5, 6, 0, 3, 0, 0], [0, 0, 6, 4, 3, 0, 5, 0, 0], [0, 0, 3, 6, 5, 0, 4, 0, 0], [0, 0, 5, 3, 4, 0, 6, 0, 0], [0, 0, 4, 5, 6, 0, 3, 0, 0]] \$

$$[y_1, 0, y_2, y_3, y_6, y_5, y_4, 0, y_5]$$

$$p = -s^3 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

\$ [[5, 4, 0, 2, 1, 0, 2, 4, 0], [6, 5, 0, 4, 0, 0, 1, 2, 0], [3, 6, 0, 5, 0, 0, 0, 4, 0], [4, 3, 0, 6, 0, 0, 0, 5, 0], [5, 4, 0, 3, 0, 0, 0, 6, 0], [6, 5, 0, 4, 0, 0, 0, 3, 0]] \$

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, y_6, 0]$$

123 . Coloring, {2, 5, 7, 9}

R: [4, 9, 4, 7, 3, 7, 5, 1, 2]

B: [2, 4, 5, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 7	6 vs 7

Omega Rank for R : cycles: {{2, 9}, {3, 4, 5, 7}} order: 4

See Matrix

\$ [[2, 1, 2, 4, 3, 0, 4, 0, 2], [0, 2, 3, 4, 4, 0, 4, 0, 1], [0, 1, 4, 3, 4, 0, 4, 0, 2], [0, 2, 4, 4, 4, 0, 3, 0, 1], [0, 1, 4, 4, 3, 0, 4, 0, 2], [0, 2, 3, 4, 4, 0, 4, 0, 1], [0, 1, 4, 3, 4, 0, 4, 0, 2]] \$

$$[2y_1 - y_2 - y_4 + 3y_5, y_1, y_2, 3y_1 - y_3 + 2y_5, y_3, 0, y_4, 0, y_5]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

Omega Rank for B : cycles: {{6, 8}} order: 6

See Matrix

$\$ [[4, 3, 0, 2, 1, 2, 2, 4, 0], [2, 4, 0, 3, 0, 4, 1, 4, 0], [1, 2, 0, 4, 0, 4, 0, 7, 0], [0, 1, 0, 2, 0, 7, 0, 8, 0], [0, 0, 0, 1, 0, 8, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0]] \$$

$$[y_4 - y_1 - y_2 - y_3 + y_6 + y_5, y_4, 0, y_1, y_2, y_3, y_6, y_5, 0]$$

$$p = -s^6 + s^7$$

124 . Coloring, {2, 5, 8, 9}

R: [4, 9, 4, 7, 3, 7, 1, 6, 2]

B: [2, 4, 5, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{1, 4, 7}, {2, 9}} order: 6

See Matrix

$\$ [[3, 1, 2, 4, 0, 2, 4, 0, 2], [4, 2, 0, 5, 0, 0, 6, 0, 1], [6, 1, 0, 4, 0, 0, 5, 0, 2], [5, 2, 0, 6, 0, 0, 4, 0, 1], [4, 1, 0, 5, 0, 0, 6, 0, 2], [6, 2, 0, 4, 0, 0, 5, 0, 1], [5, 1, 0, 6, 0, 0, 4, 0, 2]] \$$

$$[y_5, y_1, y_2, -y_5 + 5y_1 - 2y_2 - y_3 + 5y_4, 0, y_2, y_3, 0, y_4]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p' = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}} order: 4

See Matrix

$\$ [[3, 3, 0, 2, 4, 0, 2, 4, 0], [4, 3, 0, 3, 2, 0, 4, 2, 0], [2, 4, 0, 3, 4, 0, 2, 3, 0], [3, 2, 0, 4, 2, 0, 4, 3, 0], [3, 3, 0, 2, 4, 0, 2, 4, 0], [4, 3, 0, 3, 2, 0, 4, 2, 0]] \$$

$$[3y_1 - y_2 - 4y_3 + 3y_4, y_1, 0, y_2, y_3, 0, 2y_1 - 3y_3 + 2y_4, y_4, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

125 . Coloring, {2, 6, 7, 8}

R: [4, 9, 4, 7, 7, 8, 5, 6, 1]

B: [2, 4, 5, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 7

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$\$ [[1, 0, 0, 4, 3, 2, 5, 1, 2], [2, 0, 0, 1, 5, 1, 7, 2, 0], [0, 0, 0, 2, 7, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0]] \$$

$$[y_2 - y_3 + 4y_4, 0, 0, -y_1 + 4y_2 + y_4 - y_5, y_1, y_2, y_3, y_4, y_5]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {3, 5}} order: 4

See Matrix

$\$ [[5, 4, 2, 2, 1, 0, 1, 3, 0], [4, 5, 1, 4, 2, 0, 0, 2, 0], [2, 4, 2, 5, 1, 0, 0, 4, 0], [4, 2, 1, 4, 2, 0, 0, 5, 0], [5, 4, 2, 2, 1, 0, 0, 4, 0], [4, 5, 1, 4, 2, 0, 0, 2, 0], [2, 4, 2, 5, 1, 0, 0, 4, 0]] \$$

$$[2y_1 - y_2 + 3y_3, 3y_1 + 2y_3 - y_4 - y_5, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

126 . Coloring, {2, 6, 7, 9}

R: [4, 9, 4, 7, 7, 8, 5, 1, 2]

B: [2, 4, 5, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	6 vs 8

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 4, 3, 0, 5, 1, 2], [1, 2, 0, 2, 5, 0, 7, 0, 1], [0, 1, 0, 1, 7, 0, 7, 0, 2], [0, 2, 0, 0, 7, 0, 8, 0, 1], [0, 1, 0, 0, 8, 0, 7, 0, 2], [0, 2, 0, 0, 7, 0, 8, 0, 1], [0, 1, 0, 0, 8, 0, 7, 0, 2]] \$$

$$[3y_5 - y_2 + 2y_1, y_5, 0, y_4, y_3, 0, y_2, 2y_5 - y_4 - y_3 + 3y_1, y_1]$$

$$p = s^4 - s^6 \quad p' = s^4 - s^6$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 4, 6, 7, 8\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 2, 1, 2, 1, 3, 0], [1, 4, 1, 3, 2, 3, 2, 2, 0], [2, 1, 2, 4, 1, 2, 3, 3, 0], [3, 2, 1, 1, 2, 3, 2, 4, 0], [2, 3, 2, 2, 1, 4, 3, 1, 0], [3, 2, 1, 3, 2, 1, 4, 2, 0], [4, 3, 2, 2, 1, 2, 1, 3, 0], [1, 4, 1, 3, 2, 3, 2, 2, 0]] \$$

$$[3y_1 - y_3 + 2y_2 - y_4, 2y_1 + 3y_2 - y_5 - y_6, y_1, y_3, y_2, y_4, y_5, y_6, 0]$$

$$p = -s + s^7 \quad p' = -s + s^7$$

127 . Coloring, $\{2, 6, 8, 9\}$

R: $[4, 9, 4, 7, 7, 8, 1, 6, 2]$

B: $[2, 4, 5, 8, 3, 7, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 7\}, \{6, 8\}\}$ order: 6
 See Matrix

$\$ [[3, 1, 0, 4, 0, 2, 5, 1, 2], [5, 2, 0, 3, 0, 1, 4, 2, 1], [4, 1, 0, 5, 0, 2, 3, 1, 2], [3, 2, 0, 4, 0, 1, 5, 2, 1], [5, 1, 0, 3, 0, 2, 4, 1, 2], [4, 2, 0, 5, 0, 1, 3, 2, 1], [3, 1, 0, 4, 0, 2, 5, 1, 2]] \$$

$$[y_3, y_4, 0, -y_3 - y_2 + 4y_4 + 4y_1, 0, y_1, y_2, y_4, y_1]$$

$$p = -s - s^2 + s^4 + s^5 \quad p = s - s^3 - s^4 + s^6 \quad p = -s + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{3, 5\}\}$ order: 4
 See Matrix

$\$ [[3, 3, 2, 2, 4, 0, 1, 3, 0], [3, 3, 4, 3, 3, 0, 0, 2, 0], [2, 3, 3, 3, 4, 0, 0, 3, 0], [3, 2, 4, 3, 3, 0, 0, 3, 0], [3, 3, 3, 2, 4, 0, 0, 3, 0], [3, 3, 4, 3, 3, 0, 0, 2, 0], [2, 3, 3, 3, 4, 0, 0, 3, 0]] \$$

$$[9y_1 - 2y_2 - 11y_3 + 9y_5, 2y_1, 7y_1 - 9y_3 - 2y_4 + 7y_5, 2y_2, 2y_3, 0, 2y_4, 2y_5, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

128 . Coloring, $\{2, 7, 8, 9\}$

R: $[4, 9, 4, 7, 7, 7, 5, 6, 2]$

B: $[2, 4, 5, 8, 3, 8, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	3 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 2
 See Matrix

$\$ [[0, 1, 0, 4, 3, 2, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1]] \$$

$$[0, y_1 + 3y_2 - 4y_3, 0, 2y_2, y_1, y_2, 4y_1 + 12y_2 - 15y_3, 0, y_3]$$

$$p' = s^2 - s^4 \quad p = s^2 - s^4 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{3, 5\}\}$ order: 4
See Matrix

$$\$ [[6, 3, 2, 2, 1, 0, 0, 4, 0], [4, 6, 1, 3, 2, 0, 0, 2, 0], [2, 4, 2, 6, 1, 0, 0, 3, 0], [3, 2, 1, 4, 2, 0, 0, 6, 0], [6, 3, 2, 2, 1, 0, 0, 4, 0], [4, 6, 1, 3, 2, 0, 0, 2, 0]] \$$$

$$[3y_1 - y_2 + 2y_3, 2y_1 + 3y_3 - y_4, y_1, y_2, y_3, 0, 0, y_4, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

129 . Coloring, $\{3, 4, 5, 6\}$

R: $[4, 4, 5, 8, 3, 8, 1, 1, 1]$

B: $[2, 9, 4, 7, 7, 7, 5, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	3 vs 6

Omega Rank for R : cycles: $\{\{3, 5\}, \{1, 4, 8\}\}$ order: 6
See Matrix

$$\$ [[6, 0, 2, 5, 1, 0, 0, 4, 0], [4, 0, 1, 6, 2, 0, 0, 5, 0], [5, 0, 2, 4, 1, 0, 0, 6, 0], [6, 0, 1, 5, 2, 0, 0, 4, 0], [4, 0, 2, 6, 1, 0, 0, 5, 0]] \$$$

$$[y_4, 0, y_3, y_2, y_1, 0, 0, -y_4 + 5y_3 - y_2 + 5y_1, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 2
See Matrix

$$\$ [[0, 4, 0, 1, 3, 2, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 6, 0, 4]] \$$$

$$[0, y_3 - y_2, 0, y_1, -3y_1 + y_3, 2y_1, y_3, 0, y_2]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6$$

130 . Coloring, {3, 4, 5, 7}

R: [4, 4, 5, 8, 3, 7, 5, 1, 1]

B: [2, 9, 4, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}} order: 6

See Matrix

$$\$ [[3, 0, 2, 5, 4, 0, 1, 3, 0], [3, 0, 4, 3, 3, 0, 0, 5, 0], [5, 0, 3, 3, 4, 0, 0, 3, 0], [3, 0, 4, 5, 3, 0, 0, 3, 0], [3, 0, 3, 3, 4, 0, 0, 5, 0], [5, 0, 4, 3, 3, 0, 0, 3, 0]] \$$$

$$[11y_1 - 7y_2 + 11y_3 + 11y_4 - 7y_5, 0, 7y_1, 7y_2, 7y_3, 0, 7y_4, 7y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{2, 9}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 4, 0, 1, 0, 2, 5, 1, 2], [5, 5, 0, 0, 0, 1, 1, 2, 4], [1, 9, 0, 0, 0, 2, 0, 1, 5], [0, 6, 0, 0, 0, 1, 0, 2, 9], [0, 9, 0, 0, 0, 2, 0, 1, 6], [0, 6, 0, 0, 0, 1, 0, 2, 9], [0, 9, 0, 0, 0, 2, 0, 1, 6]] \$$$

$$[y_1, 4y_2 - y_3 + y_4, 0, -y_1 + y_2 + 4y_4 - y_5, 0, y_2, y_3, y_4, y_5]$$

$$p' = s^4 - s^6 \quad p = s^4 - s^6$$

131 . Coloring, {3, 4, 5, 8}

R: [4, 4, 5, 8, 3, 7, 1, 6, 1]

B: [2, 9, 4, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 6, 7, 8\}, \{3, 5\}\}$

See Matrix

$\$ [[4, 0, 2, 5, 1, 2, 1, 3, 0], [1, 0, 1, 4, 2, 3, 2, 5, 0], [2, 0, 2, 1, 1, 5, 3, 4, 0], [3, 0, 1, 2, 2, 4, 5, 1, 0], [5, 0, 2, 3, 1, 1, 4, 2, 0], [4, 0, 1, 5, 2, 2, 1, 3, 0], [1, 0, 2, 4, 1, 3, 2, 5, 0]] \$$

$$[5y_1 - y_2 + 5y_4 - y_3 - y_5 - y_6, 0, y_1, y_2, y_4, y_3, y_5, y_6, 0]$$

$$p = -s - s^2 + s^6 + s^7$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 4

See Matrix

$\$ [[2, 4, 0, 1, 3, 0, 5, 1, 2], [1, 4, 0, 0, 5, 0, 4, 0, 4], [0, 5, 0, 0, 4, 0, 5, 0, 4], [0, 4, 0, 0, 5, 0, 4, 0, 5], [0, 5, 0, 0, 4, 0, 5, 0, 4], [0, 4, 0, 0, 5, 0, 4, 0, 5], [0, 5, 0, 0, 4, 0, 5, 0, 4]] \$$

$$[y_1, -y_3 + y_4, 0, y_3, y_1 - y_3 + y_2, 0, y_4, y_3, y_2]$$

$$p' = s^4 - s^6 \quad p' = s^3 - s^5 \quad p = s^3 - s^7$$

132 . Coloring, $\{3, 4, 5, 9\}$

R: [4, 4, 5, 8, 3, 7, 1, 1, 2]

B: [2, 9, 4, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 5, 1, 0, 1, 3, 0], [4, 0, 1, 6, 2, 0, 0, 5, 0], [5, 0, 2, 4, 1, 0, 0, 6, 0], [6, 0, 1, 5, 2, 0, 0, 4, 0], [4, 0, 2, 6, 1, 0, 0, 5, 0], [5, 0, 1, 4, 2, 0, 0, 6, 0], [6, 0, 2, 5, 1, 0, 0, 4, 0]] \$$

$$[-2y_4 + 5y_1 - y_2 + 5y_3 - y_5, y_4, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[1, 3, 0, 1, 3, 2, 5, 1, 2], [2, 1, 0, 0, 5, 1, 4, 2, 3], [3, 2, 0, 0, 4, 2, 5, 1, 1], [1, 3, 0, 0, 5, 1, 4, 2, 2], [2, 1, 0, 0, 4, 2, 5, 1, 3], [3, 2, 0, 0, 5, 1, 4, 2, 1], [1, 3, 0, 0, 4, 2, 5, 1, 2], [2, 1, 0, 0, 5, 1, 4, 2, 3]] \$$

$$[-y_1 + 2y_3 + 2y_4 - y_5, y_1, 0, -y_2 + y_3 + 2y_4, y_2, y_3, 2y_3 + y_4, y_4, y_5]$$

$$p = -s^2 + s^8 \quad p = -s^2 + s^4 + s^5 - s^7 \quad p = s^2 + s^3 - s^5 - s^6$$

133 . Coloring, $\{3, 4, 6, 7\}$

$$\Omega p(\Delta)=0: \quad p = s^3 - 3s^4 + 8s^7$$

R: [4, 4, 5, 8, 7, 8, 5, 1, 1]

B: [2, 9, 4, 7, 3, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	6 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[3, 0, 0, 5, 4, 0, 2, 4, 0], [4, 0, 0, 3, 2, 0, 4, 5, 0], [5, 0, 0, 4, 4, 0, 2, 3, 0], [3, 0, 0, 5, 2, 0, 4, 4, 0], [4, 0, 0, 3, 4, 0, 2, 5, 0]] \$$

$$[y_4, 0, 0, y_3, y_2, 0, y_1, -y_4 - y_3 + 2y_2 + 2y_1, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

$$\$ [[3, 4, 2, 1, 0, 2, 4, 0, 2], [4, 5, 0, 2, 0, 0, 3, 0, 4], [3, 8, 0, 0, 0, 0, 2, 0, 5], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$$

$$[y_1, y_2, y_4, y_3, 0, y_4, y_5, 0, y_6]$$

$$p = s^5 - s^7$$

134 . Coloring, {3, 4, 6, 8}

R: [4, 4, 5, 8, 7, 8, 1, 6, 1]

B: [2, 9, 4, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 7

Omega Rank for R : cycles: {{6, 8}} order: 6

See Matrix

$$\$ [[4, 0, 0, 5, 1, 2, 2, 4, 0], [2, 0, 0, 4, 0, 4, 1, 7, 0], [1, 0, 0, 2, 0, 7, 0, 8, 0], [0, 0, 0, 1, 0, 8, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0]] \$$$

$$[y_2, 0, 0, y_2 + y_1 - y_3 - y_4 + y_5, y_1, y_3, y_4, y_5, 0]$$

$$p = -s^5 + s^6$$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {2, 9}} order: 4

See Matrix

$$\$ [[2, 4, 2, 1, 3, 0, 4, 0, 2], [0, 4, 3, 2, 4, 0, 1, 0, 4], [0, 4, 4, 3, 1, 0, 2, 0, 4], [0, 4, 1, 4, 2, 0, 3, 0, 4], [0, 4, 2, 1, 3, 0, 4, 0, 4], [0, 4, 3, 2, 4, 0, 1, 0, 4], [0, 4, 4, 3, 1, 0, 2, 0, 4]] \$$$

$$[2 y_4, 2 y_4 + 2 y_5, 2 y_3, 2 y_2, 2 y_1, 0, 5 y_4 - 2 y_3 - 2 y_2 - 2 y_1 + 5 y_5, 0, 2 y_5]$$

$$p' = s^2 - s^6 \quad p = s^2 - s^6$$

135 . Coloring, {3, 4, 6, 9}

R: [4, 4, 5, 8, 7, 8, 1, 1, 2]

B: [2, 9, 4, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 6	6 vs 8

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

$$\$ [[5, 1, 0, 5, 1, 0, 2, 4, 0], [6, 0, 0, 6, 0, 0, 1, 5, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0], [6, 0, 0, 6, 0, 0, 0, 6, 0]] \$$$

$$[y_1, -y_1 + y_3 + y_2, 0, y_1, -y_1 + y_3 + y_2, 0, y_3, y_2, 0]$$

$$p' = -s^3 + s^5 \quad p = s^3 - s^4 \quad p' = -s^3 + s^4$$

Omega Rank for B : cycles: {{3, 4, 5, 7}, {1, 2, 9}}

See Matrix

$$\$ [[1, 3, 2, 1, 3, 2, 4, 0, 2], [2, 1, 3, 2, 4, 0, 3, 0, 3], [3, 2, 4, 3, 3, 0, 2, 0, 1], [1, 3, 3, 4, 2, 0, 3, 0, 2], [2, 1, 2, 3, 3, 0, 4, 0, 3], [3, 2, 3, 2, 4, 0, 3, 0, 1], [1, 3, 4, 3, 3, 0, 2, 0, 2], [2, 1, 3, 4, 2, 0, 3, 0, 3]] \$$$

$$[-y_4 + y_1 + y_2 + y_3 - y_5, y_4, -y_6 + y_1 + y_2 + y_3, y_1, y_2, y_3, y_6, 0, y_5]$$

$$p' = -s^2 - s^4 + s^5 + s^7 \quad p = -s^2 - s^4 + s^5 + s^7$$

136 . Coloring, {3, 4, 7, 8}

R: [4, 4, 5, 8, 7, 7, 5, 6, 1]

B: [2, 9, 4, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 6	6 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 0, 0, 5, 4, 2, 3, 3, 0], [0, 0, 0, 1, 3, 3, 6, 5, 0], [0, 0, 0, 0, 6, 5, 6, 1, 0], [0, 0, 0, 0, 6, 1, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$$

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 6

See Matrix

$\$ [[5, 4, 2, 1, 0, 0, 3, 1, 2], [4, 7, 0, 2, 0, 0, 1, 0, 4], [1, 8, 0, 0, 0, 0, 2, 0, 7], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$

$$[y_2, y_3, 2y_6, y_1, 0, 0, y_4, y_6, y_5]$$

$$p = s^5 - s^7$$

137 . Coloring, $\{3, 4, 7, 9\}$

R: [4, 4, 5, 8, 7, 7, 5, 1, 2]

B: [2, 9, 4, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	7 vs 8

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 5, 4, 0, 3, 3, 0], [3, 0, 0, 3, 3, 0, 4, 5, 0], [5, 0, 0, 3, 4, 0, 3, 3, 0], [3, 0, 0, 5, 3, 0, 4, 3, 0], [3, 0, 0, 3, 4, 0, 3, 5, 0], [5, 0, 0, 3, 3, 0, 4, 3, 0]] \$$

$$[-7y_1 - 7y_2 + 11y_3 + 11y_4 - 7y_5, 7y_1, 0, 7y_2, 7y_3, 0, 7y_4, 7y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 1, 0, 2, 3, 1, 2], [5, 4, 0, 2, 0, 1, 1, 2, 3], [4, 5, 0, 0, 0, 2, 2, 1, 4], [6, 4, 0, 0, 0, 1, 0, 2, 5], [5, 6, 0, 0, 0, 2, 0, 1, 4], [4, 5, 0, 0, 0, 1, 0, 2, 6], [6, 4, 0, 0, 0, 2, 0, 1, 5], [5, 6, 0, 0, 0, 1, 0, 2, 4]] \$$

$$[-y_1 - y_2 - y_3 + 5y_4 - y_5 + 5y_6 - y_7, y_1, y_2, y_3, 0, y_4, y_5, y_6, y_7]$$

$$p = -s^4 - s^5 + s^7 + s^8$$

138 . Coloring, $\{3, 4, 8, 9\}$

R: $[4, 4, 5, 8, 7, 7, 1, 6, 2]$

B: $[2, 9, 4, 7, 3, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	5 vs 8

Omega Rank for R : cycles: $\{\{1, 4, 6, 7, 8\}\}$ order: 5

See Matrix

$\$ [[3, 1, 0, 5, 1, 2, 3, 3, 0], [3, 0, 0, 4, 0, 3, 3, 5, 0], [3, 0, 0, 3, 0, 5, 3, 4, 0], [3, 0, 0, 3, 0, 4, 5, 3, 0], [5, 0, 0, 3, 0, 3, 4, 3, 0], [4, 0, 0, 5, 0, 3, 3, 3, 0], [3, 0, 0, 4, 0, 3, 3, 5, 0]] \$$

$$[y_1, y_5, 0, y_2, y_5, y_6, y_3, y_4, 0]$$

$$p = -s^2 + s^7$$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 4, 5, 7}}

See Matrix

$$\$ [[3, 3, 2, 1, 3, 0, 3, 1, 2], [3, 3, 3, 2, 3, 0, 1, 0, 3], [3, 3, 3, 3, 1, 0, 2, 0, 3], [3, 3, 1, 3, 2, 0, 3, 0, 3], [3, 3, 2, 1, 3, 0, 3, 0, 3], [3, 3, 3, 2, 3, 0, 1, 0, 3], [3, 3, 3, 3, 1, 0, 2, 0, 3], [3, 3, 1, 3, 2, 0, 3, 0, 3]] \$$$

$$[y_5 + y_4, y_5 + y_4, 3y_5 + 3y_4 - y_1 - y_2 - y_3, y_1, y_2, 0, y_3, y_5, y_4]$$

$$p = -s^2 + s^6 \quad p' = -s^3 + s^7 \quad p'' = -s^2 + s^6$$

139 . Coloring, {3, 5, 6, 7}

R: [4, 4, 5, 7, 3, 8, 5, 1, 1]

