

New Graph

[3, 4, 4, 3, 2, 5], [6, 1, 2, 6, 1, 3]

$$\pi = [3, 4, 6, 5, 2, 4]$$

POSSIBLE RANKS

1 x 24

2 x 12

3 x 8

4 x 6

BASE DETERMINANT 55/256, .2148437500

NullSpace of Δ

{3, 5, 6}, {1, 2, 4}

Nullspace of A

[[5, 6], {3}]

1 . Coloring, {}

R: [3, 4, 4, 3, 2, 5]

B: [6, 1, 2, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

\$ [[0, 2, 8, 10, 4, 0] , [0, 4, 10, 10, 0, 0] , [0, 0, 10, 14, 0, 0] , [0, 0, 14, 10, 0, 0]] \$

$$[0, y_1, y_2, y_3, y_4, 0]$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$\$ [[6, 6, 4, 0, 0, 8] , [6, 4, 8, 0, 0, 6] , [4, 8, 6, 0, 0, 6] , [8, 6, 6, 0, 0, 4]] \$$$

$$[y_1, y_2, y_3, 0, 0, y_4]$$

2 . Coloring, {2}

R: [3, 1, 4, 3, 2, 5]

B: [6, 4, 2, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	5 vs 5	5 vs 5

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$\$ [[4, 2, 8, 6, 4, 0] , [2, 4, 10, 8, 0, 0] , [4, 0, 10, 10, 0, 0] , [0, 0, 14, 10, 0, 0] , [0, 0, 10, 14, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, y_5, 0]$$

Omega Rank for B : cycles: {{2, 3, 4, 6}} order: 4

See Matrix

$$\$ [[2, 6, 4, 4, 0, 8] , [0, 4, 8, 6, 0, 6] , [0, 8, 6, 4, 0, 6] , [0, 6, 6, 8, 0, 4] , [0, 6, 4, 6, 0, 8]] \$$$

$$[y_3, y_2, y_1, y_4, 0, y_5]$$

3 . Coloring, {3}

R: [3, 4, 2, 3, 2, 5]

B: [6, 1, 4, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R : cycles: $\{\{2, 3, 4\}\}$ order: 3

See Matrix

$$\$ [[0, 8, 8, 4, 4, 0], [0, 12, 4, 8, 0, 0], [0, 4, 8, 12, 0, 0], [0, 8, 12, 4, 0, 0]] \$$$

$$[0, y_1, y_2, y_3, y_4, 0]$$

Omega Rank for B : cycles: $\{\{3, 4, 6\}\}$ order: 3

See Matrix

$$\$ [[6, 0, 4, 6, 0, 8], [0, 0, 8, 4, 0, 12], [0, 0, 12, 8, 0, 4], [0, 0, 4, 12, 0, 8]] \$$$

$$[y_1, 0, y_2, y_4, 0, y_3]$$

4 . Coloring, $\{4\}$

R: [3, 4, 4, 6, 2, 5]

B: [6, 1, 2, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 4

Omega Rank for R : cycles: $\{\{2, 4, 5, 6\}\}$ order: 4

See Matrix

$$\$ [[0, 2, 3, 10, 4, 5], [0, 4, 0, 5, 5, 10], [0, 5, 0, 4, 10, 5], [0, 10, 0, 5, 5, 4], [0, 5, 0, 10, 4, 5]] \$$$

$$[0, y_5, y_4, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$\$ [[6, 6, 9, 0, 0, 3], [6, 9, 3, 0, 0, 6], [9, 3, 6, 0, 0, 6], [3, 6, 6, 0, 0, 9]] \$$$

$$[y_1, y_2, y_3, 0, 0, y_4]$$

5 . Coloring, {5}

$$\Omega p(\Delta)=0: \quad p = s^3 - 2s^4$$

R: [3, 4, 4, 3, 1, 5]

B: [6, 1, 2, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	3 vs 4	2 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$\$ [[2, 0, 8, 10, 4, 0], [4, 0, 12, 8, 0, 0], [0, 0, 12, 12, 0, 0], [0, 0, 12, 12, 0, 0]] \$$$

$$[y_1, 0, y_1 + y_3 - y_2, y_3, y_2, 0]$$

$$p = s^3 - s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$\$ [[4, 8, 4, 0, 0, 8], [8, 4, 8, 0, 0, 4], [4, 8, 4, 0, 0, 8], [8, 4, 8, 0, 0, 4]] \$$$

$$[y_1, y_2, y_1, 0, 0, y_2]$$

$$p' = -s + s^3 \quad p = -s + s^3$$

M N

\$ [[0, 0, 3, 0, 0, 0], [0, 0, 0, 0, 0, 4], [3, 0, 0, 3, 0, 0], [0, 0, 3, 0, 2, 0], [0, 0, 0, 2, 0, 0], [0, 4, 0, 0, 0, 0]]
\$ \$ [[0, 2, 3, 0, 3, 1], [2, 0, 1, 2, 1, 3], [3, 1, 0, 3, 0, 2], [0, 2, 3, 0, 3, 1], [3, 1, 0, 3, 0, 2], [1, 3, 2, 1, 2, 0]] \$

