

# New Graph

[4, 3, 1, 2], [3, 4, 4, 3]

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$$\pi = [1, 1, 2, 2]$$

POSSIBLE RANKS

1 x 6  
2 x 3

BASE DETERMINANT 3/16, .1875000000

*NullSpace* of  $\Delta$

{1, 2, 3, 4}

Nullspace of A

[[2, 4], [1, 3]]

1 . Coloring, {}

$$\Omega p(\Delta)=0: \quad p' = s + 2s^2 \quad p = s - 4s^3$$

**R:** [4, 3, 1, 2]

**B:** [3, 4, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	<b>R</b>	<b>B</b>
1 vs 3	2 vs 4	2 vs 4	2 vs 4	1 vs 2

Omega Rank for R : cycles: {{1, 2, 3, 4}} order: 4

See Matrix

\$ [ [2, 2, 1, 1], [1, 1, 2, 2], [2, 2, 1, 1], [1, 1, 2, 2] ] \$

$$[y_1, y_1, y_2, y_2]$$

$$p' = -1 + s^2 \quad p' = -s + s^3$$

Omega Rank for B : cycles:  $\{\{3, 4\}\}$  order: 2

See Matrix

$$\$ [ [0, 0, 3, 3], [0, 0, 3, 3] ] \$$$

$$[0, 0, y_1, y_1]$$

$$p = s - s^2$$

M          N

$$\$ [ [0, 1, 0, 0], [1, 0, 0, 0], [0, 0, 0, 2], [0, 0, 2, 0] ] \$ \quad \$ [ [0, 3, 2, 1], [3, 0, 1, 2], [2, 1, 0, 3], [1, 2, 3, 0] ] \$$$

$$\tau = 8, r' = 1/2$$

$$\mathbf{R}: [4, 3, 1, 2]$$

$$\mathbf{B}: [3, 4, 4, 3]$$

Ranges

Action of R on ranges,  $[[2], [1]]$

Action of B on ranges,  $[[2], [2]]$

Cycles: R,  $\{\{1, 2, 3, 4\}\}$ , B,  $\{\{3, 4\}\}$

$$\beta(\{1, 2\}) = 1/3$$

$$\beta(\{3, 4\}) = 2/3$$

Partitions

Action of R on partitions,  $[[2], [1]]$

Action of B on partitions,  $[[1], [1]]$

$$\alpha(\{\{1, 4\}, \{2, 3\}\}) = 2/3$$

$$\alpha(\{\{1, 3\}, \{2, 4\}\}) = 1/3$$

$$b_1 = \{1, 3\} \text{ ' , ' } b_2 = \{1, 4\} \text{ ' , ' } b_3 = \{2, 3\} \text{ ' , ' } b_4 = \{2, 4\}$$

Action of R and B on the blocks of the partitions:  $= [3, 1, 4, 2] [2, 3, 2, 3]$

with invariant measure  $[1, 2, 2, 1]$

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
<b>Coloring</b>	{}
<b>Rank</b>	2
<b>R,B</b>	[4, 3, 1, 2], [3, 4, 4, 3]
$\pi_2$	[1, 0, 0, 0, 0, 2]
$u_2$	[3, 2, 1, 1, 2, 3] (dim 1)
<b>wpp</b>	[2, 2, 2, 2]

2 . Coloring, {2}

**R:** [4, 4, 1, 2]

**B:** [3, 3, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	3 vs 3	3 vs 3	3 vs 3	2 vs 2

Omega Rank for R : cycles: {{2, 4}} order: 2

See Matrix

$$\$ [ [2, 2, 0, 2], [0, 2, 0, 4], [0, 4, 0, 2] ] \$$$

$$[y_1, y_2, 0, y_3]$$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$\$ [ [0, 0, 4, 2], [0, 0, 2, 4] ] \$$$