B: [2, 9, 4, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 6	7 vs 7

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

$$\$ [[3, 0, 2, 5, 4, 0, 3, 1, 0], [1, 0, 4, 3, 5, 0, 5, 0, 0], [0, 0, 5, 1, 9, 0, 3, 0, 0], [0, 0, 9, 0, 8, 0, 1, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0]] \$$$

$$[y_6, 0, y_4, y_5, y_2, 0, y_3, y_1, 0]$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

$$\$ [[3, 4, 0, 1, 0, 2, 3, 3, 2], [3, 5, 0, 0, 0, 3, 2, 1, 4], [2, 7, 0, 0, 0, 1, 3, 0, 5], [3, 7, 0, 0, 0, 0, 1, 0, 7], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$$

$$[y_1, y_3, 0, y_2, 0, y_6, y_5, y_4, y_7]$$

140 . Coloring, {3, 5, 6, 8}

R: [4, 4, 5, 7, 3, 8, 1, 6, 1]

B: [2, 9, 4, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 7	5 vs 7

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}, {6, 8}} order: 6

See Matrix

$\$ [[4, 0, 2, 5, 1, 2, 3, 1, 0], [3, 0, 1, 4, 2, 1, 5, 2, 0], [5, 0, 2, 3, 1, 2, 4, 1, 0], [4, 0, 1, 5, 2, 1, 3, 2, 0], [3, 0, 2, 4, 1, 2, 5, 1, 0], [5, 0, 1, 3, 2, 1, 4, 2, 0], [4, 0, 2, 5, 1, 2, 3, 1, 0]] \$$

$$[-y_2 + 4y_4 + 4y_1 - y_3, 0, y_1, y_2, y_4, y_1, y_3, y_4, 0]$$

$$p = -s - s^2 + s^4 + s^5 \quad p' = -s - s^2 + s^4 + s^5 \quad p = -s + s^7$$

Omega Rank for B : cycles: {{5, 7}, {2, 9}} order: 4

See Matrix

$\$ [[2, 4, 0, 1, 3, 0, 3, 3, 2], [3, 4, 0, 0, 3, 0, 3, 1, 4], [1, 7, 0, 0, 3, 0, 3, 0, 4], [0, 5, 0, 0, 3, 0, 3, 0, 7], [0, 7, 0, 0, 3, 0, 3, 0, 5], [0, 5, 0, 0, 3, 0, 3, 0, 7], [0, 7, 0, 0, 3, 0, 3, 0, 5]] \$$

$$[y_1, y_2, 0, -y_1 - y_2 + 4y_3 - y_4 - y_5, y_3, 0, y_3, y_4, y_5]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

141 . Coloring, {3, 5, 6, 9}

R: [4, 4, 5, 7, 3, 8, 1, 1, 2]

B: [2, 9, 4, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	6 vs 8

Omega Rank for R : cycles: $\{\{3, 5\}, \{1, 4, 7\}\}$ order: 6
See Matrix

$\$ [[5, 1, 2, 5, 1, 0, 3, 1, 0], [4, 0, 1, 6, 2, 0, 5, 0, 0], [5, 0, 2, 4, 1, 0, 6, 0, 0], [6, 0, 1, 5, 2, 0, 4, 0, 0], [4, 0, 2, 6, 1, 0, 5, 0, 0], [5, 0, 1, 4, 2, 0, 6, 0, 0], [6, 0, 2, 5, 1, 0, 4, 0, 0]] \$$

$$[-2y_2 + 5y_1 - y_4 + 5y_5 - y_3, y_2, y_1, y_4, y_5, 0, y_3, y_2, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}\}$ order: 6
See Matrix

$\$ [[1, 3, 0, 1, 3, 2, 3, 3, 2], [2, 1, 0, 0, 3, 3, 5, 1, 3], [3, 2, 0, 0, 5, 1, 6, 0, 1], [1, 3, 0, 0, 6, 0, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 6, 0, 1], [1, 3, 0, 0, 6, 0, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3]] \$$

$$[-y_1 + y_4 + y_5 + y_2 - y_6, y_1, 0, y_4, y_5, y_2, y_3, y_4 + y_5 + y_2 - y_3, y_6]$$

$$p' = -s^4 + s^7 \quad p = -s^4 + s^7$$

142 . Coloring, $\{3, 5, 7, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^4 - 16s^7$$

R: [4, 4, 5, 7, 3, 7, 5, 6, 1]

B: [2, 9, 4, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 4
See Matrix

\$ [[1, 0, 2, 5, 4, 2, 4, 0, 0] , [0, 0, 4, 1, 6, 0, 7, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] ,
[0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0]] \$

$$[y_1, 0, y_5, y_4, y_3, 2y_1, y_2, 0, 0]$$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{2, 9}} order: 4
See Matrix

\$ [[5, 4, 0, 1, 0, 0, 2, 4, 2] , [6, 7, 0, 0, 0, 0, 0, 1, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] ,
[0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$

$$[y_1, y_2, 0, y_3, 0, 0, 2y_3, y_5, y_4]$$

$$p = -s^4 + s^6$$

143 . Coloring, {3, 5, 7, 9}

R: [4, 4, 5, 7, 3, 7, 5, 1, 2]

B: [2, 9, 4, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	5 vs 7

Omega Rank for R : cycles: {{3, 5}} order: 4
See Matrix

\$ [[2, 1, 2, 5, 4, 0, 4, 0, 0] , [0, 0, 4, 3, 6, 0, 5, 0, 0] , [0, 0, 6, 0, 9, 0, 3, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0] , [0,
0, 9, 0, 9, 0, 0, 0, 0, 0] , [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$

$$[2y_1, y_1, -3y_1 + y_2 + y_3 - y_4, y_2, y_3, 0, y_4, 0, 0]$$

$$p = s^4 - s^6 \quad p' = s^4 - s^5$$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}} order: 6
See Matrix

$$\$ [[4, 3, 0, 1, 0, 2, 2, 4, 2], [4, 4, 0, 0, 0, 4, 0, 3, 3], [3, 4, 0, 0, 0, 3, 0, 4, 4], [4, 3, 0, 0, 0, 4, 0, 3, 4], [4, 4, 0, 0, 0, 3, 0, 4, 3], [3, 4, 0, 0, 0, 4, 0, 3, 4], [4, 3, 0, 0, 0, 3, 0, 4, 4]] \$$$

$$[3 y_3, 3 y_2, 0, -7 y_3 - 7 y_2 + 11 y_1 + 11 y_5 - 7 y_4, 0, 3 y_1, -14 y_3 - 14 y_2 + 22 y_1 + 22 y_5 - 14 y_4, 3 y_5, 3 y_4]$$

$$p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 - s^3 + s^5 + s^6$$

144 . Coloring, {3, 5, 8, 9}

R: [4, 4, 5, 7, 3, 7, 1, 6, 2]

B: [2, 9, 4, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	6 vs 7

Omega Rank for R : cycles: {{1, 4, 7}, {3, 5}} order: 6
See Matrix

$$\$ [[3, 1, 2, 5, 1, 2, 4, 0, 0], [4, 0, 1, 4, 2, 0, 7, 0, 0], [7, 0, 2, 4, 1, 0, 4, 0, 0], [4, 0, 1, 7, 2, 0, 4, 0, 0], [4, 0, 2, 4, 1, 0, 7, 0, 0], [7, 0, 1, 4, 2, 0, 4, 0, 0], [4, 0, 2, 7, 1, 0, 4, 0, 0]] \$$$

$$[y_2, y_3, y_4, y_5, y_1, 2 y_3, -y_2 - 3 y_3 + 5 y_4 - y_5 + 5 y_1, 0, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p' = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}} order: 6
See Matrix

$$\$ [[3, 3, 0, 1, 3, 0, 2, 4, 2], [6, 3, 0, 0, 2, 0, 3, 1, 3], [4, 6, 0, 0, 3, 0, 2, 0, 3], [3, 4, 0, 0, 2, 0, 3, 0, 6], [6, 3, 0, 0, 3, 0, 2, 0, 4], [4, 6, 0, 0, 2, 0, 3, 0, 3], [3, 4, 0, 0, 3, 0, 2, 0, 6]] \$$$

$$[-5 y_1 - 5 y_2 + 13 y_3 + 13 y_4 - 5 y_5 - 5 y_6, 5 y_1, 0, 5 y_2, 5 y_3, 0, 5 y_4, 5 y_5, 5 y_6]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

145 . Coloring, {3, 6, 7, 8}

R: [4, 4, 5, 7, 7, 8, 5, 6, 1]

B: [2, 9, 4, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	6 vs 7

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[1, 0, 0, 5, 4, 2, 5, 1, 0], [0, 0, 0, 1, 5, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0]] \$$$

$$[y_2 + 4 y_4 - y_1, 0, 0, -y_3 + 4 y_2 + y_4, y_3, y_2, y_1, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

$$\$ [[5, 4, 2, 1, 0, 0, 1, 3, 2], [4, 7, 0, 2, 0, 0, 0, 1, 4], [1, 8, 0, 0, 0, 0, 0, 2, 7], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$$

$$[y_5, y_4, 2 y_2, y_3, 0, 0, y_2, y_1, y_6]$$

$$p = s^5 - s^7$$

146 . Coloring, {3, 6, 7, 9}

R: [4, 4, 5, 7, 7, 8, 5, 1, 2]

B: [2, 9, 4, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 6	8 vs 8

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 5, 4, 0, 5, 1, 0], [1, 0, 0, 3, 5, 0, 9, 0, 0], [0, 0, 0, 1, 9, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$

$[y_1, y_5, 0, y_2, y_3, 0, y_4, y_5, 0]$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 1, 0, 2, 1, 3, 2], [3, 4, 0, 2, 0, 3, 2, 1, 3], [5, 3, 0, 0, 0, 1, 3, 2, 4], [7, 5, 0, 0, 0, 2, 1, 0, 3], [4, 7, 0, 0, 0, 2, 0, 5], [7, 4, 0, 0, 0, 0, 0, 7], [7, 7, 0, 0, 0, 0, 0, 4], [4, 7, 0, 0, 0, 0, 0, 7]] \$$

$[y_3, y_4, y_5, y_2, 0, y_8, y_1, y_7, y_6]$

147 . Coloring, $\{3, 6, 8, 9\}$

R: [4, 4, 5, 7, 7, 8, 1, 6, 2]

B: [2, 9, 4, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	8 vs 8

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}} order: 6

See Matrix

\$ [[3, 1, 0, 5, 1, 2, 5, 1, 0], [5, 0, 0, 4, 0, 1, 6, 2, 0], [6, 0, 0, 5, 0, 2, 4, 1, 0], [4, 0, 0, 6, 0, 1, 5, 2, 0], [5, 0, 0, 4, 0, 2, 6, 1, 0], [6, 0, 0, 5, 0, 1, 4, 2, 0], [4, 0, 0, 6, 0, 2, 5, 1, 0]] \$

$$[-2y_4 - y_1 + 5y_3 - y_2 + 5y_5, y_4, 0, y_1, y_4, y_3, y_2, y_5, 0]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6

See Matrix

\$ [[3, 3, 2, 1, 3, 0, 1, 3, 2], [5, 3, 3, 2, 1, 0, 0, 1, 3], [4, 5, 1, 3, 0, 0, 0, 2, 3], [5, 4, 0, 1, 0, 0, 0, 3, 5], [8, 5, 0, 0, 0, 0, 0, 1, 4], [5, 8, 0, 0, 0, 0, 0, 0, 5], [5, 5, 0, 0, 0, 0, 0, 0, 8], [8, 5, 0, 0, 0, 0, 0, 0, 5]] \$

$$[y_1, y_5, y_4, y_3, y_2, 0, y_8, y_7, y_6]$$

148 . Coloring, {3, 7, 8, 9}

R: [4, 4, 5, 7, 7, 7, 5, 6, 2]

B: [2, 9, 4, 8, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	6 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[0, 1, 0, 5, 4, 2, 6, 0, 0], [0, 0, 0, 1, 6, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$

$$[0, y_4, 0, y_2, y_3, 2y_4, y_1, 0, 0]$$

$$p = s^3 - s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 6
 See Matrix

\$ [[6, 3, 2, 1, 0, 0, 0, 4, 2] , [6, 6, 0, 2, 0, 0, 0, 1, 3] , [4, 6, 0, 0, 0, 0, 0, 2, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8]] \$

$$[y_1, y_2, y_4, y_3, 0, 0, 0, y_5, y_6]$$

149 . Coloring, {4, 5, 6, 7}

R: [4, 4, 4, 8, 3, 8, 5, 1, 1]

B: [2, 9, 5, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 5	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
 See Matrix

\$ [[3, 0, 2, 6, 3, 0, 0, 4, 0] , [4, 0, 3, 5, 0, 0, 0, 6, 0] , [6, 0, 0, 7, 0, 0, 0, 5, 0] , [5, 0, 0, 6, 0, 0, 0, 7, 0] , [7, 0, 0, 5, 0, 0, 0, 6, 0]] \$

$$[y_4, 0, y_2, y_3, y_1, 0, 0, y_5, 0]$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 4
 See Matrix

\$ [[3, 4, 0, 0, 1, 2, 6, 0, 2] , [6, 5, 0, 0, 0, 0, 3, 0, 4] , [3, 10, 0, 0, 0, 0, 0, 0, 5] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$

$$[y_1, y_2, 0, 0, y_3, 2y_3, y_4, 0, y_5]$$

$$p = -s^4 + s^6$$

150 . Coloring, {4, 5, 6, 8}

$$\Omega p(\Delta)=0: \quad p = s^3 - s^4 + 4s^5 - 8s^7$$

R: [4, 4, 4, 8, 3, 8, 1, 6, 1]

B: [2, 9, 5, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4

See Matrix

$$\$ [[4, 0, 2, 6, 0, 2, 0, 4, 0], [0, 0, 0, 6, 0, 4, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0]] \$$$

$$[2y_1, 0, y_1, y_3, 0, y_2, 0, y_4, 0]$$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: {{5, 7}, {2, 9}} order: 2

See Matrix

$$\$ [[2, 4, 0, 0, 4, 0, 6, 0, 2], [0, 4, 0, 0, 6, 0, 4, 0, 4], [0, 4, 0, 0, 4, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 4, 0, 4], [0, 4, 0, 0, 4, 0, 6, 0, 4]] \$$$

$$[2y_1 - 2y_3, 2y_1, 0, 0, 5y_1 - 2y_2, 0, 2y_2, 0, 2y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

151 . Coloring, {4, 5, 6, 9}

R: [4, 4, 4, 8, 3, 8, 1, 1, 2]

B: [2, 9, 5, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 5	4 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
See Matrix

$\$ [[5, 1, 2, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0], [6, 0, 0, 4, 0, 0, 0, 8, 0], [8, 0, 0, 6, 0, 0, 0, 4, 0], [4, 0, 0, 8, 0, 0, 0, 6, 0]] \$$

$$[y_1, y_2, 2y_2, y_4, 0, 0, 0, y_3, 0]$$

$$p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}\}$ order: 6
See Matrix

$\$ [[1, 3, 0, 0, 4, 2, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 6, 0, 1], [1, 3, 0, 0, 6, 0, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 6, 0, 1]] \$$

$$[-y_1 + y_4 - y_2, y_1, 0, 0, y_4 - y_3, y_3, y_4, 0, y_2]$$

$$p = s^2 - s^5 \quad p' = -s^2 + s^5$$

152 . Coloring, $\{4, 5, 7, 8\}$

R: $[4, 4, 4, 8, 3, 7, 5, 6, 1]$

B: $[2, 9, 5, 7, 7, 8, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 6, 7, 8\}\}$ order: 6
See Matrix

\$ [[1, 0, 2, 6, 3, 2, 1, 3, 0] , [0, 0, 3, 3, 1, 3, 2, 6, 0] , [0, 0, 1, 3, 2, 6, 3, 3, 0] , [0, 0, 2, 1, 3, 3, 6, 3, 0] , [0, 0, 3, 2, 6, 3, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 1, 3, 0]] \$

$$[y_1, 0, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

Omega Rank for B : cycles: {{2, 9}} order: 4
See Matrix

\$ [[5, 4, 0, 0, 1, 0, 5, 1, 2] , [6, 7, 0, 0, 0, 0, 1, 0, 4] , [1, 10, 0, 0, 0, 0, 0, 0, 7] , [0, 8, 0, 0, 0, 0, 0, 0, 10] , [0, 10, 0, 0, 0, 0, 0, 0, 8] , [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$

$$[y_2, y_1, 0, 0, y_3, 0, y_5, y_3, y_4]$$

$$p = -s^4 + s^6$$

153 . Coloring, {4, 5, 7, 9}

R: [4, 4, 4, 8, 3, 7, 5, 1, 2]

B: [2, 9, 5, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 7	6 vs 7

Omega Rank for R : cycles: {{1, 4, 8}} order: 6
See Matrix

\$ [[2, 1, 2, 6, 3, 0, 1, 3, 0] , [3, 0, 3, 5, 1, 0, 0, 6, 0] , [6, 0, 1, 6, 0, 0, 0, 5, 0] , [5, 0, 0, 7, 0, 0, 0, 6, 0] , [6, 0, 0, 5, 0, 0, 0, 7, 0] , [7, 0, 0, 6, 0, 0, 0, 5, 0] , [5, 0, 0, 7, 0, 0, 0, 6, 0]] \$

$$[y_1, y_2, y_4, y_5, y_6, 0, y_2, y_3, 0]$$

$$p = -s^4 + s^7$$

Omega Rank for B : cycles: {{6, 8}, {1, 2, 9}} order: 6
See Matrix

\$ [[4, 3, 0, 0, 1, 2, 5, 1, 2], [7, 4, 0, 0, 0, 1, 1, 2, 3], [4, 7, 0, 0, 0, 2, 0, 1, 4], [4, 4, 0, 0, 0, 1, 0, 2, 7], [7, 4, 0, 0, 0, 2, 0, 1, 4], [4, 7, 0, 0, 0, 1, 0, 2, 4], [4, 4, 0, 0, 0, 2, 0, 1, 7]] \$

$$[y_1, -y_1 - y_3 + 5y_2 - y_6 + 5y_5 - y_4, 0, 0, y_3, y_2, y_6, y_5, y_4]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

154 . Coloring, {4, 5, 8, 9}

R: [4, 4, 4, 8, 3, 7, 1, 6, 2]

B: [2, 9, 5, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	3 vs 6

Omega Rank for R : cycles: {{1, 4, 6, 7, 8}} order: 5

See Matrix

\$ [[3, 1, 2, 6, 0, 2, 1, 3, 0], [1, 0, 0, 6, 0, 3, 2, 6, 0], [2, 0, 0, 1, 0, 6, 3, 6, 0], [3, 0, 0, 2, 0, 6, 6, 1, 0], [6, 0, 0, 3, 0, 1, 6, 2, 0], [6, 0, 0, 6, 0, 2, 1, 3, 0], [1, 0, 0, 6, 0, 3, 2, 6, 0]] \$

$$[y_5, y_6, 2y_6, y_4, 0, y_3, y_1, y_2, 0]$$

$$p = s^2 - s^7$$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}} order: 6

See Matrix

\$ [[3, 3, 0, 0, 4, 0, 5, 1, 2], [3, 3, 0, 0, 5, 0, 4, 0, 3], [3, 3, 0, 0, 4, 0, 5, 0, 3], [3, 3, 0, 0, 5, 0, 4, 0, 3], [3, 3, 0, 0, 4, 0, 5, 0, 3], [3, 3, 0, 0, 5, 0, 4, 0, 3]] \$

$$[y_3 + y_2, y_3 + y_2, 0, 0, 3y_3 + 3y_2 - y_1, 0, y_1, y_3, y_2]$$

$$p' = -s^3 + s^5 \quad p' = -s^2 + s^4 \quad p = s^2 - s^4$$

155 . Coloring, {4, 6, 7, 8}

R: [4, 4, 4, 8, 7, 8, 5, 6, 1]

B: [2, 9, 5, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$\$ [[1, 0, 0, 6, 3, 2, 2, 4, 0], [0, 0, 0, 1, 2, 4, 3, 8, 0], [0, 0, 0, 0, 3, 8, 2, 5, 0], [0, 0, 0, 0, 2, 5, 3, 8, 0], [0, 0, 0, 0, 3, 8, 2, 5, 0], [0, 0, 0, 0, 2, 5, 3, 8, 0]] \$$

$$[-14 y_1 + 39 y_2 - 14 y_3 - y_4, 0, 0, y_1, y_2, y_3, -5 y_1 + 14 y_2 - 5 y_3, y_4, 0]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

Omega Rank for B : cycles: {{2, 9}, {3, 5}} order: 4

See Matrix

$\$ [[5, 4, 2, 0, 1, 0, 4, 0, 2], [4, 7, 1, 0, 2, 0, 0, 0, 4], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8], [0, 8, 2, 0, 1, 0, 0, 0, 7], [0, 7, 1, 0, 2, 0, 0, 0, 8]] \$$

$$[y_4, y_3, y_2, 0, y_1, 0, -y_3 + 3 y_2 + 2 y_1, 0, -y_4 + 2 y_2 + 3 y_1]$$

$$p = -s^3 + s^5 \quad p' = s^3 - s^5$$

156 . Coloring, {4, 6, 7, 9}

R: [4, 4, 4, 8, 7, 8, 5, 1, 2]

B: [2, 9, 5, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 6	6 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 6, 3, 0, 2, 4, 0], [4, 0, 0, 3, 2, 0, 3, 6, 0], [6, 0, 0, 4, 3, 0, 2, 3, 0], [3, 0, 0, 6, 2, 0, 3, 4, 0], [4, 0, 0, 3, 3, 0, 2, 6, 0], [6, 0, 0, 4, 2, 0, 3, 3, 0]] \$$

$$[-5y_1 - 5y_2 + 13y_3 + 13y_4 - 5y_5, 5y_1, 0, 5y_2, 5y_3, 0, 5y_4, 5y_5, 0]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 0, 1, 2, 4, 0, 2], [6, 4, 1, 0, 2, 0, 2, 0, 3], [5, 6, 2, 0, 1, 0, 0, 0, 4], [4, 5, 1, 0, 2, 0, 0, 0, 6], [6, 4, 2, 0, 1, 0, 0, 0, 5], [5, 6, 1, 0, 2, 0, 0, 0, 4], [4, 5, 2, 0, 1, 0, 0, 0, 6]] \$$

$$[-y_1 + 5y_2 + 5y_3 - y_4 - y_5 - y_6, y_1, y_2, 0, y_3, y_4, y_5, 0, y_6]$$

$$p = s^3 + s^4 - s^6 - s^7$$

157 . Coloring, $\{4, 6, 8, 9\}$

R: $[4, 4, 4, 8, 7, 8, 1, 6, 2]$

B: $[2, 9, 5, 7, 3, 7, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{6, 8}} order: 4
See Matrix

\$ [[3, 1, 0, 6, 0, 2, 2, 4, 0] , [2, 0, 0, 4, 0, 4, 0, 8, 0] , [0, 0, 0, 2, 0, 8, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0] , [0, 0, 0, 0, 0, 10, 0, 8, 0] , [0, 0, 0, 0, 0, 8, 0, 10, 0]] \$

$$[y_4, y_5, 0, y_3, 0, y_2, 2y_5, y_1, 0]$$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 5}} order: 6
See Matrix

\$ [[3, 3, 2, 0, 4, 0, 4, 0, 2] , [2, 3, 4, 0, 6, 0, 0, 0, 3] , [3, 2, 6, 0, 4, 0, 0, 0, 3] , [3, 3, 4, 0, 6, 0, 0, 0, 2] , [2, 3, 6, 0, 4, 0, 0, 0, 3] , [3, 2, 4, 0, 6, 0, 0, 0, 3]] \$

$$[4y_5, 4y_4, 4y_3, 0, 4y_2, 0, 5y_5 + 5y_4 - 4y_3 - 4y_2 + 5y_1, 0, 4y_1]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