$\tau = 18, r' = 1/2$

R: [3, 4, 4, 3, 1, 5]

B: [6, 1, 2, 6, 2, 3]

Ranges

Action of R on ranges, [[3], [4], [3], [1]]

Action of B on ranges, [[2], [1], [2], [2]]

Cycles: R, {{3, 4}}, B, {{1, 2, 3, 6}}

$\beta(\{1, 3\}) = 1/4$

$\beta(\{2, 6\}) = 1/3$

$\beta(\{3, 4\}) = 1/4$

$\beta(\{4, 5\}) = 1/6$

Partitions

Action of R on partitions, [[1], [1]]

Action of B on partitions, [[2], [1]]

$\alpha(\{\{1, 4, 6\}, \{2, 3, 5\}\}) = 2/3$

$\alpha(\{\{1, 2, 4\}, \{3, 5, 6\}\}) = 1/3$

$b_1 = \{1, 2, 4\}, b_2 = \{1, 4, 6\}, b_3 = \{2, 3, 5\}, b_4 = \{3, 5, 6\}$

Action of R and B on the blocks of the partitions: = [3, 3, 2, 2] [3, 1, 4, 2]
with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

'

Sandwich	
Coloring	{5}
Rank	2
R,B	[3, 4, 4, 3, 1, 5], [6, 1, 2, 6, 2, 3]
π_2	[0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]
u_2	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

6. Coloring, {6}

R: [3, 4, 4, 3, 2, 3]

B: [6, 1, 2, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 2

See Matrix

\$ [[0, 2, 12, 10, 0, 0] , [0, 0, 10, 14, 0, 0] , [0, 0, 14, 10, 0, 0]] \$

[0, y_3 , y_2 , y_1 , 0, 0]

Omega Rank for B : cycles: {{1, 5, 6}} order: 3

See Matrix

\$ [[6, 6, 0, 0, 4, 8] , [10, 0, 0, 0, 8, 6] , [8, 0, 0, 0, 6, 10] , [6, 0, 0, 0, 10, 8]] \$

[y_1 , y_2 , 0, 0, y_3 , y_4]

7 . Coloring, {2, 3}

R: [3, 1, 2, 3, 2, 5]

B: [6, 4, 4, 6, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

\$ [[4, 8, 8, 0, 4, 0] , [8, 12, 4, 0, 0, 0] , [12, 4, 8, 0, 0, 0] , [4, 8, 12, 0, 0, 0]] \$

$[y_1, y_2, y_4, 0, y_3, 0]$

Omega Rank for B : cycles: {{3, 4, 6}} order: 3

See Matrix

\$ [[2, 0, 4, 10, 0, 8] , [0, 0, 8, 4, 0, 12] , [0, 0, 12, 8, 0, 4] , [0, 0, 4, 12, 0, 8]] \$

$[y_1, 0, y_2, y_3, 0, y_4]$

8 . Coloring, {2, 4}

R: [3, 1, 4, 6, 2, 5]

B: [6, 4, 2, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	6 vs 6	5 vs 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

\$ [[4, 2, 3, 6, 4, 5] , [2, 4, 4, 3, 5, 6] , [4, 5, 2, 4, 6, 3] , [5, 6, 4, 2, 3, 4] , [6, 3, 5, 4, 4, 2] , [3, 4, 6, 5, 2, 4]] \$

$[y_1, y_2, y_3, y_4, y_5, y_6]$

Omega Rank for B : cycles: {{2, 3, 4}} order: 3

See Matrix

\$ [[2, 6, 9, 4, 0, 3] , [0, 9, 7, 6, 0, 2] , [0, 7, 8, 9, 0, 0] , [0, 8, 9, 7, 0, 0] , [0, 9, 7, 8, 0, 0]] \$

$[y_3, y_4, y_2, y_1, 0, y_5]$

9. Coloring, {2, 5}

$\Omega p(\Delta)=0: p = s^3 - 2s^4$

R: [3, 1, 4, 3, 1, 5]

B: [6, 4, 2, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	3 vs 4	2 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

\$ [[6, 0, 8, 6, 4, 0] , [4, 0, 12, 8, 0, 0] , [0, 0, 12, 12, 0, 0] , [0, 0, 12, 12, 0, 0]] \$

$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3, 0]$

$p = -s^3 + s^4$

Omega Rank for B : cycles: {{2, 3, 4, 6}} order: 4

See Matrix

\$ [[0, 8, 4, 4, 0, 8] , [0, 4, 8, 8, 0, 4] , [0, 8, 4, 4, 0, 8] , [0, 4, 8, 8, 0, 4]] \$