$$[0, 0, y_2, y_1]$$

3 . Coloring, {3}

**R:** [4, 3, 4, 2]

**B:** [3, 4, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	4 vs 4	3 vs 4	3 vs 3	2 vs 3

Omega Rank for R : cycles: {{2, 3, 4}} order: 3

See Matrix

$$\$ [ [0, 2, 1, 3], [0, 3, 2, 1], [0, 1, 3, 2] ] \$$$

$$[0, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 3}} order: 2

See Matrix

$$\$ [ [2, 0, 3, 1], [3, 0, 3, 0], [3, 0, 3, 0] ] \$$$

$$[y_2 - y_1, 0, y_2, y_1]$$

$$p = s^2 - s^3$$

4 . Coloring, {4}

**R:** [4, 3, 1, 3]

**B:** [3, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	4 vs 4	3 vs 4	3 vs 3	2 vs 3

Omega Rank for R : cycles:  $\{\{1, 3, 4\}\}$  order: 3  
See Matrix

$$\$ [ [2, 0, 3, 1], [3, 0, 1, 2], [1, 0, 2, 3] ] \$$$

$$[y_3, 0, y_1, y_2]$$

Omega Rank for B : cycles:  $\{\{2, 4\}\}$  order: 2  
See Matrix

$$\$ [ [0, 2, 1, 3], [0, 3, 0, 3], [0, 3, 0, 3] ] \$$$

$$[0, y_1, -y_1 + y_2, y_2]$$

$$p = s^2 - s^3$$

5 . Coloring,  $\{2, 3\}$

**R**: [4, 4, 4, 2]

**B**: [3, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	3 vs 3	3 vs 3	2 vs 2	2 vs 2

Omega Rank for R : cycles:  $\{\{2, 4\}\}$  order: 2  
See Matrix

$$\$ [ [0, 2, 0, 4], [0, 4, 0, 2] ] \$$$

$$[0, y_1, 0, y_2]$$

Omega Rank for B : cycles:  $\{\{1, 3\}\}$  order: 2

See Matrix

$$\$ [ [2, 0, 4, 0], [4, 0, 2, 0] ] \$$$

$$[y_1, 0, y_2, 0]$$

6 . Coloring,  $\{2, 4\}$

$$\Omega p(\Delta)=0: \quad p = s^2 \quad p' = s^2$$

**R:**  $[4, 4, 1, 3]$

**B:**  $[3, 3, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
1 vs 3	1 vs 3	1 vs 3	1 vs 3	1 vs 3

Omega Rank for R : cycles:  $\{\{1, 3, 4\}\}$  order: 3

See Matrix

$$\$ [ [2, 0, 2, 2], [2, 0, 2, 2], [2, 0, 2, 2] ] \$$$

$$[y_1, 0, y_1, y_1]$$

$$p = -s + s^2 \quad p = -s + s^3$$

Omega Rank for B : cycles:  $\{\{2, 3, 4\}\}$  order: 3

See Matrix

$$\$ [ [0, 2, 2, 2], [0, 2, 2, 2], [0, 2, 2, 2] ] \$$$

$$[0, y_1, y_1, y_1]$$

$$p = -s + s^2 \quad p = -s + s^3$$

‘ See 3-level graph

‘

$$\begin{array}{cc} & \text{M} & \text{N} \\ \$ [ [0, 0, 1, 1], [0, 0, 1, 1], [1, 1, 0, 2], [1, 1, 2, 0] ] \$ & & \$ [ [0, 0, 1, 1], [0, 0, 1, 1], [1, 1, 0, 1], [1, 1, 1, 0] ] \$ \end{array}$$

$$\tau = 6, r' = 2/3$$

$$\mathbf{R}: [4, 4, 1, 3]$$

$$\mathbf{B}: [3, 3, 4, 2]$$

Ranges

Action of R on ranges, [[1], [1]]

Action of B on ranges, [[2], [2]]