158 . Coloring, {4, 7, 8, 9}

R: [4, 4, 4, 8, 7, 7, 5, 6, 2]

B: [2, 9, 5, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 6	5 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 6
See Matrix

\$ [[0, 1, 0, 6, 3, 2, 3, 3, 0] , [0, 0, 0, 1, 3, 3, 5, 6, 0] , [0, 0, 0, 0, 5, 6, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 11, 0, 0] , [0, 0, 0, 0, 11, 0, 7, 0, 0] , [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$

$$[0, y_5, 0, y_4, y_3, y_1, y_2, y_6, 0]$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[6, 3, 2, 0, 1, 0, 3, 1, 2], [6, 6, 1, 0, 2, 0, 0, 0, 3], [3, 6, 2, 0, 1, 0, 0, 0, 6], [6, 3, 1, 0, 2, 0, 0, 0, 6], [6, 6, 2, 0, 1, 0, 0, 0, 3], [3, 6, 1, 0, 2, 0, 0, 0, 6], [6, 3, 2, 0, 1, 0, 0, 0, 6]] \$$

$$[-y_5 + 5y_4 + 5y_3 - 4y_2 - y_1, y_5, y_4, 0, y_3, 0, 3y_2, y_2, y_1]$$

$$p' = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 + s^3 - s^5 - s^6$$

159 . Coloring, $\{5, 6, 7, 8\}$

R: $[4, 4, 4, 7, 3, 8, 5, 6, 1]$

B: $[2, 9, 5, 8, 7, 7, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{6, 8\}, \{3, 4, 5, 7\}\}$ order: 4

See Matrix

$\$ [[1, 0, 2, 6, 3, 2, 3, 1, 0], [0, 0, 3, 3, 3, 1, 6, 2, 0], [0, 0, 3, 3, 6, 2, 3, 1, 0], [0, 0, 6, 3, 3, 1, 3, 2, 0], [0, 0, 3, 6, 3, 2, 3, 1, 0], [0, 0, 3, 3, 3, 1, 6, 2, 0], [0, 0, 3, 3, 6, 2, 3, 1, 0]] \$$

$$[-y_1 + y_3 + 4y_5 - y_4, 0, y_1, -y_2 + 4y_3 + y_5, y_2, y_3, y_4, y_5, 0]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 4

See Matrix

$\$ [[5, 4, 0, 0, 1, 0, 3, 3, 2], [6, 7, 0, 0, 0, 0, 1, 0, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$

$$[y_2, y_1, 0, 0, y_3, 0, y_4, 3y_3, y_5]$$

$$p = -s^4 + s^6$$

160 . Coloring, {5, 6, 7, 9}

R: [4, 4, 4, 7, 3, 8, 5, 1, 2]

B: [2, 9, 5, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 7	6 vs 7

Omega Rank for R : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

\$ [[2, 1, 2, 6, 3, 0, 3, 1, 0], [1, 0, 3, 5, 3, 0, 6, 0, 0], [0, 0, 3, 4, 6, 0, 5, 0, 0], [0, 0, 6, 3, 5, 0, 4, 0, 0], [0, 0, 5, 6, 4, 0, 3, 0, 0], [0, 0, 4, 5, 3, 0, 6, 0, 0], [0, 0, 3, 4, 6, 0, 5, 0, 0]] \$

[$y_1, y_3, y_6, y_4, y_5, 0, y_2, y_3, 0$]

$$p = -s^3 + s^7$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6

See Matrix

\$ [[4, 3, 0, 0, 1, 2, 3, 3, 2], [5, 4, 0, 0, 0, 3, 3, 0, 3], [6, 5, 0, 0, 0, 0, 3, 0, 4], [7, 6, 0, 0, 0, 0, 0, 0, 5], [5, 7, 0, 0, 0, 0, 0, 0, 6], [6, 5, 0, 0, 0, 0, 0, 0, 7], [7, 6, 0, 0, 0, 0, 0, 0, 5]] \$

[$y_2, y_1, 0, 0, y_4, y_3, y_6, 3y_4, y_5$]

$$p = -s^4 + s^7$$

161 . Coloring, {5, 6, 8, 9}

R: [4, 4, 4, 7, 3, 8, 1, 6, 2]

B: [2, 9, 5, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 6

Omega Rank for R : cycles: {{6, 8}, {1, 4, 7}} order: 6

See Matrix

\$ [[3, 1, 2, 6, 0, 2, 3, 1, 0], [3, 0, 0, 6, 0, 1, 6, 2, 0], [6, 0, 0, 3, 0, 2, 6, 1, 0], [6, 0, 0, 6, 0, 1, 3, 2, 0], [3, 0, 0, 6, 0, 2, 6, 1, 0], [6, 0, 0, 3, 0, 1, 6, 2, 0], [6, 0, 0, 6, 0, 2, 3, 1, 0]] \$

$$[-3y_1 - y_2 + 5y_3 - y_4 + 5y_5, y_1, 2y_1, y_2, 0, y_3, y_4, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}} order: 6

See Matrix

\$ [[3, 3, 0, 0, 4, 0, 3, 3, 2], [5, 3, 0, 0, 3, 0, 4, 0, 3], [3, 5, 0, 0, 4, 0, 3, 0, 3], [3, 3, 0, 0, 3, 0, 4, 0, 5], [5, 3, 0, 0, 4, 0, 3, 0, 3], [3, 5, 0, 0, 3, 0, 4, 0, 3]] \$

$$[-7y_1 + 11y_2 + 11y_3 - 7y_4 - 7y_5, 7y_1, 0, 0, 7y_2, 0, 7y_3, 7y_4, 7y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

162 . Coloring, {5, 7, 8, 9}

R: [4, 4, 4, 7, 3, 7, 5, 6, 2]

B: [2, 9, 5, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}\}$ order: 4
 See Matrix

$\$ [[0, 1, 2, 6, 3, 2, 4, 0, 0], [0, 0, 3, 3, 4, 0, 8, 0, 0], [0, 0, 4, 3, 8, 0, 3, 0, 0], [0, 0, 8, 4, 3, 0, 3, 0, 0], [0, 0, 3, 8, 3, 0, 4, 0, 0], [0, 0, 3, 3, 4, 0, 8, 0, 0]] \$$

$$[0, y_1, y_2, y_5, y_4, 2y_1, y_3, 0, 0]$$

$$p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 3
 See Matrix

$\$ [[6, 3, 0, 0, 1, 0, 2, 4, 2], [8, 6, 0, 0, 0, 0, 1, 0, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4], [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$$

$$[y_1, y_2, 0, 0, y_3, 0, y_5, 4y_3, y_4]$$

$$p = -s^3 + s^6$$

163 . Coloring, $\{6, 7, 8, 9\}$

R: [4, 4, 4, 7, 7, 8, 5, 6, 2]

B: [2, 9, 5, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[0, 1, 0, 6, 3, 2, 5, 1, 0], [0, 0, 0, 1, 5, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0]] \$$

$$[0, y_2 + 4y_4 - y_3, 0, -y_1 + 4y_2 + y_4, y_1, y_2, y_3, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}\}$ order: 6
See Matrix

$$\$ [[6, 3, 2, 0, 1, 0, 1, 3, 2], [6, 6, 1, 0, 2, 0, 0, 0, 3], [3, 6, 2, 0, 1, 0, 0, 0, 6], [6, 3, 1, 0, 2, 0, 0, 0, 6], [6, 6, 2, 0, 1, 0, 0, 0, 3], [3, 6, 1, 0, 2, 0, 0, 0, 6], [6, 3, 2, 0, 1, 0, 0, 0, 6]] \$$$

$$[y_2, y_3, y_4, 0, y_5, 0, y_1, 3y_1, -y_2 - y_3 + 5y_4 + 5y_5 - 4y_1]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

164 . Coloring, $\{2, 3, 4, 5, 6\}$

R: [4, 9, 5, 8, 3, 8, 1, 1, 1]

B: [2, 4, 4, 7, 7, 7, 5, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 5

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{3, 5\}\}$ order: 6
See Matrix

$$\$ [[6, 0, 2, 3, 1, 0, 0, 4, 2], [6, 0, 1, 6, 2, 0, 0, 3, 0], [3, 0, 2, 6, 1, 0, 0, 6, 0], [6, 0, 1, 3, 2, 0, 0, 6, 0], [6, 0, 2, 6, 1, 0, 0, 3, 0], [3, 0, 1, 6, 2, 0, 0, 6, 0]] \$$$

$$[y_5, 0, y_3, y_4, y_2, 0, 0, y_1, -y_5 + 5y_3 - y_4 + 5y_2 - y_1]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4
See Matrix

$$\$ [[0, 4, 0, 3, 3, 2, 6, 0, 0], [0, 0, 0, 4, 6, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$$

$$[0, 2 y_3, 0, y_1, y_2, y_3, y_4, 0, 0]$$

$$p = -s^3 + s^5$$

165 . Coloring, {2, 3, 4, 5, 7}

R: [4, 9, 5, 8, 3, 7, 5, 1, 1]

B: [2, 4, 4, 7, 7, 8, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}} order: 6

See Matrix

$$\$ [[3, 0, 2, 3, 4, 0, 1, 3, 2], [5, 0, 4, 3, 3, 0, 0, 3, 0], [3, 0, 3, 5, 4, 0, 0, 3, 0], [3, 0, 4, 3, 3, 0, 0, 5, 0], [5, 0, 3, 3, 4, 0, 0, 3, 0], [3, 0, 4, 5, 3, 0, 0, 3, 0], [3, 0, 3, 3, 4, 0, 0, 5, 0]] \$$$

$$[3 y_2, 0, 3 y_1, 3 y_4, 3 y_5, 0, -7 y_2 + 11 y_1 - 7 y_4 + 11 y_5 - 7 y_3, 3 y_3, -14 y_2 + 22 y_1 - 14 y_4 + 22 y_5 - 14 y_3]$$

$$p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 4, 0, 3, 0, 2, 5, 1, 0], [5, 3, 0, 4, 0, 1, 3, 2, 0], [3, 5, 0, 3, 0, 2, 4, 1, 0], [4, 3, 0, 5, 0, 1, 3, 2, 0], [3, 4, 0, 3, 0, 2, 5, 1, 0], [5, 3, 0, 4, 0, 1, 3, 2, 0]] \$$$

$$[y_1 + 4 y_3 - y_2, 4 y_1 - y_4 + y_3, 0, y_2, 0, y_1, y_4, y_3, 0]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

166 . Coloring, {2, 3, 4, 5, 8}

R: [4, 9, 5, 8, 3, 7, 1, 6, 1]

B: [2, 4, 4, 7, 7, 8, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 8	6 vs 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 6, 7, 8}}

See Matrix

\$ [[4, 0, 2, 3, 1, 2, 1, 3, 2], [3, 0, 1, 4, 2, 3, 2, 3, 0], [2, 0, 2, 3, 1, 3, 3, 4, 0], [3, 0, 1, 2, 2, 4, 3, 3, 0], [3, 0, 2, 3, 1, 3, 4, 2, 0], [4, 0, 1, 3, 2, 2, 3, 3, 0], [3, 0, 2, 4, 1, 3, 2, 3, 0], [2, 0, 1, 3, 2, 3, 3, 4, 0]] \$

[5 y₅ - y₆ + 5 y₄ - y₂ - y₃ - y₁ - y₇, 0, y₅, y₆, y₄, y₂, y₃, y₁, y₇]

$$p = -s^2 - s^3 + s^7 + s^8$$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

\$ [[2, 4, 0, 3, 3, 0, 5, 1, 0], [1, 2, 0, 4, 5, 0, 6, 0, 0], [0, 1, 0, 2, 6, 0, 9, 0, 0], [0, 0, 0, 1, 9, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0]] \$

[y₁, y₂, 0, y₃, y₄, 0, y₅, y₆, 0]

167 . Coloring, {2, 3, 4, 5, 9}

R: [4, 9, 5, 8, 3, 7, 1, 1, 2]

B: [2, 4, 4, 7, 7, 8, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	5 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{1, 4, 8\}\}$ order: 6
 See Matrix

$\$ [[5, 1, 2, 3, 1, 0, 1, 3, 2], [4, 2, 1, 5, 2, 0, 0, 3, 1], [3, 1, 2, 4, 1, 0, 0, 5, 2], [5, 2, 1, 3, 2, 0, 0, 4, 1], [4, 1, 2, 5, 1, 0, 0, 3, 2], [3, 2, 1, 4, 2, 0, 0, 5, 1], [5, 1, 2, 3, 1, 0, 0, 4, 2], [4, 2, 1, 5, 2, 0, 0, 3, 1]] \$$

$$[4y_1 + 4y_4 - y_5 - y_3 - y_2, y_1, y_4, y_5, y_1, 0, y_3, y_2, y_4]$$

$$p = s^2 + s^3 - s^5 - s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[1, 3, 0, 3, 3, 2, 5, 1, 0], [0, 1, 0, 3, 5, 1, 6, 2, 0], [0, 0, 0, 1, 6, 2, 8, 1, 0], [0, 0, 0, 0, 8, 1, 7, 2, 0], [0, 0, 0, 0, 7, 2, 8, 1, 0], [0, 0, 0, 0, 8, 1, 7, 2, 0], [0, 0, 0, 0, 7, 2, 8, 1, 0]] \$$

$$[y_3, y_4, 0, -y_3 - y_1 + 2y_2 + 3y_5, y_1, y_2, -y_4 + 3y_2 + 2y_5, y_5, 0]$$

$$p' = -s^4 + s^6 \quad p = -s^4 + s^6$$

168 . Coloring, $\{2, 3, 4, 6, 7\}$

$$\Omega p(\Delta)=0: \quad p = s^2 + 2s^4 - 8s^5 - 16s^7$$

R: $[4, 9, 5, 8, 7, 8, 5, 1, 1]$

B: $[2, 4, 4, 7, 3, 7, 1, 6, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}\}$ order: 6
 See Matrix

$\$ [[3, 0, 0, 3, 4, 0, 2, 4, 2], [6, 0, 0, 3, 2, 0, 4, 3, 0], [3, 0, 0, 6, 4, 0, 2, 3, 0], [3, 0, 0, 3, 2, 0, 4, 6, 0], [6, 0, 0, 3, 4, 0, 2, 3, 0], [3, 0, 0, 6, 2, 0, 4, 3, 0]] \$$

$$[-y_2 + 2y_3 + 2y_1 - y_4 - y_5, 0, 0, y_2, y_3, 0, y_1, y_4, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

$$\$ [[3, 4, 2, 3, 0, 2, 4, 0, 0], [4, 3, 0, 6, 0, 0, 5, 0, 0], [5, 4, 0, 3, 0, 0, 6, 0, 0], [6, 5, 0, 4, 0, 0, 3, 0, 0], [3, 6, 0, 5, 0, 0, 4, 0, 0], [4, 3, 0, 6, 0, 0, 5, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_3, y_5, 0, 0]$$

$$p = -s^2 + s^6$$

169 . Coloring, {2, 3, 4, 6, 8}

R: [4, 9, 5, 8, 7, 8, 1, 6, 1]

B: [2, 4, 4, 7, 3, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	6 vs 6

Omega Rank for R : cycles: {{6, 8}} order: 6

See Matrix

$$\$ [[4, 0, 0, 3, 1, 2, 2, 4, 2], [4, 0, 0, 4, 0, 4, 1, 5, 0], [1, 0, 0, 4, 0, 5, 0, 8, 0], [0, 0, 0, 1, 0, 8, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 0, 9, 0, 9, 0]] \$$$

$$[y_2 + y_1 + y_3 + y_4 - y_5, 0, 0, y_2, y_1, y_3, y_4, y_5, 2y_1]$$

$$p = -s^5 + s^7 \quad p = -s^5 + s^6$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

$$\$ [[2, 4, 2, 3, 3, 0, 4, 0, 0], [0, 2, 3, 6, 4, 0, 3, 0, 0], [0, 0, 4, 5, 3, 0, 6, 0, 0], [0, 0, 3, 4, 6, 0, 5, 0, 0], [0, 0, 6, 3, 5, 0, 4, 0, 0], [0, 0, 5, 6, 4, 0, 3, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, y_6, 0, y_5, 0, 0]$$

170 . Coloring, {2, 3, 4, 6, 9}

R: [4, 9, 5, 8, 7, 8, 1, 1, 2]

B: [2, 4, 4, 7, 3, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 7	5 vs 7

Omega Rank for R : cycles: {{2, 9}, {1, 4, 8}} order: 6

See Matrix

$$\$ [[5, 1, 0, 3, 1, 0, 2, 4, 2], [6, 2, 0, 5, 0, 0, 1, 3, 1], [4, 1, 0, 6, 0, 0, 0, 5, 2], [5, 2, 0, 4, 0, 0, 0, 6, 1], [6, 1, 0, 5, 0, 0, 0, 4, 2], [4, 2, 0, 6, 0, 0, 0, 5, 1], [5, 1, 0, 4, 0, 0, 0, 6, 2]] \$$$

$$[5 y_1 - y_6 - y_5 - y_4 - y_3 + 5 y_2, y_1, 0, y_6, y_5, 0, y_4, y_3, y_2]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

$$\$ [[1, 3, 2, 3, 3, 2, 4, 0, 0], [0, 1, 3, 5, 4, 0, 5, 0, 0], [0, 0, 4, 4, 5, 0, 5, 0, 0], [0, 0, 5, 4, 5, 0, 4, 0, 0], [0, 0, 5, 5, 4, 0, 4, 0, 0], [0, 0, 4, 5, 4, 0, 5, 0, 0], [0, 0, 4, 4, 5, 0, 5, 0, 0]] \$$$

$$[y_1, 3 y_1 - y_2 + y_3 + y_4 - y_5, y_2, y_3, y_4, 2 y_1, y_5, 0, 0]$$

$$p = -s^3 + s^7 \quad p = -s^3 + s^4 - s^5 + s^6$$

171 . Coloring, {2, 3, 4, 7, 8}

R: [4, 9, 5, 8, 7, 7, 5, 6, 1]

B: [2, 4, 4, 7, 3, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	7 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 0, 0, 3, 4, 2, 3, 3, 2], [2, 0, 0, 1, 3, 3, 6, 3, 0], [0, 0, 0, 2, 6, 3, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0]] \$$

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6, y_7]$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[5, 4, 2, 3, 0, 0, 3, 1, 0], [4, 5, 0, 6, 0, 0, 3, 0, 0], [3, 4, 0, 5, 0, 0, 6, 0, 0], [6, 3, 0, 4, 0, 0, 5, 0, 0], [5, 6, 0, 3, 0, 0, 4, 0, 0], [4, 5, 0, 6, 0, 0, 3, 0, 0]] \$$

$$[y_2, y_3, 2y_5, y_1, 0, 0, y_4, y_5, 0]$$

$$p = -s^2 + s^6$$

172 . Coloring, $\{2, 3, 4, 7, 9\}$

R: [4, 9, 5, 8, 7, 7, 5, 1, 2]

B: [2, 4, 4, 7, 3, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 3, 4, 0, 3, 3, 2], [3, 2, 0, 2, 3, 0, 4, 3, 1], [3, 1, 0, 3, 4, 0, 3, 2, 2], [2, 2, 0, 3, 3, 0, 4, 3, 1], [3, 1, 0, 2, 4, 0, 3, 3, 2], [3, 2, 0, 3, 3, 0, 4, 2, 1], [2, 1, 0, 3, 4, 0, 3, 3, 2]] \$$

$$[-2y_1 + 8y_3 - 2y_2 - 8y_4, 3y_3 - 5y_4, 0, 2y_1, 2y_3, 0, 5y_3 - 7y_4, 2y_2, 2y_4]$$

$$p = -s - s^2 + s^4 + s^5 \quad p = s - s^3 - s^4 + s^6 \quad p = -s + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[4, 3, 2, 3, 0, 2, 3, 1, 0], [3, 4, 0, 5, 0, 1, 3, 2, 0], [3, 3, 0, 4, 0, 2, 5, 1, 0], [5, 3, 0, 3, 0, 1, 4, 2, 0], [4, 5, 0, 3, 0, 2, 3, 1, 0], [3, 4, 0, 5, 0, 1, 3, 2, 0], [3, 3, 0, 4, 0, 2, 5, 1, 0]] \$$

$$[-y_2 + 2y_3 + 3y_5, -y_1 + 3y_3 - y_4 + 2y_5, y_1, y_2, 0, y_3, y_4, y_5, 0]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

173 . Coloring, $\{2, 3, 4, 8, 9\}$

R: $[4, 9, 5, 8, 7, 7, 1, 6, 2]$

B: $[2, 4, 4, 7, 3, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	3 vs 8	6 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 6, 7, 8\}\}$

See Matrix

$\$ [[3, 1, 0, 3, 1, 2, 3, 3, 2], [3, 2, 0, 3, 0, 3, 3, 3, 1], [3, 1, 0, 3, 0, 3, 3, 3, 2], [3, 2, 0, 3, 0, 3, 3, 3, 1], [3, 1, 0, 3, 0, 3, 3, 3, 2], [3, 2, 0, 3, 0, 3, 3, 3, 1], [3, 1, 0, 3, 0, 3, 3, 3, 2], [3, 2, 0, 3, 0, 3, 3, 3, 1]] \$$

$$[y_2, y_2 - y_3, 0, y_2, y_2 - y_1, y_1, y_2, y_2, y_3]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^8 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4

See Matrix

$$\$ [[3, 3, 2, 3, 3, 0, 3, 1, 0], [1, 3, 3, 5, 3, 0, 3, 0, 0], [0, 1, 3, 6, 3, 0, 5, 0, 0], [0, 0, 3, 4, 5, 0, 6, 0, 0], [0, 0, 5, 3, 6, 0, 4, 0, 0], [0, 0, 6, 5, 4, 0, 3, 0, 0], [0, 0, 4, 6, 3, 0, 5, 0, 0]] \$$$

$$[y_2, y_1, y_2 - y_1 + y_6 + y_5 - y_4 - y_3, y_6, y_5, 0, y_4, y_3, 0]$$

$$p = -s^4 + s^5 - s^6 + s^7$$

174 . Coloring, {2, 3, 5, 6, 7}

R: [4, 9, 5, 7, 3, 8, 5, 1, 1]

B: [2, 4, 4, 8, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 7	4 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

$$\$ [[3, 0, 2, 3, 4, 0, 3, 1, 2], [3, 0, 4, 3, 5, 0, 3, 0, 0], [0, 0, 5, 3, 7, 0, 3, 0, 0], [0, 0, 7, 0, 8, 0, 3, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0]] \$$$

$$[y_1, 0, y_2, y_3, y_4, 0, y_5, y_6, 2y_6]$$

$$p = -s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}} order: 6

See Matrix

$$\$ [[3, 4, 0, 3, 0, 2, 3, 3, 0], [3, 3, 0, 4, 0, 3, 2, 3, 0], [2, 3, 0, 3, 0, 3, 3, 4, 0], [3, 2, 0, 3, 0, 4, 3, 3, 0], [3, 3, 0, 2, 0, 3, 4, 3, 0], [4, 3, 0, 3, 0, 3, 3, 2, 0]] \$$$

$$[y_4 + y_2 - y_1, y_3, 0, y_4, 0, -y_3 + y_4 + y_2, y_2, y_1, 0]$$

$$p = s - s^3 + s^4 - s^6 \quad p' = s - s^2 + s^4 - s^5$$

175 . Coloring, {2, 3, 5, 6, 8}

R: [4, 9, 5, 7, 3, 8, 1, 6, 1]

B: [2, 4, 4, 8, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	4 vs 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 7}, {6, 8}} order: 6