$[0, y_2, y_1, y_1, 0, y_2]$

$$p = -s + s^3 \quad p' = -s + s^3$$

M N

\$ [[0, 0, 1, 0, 2, 0] , [0, 0, 0, 0, 0, 4] , [1, 0, 0, 5, 0, 0] , [0, 0, 5, 0, 0, 0] , [2, 0, 0, 0, 0, 0] , [0, 4, 0, 0, 0, 0]]
 \$ \$ [[0, 2, 3, 0, 3, 1] , [2, 0, 1, 2, 1, 3] , [3, 1, 0, 3, 0, 2] , [0, 2, 3, 0, 3, 1] , [3, 1, 0, 3, 0, 2] , [1, 3, 2, 1, 2, 0]] \$

$$\tau = 18, r' = 1/2$$

R: [3, 1, 4, 3, 1, 5]

B: [6, 4, 2, 6, 2, 3]

Ranges

Action of R on ranges, [[4], [1], [2], [4]]

Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R , {{3, 4}}, B , {{2, 3, 4, 6}}

$$\beta(\{1, 3\}) = 1/12$$

$$\beta(\{1, 5\}) = 1/6$$

$$\beta(\{2, 6\}) = 1/3$$

$$\beta(\{3, 4\}) = 5/12$$

Partitions

Action of R on partitions, [[1], [1]]

Action of B on partitions, [[2], [1]]

$$\alpha(\{\{1, 4, 6\}, \{2, 3, 5\}\}) = 2/3$$

$$\alpha(\{\{1, 2, 4\}, \{3, 5, 6\}\}) = 1/3$$

$$b1 = \{1, 2, 4\} \text{ ' , ' } b2 = \{1, 4, 6\} \text{ ' , ' } b3 = \{2, 3, 5\} \text{ ' , ' } b4 = \{3, 5, 6\}$$

Action of R and B on the blocks of the partitions: = [3, 3, 2, 2] [3, 1, 4, 2]
 with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

Sandwich	
Coloring	{2, 5}
Rank	2
R,B	[3, 1, 4, 3, 1, 5], [6, 4, 2, 6, 2, 3]
π_2	[0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0]
u_2	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

10 . Coloring, {2, 6}

R: [3, 1, 4, 3, 2, 3]

B: [6, 4, 2, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 5	5 vs 5	4 vs 4	5 vs 5

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

\$ [[4, 2, 12, 6, 0, 0] , [2, 0, 10, 12, 0, 0] , [0, 0, 14, 10, 0, 0] , [0, 0, 10, 14, 0, 0]] \$

[$y_3, y_1, y_2, y_4, 0, 0$]

Omega Rank for B : cycles: {{1, 5, 6}} order: 3

See Matrix

\$ [[2, 6, 0, 4, 4, 8] , [4, 0, 0, 6, 8, 6] , [8, 0, 0, 0, 6, 10] , [6, 0, 0, 0, 10, 8] , [10, 0, 0, 0, 8, 6]] \$

$$[y_3, y_1, 0, y_2, y_4, y_5]$$

11 . Coloring, {3, 4}

R: [3, 4, 2, 6, 2, 5]

B: [6, 1, 4, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 4

Omega Rank for R : cycles: {{2, 4, 5, 6}} order: 4

See Matrix

$$\$ [[0, 8, 3, 4, 4, 5], [0, 7, 0, 8, 5, 4], [0, 5, 0, 7, 4, 8], [0, 4, 0, 5, 8, 7], [0, 8, 0, 4, 7, 5]] \$$$

$$[0, y_1, y_2, y_4, y_3, y_5]$$

Omega Rank for B : cycles: {{3, 4}} order: 4

See Matrix

$$\$ [[6, 0, 9, 6, 0, 3], [0, 0, 9, 9, 0, 6], [0, 0, 15, 9, 0, 0], [0, 0, 9, 15, 0, 0]] \$$$

$$[y_1, 0, y_2, y_3, 0, y_4]$$

12 . Coloring, {3, 5}

R: [3, 4, 2, 3, 1, 5]

B: [6, 1, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	4 vs 5	5 vs 5	5 vs 5

Omega Rank for R : cycles: $\{\{2, 3, 4\}\}$ order: 3
 See Matrix

$$\$ [[2, 6, 8, 4, 4, 0], [4, 8, 6, 6, 0, 0], [0, 6, 10, 8, 0, 0], [0, 10, 8, 6, 0, 0], [0, 8, 6, 10, 0, 0]] \$$$

$$[y_4, y_2, y_3, y_1, y_5, 0]$$

Omega Rank for B : cycles: $\{\{3, 4, 6\}\}$ order: 3
 See Matrix

$$\$ [[4, 2, 4, 6, 0, 8], [2, 0, 8, 4, 0, 10], [0, 0, 10, 8, 0, 6], [0, 0, 6, 10, 0, 8], [0, 0, 8, 6, 0, 10]] \$$$

$$[y_1, y_3, y_4, y_5, 0, y_2]$$

13 . Coloring, $\{3, 6\}$

R: [3, 4, 2, 3, 2, 3]