Cycles: R, {{1, 3, 4}}, B, {{2, 3, 4}}

$$\beta(\{1, 3, 4\}) = 1/2$$

$$\beta(\{2, 3, 4\}) = 1/2$$

Partitions

$$\alpha(\{\{1, 2\}, \{3\}, \{4\}\}) = 1/1$$

$$b1 = \{1, 2\}, b2 = \{3\}, b3 = \{4\}$$

Action of R and B on the blocks of the partitions: = [2, 3, 1] [3, 1, 2]  
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
<b>Coloring</b>	{2, 4}
<b>Rank</b>	3
<b>R,B</b>	[4, 4, 1, 3], [3, 3, 4, 2]
$\pi_2$	[0, 1, 1, 1, 1, 2]
$u_2$	[0, 1, 1, 1, 1, 1] (dim 1)
<b>wpp</b>	[2, 2, 1, 1]
$\pi_3$	[0, 0, 1, 1]
$u_3$	[0, 0, 1, 1]

7. Coloring, {3, 4}

$$\Omega p(\Delta)=0: \quad p = s - 4s^3 \quad p' = s - 2s^2$$

**R:** [4, 3, 4, 3]

**B:** [3, 4, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
1 vs 3	2 vs 4	2 vs 4	1 vs 2	2 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 2

See Matrix

$$\$ [ [0, 0, 3, 3], [0, 0, 3, 3] ] \$$$

$$[0, 0, y_1, y_1]$$

$$p = -s + s^2$$

Omega Rank for B : cycles:  $\{\{1, 3\}, \{2, 4\}\}$  order: 2  
 See Matrix

$$\$ [ [2, 2, 1, 1], [1, 1, 2, 2], [2, 2, 1, 1], [1, 1, 2, 2] ] \$$$

$$[y_1, y_1, y_2, y_2]$$

$$p' = -s + s^3 \quad p' = -1 + s^2$$

M          N

$$\$ [ [0, 1, 0, 0], [1, 0, 0, 0], [0, 0, 0, 2], [0, 0, 2, 0] ] \$ \quad \$ [ [0, 1, 0, 1], [1, 0, 1, 0], [0, 1, 0, 1], [1, 0, 1, 0] ] \$$$

$$\tau = 8, r' = 1/2$$

**R:** [4, 3, 4, 3]

**B:** [3, 4, 1, 2]

Ranges

Action of R on ranges, [[2], [2]]

Action of B on ranges, [[2], [1]]

Cycles: R,  $\{\{3, 4\}\}$ , B,  $\{\{1, 3\}, \{2, 4\}\}$

$$\beta(\{1, 2\}) = 1/3$$

$$\beta(\{3, 4\}) = 2/3$$

Partitions

$$\alpha(\{\{1, 3\}, \{2, 4\}\}) = 1/1$$

$$b_1 = \{1, 3\}, b_2 = \{2, 4\}$$

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]  
 with invariant measure [1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

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Right Group	
<b>Coloring</b>	{3, 4}
<b>Rank</b>	2
<b>R,B</b>	[4, 3, 4, 3], [3, 4, 1, 2]
$\pi_2$	[1, 0, 0, 0, 0, 2]
$u_2$	[1, 0, 1, 1, 0, 1] (dim 1)
<b>wpp</b>	[2, 2, 2, 2]

8. Coloring, {2, 3, 4}

**R:** [4, 4, 4, 3]

**B:** [3, 3, 1, 2]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 3	3 vs 3	3 vs 3	2 vs 2	3 vs 3

Omega Rank for R : cycles: {{3, 4}} order: 2

See Matrix

$$\$ [ [0, 0, 2, 4], [0, 0, 4, 2] ] \$$$

$$[0, 0, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 3}} order: 2

See Matrix

$$\$ [ [2, 2, 2, 0], [2, 0, 4, 0], [4, 0, 2, 0] ] \$$$

$$[y_3, y_2, y_1, 0]$$

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SUMMARY	
<b>Graph Type</b>	CC
$v(A)$	1
$v(\Delta)$	1
$\pi$	[1, 1, 2, 2]
<b>Dbly Stoch</b>	false

SANDWICH		Total 1
No .	Coloring	Rank
<b>1</b>	{}	2

RT GROUPS		Total 2	
No .	Coloring	Rank	Solv
<b>1</b>	{3, 4}	2	Solvable
<b>2</b>	{2, 4}	3	Solvable

$\Delta$ -RANK'D	SC'D !RK'D	$\tau$ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	$2^{n-1}$
5	0	5, 3	5, 3	3	8	8

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