See Matrix

\$ [[4, 0, 2, 3, 1, 2, 3, 1, 2], [5, 0, 1, 4, 2, 1, 3, 2, 0], [3, 0, 2, 5, 1, 2, 4, 1, 0], [4, 0, 1, 3, 2, 1, 5, 2, 0], [5, 0, 2, 4, 1, 2, 3, 1, 0], [3, 0, 1, 5, 2, 1, 4, 2, 0], [4, 0, 2, 3, 1, 2, 5, 1, 0], [5, 0, 1, 4, 2, 1, 3, 2, 0]] \$

$$[y_4, 0, y_3, y_1, y_2, y_3, -y_4 + 4y_3 - y_1 + 4y_2 - y_5, y_2, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p' = -s^2 - s^3 + s^5 + s^6 \quad p = -s^2 + s^8$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}} order: 4

See Matrix

\$ [[2, 4, 0, 3, 3, 0, 3, 3, 0], [3, 2, 0, 4, 3, 0, 3, 3, 0], [3, 3, 0, 2, 3, 0, 3, 4, 0], [4, 3, 0, 3, 3, 0, 3, 2, 0], [2, 4, 0, 3, 3, 0, 3, 3, 0], [3, 2, 0, 4, 3, 0, 3, 3, 0]] \$

$$[-y_1 - y_2 + 4y_3 - y_4, y_1, 0, y_2, y_3, 0, y_3, y_4, 0]$$

$$p = s - s^5 \quad p' = s - s^5$$

176 . Coloring, {2, 3, 5, 6, 9}

R: [4, 9, 5, 7, 3, 8, 1, 1, 2]

B: [2, 4, 4, 8, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	6 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{1, 4, 7\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 3, 1, 0, 3, 1, 2], [4, 2, 1, 5, 2, 0, 3, 0, 1], [3, 1, 2, 4, 1, 0, 5, 0, 2], [5, 2, 1, 3, 2, 0, 4, 0, 1], [4, 1, 2, 5, 1, 0, 3, 0, 2], [3, 2, 1, 4, 2, 0, 5, 0, 1], [5, 1, 2, 3, 1, 0, 4, 0, 2], [4, 2, 1, 5, 2, 0, 3, 0, 1]] \$$

$$[4y_4 - y_3 + 4y_2 - y_1 - y_5, y_2, y_4, y_3, y_2, 0, y_1, y_5, y_4]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 3, 0, 3, 3, 2, 3, 3, 0], [0, 1, 0, 3, 3, 3, 5, 3, 0], [0, 0, 0, 1, 5, 3, 6, 3, 0], [0, 0, 0, 0, 6, 3, 8, 1, 0], [0, 0, 0, 0, 8, 1, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[y_1, y_1 + y_6 + y_4 + y_5 - y_3 - y_2, 0, y_6, y_4, y_5, y_3, y_2, 0]$$

$$p = -s^6 + s^7$$

177 . Coloring, $\{2, 3, 5, 7, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^3 + s^4 + 4s^5 + 8s^7$$

R: [4, 9, 5, 7, 3, 7, 5, 6, 1]

B: [2, 4, 4, 8, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	6 vs 7	5 vs 5

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

\$ [[1, 0, 2, 3, 4, 2, 4, 0, 2], [2, 0, 4, 1, 6, 0, 5, 0, 0], [0, 0, 6, 2, 9, 0, 1, 0, 0], [0, 0, 9, 0, 7, 0, 2, 0, 0], [0, 0, 7, 0, 11, 0, 0, 0, 0], [0, 0, 11, 0, 7, 0, 0, 0, 0], [0, 0, 7, 0, 11, 0, 0, 0, 0]] \$

$$[y_1, 0, y_2, y_3, y_4, y_6, y_5, 0, y_6]$$

$$p = s^5 - s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

\$ [[5, 4, 0, 3, 0, 0, 2, 4, 0], [6, 5, 0, 4, 0, 0, 0, 3, 0], [3, 6, 0, 5, 0, 0, 0, 4, 0], [4, 3, 0, 6, 0, 0, 0, 5, 0], [5, 4, 0, 3, 0, 0, 0, 6, 0]] \$

$$[y_1, y_2, 0, y_3, 0, 0, y_4, y_5, 0]$$

178 . Coloring, {2, 3, 5, 7, 9}

R: [4, 9, 5, 7, 3, 7, 5, 1, 2]

B: [2, 4, 4, 8, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: {{2, 9}, {3, 5}} order: 4

See Matrix

\$ [[2, 1, 2, 3, 4, 0, 4, 0, 2], [0, 2, 4, 2, 6, 0, 3, 0, 1], [0, 1, 6, 0, 7, 0, 2, 0, 2], [0, 2, 7, 0, 8, 0, 0, 0, 1], [0, 1, 8, 0, 7, 0, 0, 0, 2], [0, 2, 7, 0, 8, 0, 0, 0, 1], [0, 1, 8, 0, 7, 0, 0, 0, 2]] \$

$$[y_2, y_1, -y_2 + 2y_1 - y_3 + 3y_5, 3y_1 - y_4 + 2y_5, y_4, 0, y_3, 0, y_5]$$

$$p' = -s^4 + s^6 \quad p = s^4 - s^6$$

Omega Rank for B : cycles: {{6, 8}} order: 6

See Matrix

\$ [[4, 3, 0, 3, 0, 2, 2, 4, 0], [2, 4, 0, 3, 0, 4, 0, 5, 0], [0, 2, 0, 4, 0, 5, 0, 7, 0], [0, 0, 0, 2, 0, 7, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0]] \$

$$[y_1, y_1 + y_2 + y_3 - y_5 - y_4, 0, y_2, 0, y_3, y_5, y_4, 0]$$

$$p = -s^5 + s^6$$

179 . Coloring, {2, 3, 5, 8, 9}

R: [4, 9, 5, 7, 3, 7, 1, 6, 2]

B: [2, 4, 4, 8, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 8	4 vs 6

Omega Rank for R : cycles: {{2, 9}, {3, 5}, {1, 4, 7}} order: 6

See Matrix

\$ [[3, 1, 2, 3, 1, 2, 4, 0, 2], [4, 2, 1, 3, 2, 0, 5, 0, 1], [5, 1, 2, 4, 1, 0, 3, 0, 2], [3, 2, 1, 5, 2, 0, 4, 0, 1], [4, 1, 2, 3, 1, 0, 5, 0, 2], [5, 2, 1, 4, 2, 0, 3, 0, 1], [3, 1, 2, 5, 1, 0, 4, 0, 2], [4, 2, 1, 3, 2, 0, 5, 0, 1]] \$

$$[4y_5 - y_1 + 4y_2 - y_3 - y_4, y_2, y_5, y_1, y_2, y_3, y_4, 0, y_5]$$

$$p = -s^2 + s^8 \quad p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}, {5, 7}} order: 4

See Matrix

\$ [[3, 3, 0, 3, 3, 0, 2, 4, 0], [4, 3, 0, 3, 2, 0, 3, 3, 0], [3, 4, 0, 3, 3, 0, 2, 3, 0], [3, 3, 0, 4, 2, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 2, 4, 0], [4, 3, 0, 3, 2, 0, 3, 3, 0]] \$

$$[9y_1 - 4y_3 - 13y_2 + 9y_4, 4y_1, 0, 4y_3, 4y_2, 0, 5y_1 - 9y_2 + 5y_4, 4y_4, 0]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

180 . Coloring, {2, 3, 6, 7, 8}

R: [4, 9, 5, 7, 7, 8, 5, 6, 1]

B: [2, 4, 4, 8, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[1, 0, 0, 3, 4, 2, 5, 1, 2], [2, 0, 0, 1, 5, 1, 7, 2, 0], [0, 0, 0, 2, 7, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0], [0, 0, 0, 0, 6, 1, 9, 2, 0], [0, 0, 0, 0, 9, 2, 6, 1, 0]] \$$$

$$[y_5, 0, 0, y_3, y_4, y_2, -y_5 + y_2 + 4y_1, y_1, -y_3 - y_4 + 4y_2 + y_1]$$

$$p' = s^4 - s^6 \quad p = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

$$\$ [[5, 4, 2, 3, 0, 0, 1, 3, 0], [4, 5, 0, 6, 0, 0, 0, 3, 0], [3, 4, 0, 5, 0, 0, 0, 6, 0], [6, 3, 0, 4, 0, 0, 0, 5, 0], [5, 6, 0, 3, 0, 0, 0, 4, 0], [4, 5, 0, 6, 0, 0, 0, 3, 0]] \$$$

$$[y_1, y_2, 2y_4, y_3, 0, 0, y_4, y_5, 0]$$

$$p = -s^2 + s^6$$

181 . Coloring, {2, 3, 6, 7, 9}

R: [4, 9, 5, 7, 7, 8, 5, 1, 2]

B: [2, 4, 4, 8, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	6 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 3, 4, 0, 5, 1, 2], [1, 2, 0, 2, 5, 0, 7, 0, 1], [0, 1, 0, 1, 7, 0, 7, 0, 2], [0, 2, 0, 0, 7, 0, 8, 0, 1], [0, 1, 0, 0, 8, 0, 7, 0, 2], [0, 2, 0, 0, 7, 0, 8, 0, 1], [0, 1, 0, 0, 8, 0, 7, 0, 2]] \$$

$$[y_5, y_3, 0, y_4, y_2, 0, -y_5 + 3y_3 + 2y_1, 2y_3 - y_4 - y_2 + 3y_1, y_1]$$

$$p' = s^4 - s^6 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 6, 7, 8\}\}$ order: 6

See Matrix

$\$ [[4, 3, 2, 3, 0, 2, 1, 3, 0], [1, 4, 0, 5, 0, 3, 2, 3, 0], [2, 1, 0, 4, 0, 3, 3, 5, 0], [3, 2, 0, 1, 0, 5, 3, 4, 0], [3, 3, 0, 2, 0, 4, 5, 1, 0], [5, 3, 0, 3, 0, 1, 4, 2, 0], [4, 5, 0, 3, 0, 2, 1, 3, 0]] \$$

$$[y_2 + y_3 - y_4 - y_1 + y_5 + y_6, y_2, y_3, y_4, 0, y_1, y_5, y_6, 0]$$

$$p = -s^2 + s^3 - s^4 + s^5 - s^6 + s^7$$

182 . Coloring, $\{2, 3, 6, 8, 9\}$

R: [4, 9, 5, 7, 7, 8, 1, 6, 2]

B: [2, 4, 4, 8, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	6 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{2, 9\}, \{6, 8\}\}$ order: 6
 See Matrix

$\$ [[3, 1, 0, 3, 1, 2, 5, 1, 2], [5, 2, 0, 3, 0, 1, 4, 2, 1], [4, 1, 0, 5, 0, 2, 3, 1, 2], [3, 2, 0, 4, 0, 1, 5, 2, 1], [5, 1, 0, 3, 0, 2, 4, 1, 2], [4, 2, 0, 5, 0, 1, 3, 2, 1], [3, 1, 0, 4, 0, 2, 5, 1, 2], [5, 2, 0, 3, 0, 1, 4, 2, 1]] \$$

$$[4y_5 - y_1 - y_2 - y_3 + 4y_4, y_5, 0, y_1, y_2, y_4, y_3, y_5, y_4]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
 See Matrix

$\$ [[3, 3, 2, 3, 3, 0, 1, 3, 0], [3, 3, 3, 5, 1, 0, 0, 3, 0], [3, 3, 1, 6, 0, 0, 0, 5, 0], [5, 3, 0, 4, 0, 0, 0, 6, 0], [6, 5, 0, 3, 0, 0, 0, 4, 0], [4, 6, 0, 5, 0, 0, 0, 3, 0], [3, 4, 0, 6, 0, 0, 0, 5, 0]] \$$

$$[y_1 + y_2 - y_3 - y_4 + y_5 + y_6, y_1, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

$$p = s^4 - s^5 + s^6 - s^7$$

183 . Coloring, $\{2, 3, 7, 8, 9\}$

R: $[4, 9, 5, 7, 7, 7, 5, 6, 2]$

B: $[2, 4, 4, 8, 3, 8, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 6	4 vs 5

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 2
 See Matrix

$\$ [[0, 1, 0, 3, 4, 2, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1]] \$$

$$[0, 2y_1 + 5y_2 - 8y_3, 0, 3y_2, 2y_1, 2y_2, 8y_1 + 20y_2 - 30y_3, 0, 2y_3]$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
See Matrix

$$\$ [[6, 3, 2, 3, 0, 0, 0, 4, 0], [4, 6, 0, 5, 0, 0, 0, 3, 0], [3, 4, 0, 6, 0, 0, 0, 5, 0], [5, 3, 0, 4, 0, 0, 0, 6, 0], [6, 5, 0, 3, 0, 0, 0, 4, 0]] \$$$

$$[y_2, y_3, y_1, -y_2 + y_3 + y_1 + y_4, 0, 0, 0, y_4, 0]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

184 . Coloring, $\{2, 4, 5, 6, 7\}$

R: [4, 9, 4, 8, 3, 8, 5, 1, 1]

B: [2, 4, 5, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}\}$ order: 3
See Matrix

$$\$ [[3, 0, 2, 4, 3, 0, 0, 4, 2], [6, 0, 3, 5, 0, 0, 0, 4, 0], [4, 0, 0, 9, 0, 0, 0, 5, 0], [5, 0, 0, 4, 0, 0, 0, 9, 0], [9, 0, 0, 5, 0, 0, 0, 4, 0], [4, 0, 0, 9, 0, 0, 0, 5, 0]] \$$$

$$[2 y_3, 0, 2 y_1, 2 y_2, 3 y_5, 0, 0, 2 y_4, 2 y_5]$$

$$p = -s^3 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4
See Matrix

$$\$ [[3, 4, 0, 2, 1, 2, 6, 0, 0], [6, 3, 0, 4, 0, 0, 5, 0, 0], [5, 6, 0, 3, 0, 0, 4, 0, 0], [4, 5, 0, 6, 0, 0, 3, 0, 0], [3, 4, 0, 5, 0, 0, 6, 0, 0], [6, 3, 0, 4, 0, 0, 5, 0, 0]] \$$$

$$[y_1, y_2, 0, y_3, y_4, 2y_4, y_5, 0, 0]$$

$$p = -s^2 + s^6$$

185 . Coloring, {2, 4, 5, 6, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 + 6s^4 + 16s^7$$

R: [4, 9, 4, 8, 3, 8, 1, 6, 1]

B: [2, 4, 5, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4

See Matrix

$$\begin{aligned} \$ [[4, 0, 2, 4, 0, 2, 0, 4, 2], [2, 0, 0, 6, 0, 4, 0, 6, 0], [0, 0, 0, 2, 0, 6, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0], \\ [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0]] \$ \end{aligned}$$

$$[y_1, 0, y_5, y_2, 0, y_3, 0, y_4, y_5]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$\begin{aligned} \$ [[2, 4, 0, 2, 4, 0, 6, 0, 0], [0, 2, 0, 4, 6, 0, 6, 0, 0], [0, 0, 0, 2, 6, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], \\ [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$ \end{aligned}$$

$$[y_4, y_3, 0, y_2, y_1, 0, y_5, 0, 0]$$

186 . Coloring, {2, 4, 5, 6, 9}

R: [4, 9, 4, 8, 3, 8, 1, 1, 2]

B: [2, 4, 5, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{2, 9\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 4, 0, 0, 0, 4, 2], [4, 2, 0, 7, 0, 0, 0, 4, 1], [4, 1, 0, 4, 0, 0, 0, 7, 2], [7, 2, 0, 4, 0, 0, 0, 4, 1], [4, 1, 0, 7, 0, 0, 0, 4, 2], [4, 2, 0, 4, 0, 0, 0, 7, 1]] \$$

$[y_3, y_1, -y_3 + 5y_1 - y_2 - y_4 + 5y_5, y_2, 0, 0, 0, y_4, y_5]$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[1, 3, 0, 2, 4, 2, 6, 0, 0], [0, 1, 0, 3, 6, 0, 8, 0, 0], [0, 0, 0, 1, 8, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$[y_1, 3y_1 + y_2 + y_4 - y_3, 0, y_2, y_4, 2y_1, y_3, 0, 0]$

$$p = -s^4 + s^5 \quad p = -s^4 + s^6$$

187 . Coloring, $\{2, 4, 5, 7, 8\}$

R: $[4, 9, 4, 8, 3, 7, 5, 6, 1]$

B: $[2, 4, 5, 7, 7, 8, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	8 vs 8	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 6, 7, 8\}\}$ order: 6

See Matrix

$\$ [[1, 0, 2, 4, 3, 2, 1, 3, 2], [2, 0, 3, 3, 1, 3, 2, 4, 0], [0, 0, 1, 5, 2, 4, 3, 3, 0], [0, 0, 2, 1, 3, 3, 4, 5, 0], [0, 0, 3, 2, 4, 5, 3, 1, 0], [0, 0, 4, 3, 3, 1, 5, 2, 0], [0, 0, 3, 4, 5, 2, 1, 3, 0], [0, 0, 5, 3, 1, 3, 2, 4, 0]] \$$

$[y_5, 0, y_4, y_1, y_2, y_3, y_8, y_7, y_6]$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[5, 4, 0, 2, 1, 0, 5, 1, 0], [6, 5, 0, 4, 0, 0, 3, 0, 0], [3, 6, 0, 5, 0, 0, 4, 0, 0], [4, 3, 0, 6, 0, 0, 5, 0, 0], [5, 4, 0, 3, 0, 0, 6, 0, 0], [6, 5, 0, 4, 0, 0, 3, 0, 0]] \$$

$[y_1, y_2, 0, y_5, y_4, 0, y_3, y_4, 0]$

$$p = s^2 - s^6$$

188 . Coloring, $\{2, 4, 5, 7, 9\}$

R: $[4, 9, 4, 8, 3, 7, 5, 1, 2]$

B: $[2, 4, 5, 7, 7, 8, 1, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	7 vs 8	5 vs 7

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 2, 4, 3, 0, 1, 3, 2], [3, 2, 3, 4, 1, 0, 0, 4, 1], [4, 1, 1, 6, 0, 0, 0, 4, 2], [4, 2, 0, 5, 0, 0, 0, 6, 1], [6, 1, 0, 4, 0, 0, 0, 5, 2], [5, 2, 0, 6, 0, 0, 0, 4, 1], [4, 1, 0, 5, 0, 0, 0, 6, 2], [6, 2, 0, 4, 0, 0, 0, 5, 1]] \$$

$[5y_1 - y_2 - y_3 - y_4 - y_5 - y_6 + 5y_7, y_1, y_2, y_3, y_4, 0, y_5, y_6, y_7]$

$$p = -s^4 - s^5 + s^7 + s^8$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[4, 3, 0, 2, 1, 2, 5, 1, 0], [5, 4, 0, 3, 0, 1, 3, 2, 0], [3, 5, 0, 4, 0, 2, 3, 1, 0], [3, 3, 0, 5, 0, 1, 4, 2, 0], [4, 3, 0, 3, 0, 2, 5, 1, 0], [5, 4, 0, 3, 0, 1, 3, 2, 0], [3, 5, 0, 4, 0, 2, 3, 1, 0]] \$$

$$[y_4, y_5, 0, -y_4 - y_3 + 2y_2 + 3y_1, y_3, y_2, -y_5 + 3y_2 + 2y_1, y_1, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

189 . Coloring, $\{2, 4, 5, 8, 9\}$

R: $[4, 9, 4, 8, 3, 7, 1, 6, 2]$

B: $[2, 4, 5, 7, 7, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 8	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 6, 7, 8\}, \{2, 9\}\}$

See Matrix

$\$ [[3, 1, 2, 4, 0, 2, 1, 3, 2], [1, 2, 0, 5, 0, 3, 2, 4, 1], [2, 1, 0, 1, 0, 4, 3, 5, 2], [3, 2, 0, 2, 0, 5, 4, 1, 1], [4, 1, 0, 3, 0, 1, 5, 2, 2], [5, 2, 0, 4, 0, 2, 1, 3, 1], [1, 1, 0, 5, 0, 3, 2, 4, 2], [2, 2, 0, 1, 0, 4, 3, 5, 1]] \$$

$$[y_4, y_2, y_3, y_1, 0, -y_4 + 5y_2 - y_3 - y_1 - y_5 - y_6 + 5y_7, y_5, y_6, y_7]$$

$$p = -s^2 - s^3 + s^7 + s^8$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

$\$ [[3, 3, 0, 2, 4, 0, 5, 1, 0], [1, 3, 0, 3, 5, 0, 6, 0, 0], [0, 1, 0, 3, 6, 0, 8, 0, 0], [0, 0, 0, 1, 8, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = -s^5 + s^6$$

190 . Coloring, {2, 4, 6, 7, 8}

R: [4, 9, 4, 8, 7, 8, 5, 6, 1]

B: [2, 4, 5, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

\$ [[1, 0, 0, 4, 3, 2, 2, 4, 2], [2, 0, 0, 1, 2, 4, 3, 6, 0], [0, 0, 0, 2, 3, 6, 2, 5, 0], [0, 0, 0, 0, 2, 5, 3, 8, 0], [0, 0, 0, 0, 3, 8, 2, 5, 0], [0, 0, 0, 0, 2, 5, 3, 8, 0], [0, 0, 0, 0, 3, 8, 2, 5, 0]] \$

$$[y_1, 0, 0, -14y_1 - y_2 + 39y_3 - 14y_4 - y_5, -5y_1 + 14y_3 - 5y_4, y_2, y_3, y_4, y_5]$$

$$p' = s^4 - s^6 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}, {3, 5}} order: 4

See Matrix

\$ [[5, 4, 2, 2, 1, 0, 4, 0, 0], [4, 5, 1, 4, 2, 0, 2, 0, 0], [2, 4, 2, 5, 1, 0, 4, 0, 0], [4, 2, 1, 4, 2, 0, 5, 0, 0], [5, 4, 2, 2, 1, 0, 4, 0, 0], [4, 5, 1, 4, 2, 0, 2, 0, 0]] \$

$$[y_2, 3y_1 + 2y_4 - y_3, y_1, -y_2 + 2y_1 + 3y_4, y_4, 0, y_3, 0, 0]$$

$$p = s - s^5 \quad p' = -s + s^5$$

191 . Coloring, {2, 4, 6, 7, 9}

R: [4, 9, 4, 8, 7, 8, 5, 1, 2]

B: [2, 4, 5, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	4 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}, \{1, 4, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 4, 3, 0, 2, 4, 2], [4, 2, 0, 2, 2, 0, 3, 4, 1], [4, 1, 0, 4, 3, 0, 2, 2, 2], [2, 2, 0, 4, 2, 0, 3, 4, 1], [4, 1, 0, 2, 3, 0, 2, 4, 2], [4, 2, 0, 4, 2, 0, 3, 2, 1], [2, 1, 0, 4, 3, 0, 2, 4, 2]] \$$

$$[y_2, 3y_1 - 4y_3, 0, -y_2 + 10y_1 - y_4 - 10y_3, y_1, 0, 4y_1 - 5y_3, y_4, y_3]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p' = s - s^3 - s^4 + s^6 \quad p = s - s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{3, 5\}\}$ order: 4

See Matrix

$\$ [[4, 3, 2, 2, 1, 2, 4, 0, 0], [4, 4, 1, 3, 2, 0, 4, 0, 0], [4, 4, 2, 4, 1, 0, 3, 0, 0], [3, 4, 1, 4, 2, 0, 4, 0, 0], [4, 3, 2, 4, 1, 0, 4, 0, 0], [4, 4, 1, 3, 2, 0, 4, 0, 0], [4, 4, 2, 4, 1, 0, 3, 0, 0]] \$$

$$[y_4, y_5, y_3, y_2, y_1, -y_4 + 3y_3 - y_2 + 2y_1, -y_5 + 2y_3 + 3y_1, 0, 0]$$

$$p = -s^2 + s^6 \quad p' = s^2 - s^6$$

192 . Coloring, $\{2, 4, 6, 8, 9\}$

R: [4, 9, 4, 8, 7, 8, 1, 6, 2]