B: [6, 1, 4, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: $\{\{2, 3, 4\}\}$ order: 3
 See Matrix

$$\$ [[0, 8, 12, 4, 0, 0], [0, 12, 4, 8, 0, 0], [0, 4, 8, 12, 0, 0]] \$$$

$$[0, y_2, y_3, y_1, 0, 0]$$

Omega Rank for B : cycles: $\{\{1, 5, 6\}\}$ order: 3
 See Matrix

$$\$ [[6, 0, 0, 6, 4, 8], [4, 0, 0, 0, 8, 12], [8, 0, 0, 0, 12, 4], [12, 0, 0, 0, 4, 8]] \$$$

$$[y_1, 0, 0, y_4, y_3, y_2]$$

14 . Coloring, {4, 5}

R: [3, 4, 4, 6, 1, 5]

B: [6, 1, 2, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 4

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

$$\$ [[2, 0, 3, 10, 4, 5], [4, 0, 2, 3, 5, 10], [5, 0, 4, 2, 10, 3], [10, 0, 5, 4, 3, 2], [3, 0, 10, 5, 2, 4]] \$$$

$$[y_4, 0, y_5, y_1, y_2, y_3]$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$\$ [[4, 8, 9, 0, 0, 3], [8, 9, 3, 0, 0, 4], [9, 3, 4, 0, 0, 8], [3, 4, 8, 0, 0, 9]] \$$$

$$[y_2, y_1, y_4, 0, 0, y_3]$$

15 . Coloring, {4, 6}

R: [3, 4, 4, 6, 2, 3]

B: [6, 1, 2, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 6	6 vs 6	4 vs 4	5 vs 5

Omega Rank for R : cycles: {{3, 4, 6}} order: 3
See Matrix

$$\$ [[0, 2, 7, 10, 0, 5], [0, 0, 5, 9, 0, 10], [0, 0, 10, 5, 0, 9], [0, 0, 9, 10, 0, 5]] \$$$

$$[0, y_2, y_3, y_4, 0, y_1]$$

Omega Rank for B : cycles: {{1, 5, 6}} order: 3
See Matrix

$$\$ [[6, 6, 5, 0, 4, 3], [10, 5, 0, 0, 3, 6], [8, 0, 0, 0, 6, 10], [6, 0, 0, 0, 10, 8], [10, 0, 0, 0, 8, 6]] \$$$

$$[y_1, y_2, y_5, 0, y_4, y_3]$$

16 . Coloring, {5, 6}

$$\Omega p(\Delta)=0: \quad p = s^3 - 2s^4$$

R: [3, 4, 4, 3, 1, 3]

B: [6, 1, 2, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	2 vs 3	2 vs 4

Omega Rank for R : cycles: {{3, 4}} order: 2
See Matrix

$$\$ [[2, 0, 12, 10, 0, 0], [0, 0, 12, 12, 0, 0], [0, 0, 12, 12, 0, 0]] \$$$

$$[y_2, 0, y_1, -y_2 + y_1, 0, 0]$$

$$p = -s^2 + s^3$$

Omega Rank for B : cycles: {{1, 2, 5, 6}} order: 4
See Matrix

$\$ [[4, 8, 0, 0, 4, 8], [8, 4, 0, 0, 8, 4], [4, 8, 0, 0, 4, 8], [8, 4, 0, 0, 8, 4]] \$$

$[y_2, y_1, 0, 0, y_2, y_1]$

$$p' = s - s^3 \quad p = s - s^3$$

M N

$\$ [[0, 0, 1, 0, 2, 0], [0, 0, 0, 0, 0, 4], [1, 0, 0, 5, 0, 0], [0, 0, 5, 0, 0, 0], [2, 0, 0, 0, 0, 0], [0, 4, 0, 0, 0, 0]]$
 $\$ \quad \$ [[0, 2, 3, 0, 3, 1], [2, 0, 1, 2, 1, 3], [3, 1, 0, 3, 0, 2], [0, 2, 3, 0, 3, 1], [3, 1, 0, 3, 0, 2], [1, 3, 2, 1, 2, 0]] \$$

$$\tau = 18, r' = 1/2$$

R: [3, 4, 4, 3, 1, 3]

B: [6, 1, 2, 6, 2, 5]

Ranges

Action of R on ranges, [[4], [1], [4], [4]]

Action of B on ranges, [[3], [3], [2], [3]]

Cycles: R, {{3, 4}}, B, {{1, 2, 5, 6}}

$$\beta(\{1, 3\}) = 1/12$$

$$\beta(\{1, 5\}) = 1/6$$

$$\beta(\{2, 6\}) = 1/3$$

$$\beta(\{3, 4\}) = 5/12$$

Partitions

Action of R on partitions, [[1], [1]]

Action of B on partitions, [[2], [1]]

$$\alpha(\{\{1, 4, 6\}, \{2, 3, 5\}\}) = 2/3$$

$$\alpha(\{\{1, 2, 4\}, \{3, 5, 6\}\}) = 1/3$$

$$b1 = \{1, 2, 4\}, b2 = \{1, 4, 6\}, b3 = \{2, 3, 5\}, b4 = \{3, 5, 6\}$$

Action of R and B on the blocks of the partitions: = [3, 3, 2, 2] [3, 1, 4, 2]
 with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

Sandwich	
Coloring	{5, 6}
Rank	2
R,B	[3, 4, 4, 3, 1, 3], [6, 1, 2, 6, 2, 5]
π_2	[0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0]
u_2	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

17 . Coloring, {2, 3, 4}

R: [3, 1, 2, 6, 2, 5]

B: [6, 4, 4, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 4

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

\$ [[4, 8, 3, 0, 4, 5] , [8, 7, 4, 0, 5, 0] , [7, 9, 8, 0, 0, 0] , [9, 8, 7, 0, 0, 0] , [8, 7, 9, 0, 0, 0]] \$

[$y_1, y_3, y_2, 0, y_5, y_4$]

Omega Rank for B : cycles: {{3, 4}} order: 4

See Matrix

\$ [[2, 0, 9, 10, 0, 3] , [0, 0, 13, 9, 0, 2] , [0, 0, 11, 13, 0, 0] , [0, 0, 13, 11, 0, 0]] \$

$$[y_1, 0, y_3, y_2, 0, y_4]$$

18 . Coloring, {2, 3, 5}

R: [3, 1, 2, 3, 1, 5]

B: [6, 4, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	4 vs 5	4 vs 4	4 vs 4

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

$$\$ [[6, 6, 8, 0, 4, 0], [10, 8, 6, 0, 0, 0], [8, 6, 10, 0, 0, 0], [6, 10, 8, 0, 0, 0]] \$$$

$$[y_4, y_2, y_3, 0, y_1, 0]$$

Omega Rank for B : cycles: {{3, 4, 6}} order: 3

See Matrix

$$\$ [[0, 2, 4, 10, 0, 8], [0, 0, 8, 6, 0, 10], [0, 0, 10, 8, 0, 6], [0, 0, 6, 10, 0, 8]] \$$$

$$[0, y_2, y_1, y_3, 0, y_4]$$

19 . Coloring, {2, 3, 6}

R: [3, 1, 2, 3, 2, 3]

B: [6, 4, 4, 6, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: $\{\{1, 2, 3\}\}$ order: 3
 See Matrix

$$\$ [[4, 8, 12, 0, 0, 0], [8, 12, 4, 0, 0, 0], [12, 4, 8, 0, 0, 0]] \$$$

$$[y_1, y_2, y_3, 0, 0, 0]$$

Omega Rank for B : cycles: $\{\{1, 5, 6\}\}$ order: 3
 See Matrix

$$\$ [[2, 0, 0, 10, 4, 8], [4, 0, 0, 0, 8, 12], [8, 0, 0, 0, 12, 4], [12, 0, 0, 0, 4, 8]] \$$$

$$[y_3, 0, 0, y_2, y_1, y_4]$$

20 . Coloring, $\{2, 4, 5\}$

R: [3, 1, 4, 6, 1, 5]

B: [6, 4, 2, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 4

Omega Rank for R : cycles: $\{\{1, 3, 4, 5, 6\}\}$ order: 5
 See Matrix

$$\$ [[6, 0, 3, 6, 4, 5], [4, 0, 6, 3, 5, 6], [5, 0, 4, 6, 6, 3], [6, 0, 5, 4, 3, 6], [3, 0, 6, 5, 6, 4]] \$$$

$$[y_1, 0, y_3, y_4, y_5, y_2]$$

Omega Rank for B : cycles: $\{\{2, 3, 4\}\}$ order: 3
 See Matrix

$$\$ [[0, 8, 9, 4, 0, 3], [0, 9, 7, 8, 0, 0], [0, 7, 8, 9, 0, 0], [0, 8, 9, 7, 0, 0]] \$$$

$$[0, y_4, y_2, y_3, 0, y_1]$$

21 . Coloring, {2, 4, 6}

R: [3, 1, 4, 6, 2, 3]

B: [6, 4, 2, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 6	6 vs 6	5 vs 5	3 vs 6

Omega Rank for R : cycles: {{3, 4, 6}} order: 3

See Matrix

$$\$ [[4, 2, 7, 6, 0, 5], [2, 0, 9, 7, 0, 6], [0, 0, 8, 9, 0, 7], [0, 0, 7, 8, 0, 9], [0, 0, 9, 7, 0, 8]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