B: [2, 4, 5, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[3, 1, 0, 4, 0, 2, 2, 4, 2], [2, 2, 0, 3, 0, 4, 0, 6, 1], [0, 1, 0, 2, 0, 6, 0, 7, 2], [0, 2, 0, 0, 0, 7, 0, 8, 1], [0, 1, 0, 0, 0, 8, 0, 7, 2], [0, 2, 0, 0, 0, 7, 0, 8, 1], [0, 1, 0, 0, 0, 8, 0, 7, 2]] \$$

$$[3y_1 - y_2 + 2y_3, y_1, 0, 2y_1 - y_4 - y_5 + 3y_3, 0, y_4, y_5, y_2, y_3]$$

$$p' = s^4 - s^6 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{3, 5\}\}$ order: 6
 See Matrix

$\$ [[3, 3, 2, 2, 4, 0, 4, 0, 0], [0, 3, 4, 3, 6, 0, 2, 0, 0], [0, 0, 6, 3, 6, 0, 3, 0, 0], [0, 0, 6, 0, 9, 0, 3, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0], [0, 0, 9, 0, 9, 0, 0, 0, 0]] \$$

$$[y_1 + y_2 - y_3 - y_4 + y_5, y_1, y_2, y_3, y_4, 0, y_5, 0, 0]$$

$$p = s^5 - s^6$$

193 . Coloring, $\{2, 4, 7, 8, 9\}$

R: $[4, 9, 4, 8, 7, 7, 5, 6, 2]$

B: $[2, 4, 5, 7, 3, 8, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}\}$ order: 4
 See Matrix

$\$ [[0, 1, 0, 4, 3, 2, 3, 3, 2], [0, 2, 0, 0, 3, 3, 5, 4, 1], [0, 1, 0, 0, 5, 4, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2]] \$$

$$[0, y_1 + y_2 + y_3 - 4y_5, 0, y_1, y_2, y_3, 4y_1 + 4y_2 + 4y_3 - 15y_5 - y_4, y_4, y_5]$$

$$p' = s^4 - s^6 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{3, 5\}, \{1, 2, 4, 7\}\}$ order: 4

See Matrix

$$\$ [[6, 3, 2, 2, 1, 0, 3, 1, 0], [4, 6, 1, 3, 2, 0, 2, 0, 0], [2, 4, 2, 6, 1, 0, 3, 0, 0], [3, 2, 1, 4, 2, 0, 6, 0, 0], [6, 3, 2, 2, 1, 0, 4, 0, 0], [4, 6, 1, 3, 2, 0, 2, 0, 0], [2, 4, 2, 6, 1, 0, 3, 0, 0]] \$$$

$$[y_3, y_4, y_5, -y_3 + 3y_5 + 2y_2, y_2, 0, -y_4 + 2y_5 + 3y_2 - y_1, y_1, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

194 . Coloring, $\{2, 5, 6, 7, 8\}$

R: [4, 9, 4, 7, 3, 8, 5, 6, 1]

B: [2, 4, 5, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 8	6 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$$\$ [[1, 0, 2, 4, 3, 2, 3, 1, 2], [2, 0, 3, 3, 3, 1, 4, 2, 0], [0, 0, 3, 5, 4, 2, 3, 1, 0], [0, 0, 4, 3, 3, 1, 5, 2, 0], [0, 0, 3, 4, 5, 2, 3, 1, 0], [0, 0, 5, 3, 3, 1, 4, 2, 0], [0, 0, 3, 5, 4, 2, 3, 1, 0], [0, 0, 4, 3, 3, 1, 5, 2, 0]] \$$$

$$[-y_2 + y_5 - y_3 + 4y_4, 0, y_2, y_1, -y_1 + 4y_5 + y_4 - y_6, y_5, y_3, y_4, y_6]$$

$$p = -s^3 + s^7 \quad p' = -s^3 + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4

See Matrix

$$\$ [[5, 4, 0, 2, 1, 0, 3, 3, 0], [6, 5, 0, 4, 0, 0, 1, 2, 0], [3, 6, 0, 5, 0, 0, 0, 4, 0], [4, 3, 0, 6, 0, 0, 0, 5, 0], [5, 4, 0, 3, 0, 0, 0, 6, 0], [6, 5, 0, 4, 0, 0, 0, 3, 0]] \$$$

$$[y_1, y_2, 0, y_3, y_5, 0, y_4, y_6, 0]$$

195 . Coloring, {2, 5, 6, 7, 9}

R: [4, 9, 4, 7, 3, 8, 5, 1, 2]

B: [2, 4, 5, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 8	6 vs 7

Omega Rank for R : cycles: {{2, 9}, {3, 4, 5, 7}} order: 4

See Matrix

$\$ [[2, 1, 2, 4, 3, 0, 3, 1, 2], [1, 2, 3, 4, 3, 0, 4, 0, 1], [0, 1, 3, 4, 4, 0, 4, 0, 2], [0, 2, 4, 3, 4, 0, 4, 0, 1], [0, 1, 4, 4, 4, 0, 3, 0, 2], [0, 2, 4, 4, 3, 0, 4, 0, 1], [0, 1, 3, 4, 4, 0, 4, 0, 2], [0, 2, 4, 3, 4, 0, 4, 0, 1]] \$$

$$[y_4, y_5, y_2, y_3, 2y_5 - y_3 - y_6 + 3y_1, 0, -y_4 + 3y_5 - y_2 + 2y_1, y_6, y_1]$$

$$p' = -s^3 + s^7 \quad p = -s^3 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 6, 7, 8}} order: 6

See Matrix

$\$ [[4, 3, 0, 2, 1, 2, 3, 3, 0], [3, 4, 0, 3, 0, 3, 3, 2, 0], [3, 3, 0, 4, 0, 2, 3, 3, 0], [3, 3, 0, 3, 0, 3, 2, 4, 0], [2, 3, 0, 3, 0, 4, 3, 3, 0], [3, 2, 0, 3, 0, 3, 4, 3, 0], [4, 3, 0, 2, 0, 3, 3, 3, 0]] \$$

$$[y_1, y_1 + y_6 + y_5 + y_3 - y_4 - y_2, 0, y_6, y_5, y_3, y_4, y_2, 0]$$

$$p = s^2 - s^3 + s^4 - s^5 + s^6 - s^7$$

196 . Coloring, {2, 5, 6, 8, 9}

R: [4, 9, 4, 7, 3, 8, 1, 6, 2]

B: [2, 4, 5, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	4 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{6, 8\}, \{1, 4, 7\}\}$ order: 6

See Matrix

$\$ [[3, 1, 2, 4, 0, 2, 3, 1, 2], [3, 2, 0, 5, 0, 1, 4, 2, 1], [4, 1, 0, 3, 0, 2, 5, 1, 2], [5, 2, 0, 4, 0, 1, 3, 2, 1], [3, 1, 0, 5, 0, 2, 4, 1, 2], [4, 2, 0, 3, 0, 1, 5, 2, 1], [5, 1, 0, 4, 0, 2, 3, 1, 2], [3, 2, 0, 5, 0, 1, 4, 2, 1]] \$$

$$[y_4, y_3, y_2, y_1, 0, y_5, -y_4 + 4y_3 - y_2 - y_1 + 4y_5, y_3, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = -s^2 + s^8 \quad p' = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[3, 3, 0, 2, 4, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 4, 2, 0], [2, 3, 0, 3, 4, 0, 3, 3, 0], [3, 2, 0, 3, 3, 0, 4, 3, 0], [3, 3, 0, 2, 4, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 4, 2, 0]] \$$

$$[2y_3, 9y_3 + 9y_2 - 11y_1 - 2y_4, 0, 2y_2, 7y_3 + 7y_2 - 9y_1, 0, 2y_1, 2y_4, 0]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

197 . Coloring, $\{2, 5, 7, 8, 9\}$

R: [4, 9, 4, 7, 3, 7, 5, 6, 2]

B: [2, 4, 5, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}, \{2, 9\}\}$ order: 4
 See Matrix

$\$ [[0, 1, 2, 4, 3, 2, 4, 0, 2], [0, 2, 3, 2, 4, 0, 6, 0, 1], [0, 1, 4, 3, 6, 0, 2, 0, 2], [0, 2, 6, 4, 2, 0, 3, 0, 1], [0, 1, 2, 6, 3, 0, 4, 0, 2], [0, 2, 3, 2, 4, 0, 6, 0, 1], [0, 1, 4, 3, 6, 0, 2, 0, 2]] \$$

$$[0, -4y_5 + y_1 + y_2 + y_3, -15y_5 + 4y_1 + 4y_2 + 4y_3 - y_4, y_1, y_2, y_3, y_4, 0, y_5]$$

$$p' = -s^2 + s^6 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
 See Matrix

$\$ [[6, 3, 0, 2, 1, 0, 2, 4, 0], [6, 6, 0, 3, 0, 0, 1, 2, 0], [3, 6, 0, 6, 0, 0, 0, 3, 0], [3, 3, 0, 6, 0, 0, 0, 6, 0], [6, 3, 0, 3, 0, 0, 0, 6, 0], [6, 6, 0, 3, 0, 0, 0, 3, 0]] \$$

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = -s^3 + s^4 - s^5 + s^6$$

198 . Coloring, $\{2, 6, 7, 8, 9\}$

R: $[4, 9, 4, 7, 7, 8, 5, 6, 2]$

B: $[2, 4, 5, 8, 3, 7, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 7	5 vs 7

Omega Rank for R : cycles: $\{\{5, 7\}, \{2, 9\}, \{6, 8\}\}$ order: 2
 See Matrix

$\$ [[0, 1, 0, 4, 3, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2]] \$$

$$[0, y_2, 0, y_2 - y_1 + 3y_3, y_1, y_3, 3y_2 + y_3, y_2, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{3, 5\}\}$ order: 4
See Matrix

$$\$ [[6, 3, 2, 2, 1, 0, 1, 3, 0], [4, 6, 1, 3, 2, 0, 0, 2, 0], [2, 4, 2, 6, 1, 0, 0, 3, 0], [3, 2, 1, 4, 2, 0, 0, 6, 0], [6, 3, 2, 2, 1, 0, 0, 4, 0], [4, 6, 1, 3, 2, 0, 0, 2, 0], [2, 4, 2, 6, 1, 0, 0, 3, 0]] \$$$

$$[3y_1 - y_2 + 2y_3, 2y_1 + 3y_3 - y_4 - y_5, y_1, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = s^2 - s^6 \quad p' = -s^2 + s^6$$

199 . Coloring, $\{3, 4, 5, 6, 7\}$

$$\Omega p(\Delta)=0: \quad p = -s^2 + 6s^4 - 16s^7$$

R: [4, 4, 5, 8, 3, 8, 5, 1, 1]

B: [2, 9, 4, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 5	5 vs 6

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{3, 5\}\}$ order: 6
See Matrix

$$\$ [[3, 0, 2, 5, 4, 0, 0, 4, 0], [4, 0, 4, 3, 2, 0, 0, 5, 0], [5, 0, 2, 4, 4, 0, 0, 3, 0], [3, 0, 4, 5, 2, 0, 0, 4, 0], [4, 0, 2, 3, 4, 0, 0, 5, 0]] \$$$

$$[y_3, 0, y_4, y_1, y_2, 0, 0, -y_3 + 2y_4 - y_1 + 2y_2, 0]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 4
See Matrix

$$\$ [[3, 4, 0, 1, 0, 2, 6, 0, 2], [6, 5, 0, 0, 0, 0, 3, 0, 4], [3, 10, 0, 0, 0, 0, 0, 0, 5], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_1, y_3, 0, y_2, 0, 2y_2, y_5, 0, y_4]$$

$$p = -s^4 + s^6$$

200 . Coloring, {3, 4, 5, 6, 8}

R: [4, 4, 5, 8, 3, 8, 1, 6, 1]

B: [2, 9, 4, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	3 vs 6

Omega Rank for R : cycles: {{3, 5}, {6, 8}} order: 4

See Matrix

\$ [[4, 0, 2, 5, 1, 2, 0, 4, 0], [0, 0, 1, 4, 2, 4, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0]] \$

$$[3y_1 + 2y_2 - y_4, 0, y_1, 2y_1 + 3y_2 - y_3, y_2, y_3, 0, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{5, 7}, {2, 9}} order: 2

See Matrix

\$ [[2, 4, 0, 1, 3, 0, 6, 0, 2], [0, 4, 0, 0, 6, 0, 4, 0, 4], [0, 4, 0, 0, 4, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 4, 0, 4], [0, 4, 0, 0, 4, 0, 6, 0, 4], [0, 4, 0, 0, 6, 0, 4, 0, 4]] \$

$$[4y_2, 2y_3, 0, 2y_2, 2y_1, 0, -2y_2 - 2y_1 + 5y_3, 0, 2y_3 - 4y_2]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6$$

201 . Coloring, {3, 4, 5, 6, 9}

R: [4, 4, 5, 8, 3, 8, 1, 1, 2]

B: [2, 9, 4, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 8\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[5, 1, 2, 5, 1, 0, 0, 4, 0], [4, 0, 1, 6, 2, 0, 0, 5, 0], [5, 0, 2, 4, 1, 0, 0, 6, 0], [6, 0, 1, 5, 2, 0, 0, 4, 0], [4, 0, 2, 6, 1, 0, 0, 5, 0], [5, 0, 1, 4, 2, 0, 0, 6, 0]] \$$

$$[-y_1 + 5y_2 - y_3 + 5y_4 - y_5, y_1, y_2, y_3, y_4, 0, 0, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 3, 0, 1, 3, 2, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 6, 0, 1], [1, 3, 0, 0, 6, 0, 6, 0, 2], [2, 1, 0, 0, 6, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 6, 0, 1], [1, 3, 0, 0, 6, 0, 6, 0, 2]] \$$

$$[y_2, -y_2 + y_3 - y_4, 0, y_1, -3y_1 + y_3, 2y_1, y_3, 0, y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5 \quad p'' = -s^3 + s^6$$

202 . Coloring, $\{3, 4, 5, 7, 8\}$

R: [4, 4, 5, 8, 3, 7, 5, 6, 1]

B: [2, 9, 4, 7, 7, 8, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	7 vs 7	5 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

\$ [[1, 0, 2, 5, 4, 2, 1, 3, 0], [0, 0, 4, 1, 3, 3, 2, 5, 0], [0, 0, 3, 0, 6, 5, 3, 1, 0], [0, 0, 6, 0, 6, 1, 5, 0, 0], [0, 0, 6, 0, 11, 0, 1, 0, 0], [0, 0, 11, 0, 7, 0, 0, 0, 0], [0, 0, 7, 0, 11, 0, 0, 0, 0]] \$

$[y_1, 0, y_4, y_2, y_3, y_7, y_5, y_6, 0]$

Omega Rank for B : cycles: {{2, 9}} order: 4

See Matrix

\$ [[5, 4, 0, 1, 0, 0, 5, 1, 2], [6, 7, 0, 0, 0, 0, 1, 0, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$

$[y_5, y_4, 0, y_3, 0, 0, y_2, y_3, y_1]$

$$p = -s^4 + s^6$$

203 . Coloring, {3, 4, 5, 7, 9}

R: [4, 4, 5, 8, 3, 7, 5, 1, 2]

B: [2, 9, 4, 7, 7, 8, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	6 vs 7

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}} order: 6

See Matrix

\$ [[2, 1, 2, 5, 4, 0, 1, 3, 0], [3, 0, 4, 3, 3, 0, 0, 5, 0], [5, 0, 3, 3, 4, 0, 0, 3, 0], [3, 0, 4, 5, 3, 0, 0, 3, 0], [3, 0, 3, 3, 4, 0, 0, 5, 0], [5, 0, 4, 3, 3, 0, 0, 3, 0], [3, 0, 3, 5, 4, 0, 0, 3, 0]] \$

$[4 y_4, 7 y_4 - 11 y_3 + 7 y_1 - 11 y_2 + 7 y_5, 4 y_3, 4 y_1, 4 y_2, 0, 7 y_4 - 11 y_3 + 7 y_1 - 11 y_2 + 7 y_5, 4 y_5, 0]$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p = s^2 - s^4 - s^5 + s^7$$

Omega Rank for B : cycles: $\{\{6, 8\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[4, 3, 0, 1, 0, 2, 5, 1, 2], [7, 4, 0, 0, 0, 1, 1, 2, 3], [4, 7, 0, 0, 0, 2, 0, 1, 4], [4, 4, 0, 0, 0, 1, 0, 2, 7], [7, 4, 0, 0, 0, 2, 0, 1, 4], [4, 7, 0, 0, 0, 1, 0, 2, 4], [4, 4, 0, 0, 0, 2, 0, 1, 7]] \$$

$$[-y_1 - y_2 + 5y_3 - y_4 + 5y_5 - y_6, y_1, 0, y_2, 0, y_3, y_4, y_5, y_6]$$

$$p = s^3 + s^4 - s^6 - s^7$$

204 . Coloring, $\{3, 4, 5, 8, 9\}$

R: $[4, 4, 5, 8, 3, 7, 1, 6, 2]$

B: $[2, 9, 4, 7, 7, 8, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 8	3 vs 7

Omega Rank for R : cycles: $\{\{3, 5\}, \{1, 4, 6, 7, 8\}\}$

See Matrix

$\$ [[3, 1, 2, 5, 1, 2, 1, 3, 0], [1, 0, 1, 4, 2, 3, 2, 5, 0], [2, 0, 2, 1, 1, 5, 3, 4, 0], [3, 0, 1, 2, 2, 4, 5, 1, 0], [5, 0, 2, 3, 1, 1, 4, 2, 0], [4, 0, 1, 5, 2, 2, 1, 3, 0], [1, 0, 2, 4, 1, 3, 2, 5, 0], [2, 0, 1, 1, 2, 5, 3, 4, 0]] \$$

$$[-y_1 + 5y_2 - y_3 + 5y_4 - y_5 - y_6 - y_7, y_1, y_2, y_3, y_4, y_5, y_6, y_7, 0]$$

$$p = -s^2 - s^3 + s^7 + s^8$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 2, 9\}\}$ order: 6

See Matrix

$\$ [[3, 3, 0, 1, 3, 0, 5, 1, 2], [3, 3, 0, 0, 5, 0, 4, 0, 3], [3, 3, 0, 0, 4, 0, 5, 0, 3], [3, 3, 0, 0, 5, 0, 4, 0, 3], [3, 3, 0, 0, 4, 0, 5, 0, 3], [3, 3, 0, 0, 5, 0, 4, 0, 3], [3, 3, 0, 0, 4, 0, 5, 0, 3]] \$$

$$[y_1 + y_3, y_1 + y_3, 0, y_1, 2y_1 + 3y_3 - y_2, 0, y_2, y_1, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

205 . Coloring, {3, 4, 6, 7, 8}

$$\Omega p(\Delta)=0: \quad p = -s^3 + s^4 + 4s^5 - 8s^7$$

R: [4, 4, 5, 8, 7, 8, 5, 6, 1]

B: [2, 9, 4, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 6	6 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

$$\$ [[1, 0, 0, 5, 4, 2, 2, 4, 0], [0, 0, 0, 1, 2, 4, 4, 7, 0], [0, 0, 0, 0, 4, 7, 2, 5, 0], [0, 0, 0, 0, 2, 5, 4, 7, 0], [0, 0, 0, 0, 4, 7, 2, 5, 0], [0, 0, 0, 0, 2, 5, 4, 7, 0]] \$$$

$$[3y_1 - 4y_4 + 3y_2 - y_3, 0, 0, y_1, y_4, y_2, 2y_1 - 3y_4 + 2y_2, y_3, 0]$$

$$p' = s^3 - s^5 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 6

See Matrix

$$\$ [[5, 4, 2, 1, 0, 0, 4, 0, 2], [4, 7, 0, 2, 0, 0, 1, 0, 4], [1, 8, 0, 0, 0, 0, 2, 0, 7], [2, 8, 0, 0, 0, 0, 0, 0, 8], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$$

$$[y_4, y_1, y_2, y_3, 0, 0, y_5, 0, y_6]$$

206 . Coloring, {3, 4, 6, 7, 9}

$$\Omega p(\Delta)=0: \quad p' = s^2 + 4s^4 + 4s^5 + 8s^6 \quad p = s^2 + 4s^4 + 4s^5 + 8s^6$$

R: [4, 4, 5, 8, 7, 8, 5, 1, 2]

B: [2, 9, 4, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 7	8 vs 8	8 vs 8	5 vs 6	4 vs 7

Omega Rank for R : cycles: {{5, 7}, {1, 4, 8}} order: 6

See Matrix

\$ [[2, 1, 0, 5, 4, 0, 2, 4, 0], [4, 0, 0, 3, 2, 0, 4, 5, 0], [5, 0, 0, 4, 4, 0, 2, 3, 0], [3, 0, 0, 5, 2, 0, 4, 4, 0], [4, 0, 0, 3, 4, 0, 2, 5, 0], [5, 0, 0, 4, 2, 0, 4, 3, 0]] \$

$$[y_3, y_2, 0, -y_3 - y_2 + 2y_1 + 2y_5 - y_4, y_1, 0, y_5, y_4, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6

See Matrix

\$ [[4, 3, 2, 1, 0, 2, 4, 0, 2], [6, 4, 0, 2, 0, 0, 3, 0, 3], [6, 6, 0, 0, 0, 0, 2, 0, 4], [6, 6, 0, 0, 0, 0, 0, 0, 6], [6, 6, 0, 0, 0, 0, 0, 0, 6], [6, 6, 0, 0, 0, 0, 0, 0, 6], [6, 6, 0, 0, 0, 0, 0, 0, 6]] \$

$$[-y_3 + y_2 + y_4, y_1, y_3, -y_1 - y_3 + y_2 + y_4, 0, y_3, y_2, 0, y_4]$$

$$p = -s^4 + s^5 \quad p = -s^4 + s^6 \quad p = -s^4 + s^7$$

207 . Coloring, {3, 4, 6, 8, 9}

R: [4, 4, 5, 8, 7, 8, 1, 6, 2]

B: [2, 9, 4, 7, 3, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	6 vs 7

Omega Rank for R : cycles: {{6, 8}} order: 6

See Matrix

\$ [[3, 1, 0, 5, 1, 2, 2, 4, 0], [2, 0, 0, 4, 0, 4, 1, 7, 0], [1, 0, 0, 2, 0, 7, 0, 8, 0], [0, 0, 0, 1, 0, 8, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0], [0, 0, 0, 0, 9, 0, 9, 0]] \$

$$[y_5 - 2y_4 + y_3 + y_2 - y_1, y_4, 0, y_5, y_4, y_3, y_2, y_1, 0]$$

$$p = -s^5 + s^6 \quad p = -s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 9}, {3, 4, 5, 7}}

See Matrix

\$ [[3, 3, 2, 1, 3, 0, 4, 0, 2], [2, 3, 3, 2, 4, 0, 1, 0, 3], [3, 2, 4, 3, 1, 0, 2, 0, 3], [3, 3, 1, 4, 2, 0, 3, 0, 2], [2, 3, 2, 1, 3, 0, 4, 0, 3], [3, 2, 3, 2, 4, 0, 1, 0, 3], [3, 3, 4, 3, 1, 0, 2, 0, 2]] \$

$$[4y_2, 4y_3, 5y_2 + 5y_3 - 4y_1 - 4y_4 - 4y_5 + 5y_6, 4y_1, 4y_4, 0, 4y_5, 0, 4y_6]$$

$$p = -s - s^2 - s^3 + s^5 + s^6 + s^7$$

208 . Coloring, {3, 4, 7, 8, 9}

R: [4, 4, 5, 8, 7, 7, 5, 6, 2]

B: [2, 9, 4, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 6	6 vs 7

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

\$ [[0, 1, 0, 5, 4, 2, 3, 3, 0], [0, 0, 0, 1, 3, 3, 6, 5, 0], [0, 0, 0, 0, 6, 5, 6, 1, 0], [0, 0, 0, 0, 6, 1, 11, 0, 0], [0, 0, 0, 0, 11, 0, 7, 0, 0], [0, 0, 0, 0, 7, 0, 11, 0, 0]] \$

$$[0, y_1, 0, y_3, y_4, y_2, y_5, y_6, 0]$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 6
 See Matrix