Omega Rank for B : cycles: {{1, 5, 6}, {2, 3, 4}} order: 3

See Matrix

$$\$ [[2, 6, 5, 4, 4, 3], [4, 5, 4, 6, 3, 2], [3, 4, 6, 5, 2, 4], [2, 6, 5, 4, 4, 3], [4, 5, 4, 6, 3, 2], [3, 4, 6, 5, 2, 4]] \$$$

$$[9y_1 - 2y_2 - 11y_3, 2y_1 + 2y_2 - 2y_3, 2y_1, 11y_1 - 2y_2 - 13y_3, 2y_2, 2y_3]$$

$$p' = -s^2 + s^5 \quad p' = -s + s^4 \quad p' = -1 + s^3$$

22 . Coloring, {2, 5, 6}

$$\Omega p(\Delta)=0: \quad p = s^3 - 2s^4$$

R: [3, 1, 4, 3, 1, 3]

B: [6, 4, 2, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	2 vs 3	2 vs 4

Omega Rank for R : cycles: $\{\{3, 4\}\}$ order: 2
See Matrix

$$\$ [[6, 0, 12, 6, 0, 0], [0, 0, 12, 12, 0, 0], [0, 0, 12, 12, 0, 0]] \$$$

$$[y_2, 0, y_2 + y_1, y_1, 0, 0]$$

$$p = s^2 - s^3$$

Omega Rank for B : cycles: $\{\{2, 4, 5, 6\}\}$ order: 4
See Matrix

$$\$ [[0, 8, 0, 4, 4, 8], [0, 4, 0, 8, 8, 4], [0, 8, 0, 4, 4, 8], [0, 4, 0, 8, 8, 4]] \$$$

$$[0, y_2, 0, y_1, y_1, y_2]$$

$$p' = s - s^3 \quad p = s - s^3$$

M N

$$\$ [[0, 0, 3, 0, 0, 0], [0, 0, 0, 0, 0, 4], [3, 0, 0, 3, 0, 0], [0, 0, 3, 0, 2, 0], [0, 0, 0, 2, 0, 0], [0, 4, 0, 0, 0, 0]]$$

$$\$ \quad \$ [[0, 2, 3, 0, 3, 1], [2, 0, 1, 2, 1, 3], [3, 1, 0, 3, 0, 2], [0, 2, 3, 0, 3, 1], [3, 1, 0, 3, 0, 2], [1, 3, 2, 1, 2, 0]] \$$$

$$\tau = 18, r' = 1/2$$

$$\mathbf{R}: [3, 1, 4, 3, 1, 3]$$

$$\mathbf{B}: [6, 4, 2, 6, 2, 5]$$

Ranges

Action of R on ranges, $[[3], [1], [3], [1]]$

Action of B on ranges, $[[2], [4], [2], [2]]$

Cycles: R , $\{\{3, 4\}\}$, B , $\{\{2, 4, 5, 6\}\}$

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 6\}) = 1/3$$

$$\beta(\{3, 4\}) = 1/4$$

$$\beta(\{4, 5\}) = 1/6$$

Partitions

Action of R on partitions, [[1], [1]]

Action of B on partitions, [[2], [1]]

$$\alpha(\{\{1, 4, 6\}, \{2, 3, 5\}\}) = 2/3$$

$$\alpha(\{\{1, 2, 4\}, \{3, 5, 6\}\}) = 1/3$$

$$b_1 = \{1, 2, 4\} \text{ , , } b_2 = \{1, 4, 6\} \text{ , , } b_3 = \{2, 3, 5\} \text{ , , } b_4 = \{3, 5, 6\}$$

Action of R and B on the blocks of the partitions: = [3, 3, 2, 2] [3, 1, 4, 2]
with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 5, 6}
Rank	2
R,B	[3, 1, 4, 3, 1, 3], [6, 4, 2, 6, 2, 5]
π_2	[0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]
u_2	[2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

23 . Coloring, {3, 4, 5}

R: [3, 4, 2, 6, 1, 5]

B: [6, 1, 4, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	5 vs 6	6 vs 6	5 vs 5

Omega Rank for R : cycles: $\{\{1, 2, 3, 4, 5, 6\}\}$ order: 6

See Matrix

$\$ [[2, 6, 3, 4, 4, 5], [4, 3, 2, 6, 5, 4], [5, 2, 4, 3, 4, 6], [4, 4, 5, 2, 6, 3], [6, 5, 4, 4, 3, 2], [3, 4, 6, 5, 2, 4]] \$$

$[y_1, y_4, y_5, y_6, y_2, y_3]$

Omega Rank for B : cycles: $\{\{3, 4\}\}$ order: 4

See Matrix

$\$ [[4, 2, 9, 6, 0, 3], [2, 0, 9, 9, 0, 4], [0, 0, 13, 9, 0, 2], [0, 0, 11, 13, 0, 0], [0, 0, 13, 11, 0, 0]] \$$

$[y_3, y_2, y_1, y_5, 0, y_4]$

24 . Coloring, $\{3, 4, 6\}$

R: $[3, 4, 2, 6, 2, 3]$

B: $[6, 1, 4, 3, 1, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	4 vs 4	4 vs 5

Omega Rank for R : cycles: $\{\{2, 3, 4, 6\}\}$ order: 4

See Matrix

$\$ [[0, 8, 7, 4, 0, 5], [0, 7, 5, 8, 0, 4], [0, 5, 4, 7, 0, 8], [0, 4, 8, 5, 0, 7]] \$$

$[0, y_1, y_4, y_2, 0, y_3]$

Omega Rank for B : cycles: $\{\{3, 4\}, \{1, 5, 6\}\}$ order: 6

See Matrix

$\$ [[6, 0, 5, 6, 4, 3] , [4, 0, 6, 5, 3, 6] , [3, 0, 5, 6, 6, 4] , [6, 0, 6, 5, 4, 3] , [4, 0, 5, 6, 3, 6]] \$$

$$[11 y_3, 0, 11 y_2, 11 y_1, -11 y_3 + 13 y_2 + 13 y_1 - 11 y_4, 11 y_4]$$

$$p = s + s^2 - s^4 - s^5$$

25 . Coloring, {3, 5, 6}

R: [3, 4, 2, 3, 1, 3]

B: [6, 1, 4, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	5 vs 5

Omega Rank for R : cycles: {{2, 3, 4}} order: 3

See Matrix

$\$ [[2, 6, 12, 4, 0, 0] , [0, 12, 6, 6, 0, 0] , [0, 6, 6, 12, 0, 0] , [0, 6, 12, 6, 0, 0]] \$$

$$[y_1, y_2, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{1, 2, 5, 6}} order: 4

See Matrix

$\$ [[4, 2, 0, 6, 4, 8] , [2, 4, 0, 0, 8, 10] , [4, 8, 0, 0, 10, 2] , [8, 10, 0, 0, 2, 4] , [10, 2, 0, 0, 4, 8]] \$$

$$[y_2, y_1, 0, y_5, y_4, y_3]$$

26 . Coloring, {4, 5, 6}

R: [3, 4, 4, 6, 1, 3]

B: [6, 1, 2, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	4 vs 4	5 vs 5

Omega Rank for R : cycles: $\{\{3, 4, 6\}\}$ order: 3
See Matrix

$$\$ [[2, 0, 7, 10, 0, 5], [0, 0, 7, 7, 0, 10], [0, 0, 10, 7, 0, 7], [0, 0, 7, 10, 0, 7]] \$$$

$$[y_3, 0, y_4, y_1, 0, y_2]$$

Omega Rank for B : cycles: $\{\{1, 2, 5, 6\}\}$ order: 4
See Matrix

$$\$ [[4, 8, 5, 0, 4, 3], [8, 9, 0, 0, 3, 4], [9, 3, 0, 0, 4, 8], [3, 4, 0, 0, 8, 9], [4, 8, 0, 0, 9, 3]] \$$$

$$[y_4, y_1, y_2, 0, y_3, y_5]$$

27 . Coloring, $\{2, 3, 4, 5\}$

R: [3, 1, 2, 6, 1, 5]

B: [6, 4, 4, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	5 vs 6	5 vs 5	3 vs 4

Omega Rank for R : cycles: $\{\{1, 2, 3\}\}$ order: 3
See Matrix

$$\$ [[6, 6, 3, 0, 4, 5], [10, 3, 6, 0, 5, 0], [8, 6, 10, 0, 0, 0], [6, 10, 8, 0, 0, 0], [10, 8, 6, 0, 0, 0]] \$$$

$$[y_2, y_1, y_4, 0, y_5, y_3]$$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$\$ [[0, 2, 9, 10, 0, 3], [0, 0, 13, 11, 0, 0], [0, 0, 11, 13, 0, 0], [0, 0, 13, 11, 0, 0]] \$$$

$$[0, 2 y_1, 2 y_2, 2 y_3, 0, 3 y_1]$$

$$p = -s^2 + s^4$$

28 . Coloring, {2, 3, 4, 6}

R: [3, 1, 2, 6, 2, 3]

B: [6, 4, 4, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	4 vs 4	4 vs 5

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

$$\$ [[4, 8, 7, 0, 0, 5], [8, 7, 9, 0, 0, 0], [7, 9, 8, 0, 0, 0], [9, 8, 7, 0, 0, 0]] \$$$

$$[y_3, y_2, y_1, 0, 0, y_4]$$

Omega Rank for B : cycles: {{3, 4}, {1, 5, 6}} order: 6

See Matrix

$$\$ [[2, 0, 5, 10, 4, 3], [4, 0, 10, 5, 3, 2], [3, 0, 5, 10, 2, 4], [2, 0, 10, 5, 4, 3], [4, 0, 5, 10, 3, 2]] \$$$

$$[3 y_3, 0, 3 y_4, 5 y_3 - 3 y_4 + 5 y_1 + 5 y_2, 3 y_1, 3 y_2]$$

$$p = -s - s^2 + s^4 + s^5$$

29 . Coloring, {2, 3, 5, 6}

R: [3, 1, 2, 3, 1, 3]