$\$ [[6, 3, 2, 1, 0, 0, 3, 1, 2], [6, 6, 0, 2, 0, 0, 1, 0, 3], [4, 6, 0, 0, 0, 0, 2, 0, 6], [8, 4, 0, 0, 0, 0, 0, 0, 6], [6, 8, 0, 0, 0, 0, 0, 0, 4], [4, 6, 0, 0, 0, 0, 0, 0, 8], [8, 4, 0, 0, 0, 0, 0, 0, 6]] \$$

$$[y_1, y_2, 2y_6, y_3, 0, 0, y_4, y_6, y_5]$$

$$p = -s^4 + s^7$$

209 . Coloring, $\{3, 5, 6, 7, 8\}$

R: $[4, 4, 5, 7, 3, 8, 5, 6, 1]$

B: $[2, 9, 4, 8, 7, 7, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[1, 0, 2, 5, 4, 2, 3, 1, 0], [0, 0, 4, 1, 5, 1, 5, 2, 0], [0, 0, 5, 0, 9, 2, 1, 1, 0], [0, 0, 9, 0, 6, 1, 0, 2, 0], [0, 0, 6, 0, 9, 2, 0, 1, 0], [0, 0, 9, 0, 6, 1, 0, 2, 0], [0, 0, 6, 0, 9, 2, 0, 1, 0]] \$$

$$[y_1, 0, -y_1 + y_3 - y_2 + 4y_5, y_4, 4y_3 + y_5 - y_4, y_3, y_2, y_5, 0]$$

$$p = s^4 - s^6 \quad p' = s^4 - s^6$$

Omega Rank for B : cycles: $\{\{2, 9\}\}$ order: 4
 See Matrix

$\$ [[5, 4, 0, 1, 0, 0, 3, 3, 2], [6, 7, 0, 0, 0, 0, 0, 1, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8], [0, 8, 0, 0, 0, 0, 0, 0, 10]] \$$

$$[y_3, y_2, 0, y_1, 0, 0, 3y_1, y_5, y_4]$$

$$p = s^4 - s^6$$

210 . Coloring, {3, 5, 6, 7, 9}

R: [4, 4, 5, 7, 3, 8, 5, 1, 2]

B: [2, 9, 4, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 7	7 vs 7

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

\$ [[2, 1, 2, 5, 4, 0, 3, 1, 0], [1, 0, 4, 3, 5, 0, 5, 0, 0], [0, 0, 5, 1, 9, 0, 3, 0, 0], [0, 0, 9, 0, 8, 0, 1, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0], [0, 0, 10, 0, 8, 0, 0, 0, 0], [0, 0, 8, 0, 10, 0, 0, 0, 0]] \$

[$y_4, y_5, y_6, y_3, y_1, 0, y_2, y_5, 0$]

$$p = -s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6

See Matrix

\$ [[4, 3, 0, 1, 0, 2, 3, 3, 2], [5, 4, 0, 0, 0, 3, 2, 1, 3], [5, 5, 0, 0, 0, 1, 3, 0, 4], [7, 5, 0, 0, 0, 0, 1, 0, 5], [6, 7, 0, 0, 0, 0, 0, 0, 5], [5, 6, 0, 0, 0, 0, 0, 0, 7], [7, 5, 0, 0, 0, 0, 0, 0, 6]] \$

[$y_6, y_7, 0, y_4, 0, y_5, y_1, y_2, y_3$]

211 . Coloring, {3, 5, 6, 8, 9}

R: [4, 4, 5, 7, 3, 8, 1, 6, 2]

B: [2, 9, 4, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 9	9 vs 9	5 vs 8	5 vs 7

Omega Rank for R : cycles: $\{\{1, 4, 7\}, \{3, 5\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[3, 1, 2, 5, 1, 2, 3, 1, 0], [3, 0, 1, 4, 2, 1, 5, 2, 0], [5, 0, 2, 3, 1, 2, 4, 1, 0], [4, 0, 1, 5, 2, 1, 3, 2, 0], [3, 0, 2, 4, 1, 2, 5, 1, 0], [5, 0, 1, 3, 2, 1, 4, 2, 0], [4, 0, 2, 5, 1, 2, 3, 1, 0], [3, 0, 1, 4, 2, 1, 5, 2, 0]] \$$

$$[-y_1 + 4y_3 - y_2 + 4y_5 - y_4, y_1, y_3, y_2, y_5, y_3, y_4, y_5, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[3, 3, 0, 1, 3, 0, 3, 3, 2], [5, 3, 0, 0, 3, 0, 3, 1, 3], [4, 5, 0, 0, 3, 0, 3, 0, 3], [3, 4, 0, 0, 3, 0, 3, 0, 5], [5, 3, 0, 0, 3, 0, 3, 0, 4], [4, 5, 0, 0, 3, 0, 3, 0, 3], [3, 4, 0, 0, 3, 0, 3, 0, 5]] \$$

$$[-y_1 - y_2 + 4y_3 - y_4 - y_5, y_1, 0, y_2, y_3, 0, y_3, y_4, y_5]$$

$$p = -s^3 + s^6 \quad p' = -s^3 + s^6$$

212 . Coloring, $\{3, 5, 7, 8, 9\}$

R: [4, 4, 5, 7, 3, 7, 5, 6, 2]

B: [2, 9, 4, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 4
See Matrix

\$ [[0, 1, 2, 5, 4, 2, 4, 0, 0] , [0, 0, 4, 1, 6, 0, 7, 0, 0] , [0, 0, 6, 0, 11, 0, 1, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0] ,
[0, 0, 7, 0, 11, 0, 0, 0, 0] , [0, 0, 11, 0, 7, 0, 0, 0, 0]] \$

$$[0, y_4, y_5, y_1, y_2, 2y_4, y_3, 0, 0]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 3
See Matrix

\$ [[6, 3, 0, 1, 0, 0, 2, 4, 2] , [8, 6, 0, 0, 0, 0, 0, 1, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8,
6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$

$$[y_3, y_2, 0, y_1, 0, 0, 2y_1, y_5, y_4]$$

$$p = s^3 - s^6$$

213 . Coloring, {3, 6, 7, 8, 9}

R: [4, 4, 5, 7, 7, 8, 5, 6, 2]

B: [2, 9, 4, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 6	6 vs 7

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4
See Matrix

\$ [[0, 1, 0, 5, 4, 2, 5, 1, 0] , [0, 0, 0, 1, 5, 1, 9, 2, 0] , [0, 0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0] , [0,
0, 0, 0, 9, 2, 6, 1, 0] , [0, 0, 0, 0, 6, 1, 9, 2, 0]] \$

$$[0, y_2 - y_3 + 4y_4, 0, -y_1 + 4y_2 + y_4, y_1, y_2, y_3, y_4, 0]$$

$$p' = s^3 - s^5 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6
See Matrix

$$\$ [[6, 3, 2, 1, 0, 0, 1, 3, 2], [6, 6, 0, 2, 0, 0, 0, 1, 3], [4, 6, 0, 0, 0, 0, 0, 2, 6], [8, 4, 0, 0, 0, 0, 0, 0, 6], [6, 8, 0, 0, 0, 0, 0, 0, 4], [4, 6, 0, 0, 0, 0, 0, 0, 8], [8, 4, 0, 0, 0, 0, 0, 0, 6]] \$$$

$$[y_3, y_1, 2y_6, y_2, 0, 0, y_6, y_5, y_4]$$

$$p = -s^4 + s^7$$

214 . Coloring, {4, 5, 6, 7, 8}

R: [4, 4, 4, 8, 3, 8, 5, 6, 1]

B: [2, 9, 5, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4
See Matrix

$$\$ [[1, 0, 2, 6, 3, 2, 0, 4, 0], [0, 0, 3, 3, 0, 4, 0, 8, 0], [0, 0, 0, 3, 0, 8, 0, 7, 0], [0, 0, 0, 0, 0, 7, 0, 11, 0], [0, 0, 0, 0, 0, 11, 0, 7, 0], [0, 0, 0, 0, 0, 7, 0, 11, 0]] \$$$

$$[y_1, 0, y_2, y_3, 3y_1, y_4, 0, y_5, 0]$$

$$p = -s^4 + s^6$$

Omega Rank for B : cycles: {{2, 9}} order: 4
See Matrix

$$\$ [[5, 4, 0, 0, 1, 0, 6, 0, 2], [6, 7, 0, 0, 0, 0, 1, 0, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$$

$$[y_1, y_3, 0, 0, y_2, 0, y_4, 0, y_5]$$

215 . Coloring, {4, 5, 6, 7, 9}

R: [4, 4, 4, 8, 3, 8, 5, 1, 2]

B: [2, 9, 5, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{1, 4, 8}} order: 3

See Matrix

\$ [[2, 1, 2, 6, 3, 0, 0, 4, 0], [4, 0, 3, 5, 0, 0, 0, 6, 0], [6, 0, 0, 7, 0, 0, 0, 5, 0], [5, 0, 0, 6, 0, 0, 0, 7, 0], [7, 0, 0, 5, 0, 0, 0, 6, 0], [6, 0, 0, 7, 0, 0, 0, 5, 0]] \$

$$[y_1, y_2, y_3, y_4, 3y_2, 0, 0, y_5, 0]$$

$$p = -s^3 + s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

\$ [[4, 3, 0, 0, 1, 2, 6, 0, 2], [8, 4, 0, 0, 0, 0, 3, 0, 3], [6, 8, 0, 0, 0, 0, 0, 0, 4], [4, 6, 0, 0, 0, 0, 0, 0, 8], [8, 4, 0, 0, 0, 0, 0, 0, 6], [6, 8, 0, 0, 0, 0, 0, 0, 4]] \$

$$[y_1, y_2, 0, 0, y_3, 2y_3, y_4, 0, y_5]$$

$$p = -s^3 + s^6$$

216 . Coloring, {4, 5, 6, 8, 9}

R: [4, 4, 4, 8, 3, 8, 1, 6, 2]

B: [2, 9, 5, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	4 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4

See Matrix

\$ [[3, 1, 2, 6, 0, 2, 0, 4, 0], [0, 0, 0, 6, 0, 4, 0, 8, 0], [0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0],
[0, 0, 0, 0, 0, 8, 0, 10, 0], [0, 0, 0, 0, 0, 10, 0, 8, 0]] \$

[3 y₂, y₂, 2 y₂, y₁, 0, y₄, 0, y₃, 0]

$$p' = s^3 - s^5 \quad p = s^3 - s^5$$

Omega Rank for B : cycles: {{5, 7}, {1, 2, 9}} order: 6

See Matrix

\$ [[3, 3, 0, 0, 4, 0, 6, 0, 2], [2, 3, 0, 0, 6, 0, 4, 0, 3], [3, 2, 0, 0, 4, 0, 6, 0, 3], [3, 3, 0, 0, 6, 0, 4, 0, 2], [2,
3, 0, 0, 4, 0, 6, 0, 3]] \$

[4 y₃, 4 y₂, 0, 0, 4 y₁, 0, 5 y₃ + 5 y₂ - 4 y₁ + 5 y₄, 0, 4 y₄]

$$p = -s - s^2 + s^4 + s^5$$

217 . Coloring, {4, 5, 7, 8, 9}

R: [4, 4, 4, 8, 3, 7, 5, 6, 2]

B: [2, 9, 5, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	7 vs 7	5 vs 6

Omega Rank for R : cycles: {{3, 4, 5, 6, 7, 8}} order: 6

See Matrix

\$ [[0, 1, 2, 6, 3, 2, 1, 3, 0] , [0, 0, 3, 3, 1, 3, 2, 6, 0] , [0, 0, 1, 3, 2, 6, 3, 3, 0] , [0, 0, 2, 1, 3, 3, 6, 3, 0] , [0, 0, 3, 2, 6, 3, 3, 1, 0] , [0, 0, 6, 3, 3, 1, 3, 2, 0] , [0, 0, 3, 6, 3, 2, 1, 3, 0]] \$

[0, y₁, y₇, y₅, y₆, y₄, y₂, y₃, 0]

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

\$ [[6, 3, 0, 0, 1, 0, 5, 1, 2] , [8, 6, 0, 0, 0, 0, 1, 0, 3] , [4, 8, 0, 0, 0, 0, 0, 0, 6] , [6, 4, 0, 0, 0, 0, 0, 0, 8] , [8, 6, 0, 0, 0, 0, 0, 0, 4] , [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$

[y₁, y₂, 0, 0, y₄, 0, y₃, y₄, y₅]

$$p = -s^3 + s^6$$

218 . Coloring, {4, 6, 7, 8, 9}

R: [4, 4, 4, 8, 7, 8, 5, 6, 2]

B: [2, 9, 5, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

\$ [[0, 1, 0, 6, 3, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0] , [0, 0, 0, 0, 3, 8, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 3, 8, 0]] \$

[0, y₁, 0, -14 y₁ - y₂ + 39 y₃ - 14 y₄, -5 y₁ + 14 y₃ - 5 y₄, y₂, y₃, y₄, 0]

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[6, 3, 2, 0, 1, 0, 4, 0, 2], [6, 6, 1, 0, 2, 0, 0, 0, 3], [3, 6, 2, 0, 1, 0, 0, 0, 6], [6, 3, 1, 0, 2, 0, 0, 0, 6], [6, 6, 2, 0, 1, 0, 0, 0, 3], [3, 6, 1, 0, 2, 0, 0, 0, 6]] \$$

$$[-y_1 + 5y_2 + 5y_3 - y_4 - y_5, y_1, y_2, 0, y_3, 0, y_4, 0, y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

219 . Coloring, $\{5, 6, 7, 8, 9\}$

R: $[4, 4, 4, 7, 3, 8, 5, 6, 2]$

B: $[2, 9, 5, 8, 7, 7, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 2, 6, 3, 2, 3, 1, 0], [0, 0, 3, 3, 3, 1, 6, 2, 0], [0, 0, 3, 3, 6, 2, 3, 1, 0], [0, 0, 6, 3, 3, 1, 3, 2, 0], [0, 0, 3, 6, 3, 2, 3, 1, 0], [0, 0, 3, 3, 3, 1, 6, 2, 0], [0, 0, 3, 3, 6, 2, 3, 1, 0]] \$$

$$[0, y_2, y_3, y_1, -y_1 + 4y_2 + 4y_3 + 4y_4 - 15y_5, y_2 + y_3 + y_4 - 4y_5, y_4, y_5, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 3

See Matrix

$\$ [[6, 3, 0, 0, 1, 0, 3, 3, 2], [8, 6, 0, 0, 0, 0, 1, 0, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4], [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$$

$$[y_1, y_2, 0, 0, y_3, 0, y_4, 3y_3, y_5]$$

$$p = -s^3 + s^6$$

220 . Coloring, {2, 3, 4, 5, 6, 7}

$$\Omega p(\Delta)=0: \quad p = s^3 + s^4 - 4s^5 - 8s^7$$

R: [4, 9, 5, 8, 3, 8, 5, 1, 1]

B: [2, 4, 4, 7, 7, 7, 1, 6, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 5

Omega Rank for R : cycles: {{1, 4, 8}, {3, 5}} order: 6

See Matrix

$$\$ [[3, 0, 2, 3, 4, 0, 0, 4, 2], [6, 0, 4, 3, 2, 0, 0, 3, 0], [3, 0, 2, 6, 4, 0, 0, 3, 0], [3, 0, 4, 3, 2, 0, 0, 6, 0], [6, 0, 2, 3, 4, 0, 0, 3, 0], [3, 0, 4, 6, 2, 0, 0, 3, 0]] \$$$

$$[2y_1 - y_2 + 2y_3 - y_4 - y_5, 0, y_1, y_2, y_3, 0, 0, y_4, y_5]$$

$$p = s^2 + s^3 - s^5 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

$$\$ [[3, 4, 0, 3, 0, 2, 6, 0, 0], [6, 3, 0, 4, 0, 0, 5, 0, 0], [5, 6, 0, 3, 0, 0, 4, 0, 0], [4, 5, 0, 6, 0, 0, 3, 0, 0], [3, 4, 0, 5, 0, 0, 6, 0, 0]] \$$$

$$[y_1, y_2, 0, y_5, 0, y_3, y_4, 0, 0]$$

221 . Coloring, {2, 3, 4, 5, 6, 8}

R: [4, 9, 5, 8, 3, 8, 1, 6, 1]

B: [2, 4, 4, 7, 7, 7, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 5

Omega Rank for R : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[4, 0, 2, 3, 1, 2, 0, 4, 2], [2, 0, 1, 4, 2, 4, 0, 5, 0], [0, 0, 2, 2, 1, 5, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0]] \$$

$$[y_3, 0, y_4, y_5, y_2, 2y_4 - y_5 + 3y_2 - y_1, 0, -y_3 + 3y_4 + 2y_2, y_1]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 4, 0, 3, 3, 0, 6, 0, 0], [0, 2, 0, 4, 6, 0, 6, 0, 0], [0, 0, 0, 2, 6, 0, 10, 0, 0], [0, 0, 0, 0, 10, 0, 8, 0, 0], [0, 0, 0, 0, 8, 0, 10, 0, 0]] \$$

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, 0, 0]$$

222 . Coloring, $\{2, 3, 4, 5, 6, 9\}$

R: [4, 9, 5, 8, 3, 8, 1, 1, 2]

B: [2, 4, 4, 7, 7, 7, 5, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 7	4 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{1, 4, 8\}\}$ order: 6
 See Matrix

$\$ [[5, 1, 2, 3, 1, 0, 0, 4, 2], [4, 2, 1, 5, 2, 0, 0, 3, 1], [3, 1, 2, 4, 1, 0, 0, 5, 2], [5, 2, 1, 3, 2, 0, 0, 4, 1], [4, 1, 2, 5, 1, 0, 0, 3, 2], [3, 2, 1, 4, 2, 0, 0, 5, 1], [5, 1, 2, 3, 1, 0, 0, 4, 2]] \$$

$$[y_4, y_3, y_2, y_1, y_3, 0, 0, -y_4 + 4y_3 + 4y_2 - y_1, y_2]$$

$$p = s - s^7 \quad p' = s^2 + s^3 - s^5 - s^6 \quad p'' = s - s^3 - s^4 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4
 See Matrix

$\$ [[1, 3, 0, 3, 3, 2, 6, 0, 0], [0, 1, 0, 3, 6, 0, 8, 0, 0], [0, 0, 0, 1, 8, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[y_4, y_3, 0, y_2, y_1, 2y_4, 3y_4 - y_3 + y_2 + y_1, 0, 0]$$

$$p = -s^4 + s^5 \quad p' = -s^4 + s^6$$

223 . Coloring, $\{2, 3, 4, 5, 7, 8\}$

R: $[4, 9, 5, 8, 3, 7, 5, 6, 1]$

B: $[2, 4, 4, 7, 7, 8, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	8 vs 8	5 vs 5

Omega Rank for R : cycles: $\{\{3, 5\}\}$ order: 8
 See Matrix

$\$ [[1, 0, 2, 3, 4, 2, 1, 3, 2], [2, 0, 4, 1, 3, 3, 2, 3, 0], [0, 0, 3, 2, 6, 3, 3, 1, 0], [0, 0, 6, 0, 6, 1, 3, 2, 0], [0, 0, 6, 0, 9, 2, 1, 0, 0], [0, 0, 9, 0, 7, 0, 2, 0, 0], [0, 0, 7, 0, 11, 0, 0, 0, 0], [0, 0, 11, 0, 7, 0, 0, 0, 0]] \$$

$$[y_1, 0, y_2, y_3, y_4, y_5, y_6, y_7, y_8]$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[5, 4, 0, 3, 0, 0, 5, 1, 0], [6, 5, 0, 4, 0, 0, 3, 0, 0], [3, 6, 0, 5, 0, 0, 4, 0, 0], [4, 3, 0, 6, 0, 0, 5, 0, 0], [5, 4, 0, 3, 0, 0, 6, 0, 0]] \$$

$[y_4, y_3, 0, y_2, 0, 0, y_1, y_5, 0]$

224 . Coloring, $\{2, 3, 4, 5, 7, 9\}$

R: $[4, 9, 5, 8, 3, 7, 5, 1, 2]$

B: $[2, 4, 4, 7, 7, 8, 1, 6, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 8	4 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{1, 4, 8\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[2, 1, 2, 3, 4, 0, 1, 3, 2], [3, 2, 4, 2, 3, 0, 0, 3, 1], [3, 1, 3, 3, 4, 0, 0, 2, 2], [2, 2, 4, 3, 3, 0, 0, 3, 1], [3, 1, 3, 2, 4, 0, 0, 3, 2], [3, 2, 4, 3, 3, 0, 0, 2, 1], [2, 1, 3, 3, 4, 0, 0, 3, 2], [3, 2, 4, 2, 3, 0, 0, 3, 1]] \$$

$[2y_5, 2y_1, 2y_3, 2y_4, -7y_1 + 5y_3 + 5y_2, 0, 2y_2, -2y_5 - 8y_1 + 8y_3 - 2y_4 + 8y_2, -5y_1 + 3y_3 + 3y_2]$

$p = -s^2 - s^3 + s^5 + s^6 \quad p = s^2 - s^4 - s^5 + s^7 \quad p = -s^2 + s^8$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[4, 3, 0, 3, 0, 2, 5, 1, 0], [5, 4, 0, 3, 0, 1, 3, 2, 0], [3, 5, 0, 4, 0, 2, 3, 1, 0], [3, 3, 0, 5, 0, 1, 4, 2, 0], [4, 3, 0, 3, 0, 2, 5, 1, 0], [5, 4, 0, 3, 0, 1, 3, 2, 0]] \$$

$[y_3, y_2, 0, -y_3 + 2y_1 + 3y_4, 0, y_1, -y_2 + 3y_1 + 2y_4, y_4, 0]$

$p' = -s + s^5 \quad p = -s + s^5$

225 . Coloring, {2, 3, 4, 5, 8, 9}

R: [4, 9, 5, 8, 3, 7, 1, 6, 2]

B: [2, 4, 4, 4, 7, 7, 8, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	8 vs 9	6 vs 9	5 vs 6

Omega Rank for R : cycles: {{2, 9}, {3, 5}, {1, 4, 6, 7, 8}}

See Matrix

\$ [[3, 1, 2, 3, 1, 2, 1, 3, 2], [1, 2, 1, 3, 2, 3, 2, 3, 1], [2, 1, 2, 1, 1, 3, 3, 3, 2], [3, 2, 1, 2, 2, 3, 3, 1, 1], [3, 1, 2, 3, 1, 1, 3, 2, 2], [3, 2, 1, 3, 2, 2, 1, 3, 1], [1, 1, 2, 3, 1, 3, 2, 3, 2], [2, 2, 1, 1, 2, 3, 3, 3, 1], [3, 1, 2, 2, 1, 3, 3, 1, 2]] \$

$$[4 y_2 + 4 y_6 - y_1 - y_3 - y_4 - y_5, y_2, y_6, y_1, y_2, y_3, y_4, y_5, y_6]$$

$$p' = s^2 + s^3 - s^7 - s^8 \quad p' = s + s^2 - s^6 - s^7 \quad p' = 1 - s^2 - s^5 + s^7$$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

\$ [[3, 3, 0, 3, 3, 0, 5, 1, 0], [1, 3, 0, 3, 5, 0, 6, 0, 0], [0, 1, 0, 3, 6, 0, 8, 0, 0], [0, 0, 0, 1, 8, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$

$$[y_4, y_5, 0, y_3, y_2, 0, y_4 - y_5 + y_3 + y_2 - y_1, y_1, 0]$$

$$p = s^5 - s^6$$

226 . Coloring, {2, 3, 4, 6, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 - 2s^4 - 16s^7$$

R: [4, 9, 5, 8, 7, 8, 5, 6, 1]

B: [2, 4, 4, 4, 7, 3, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 7	5 vs 5