B: [6, 4, 4, 6, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

$$\$ [[6, 6, 12, 0, 0, 0], [6, 12, 6, 0, 0, 0], [12, 6, 6, 0, 0, 0]] \$$$

$$[y_2, y_3, y_1, 0, 0, 0]$$

Omega Rank for B : cycles: {{2, 4, 5, 6}} order: 4

See Matrix

$$\$ [[0, 2, 0, 10, 4, 8], [0, 4, 0, 2, 8, 10], [0, 8, 0, 4, 10, 2], [0, 10, 0, 8, 2, 4]] \$$$

$$[0, y_1, 0, y_2, y_3, y_4]$$

30 . Coloring, {2, 4, 5, 6}

R: [3, 1, 4, 6, 1, 3]

B: [6, 4, 2, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	4 vs 4	5 vs 5

Omega Rank for R : cycles: $\{\{3, 4, 6\}\}$ order: 3
 See Matrix

$$\$ [[6, 0, 7, 6, 0, 5] , [0, 0, 11, 7, 0, 6] , [0, 0, 6, 11, 0, 7] , [0, 0, 7, 6, 0, 11]] \$$$

$$[y_4, 0, y_3, y_2, 0, y_1]$$

Omega Rank for B : cycles: $\{\{2, 3, 4\}\}$ order: 3
 See Matrix

$$\$ [[0, 8, 5, 4, 4, 3] , [0, 9, 4, 8, 3, 0] , [0, 7, 8, 9, 0, 0] , [0, 8, 9, 7, 0, 0] , [0, 9, 7, 8, 0, 0]] \$$$

$$[0, y_3, y_4, y_5, y_2, y_1]$$

31 . Coloring, $\{3, 4, 5, 6\}$

R: $[3, 4, 2, 6, 1, 3]$

B: $[6, 1, 4, 3, 2, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 4, 6\}\}$ order: 4
 See Matrix

$$\$ [[2, 6, 7, 4, 0, 5] , [0, 7, 7, 6, 0, 4] , [0, 7, 4, 7, 0, 6] , [0, 4, 6, 7, 0, 7] , [0, 6, 7, 4, 0, 7]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

Omega Rank for B : cycles: $\{\{1, 2, 5, 6\}, \{3, 4\}\}$ order: 4
 See Matrix

$$\$ [[4, 2, 5, 6, 4, 3] , [2, 4, 6, 5, 3, 4] , [4, 3, 5, 6, 4, 2] , [3, 4, 6, 5, 2, 4] , [4, 2, 5, 6, 4, 3] , [2, 4, 6, 5, 3, 4]] \$$$

$$[-23 y_2 + 39 y_1 - 10 y_3 - 23 y_4, 10 y_2, 10 y_1, -11 y_2 + 23 y_1 - 11 y_4, 10 y_3, 10 y_4]$$

$$p' = -s + s^5 \quad p' = -1 + s^4$$

32 . Coloring, {2, 3, 4, 5, 6}

R: [3, 1, 2, 6, 1, 3]

B: [6, 4, 4, 3, 2, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	6 vs 6	5 vs 6	4 vs 4	5 vs 5

Omega Rank for R : cycles: {{1, 2, 3}} order: 3

See Matrix

$$\$ [[6, 6, 7, 0, 0, 5], [6, 7, 11, 0, 0, 0], [7, 11, 6, 0, 0, 0], [11, 6, 7, 0, 0, 0]] \$$$

$$[y_1, y_2, y_3, 0, 0, y_4]$$

Omega Rank for B : cycles: {{3, 4}} order: 4

See Matrix

$$\$ [[0, 2, 5, 10, 4, 3], [0, 4, 10, 7, 3, 0], [0, 3, 7, 14, 0, 0], [0, 0, 14, 10, 0, 0], [0, 0, 10, 14, 0, 0]] \$$$

$$[0, y_1, y_2, y_3, y_4, y_5]$$

SUMMARY	
Graph Type	NOT CC
$v(A)$	1
$v(\Delta)$	2
π	[3, 4, 6, 5, 2, 4]
Dbly Stoch	false

SANDWICH			Total 4
No .	Coloring	Rank	
1	{5}	2	
2	{5, 6}	2	
3	{2, 5, 6}	2	
4	{2, 5}	2	

RT GROUPS				Total 0
No .	Coloring	Rank	Solv	

Δ -RANK'D	SC'D !RK'D	τ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
28	0	24 , 23	28 , 23	4	32	32