Omega Rank for R : cycles: $\{\{5, 7\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[1, 0, 0, 3, 4, 2, 2, 4, 2], [2, 0, 0, 1, 2, 4, 4, 5, 0], [0, 0, 0, 2, 4, 5, 2, 5, 0], [0, 0, 0, 0, 2, 5, 4, 7, 0], [0, 0, 0, 0, 4, 7, 2, 5, 0], [0, 0, 0, 0, 2, 5, 4, 7, 0], [0, 0, 0, 0, 4, 7, 2, 5, 0]] \$$

$$[y_1, 0, 0, 3y_1 - y_2 - 4y_3 + 3y_4 - y_5, 2y_1 - 3y_3 + 2y_4, y_2, y_3, y_4, y_5]$$

$$p = -s^4 + s^6 \quad p' = s^4 - s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[5, 4, 2, 3, 0, 0, 4, 0, 0], [4, 5, 0, 6, 0, 0, 3, 0, 0], [3, 4, 0, 5, 0, 0, 6, 0, 0], [6, 3, 0, 4, 0, 0, 5, 0, 0], [5, 6, 0, 3, 0, 0, 4, 0, 0]] \$$

$$[y_1, y_2, y_3, y_4, 0, 0, y_5, 0, 0]$$

227 . Coloring, $\{2, 3, 4, 6, 7, 9\}$

R: [4, 9, 5, 8, 7, 8, 5, 1, 2]

B: [2, 4, 4, 7, 3, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 7	4 vs 6

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 4, 8\}, \{2, 9\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 3, 4, 0, 2, 4, 2], [4, 2, 0, 2, 2, 0, 4, 3, 1], [3, 1, 0, 4, 4, 0, 2, 2, 2], [2, 2, 0, 3, 2, 0, 4, 4, 1], [4, 1, 0, 2, 4, 0, 2, 3, 2], [3, 2, 0, 4, 2, 0, 4, 2, 1], [2, 1, 0, 3, 4, 0, 2, 4, 2]] \$$

$$[y_4, y_3, 0, y_2, 2y_1, 0, 2y_3, -y_4 + 3y_3 - y_2 + 3y_1, y_1]$$

$$p = -s + s^3 + s^4 - s^6 \quad p' = -s - s^2 + s^4 + s^5 \quad p = -s + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4

See Matrix

$\$ [[4, 3, 2, 3, 0, 2, 4, 0, 0], [4, 4, 0, 5, 0, 0, 5, 0, 0], [5, 4, 0, 4, 0, 0, 5, 0, 0], [5, 5, 0, 4, 0, 0, 4, 0, 0], [4, 5, 0, 5, 0, 0, 4, 0, 0], [4, 4, 0, 5, 0, 0, 5, 0, 0]] \$$

$$[y_1, y_1 + y_3 - y_2, y_4, y_3, 0, y_4, y_2, 0, 0]$$

$$p = s^2 - s^6 \quad p' = s^2 - s^3 + s^4 - s^5$$

228 . Coloring, $\{2, 3, 4, 6, 8, 9\}$

R: $[4, 9, 5, 8, 7, 8, 1, 6, 2]$

B: $[2, 4, 4, 7, 3, 7, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 8	5 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 6

See Matrix

$\$ [[3, 1, 0, 3, 1, 2, 2, 4, 2], [2, 2, 0, 3, 0, 4, 1, 5, 1], [1, 1, 0, 2, 0, 5, 0, 7, 2], [0, 2, 0, 1, 0, 7, 0, 7, 1], [0, 1, 0, 0, 0, 7, 0, 8, 2], [0, 2, 0, 0, 0, 8, 0, 7, 1], [0, 1, 0, 0, 0, 7, 0, 8, 2], [0, 2, 0, 0, 0, 8, 0, 7, 1]] \$$

$$[2y_1 - y_2 - y_5 + 3y_6, y_1, 0, 3y_1 - y_3 - y_4 + 2y_6, y_2, y_3, y_4, y_5, y_6]$$

$$p = s^5 - s^7 \quad p' = -s^5 + s^7$$

Omega Rank for B : cycles: {{3, 4, 5, 7}} order: 4
See Matrix

$$\$ [[3, 3, 2, 3, 3, 0, 4, 0, 0], [0, 3, 3, 5, 4, 0, 3, 0, 0], [0, 0, 4, 6, 3, 0, 5, 0, 0], [0, 0, 3, 4, 5, 0, 6, 0, 0], [0, 0, 5, 3, 6, 0, 4, 0, 0], [0, 0, 6, 5, 4, 0, 3, 0, 0]] \$$$

$$[y_1 + y_2 - y_3 - y_4 + y_5, y_1, y_2, y_3, y_4, 0, y_5, 0, 0]$$

$$p = -s^3 + s^4 - s^5 + s^6$$

229 . Coloring, {2, 3, 4, 7, 8, 9}

R: [4, 9, 5, 8, 7, 7, 5, 6, 2]

B: [2, 4, 4, 7, 3, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	4 vs 6

Omega Rank for R : cycles: {{5, 7}, {2, 9}} order: 4
See Matrix

$$\$ [[0, 1, 0, 3, 4, 2, 3, 3, 2], [0, 2, 0, 0, 3, 3, 6, 3, 1], [0, 1, 0, 0, 6, 3, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2], [0, 2, 0, 0, 6, 0, 9, 0, 1], [0, 1, 0, 0, 9, 0, 6, 0, 2]] \$$$

$$[0, y_5, 0, y_3, y_4, -15 y_5 - y_3 - y_4 + 4 y_1 + 4 y_2, y_1, y_2, -4 y_5 + y_1 + y_2]$$

$$p = s^4 - s^6 \quad p' = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4
See Matrix

$$\$ [[6, 3, 2, 3, 0, 0, 3, 1, 0], [4, 6, 0, 5, 0, 0, 3, 0, 0], [3, 4, 0, 6, 0, 0, 5, 0, 0], [5, 3, 0, 4, 0, 0, 6, 0, 0], [6, 5, 0, 3, 0, 0, 4, 0, 0], [4, 6, 0, 5, 0, 0, 3, 0, 0]] \$$$

$$[y_4, y_3, 2y_1, y_2, 0, 0, y_4 - y_3 + y_2 - 3y_1, y_1, 0]$$

$$p' = s^2 - s^3 + s^4 - s^5 \quad p = s^2 - s^6$$

230 . Coloring, {2, 3, 5, 6, 7, 8}

R: [4, 9, 5, 7, 3, 8, 5, 6, 1]

B: [2, 4, 4, 8, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	6 vs 8	5 vs 5

Omega Rank for R : cycles: {{3, 5}, {6, 8}} order: 6

See Matrix

\$ [[1, 0, 2, 3, 4, 2, 3, 1, 2], [2, 0, 4, 1, 5, 1, 3, 2, 0], [0, 0, 5, 2, 7, 2, 1, 1, 0], [0, 0, 7, 0, 6, 1, 2, 2, 0], [0, 0, 6, 0, 9, 2, 0, 1, 0], [0, 0, 9, 0, 6, 1, 0, 2, 0], [0, 0, 6, 0, 9, 2, 0, 1, 0], [0, 0, 9, 0, 6, 1, 0, 2, 0]] \$

$$[-y_1 + y_3 - y_4 + 4y_5, 0, y_1, -y_2 + 4y_3 + y_5 - y_6, y_2, y_3, y_4, y_5, y_6]$$

$$p = -s^5 + s^7 \quad p' = -s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

\$ [[5, 4, 0, 3, 0, 0, 3, 3, 0], [6, 5, 0, 4, 0, 0, 0, 3, 0], [3, 6, 0, 5, 0, 0, 0, 4, 0], [4, 3, 0, 6, 0, 0, 0, 5, 0], [5, 4, 0, 3, 0, 0, 0, 6, 0]] \$

$$[y_1, y_2, 0, y_3, 0, 0, y_4, y_5, 0]$$

231 . Coloring, {2, 3, 5, 6, 7, 9}

R: [4, 9, 5, 7, 3, 8, 5, 1, 2]

B: [2, 4, 4, 8, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 8	5 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}\}$ order: 6

See Matrix

$\$ [[2, 1, 2, 3, 4, 0, 3, 1, 2], [1, 2, 4, 2, 5, 0, 3, 0, 1], [0, 1, 5, 1, 7, 0, 2, 0, 2], [0, 2, 7, 0, 7, 0, 1, 0, 1], [0, 1, 7, 0, 8, 0, 0, 0, 2], [0, 2, 8, 0, 7, 0, 0, 0, 1], [0, 1, 7, 0, 8, 0, 0, 0, 2], [0, 2, 8, 0, 7, 0, 0, 0, 1]] \$$

$$[y_2, y_1, -y_2 + 3y_1 - y_5 + 2y_4, 2y_1 - y_6 - y_3 + 3y_4, y_6, 0, y_5, y_3, y_4]$$

$$p' = s^5 - s^7 \quad p = s^5 - s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 6, 7, 8\}\}$ order: 6

See Matrix

$\$ [[4, 3, 0, 3, 0, 2, 3, 3, 0], [3, 4, 0, 3, 0, 3, 2, 3, 0], [2, 3, 0, 4, 0, 3, 3, 3, 0], [3, 2, 0, 3, 0, 3, 3, 4, 0], [3, 3, 0, 2, 0, 4, 3, 3, 0], [3, 3, 0, 3, 0, 3, 4, 2, 0]] \$$

$$[y_1 - y_5 - y_4 + y_3 + y_2, y_1, 0, y_5, 0, y_4, y_3, y_2, 0]$$

$$p = -s + s^2 - s^3 + s^4 - s^5 + s^6$$

232 . Coloring, $\{2, 3, 5, 6, 8, 9\}$

$$\Omega_p(\Delta)=0: \quad p = s - 64s^7 \quad p' = s + 32s^6 \quad p' = s^2 - 16s^6 \quad p' = s^3 + 8s^6 \quad p' = s^4 - 4s^6 \quad p' = s^5 + 2s^6$$

R: [4, 9, 5, 7, 3, 8, 1, 6, 2]

B: [2, 4, 4, 8, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
1 vs 7	2 vs 9	2 vs 9	2 vs 9	1 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{6, 8\}, \{1, 4, 7\}\}$ order: 6

See Matrix

$\$ [[3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1], [3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1], [3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1], [3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1], [3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1], [3, 1, 2, 3, 1, 2, 3, 1, 2], [3, 2, 1, 3, 2, 1, 3, 2, 1]] \$$

$[y_1 + y_2, y_1, y_2, y_1 + y_2, y_1, y_2, y_1 + y_2, y_1, y_2]$

$p' = -1 + s^8 \quad p' = -1 + s^2 \quad p' = -s + s^3 \quad p' = -1 + s^4 \quad p' = -s + s^5 \quad p' = -1 + s^6 \quad p' = -s + s^7$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0], [3, 3, 0, 3, 3, 0, 3, 3, 0]] \$$

$[y_1, y_1, 0, y_1, y_1, 0, y_1, y_1, 0]$

$p' = -s^2 + s^5 \quad p = s - s^3 \quad p' = -s^2 + s^3 \quad p' = -s^2 + s^4 \quad p' = s - s^2$

‘ See 6-level graph

‘

M \ ; N

$\$ [[0, 8, 4, 15, 8, 4, 15, 8, 4], [8, 0, 0, 8, 10, 0, 8, 10, 0], [4, 0, 0, 4, 0, 5, 4, 0, 5], [15, 8, 4, 0, 8, 4, 15, 8, 4], [8, 10, 0, 8, 0, 0, 8, 10, 0], [4, 0, 5, 4, 0, 0, 4, 0, 5], [15, 8, 4, 15, 8, 4, 0, 8, 4], [8, 10, 0, 8, 10, 0, 8, 0, 0], [4, 0, 5, 4, 0, 5, 4, 0, 0]] \$ \quad \$ [[0, 3, 3, 3, 3, 3, 3, 3, 3], [3, 0, 1, 3, 3, 2, 3, 3, 3], [3, 1, 0, 3, 3, 3, 3, 2, 3], [3, 3, 3, 0, 3, 3, 3, 3, 3], [3, 3, 3, 3, 0, 1, 3, 3, 2], [3, 2, 3, 3, 1, 0, 3, 3, 3], [3, 3, 3, 3, 3, 3, 0, 3, 3], [3, 3, 2, 3, 3, 3, 3, 0, 1], [3, 3, 3, 3, 2, 3, 3, 1, 0]] \$$

$\tau = 15, r' = 5/6$

R: [4, 9, 5, 7, 3, 8, 1, 6, 2]

B: [2, 4, 4, 8, 7, 7, 5, 1, 1]

Ranges

Action of R on ranges, [[2], [1]]

Action of B on ranges, [[1], [1]]

233 . Coloring, {2, 3, 5, 7, 8, 9}

R: [4, 9, 5, 7, 3, 7, 5, 6, 2]

B: [2, 4, 4, 8, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 7	4 vs 5

Omega Rank for R : cycles: {{2, 9}, {3, 5}} order: 4

See Matrix

\$ [[0, 1, 2, 3, 4, 2, 4, 0, 2] , [0, 2, 4, 0, 6, 0, 5, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2] , [0, 2, 9, 0, 6, 0, 0, 0, 1] , [0, 1, 6, 0, 9, 0, 0, 0, 2]] \$

$$[0, 2 y_4, 2 y_3, 3 y_2, -30 y_4 - 5 y_2 + 8 y_3 + 8 y_1, 2 y_2, 2 y_1, 0, -8 y_4 + 2 y_3 + 2 y_1]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

\$ [[6, 3, 0, 3, 0, 0, 2, 4, 0] , [6, 6, 0, 3, 0, 0, 0, 3, 0] , [3, 6, 0, 6, 0, 0, 0, 3, 0] , [3, 3, 0, 6, 0, 0, 0, 6, 0] , [6, 3, 0, 3, 0, 0, 0, 6, 0]] \$

$$[y_2, y_1, 0, -y_2 + y_1 + y_4 + y_3, 0, 0, y_4, y_3, 0]$$

$$p = s^2 - s^3 + s^4 - s^5$$

234 . Coloring, {2, 3, 6, 7, 8, 9}

R: [4, 9, 5, 7, 7, 8, 5, 6, 2]

B: [2, 4, 4, 8, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	3 vs 7	4 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{5, 7\}, \{6, 8\}\}$ order: 2

See Matrix

$\$ [[0, 1, 0, 3, 4, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2], [0, 2, 0, 0, 5, 1, 7, 2, 1], [0, 1, 0, 0, 7, 2, 5, 1, 2]] \$$

$$[0, y_3, 0, y_3 - y_1 + 3y_2, y_1, y_2, 3y_3 + y_2, y_3, y_2]$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4

See Matrix

$\$ [[6, 3, 2, 3, 0, 0, 1, 3, 0], [4, 6, 0, 5, 0, 0, 0, 3, 0], [3, 4, 0, 6, 0, 0, 0, 5, 0], [5, 3, 0, 4, 0, 0, 0, 6, 0], [6, 5, 0, 3, 0, 0, 0, 4, 0], [4, 6, 0, 5, 0, 0, 0, 3, 0]] \$$

$$[y_1 - y_2 + 3y_3 + y_4, y_1, 2y_3, y_2, 0, 0, y_3, y_4, 0]$$

$$p = -s^2 + s^6 \quad p = s^2 - s^3 + s^4 - s^5$$

235 . Coloring, $\{2, 4, 5, 6, 7, 8\}$

R: [4, 9, 4, 8, 3, 8, 5, 6, 1]

B: [2, 4, 5, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 7	5 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4

See Matrix

\$ [[1, 0, 2, 4, 3, 2, 0, 4, 2] , [2, 0, 3, 3, 0, 4, 0, 6, 0] , [0, 0, 0, 5, 0, 6, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0] , [0, 0, 0, 0, 0, 7, 0, 11, 0] , [0, 0, 0, 0, 0, 11, 0, 7, 0]] \$

$$[y_2, 0, y_1, y_5, -9y_2 + 6y_1, y_4, 0, y_3, -6y_2 + 4y_1]$$

$$p = -s^4 + s^6 \quad p' = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

\$ [[5, 4, 0, 2, 1, 0, 6, 0, 0] , [6, 5, 0, 4, 0, 0, 3, 0, 0] , [3, 6, 0, 5, 0, 0, 4, 0, 0] , [4, 3, 0, 6, 0, 0, 5, 0, 0] , [5, 4, 0, 3, 0, 0, 6, 0, 0]] \$

$$[y_1, y_2, 0, y_3, y_4, 0, y_5, 0, 0]$$

236 . Coloring, {2, 4, 5, 6, 7, 9}

R: [4, 9, 4, 8, 3, 8, 5, 1, 2]

B: [2, 4, 5, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	6 vs 7	4 vs 6

Omega Rank for R : cycles: {{2, 9}, {1, 4, 8}} order: 6

See Matrix

\$ [[2, 1, 2, 4, 3, 0, 0, 4, 2] , [4, 2, 3, 4, 0, 0, 0, 4, 1] , [4, 1, 0, 7, 0, 0, 0, 4, 2] , [4, 2, 0, 4, 0, 0, 0, 7, 1] , [7, 1, 0, 4, 0, 0, 0, 4, 2] , [4, 2, 0, 7, 0, 0, 0, 4, 1] , [4, 1, 0, 4, 0, 0, 0, 7, 2]] \$

$$[y_3, y_2, y_1, -y_3 + 5y_2 - y_1 - y_6 - y_4 + 5y_5, y_6, 0, 0, y_4, y_5]$$

$$p = -s^3 - s^4 + s^6 + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4
See Matrix

$\$ [[4, 3, 0, 2, 1, 2, 6, 0, 0], [6, 4, 0, 3, 0, 0, 5, 0, 0], [5, 6, 0, 4, 0, 0, 3, 0, 0], [3, 5, 0, 6, 0, 0, 4, 0, 0], [4, 3, 0, 5, 0, 0, 6, 0, 0], [6, 4, 0, 3, 0, 0, 5, 0, 0]] \$$

$$[y_1 - y_2 - 3y_3 + y_4, y_1, 0, y_2, y_3, 2y_3, y_4, 0, 0]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^3 - s^4 + s^5$$

237 . Coloring, $\{2, 4, 5, 6, 8, 9\}$

R: $[4, 9, 4, 8, 3, 8, 1, 6, 2]$

B: $[2, 4, 5, 7, 7, 7, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 7	4 vs 5

Omega Rank for R : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 4
See Matrix

$\$ [[3, 1, 2, 4, 0, 2, 0, 4, 2], [0, 2, 0, 5, 0, 4, 0, 6, 1], [0, 1, 0, 0, 0, 6, 0, 9, 2], [0, 2, 0, 0, 0, 9, 0, 6, 1], [0, 1, 0, 0, 0, 6, 0, 9, 2], [0, 2, 0, 0, 0, 9, 0, 6, 1], [0, 1, 0, 0, 0, 6, 0, 9, 2]] \$$

$$[3y_1, 5y_1 + 2y_4 - 8y_3, 2y_1, 2y_2, 0, -2y_2 - 30y_3 + 20y_1 + 8y_4, 0, 2y_4, 2y_3]$$

$$p = s^3 - s^5 \quad p' = -s^3 + s^5 \quad p'' = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4
See Matrix

$\$ [[3, 3, 0, 2, 4, 0, 6, 0, 0], [0, 3, 0, 3, 6, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$$

$$[y_3, y_2, 0, -y_3 + y_2 - y_1 + y_4, y_1, 0, y_4, 0, 0]$$

$$p = -s^4 + s^5$$

238 . Coloring, {2, 4, 5, 7, 8, 9}

R: [4, 9, 4, 8, 3, 7, 5, 6, 2]

B: [2, 4, 5, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	7 vs 8	6 vs 8	4 vs 6

Omega Rank for R : cycles: {{2, 9}, {3, 4, 5, 6, 7, 8}} order: 6

See Matrix

\$ [[0, 1, 2, 4, 3, 2, 1, 3, 2], [0, 2, 3, 2, 1, 3, 2, 4, 1], [0, 1, 1, 3, 2, 4, 3, 2, 2], [0, 2, 2, 1, 3, 2, 4, 3, 1], [0, 1, 3, 2, 4, 3, 2, 1, 2], [0, 2, 4, 3, 2, 1, 3, 2, 1], [0, 1, 2, 4, 3, 2, 1, 3, 2], [0, 2, 3, 2, 1, 3, 2, 4, 1]] \$

$$[0, y_1, 4y_1 - y_4 - y_3 + y_2, y_1 - y_6 - y_5 + 4y_2, y_6, y_5, y_4, y_3, y_2]$$

$$p' = -s + s^7 \quad p = -s + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

\$ [[6, 3, 0, 2, 1, 0, 5, 1, 0], [6, 6, 0, 3, 0, 0, 3, 0, 0], [3, 6, 0, 6, 0, 0, 3, 0, 0], [3, 3, 0, 6, 0, 0, 6, 0, 0], [6, 3, 0, 3, 0, 0, 6, 0, 0], [6, 6, 0, 3, 0, 0, 3, 0, 0]] \$

$$[y_1 - y_2 + y_3, y_1, 0, y_2, y_4, 0, y_3, y_4, 0]$$

$$p = s^2 - s^6 \quad p' = s^2 - s^3 + s^4 - s^5$$

239 . Coloring, {2, 4, 6, 7, 8, 9}

R: [4, 9, 4, 8, 7, 8, 5, 6, 2]

B: [2, 4, 5, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	3 vs 7	4 vs 6

Omega Rank for R : cycles: $\{\{2, 9\}, \{5, 7\}, \{6, 8\}\}$ order: 2

See Matrix

$\$ [[0, 1, 0, 4, 3, 2, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 6, 1], [0, 1, 0, 0, 3, 6, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 6, 1], [0, 1, 0, 0, 3, 6, 2, 4, 2], [0, 2, 0, 0, 2, 4, 3, 6, 1], [0, 1, 0, 0, 3, 6, 2, 4, 2]] \$$

$[0, y_3, 0, y_2, -5y_3 + 4y_1, -10y_3 - y_2 + 8y_1, y_1, 2y_1, -4y_3 + 3y_1]$

$p' = s^2 - s^6 \quad p = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p' = s^4 - s^6$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}, \{3, 5\}\}$ order: 4

See Matrix

$\$ [[6, 3, 2, 2, 1, 0, 4, 0, 0], [4, 6, 1, 3, 2, 0, 2, 0, 0], [2, 4, 2, 6, 1, 0, 3, 0, 0], [3, 2, 1, 4, 2, 0, 6, 0, 0], [6, 3, 2, 2, 1, 0, 4, 0, 0], [4, 6, 1, 3, 2, 0, 2, 0, 0]] \$$

$[3y_3 - y_4 + 2y_2, 2y_3 + 3y_2 - y_1, y_3, y_4, y_2, 0, y_1, 0, 0]$

$p = s - s^5 \quad p' = s - s^5$

240 . Coloring, $\{2, 5, 6, 7, 8, 9\}$

R: $[4, 9, 4, 7, 3, 8, 5, 6, 2]$

B: $[2, 4, 5, 8, 7, 7, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	7 vs 8	4 vs 8	5 vs 6

Omega Rank for R : cycles: $\{\{3, 4, 5, 7\}, \{2, 9\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[0, 1, 2, 4, 3, 2, 3, 1, 2], [0, 2, 3, 2, 3, 1, 4, 2, 1], [0, 1, 3, 3, 4, 2, 2, 1, 2], [0, 2, 4, 3, 2, 1, 3, 2, 1], [0, 1, 2, 4, 3, 2, 3, 1, 2], [0, 2, 3, 2, 3, 1, 4, 2, 1], [0, 1, 3, 3, 4, 2, 2, 1, 2], [0, 2, 4, 3, 2, 1, 3, 2, 1]] \$$

$$[0, y_3, 3y_3 - y_2 + y_4, y_3 - y_1 + 3y_4, y_1, y_4, y_2, y_3, y_4]$$

$$p' = s - s^5 \quad p' = s^2 - s^6 \quad p = s - s^5 \quad p' = -s^3 + s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 8\}\}$ order: 4
 See Matrix

$\$ [[6, 3, 0, 2, 1, 0, 3, 3, 0], [6, 6, 0, 3, 0, 0, 1, 2, 0], [3, 6, 0, 6, 0, 0, 0, 3, 0], [3, 3, 0, 6, 0, 0, 0, 6, 0], [6, 3, 0, 3, 0, 0, 0, 6, 0], [6, 6, 0, 3, 0, 0, 0, 3, 0]] \$$

$$[y_1 - y_2 - y_3 + y_4 + y_5, y_1, 0, y_2, y_3, 0, y_4, y_5, 0]$$

$$p = -s^3 + s^4 - s^5 + s^6$$

241 . Coloring, $\{3, 4, 5, 6, 7, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^2 - 2s^4 - 8s^5 + 16s^7$$

R: $[4, 4, 5, 8, 3, 8, 5, 6, 1]$

B: $[2, 9, 4, 7, 7, 7, 1, 1, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	4 vs 6	5 vs 5

Omega Rank for R : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4
 See Matrix

$\$ [[1, 0, 2, 5, 4, 2, 0, 4, 0], [0, 0, 4, 1, 2, 4, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0]] \$$

$$[3y_1 - 4y_3 + 3y_2 - y_4, 0, 2y_1 - 3y_3 + 2y_2, y_1, y_3, y_2, 0, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 9}} order: 4

See Matrix

$$\$ [[5, 4, 0, 1, 0, 0, 6, 0, 2], [6, 7, 0, 0, 0, 0, 1, 0, 4], [1, 10, 0, 0, 0, 0, 0, 0, 7], [0, 8, 0, 0, 0, 0, 0, 0, 10], [0, 10, 0, 0, 0, 0, 0, 0, 8]] \$$$

$$[y_5, y_4, 0, y_3, 0, 0, y_2, 0, y_1]$$

242 . Coloring, {3, 4, 5, 6, 7, 9}

R: [4, 4, 5, 8, 3, 8, 5, 1, 2]

B: [2, 9, 4, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{3, 5}, {1, 4, 8}} order: 6

See Matrix

$$\$ [[2, 1, 2, 5, 4, 0, 0, 4, 0], [4, 0, 4, 3, 2, 0, 0, 5, 0], [5, 0, 2, 4, 4, 0, 0, 3, 0], [3, 0, 4, 5, 2, 0, 0, 4, 0], [4, 0, 2, 3, 4, 0, 0, 5, 0], [5, 0, 4, 4, 2, 0, 0, 3, 0]] \$$$

$$[-y_2 + 2y_1 - y_4 + 2y_5 - y_3, y_2, y_1, y_4, y_5, 0, 0, y_3, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

$$\$ [[4, 3, 0, 1, 0, 2, 6, 0, 2], [8, 4, 0, 0, 0, 0, 3, 0, 3], [6, 8, 0, 0, 0, 0, 0, 0, 4], [4, 6, 0, 0, 0, 0, 0, 0, 8], [8, 4, 0, 0, 0, 0, 0, 0, 6], [6, 8, 0, 0, 0, 0, 0, 0, 4]] \$$$

$$[y_2, y_1, 0, y_3, 0, 2y_3, y_4, 0, y_5]$$

$$p = s^3 - s^6$$

243 . Coloring, {3, 4, 5, 6, 8, 9}

R: [4, 4, 5, 8, 3, 8, 1, 6, 2]

B: [2, 9, 4, 7, 7, 7, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	7 vs 8	4 vs 7	5 vs 6

Omega Rank for R : cycles: {{3, 5}, {6, 8}} order: 4

See Matrix

$$\$ [[3, 1, 2, 5, 1, 2, 0, 4, 0], [0, 0, 1, 4, 2, 4, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0], [0, 0, 1, 0, 2, 8, 0, 7, 0], [0, 0, 2, 0, 1, 7, 0, 8, 0]] \$$$

$$[3y_4, y_4, y_3, y_2, y_1, 2y_3 - y_2 + 3y_1, 0, -4y_4 + 3y_3 + 2y_1, 0]$$

$$p = s^3 - s^7 \quad p' = s^4 - s^6 \quad p'' = -s^3 + s^5$$

Omega Rank for B : cycles: {{1, 2, 9}, {5, 7}} order: 6

See Matrix

$$\$ [[3, 3, 0, 1, 3, 0, 6, 0, 2], [2, 3, 0, 0, 6, 0, 4, 0, 3], [3, 2, 0, 0, 4, 0, 6, 0, 3], [3, 3, 0, 0, 6, 0, 4, 0, 2], [2, 3, 0, 0, 4, 0, 6, 0, 3], [3, 2, 0, 0, 6, 0, 4, 0, 3]] \$$$

$$[4y_1, 4y_2, 0, 4y_5, 5y_1 + 5y_2 - 4y_5 - 4y_4 + 5y_3, 0, 4y_4, 0, 4y_3]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

244 . Coloring, {3, 4, 5, 7, 8, 9}

R: [4, 4, 5, 8, 3, 7, 5, 6, 2]

B: [2, 9, 4, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	7 vs 7	5 vs 6

Omega Rank for R : cycles: {{3, 5}} order: 6

See Matrix

\$ [[0, 1, 2, 5, 4, 2, 1, 3, 0], [0, 0, 4, 1, 3, 3, 2, 5, 0], [0, 0, 3, 0, 6, 5, 3, 1, 0], [0, 0, 6, 0, 6, 1, 5, 0, 0], [0, 0, 6, 0, 11, 0, 1, 0, 0], [0, 0, 11, 0, 7, 0, 0, 0, 0], [0, 0, 7, 0, 11, 0, 0, 0, 0]] \$

[0, y₃, y₂, y₁, y₄, y₇, y₆, y₅, 0]

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

\$ [[6, 3, 0, 1, 0, 0, 5, 1, 2], [8, 6, 0, 0, 0, 0, 1, 0, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4], [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$

[y₁, y₂, 0, y₃, 0, 0, y₅, y₃, y₄]

$$p = -s^3 + s^6$$

245 . Coloring, {3, 4, 6, 7, 8, 9}

R: [4, 4, 5, 8, 7, 8, 5, 6, 2]

B: [2, 9, 4, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	6 vs 6

Omega Rank for R : cycles: {{5, 7}, {6, 8}} order: 4

See Matrix

\$ [[0, 1, 0, 5, 4, 2, 2, 4, 0] , [0, 0, 0, 1, 2, 4, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0] , [0, 0, 0, 0, 4, 7, 2, 5, 0] , [0, 0, 0, 0, 2, 5, 4, 7, 0]] \$

$$[0, 3y_1 - 4y_2 + 3y_3 - y_4, 0, y_1, y_2, y_3, 2y_1 - 3y_2 + 2y_3, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 6

See Matrix

\$ [[6, 3, 2, 1, 0, 0, 4, 0, 2] , [6, 6, 0, 2, 0, 0, 1, 0, 3] , [4, 6, 0, 0, 0, 0, 2, 0, 6] , [8, 4, 0, 0, 0, 0, 0, 0, 6] , [6, 8, 0, 0, 0, 0, 0, 0, 4] , [4, 6, 0, 0, 0, 0, 0, 0, 8]] \$

$$[y_4, y_5, y_1, y_2, 0, 0, y_3, 0, y_6]$$

246 . Coloring, {3, 5, 6, 7, 8, 9}

R: [4, 4, 5, 7, 3, 8, 5, 6, 2]

B: [2, 9, 4, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	5 vs 7	5 vs 6

Omega Rank for R : cycles: {{3, 5}, {6, 8}} order: 4

See Matrix

\$ [[0, 1, 2, 5, 4, 2, 3, 1, 0] , [0, 0, 4, 1, 5, 1, 5, 2, 0] , [0, 0, 5, 0, 9, 2, 1, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0] , [0, 0, 9, 0, 6, 1, 0, 2, 0] , [0, 0, 6, 0, 9, 2, 0, 1, 0]] \$

$$[0, y_5, y_4, y_3, y_2, y_1, -y_5 - y_4 - 15y_1 + 4y_3 + 4y_2, y_3 + y_2 - 4y_1, 0]$$

$$p = -s^4 + s^6 \quad p' = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

\$ [[6, 3, 0, 1, 0, 0, 3, 3, 2], [8, 6, 0, 0, 0, 0, 0, 1, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4], [4, 8, 0, 0, 0, 0, 0, 0, 6]] \$

$[y_3, y_4, 0, y_5, 0, 0, 3y_5, y_1, y_2]$

$$p = -s^3 + s^6$$

247 . Coloring, {4, 5, 6, 7, 8, 9}

R: [4, 4, 4, 8, 3, 8, 5, 6, 2]

B: [2, 9, 5, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 6	5 vs 5

Omega Rank for R : cycles: {{6, 8}} order: 4

See Matrix

\$ [[0, 1, 2, 6, 3, 2, 0, 4, 0], [0, 0, 3, 3, 0, 4, 0, 8, 0], [0, 0, 0, 3, 0, 8, 0, 7, 0], [0, 0, 0, 0, 0, 7, 0, 11, 0], [0, 0, 0, 0, 0, 11, 0, 7, 0], [0, 0, 0, 0, 0, 7, 0, 11, 0]] \$

$[0, y_2, y_1, y_4, 3y_2, y_3, 0, y_5, 0]$

$$p = s^4 - s^6$$

Omega Rank for B : cycles: {{1, 2, 9}} order: 3

See Matrix

\$ [[6, 3, 0, 0, 1, 0, 6, 0, 2], [8, 6, 0, 0, 0, 0, 1, 0, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4]] \$

$[y_1, y_3, 0, 0, y_2, 0, y_4, 0, y_5]$

248 . Coloring, {2, 3, 4, 5, 6, 7, 8}

$$\Omega p(\Delta)=0: \quad p = -s^3 + s^4 + 8s^7$$

R: [4, 9, 5, 8, 3, 8, 5, 6, 1]

B: [2, 4, 4, 7, 7, 7, 1, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 7	7 vs 7	7 vs 7	5 vs 7	4 vs 4

Omega Rank for R : cycles: {{3, 5}, {6, 8}} order: 4

See Matrix

$$\$ [[1, 0, 2, 3, 4, 2, 0, 4, 2], [2, 0, 4, 1, 2, 4, 0, 5, 0], [0, 0, 2, 2, 4, 5, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0]] \$$$

$$[y_2, 0, y_1, y_5, 2y_2 - 3y_1 + 2y_3, y_4, 0, y_3, 3y_2 - 4y_1 - y_5 - y_4 + 3y_3]$$

$$p' = -s^4 + s^6 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

$$\$ [[5, 4, 0, 3, 0, 0, 6, 0, 0], [6, 5, 0, 4, 0, 0, 3, 0, 0], [3, 6, 0, 5, 0, 0, 4, 0, 0], [4, 3, 0, 6, 0, 0, 5, 0, 0]] \$$$

$$[y_2, y_3, 0, y_1, 0, 0, y_4, 0, 0]$$

249 . Coloring, {2, 3, 4, 5, 6, 7, 9}

R: [4, 9, 5, 8, 3, 8, 5, 1, 2]

B: [2, 4, 4, 7, 7, 7, 1, 6, 1]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 7	4 vs 5

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{1, 4, 8\}\}$ order: 6
See Matrix

$\$ [[2, 1, 2, 3, 4, 0, 0, 4, 2], [4, 2, 4, 2, 2, 0, 0, 3, 1], [3, 1, 2, 4, 4, 0, 0, 2, 2], [2, 2, 4, 3, 2, 0, 0, 4, 1], [4, 1, 2, 2, 4, 0, 0, 3, 2], [3, 2, 4, 4, 2, 0, 0, 2, 1], [2, 1, 2, 3, 4, 0, 0, 4, 2]] \$$

$$[y_2, y_1, 2y_1, -y_2 + 3y_1 - y_4 + 3y_3, 2y_3, 0, 0, y_4, y_3]$$

$$p' = s^2 + s^3 - s^5 - s^6 \quad p' = s - s^3 - s^4 + s^6 \quad p = s - s^7$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4
See Matrix

$\$ [[4, 3, 0, 3, 0, 2, 6, 0, 0], [6, 4, 0, 3, 0, 0, 5, 0, 0], [5, 6, 0, 4, 0, 0, 3, 0, 0], [3, 5, 0, 6, 0, 0, 4, 0, 0], [4, 3, 0, 5, 0, 0, 6, 0, 0]] \$$

$$[y_1 - y_2 - y_3 + y_4, y_1, 0, y_2, 0, y_3, y_4, 0, 0]$$

$$p = s^2 - s^3 + s^4 - s^5$$

250 . Coloring, $\{2, 3, 4, 5, 6, 8, 9\}$

R: $[4, 9, 5, 8, 3, 8, 1, 6, 2]$

B: $[2, 4, 4, 7, 7, 7, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 8	4 vs 5

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{6, 8\}\}$ order: 4
See Matrix

\$ [[3, 1, 2, 3, 1, 2, 0, 4, 2], [0, 2, 1, 3, 2, 4, 0, 5, 1], [0, 1, 2, 0, 1, 5, 0, 7, 2], [0, 2, 1, 0, 2, 7, 0, 5, 1], [0, 1, 2, 0, 1, 5, 0, 7, 2], [0, 2, 1, 0, 2, 7, 0, 5, 1], [0, 1, 2, 0, 1, 5, 0, 7, 2], [0, 2, 1, 0, 2, 7, 0, 5, 1]] \$

$$[y_2 + 3y_4 - y_3, y_2, y_4, 3y_2 + y_4 - y_1, y_2, y_1, 0, y_3, y_4]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^7 \quad p' = -s^3 + s^7 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

\$ [[3, 3, 0, 3, 3, 0, 6, 0, 0], [0, 3, 0, 3, 6, 0, 6, 0, 0], [0, 0, 0, 3, 6, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0], [0, 0, 0, 0, 9, 0, 9, 0, 0]] \$

$$[y_2, y_2 + y_1 + y_3 - y_4, 0, y_1, y_3, 0, y_4, 0, 0]$$

$$p = -s^4 + s^5$$

251 . Coloring, {2, 3, 4, 5, 7, 8, 9}

R: [4, 9, 5, 8, 3, 7, 5, 6, 2]

B: [2, 4, 4, 7, 7, 8, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	6 vs 8	4 vs 5

Omega Rank for R : cycles: {{2, 9}, {3, 5}} order: 6

See Matrix

\$ [[0, 1, 2, 3, 4, 2, 1, 3, 2], [0, 2, 4, 0, 3, 3, 2, 3, 1], [0, 1, 3, 0, 6, 3, 3, 0, 2], [0, 2, 6, 0, 6, 0, 3, 0, 1], [0, 1, 6, 0, 9, 0, 0, 0, 2], [0, 2, 9, 0, 6, 0, 0, 0, 1], [0, 1, 6, 0, 9, 0, 0, 0, 2], [0, 2, 9, 0, 6, 0, 0, 0, 1]] \$

$$[0, y_6, y_3, y_4, y_5, -15y_6 - y_4 - y_5 + 4y_3 + 4y_2 + 4y_1, y_2, y_1, -4y_6 + y_3 + y_2 + y_1]$$

$$p' = -s^5 + s^7 \quad p = -s^5 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

\$ [[6, 3, 0, 3, 0, 0, 5, 1, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0] , [6, 3, 0, 3, 0, 0, 6, 0, 0]] \$

$$[y_1, y_1 + y_2 - y_3 - y_4, 0, y_2, 0, 0, y_3, y_4, 0]$$

$$p = s^2 - s^3 + s^4 - s^5$$

252 . Coloring, {2, 3, 4, 6, 7, 8, 9}

R: [4, 9, 5, 8, 7, 8, 5, 6, 2]

B: [2, 4, 4, 7, 3, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 7	4 vs 5

Omega Rank for R : cycles: {{2, 9}, {6, 8}, {5, 7}} order: 2

See Matrix

\$ [[0, 1, 0, 3, 4, 2, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2] , [0, 2, 0, 0, 2, 4, 4, 5, 1] , [0, 1, 0, 0, 4, 5, 2, 4, 2]] \$

$$[0, y_1 + y_2 - 2y_3, 0, y_1, 2y_3, y_2, 2y_1 + 2y_2 - 4y_3, 2y_1 + 2y_2 - 3y_3, y_3]$$

$$p' = s^2 - s^6 \quad p' = s^3 - s^5 \quad p' = s^4 - s^6 \quad p = s^2 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4

See Matrix

\$ [[6, 3, 2, 3, 0, 0, 4, 0, 0] , [4, 6, 0, 5, 0, 0, 3, 0, 0] , [3, 4, 0, 6, 0, 0, 5, 0, 0] , [5, 3, 0, 4, 0, 0, 6, 0, 0] , [6, 5, 0, 3, 0, 0, 4, 0, 0]] \$

$$[y_1 + y_2 - y_4 + y_3, y_1, y_2, y_4, 0, 0, y_3, 0, 0]$$

$$p = s^2 - s^3 + s^4 - s^5$$

253 . Coloring, {2, 3, 5, 6, 7, 8, 9}

R: [4, 9, 5, 7, 3, 8, 5, 6, 2]

B: [2, 4, 4, 8, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	9 vs 9	9 vs 9	4 vs 8	4 vs 5

Omega Rank for R : cycles: {{2, 9}, {3, 5}, {6, 8}} order: 4

See Matrix

$\$ [[0, 1, 2, 3, 4, 2, 3, 1, 2], [0, 2, 4, 0, 5, 1, 3, 2, 1], [0, 1, 5, 0, 7, 2, 0, 1, 2], [0, 2, 7, 0, 5, 1, 0, 2, 1], [0, 1, 5, 0, 7, 2, 0, 1, 2], [0, 2, 7, 0, 5, 1, 0, 2, 1], [0, 1, 5, 0, 7, 2, 0, 1, 2], [0, 2, 7, 0, 5, 1, 0, 2, 1]] \$$

$$[0, y_1, 3y_1 + y_2 - y_4, y_1 - y_3 + 3y_2, y_3, y_2, y_4, y_1, y_2]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^7 \quad p' = -s^3 + s^7$$

Omega Rank for B : cycles: {{1, 2, 4, 8}} order: 4

See Matrix

$\$ [[6, 3, 0, 3, 0, 0, 3, 3, 0], [6, 6, 0, 3, 0, 0, 0, 3, 0], [3, 6, 0, 6, 0, 0, 0, 3, 0], [3, 3, 0, 6, 0, 0, 0, 6, 0], [6, 3, 0, 3, 0, 0, 0, 6, 0]] \$$

$$[y_1, y_1 + y_4 - y_3 - y_2, 0, y_4, 0, 0, y_3, y_2, 0]$$

$$p = s^2 - s^3 + s^4 - s^5$$

254 . Coloring, {2, 4, 5, 6, 7, 8, 9}

R: [4, 9, 4, 8, 3, 8, 5, 6, 2]

B: [2, 4, 5, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	7 vs 7	7 vs 7	5 vs 7	4 vs 5

Omega Rank for R : cycles: $\{\{2, 9\}, \{6, 8\}\}$ order: 4
See Matrix

$\$ [[0, 1, 2, 4, 3, 2, 0, 4, 2], [0, 2, 3, 2, 0, 4, 0, 6, 1], [0, 1, 0, 3, 0, 6, 0, 6, 2], [0, 2, 0, 0, 0, 6, 0, 9, 1], [0, 1, 0, 0, 0, 9, 0, 6, 2], [0, 2, 0, 0, 0, 6, 0, 9, 1], [0, 1, 0, 0, 0, 9, 0, 6, 2]] \$$

$$[0, y_1, y_4, y_5, y_3, -15y_1 - y_5 - y_3 + 4y_4 + 4y_2, 0, y_2, -4y_1 + y_4 + y_2]$$

$$p' = -s^4 + s^6 \quad p = s^4 - s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 7\}\}$ order: 4
See Matrix

$\$ [[6, 3, 0, 2, 1, 0, 6, 0, 0], [6, 6, 0, 3, 0, 0, 3, 0, 0], [3, 6, 0, 6, 0, 0, 3, 0, 0], [3, 3, 0, 6, 0, 0, 6, 0, 0], [6, 3, 0, 3, 0, 0, 6, 0, 0]] \$$

$$[y_4, y_3, 0, y_2, y_1, 0, y_4 - y_3 + y_2 + y_1, 0, 0]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

255 . Coloring, $\{3, 4, 5, 6, 7, 8, 9\}$

R: [4, 4, 5, 8, 3, 8, 5, 6, 2]

B: [2, 9, 4, 7, 7, 7, 1, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	4 vs 6	5 vs 5

Omega Rank for R : cycles: $\{\{3, 5\}, \{6, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 2, 5, 4, 2, 0, 4, 0], [0, 0, 4, 1, 2, 4, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0], [0, 0, 2, 0, 4, 7, 0, 5, 0], [0, 0, 4, 0, 2, 5, 0, 7, 0]] \$$

$$[0, 3y_2 - 4y_3 + 3y_1 - y_4, 2y_2 - 3y_3 + 2y_1, y_2, y_3, y_1, 0, y_4, 0]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

Omega Rank for B : cycles: $\{\{1, 2, 9\}\}$ order: 3

See Matrix

$\$ [[6, 3, 0, 1, 0, 0, 6, 0, 2], [8, 6, 0, 0, 0, 0, 1, 0, 3], [4, 8, 0, 0, 0, 0, 0, 0, 6], [6, 4, 0, 0, 0, 0, 0, 0, 8], [8, 6, 0, 0, 0, 0, 0, 0, 4]] \$$

$$[y_1, y_2, 0, y_3, 0, 0, y_4, 0, y_5]$$

256 . Coloring, $\{2, 3, 4, 5, 6, 7, 8, 9\}$

R: $[4, 9, 5, 8, 3, 8, 5, 6, 2]$

B: $[2, 4, 4, 7, 7, 7, 1, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
7 vs 7	8 vs 8	8 vs 8	3 vs 7	3 vs 4

Omega Rank for R : cycles: $\{\{2, 9\}, \{3, 5\}, \{6, 8\}\}$ order: 2

See Matrix

$\$ [[0, 1, 2, 3, 4, 2, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 5, 1], [0, 1, 2, 0, 4, 5, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 5, 1], [0, 1, 2, 0, 4, 5, 0, 4, 2], [0, 2, 4, 0, 2, 4, 0, 5, 1], [0, 1, 2, 0, 4, 5, 0, 4, 2]] \$$

$$[0, y_1 + y_2 - 2y_3, 2y_1 + 2y_2 - 4y_3, y_1, 2y_3, y_2, 0, 2y_1 + 2y_2 - 3y_3, y_3]$$

$$p = -s^2 + s^4 \quad p' = s^4 - s^6 \quad p = -s^2 + s^6 \quad p' = s^2 - s^6$$

Omega Rank for B : cycles: {{1, 2, 4, 7}} order: 4
 See Matrix

$$\$ [[6, 3, 0, 3, 0, 0, 6, 0, 0] , [6, 6, 0, 3, 0, 0, 3, 0, 0] , [3, 6, 0, 6, 0, 0, 3, 0, 0] , [3, 3, 0, 6, 0, 0, 6, 0, 0]] \$$$

$$[y_1, y_2, 0, -y_1 + y_2 + y_3, 0, 0, y_3, 0, 0]$$

$$p = -s + s^2 - s^3 + s^4$$

SUMMARY	
Graph Type	CC
v(A)	2
v(Δ)	2
π	[3, 2, 1, 3, 2, 1, 3, 2, 1]
Dbly Stoch	false

SANDWICH		Total 4
No .	Coloring	Rank
1	{}	3
2	{2, 3, 5, 6, 8, 9}	6
3	{2, 5, 8}	3
4	{3, 6, 9}	3

RT GROUPS		Total 1	
No .	Coloring	Rank	Solv
1	{2, 4, 7, 9}	2	Not Solvable

CC Colorings		Total 1
No .	Coloring	Sandwich,Rank
1	{}	true, 3

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
213	0	243 , 247	28 , 44	5	256	256
