

New Graph

[3, 3, 1, 1, 7, 7, 5, 5], [6, 8, 8, 6, 2, 4, 4, 2]

$$\pi = [1, 1, 1, 1, 1, 1, 1, 1]$$

POSSIBLE RANKS

1 x 8
2 x 4

BASE DETERMINANT 4236243/134217728, .3156246990e-1

NullSpace of Δ

{1, 3, 6, 8}, {2, 4, 5, 7}

Nullspace of A

[[{2, 4}, {5, 7}], [{1, 3}, {6, 8}]]

1 . Coloring, {}

$$\Omega p(\Delta)=0: p' = s^3 \quad p' = s^5 \quad p' = s^4 \quad p = s^2 \quad p' = s^2$$

R: [3, 3, 1, 1, 7, 7, 5, 5]

B: [6, 8, 8, 6, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
1 vs 6	1 vs 6	1 vs 6	1 vs 4	1 vs 4

Omega Rank for R : cycles: {{5, 7}, {1, 3}} order: 2

See Matrix

\$ [[2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0]] \$

$$[y_1, 0, y_1, 0, y_1, 0, y_1, 0]$$

$$p = -s + s^4 \quad p = -s + s^2 \quad p = -s + s^3$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{2, 8\}\}$ order: 2

See Matrix

$$\$ [[0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2]] \$$$

$$[0, y_1, 0, y_1, 0, y_1, 0, y_1]$$

$$p = -s + s^4 \quad p = -s + s^2 \quad p = -s + s^3$$

‘ See 4-level graph

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$$M \quad \setminus ; \quad N$$

$$\begin{aligned} & \$ [[0, 0, 2, 0, 2, 0, 2, 0], [0, 0, 0, 2, 0, 2, 0, 2], [2, 0, 0, 0, 2, 0, 2, 0], [0, 2, 0, 0, 0, 2, 0, 2], [2, 0, 2, 0, 0, 0, 2, 0], \\ & [0, 2, 0, 2, 0, 0, 0, 2], [2, 0, 2, 0, 2, 0, 0, 0], [0, 2, 0, 2, 0, 2, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 2, 2, 2, 2], \\ & [1, 0, 1, 2, 2, 2, 2, 2], [2, 1, 0, 1, 2, 2, 2, 2], [1, 2, 1, 0, 2, 2, 2, 2], [2, 2, 2, 2, 0, 1, 2, 1], [2, 2, 2, 2, 1, 0, 1, \\ & 2], [2, 2, 2, 2, 2, 1, 0, 1], [2, 2, 2, 2, 1, 2, 1, 0]] \$ \end{aligned}$$

$$\tau = 16, r' = 3/4$$

$$\mathbf{R}: [3, 3, 1, 1, 7, 7, 5, 5]$$

$$\mathbf{B}: [6, 8, 8, 6, 2, 4, 4, 2]$$

Ranges

Action of R on ranges, $[[1], [1]]$

Action of B on ranges, $[[2], [2]]$

Cycles: R , $\{\{5, 7\}, \{1, 3\}\}$, B , $\{\{4, 6\}, \{2, 8\}\}$

$$\beta(\{1, 3, 5, 7\}) = 1/2$$

$$\beta(\{2, 4, 6, 8\}) = 1/2$$

Partitions

Action of R on partitions, $[[1], [1]]$

Action of B on partitions, $[[2], [2]]$

$$\alpha(\{1, 2\}, \{7, 8\}, \{5, 6\}, \{3, 4\}) = 1/2$$

$$\alpha(\{6, 7\}, \{5, 8\}, \{2, 3\}, \{1, 4\}) = 1/2$$

Omega Rank for R : cycles: $\{\{5, 7\}, \{1, 3\}\}$ order: 2

See Matrix

$$\$ [[2, 0, 1, 0, 2, 0, 2, 1], [1, 0, 2, 0, 3, 0, 2, 0], [2, 0, 1, 0, 2, 0, 3, 0], [1, 0, 2, 0, 3, 0, 2, 0], [2, 0, 1, 0, 2, 0, 3, 0]] \$$$

$$[-4y_3 + 3y_2, 0, y_3, 0, y_2, 0, -5y_3 + 4y_2 - y_1, y_1]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{2, 3, 8\}\}$ order: 6

See Matrix

$$\$ [[0, 2, 1, 2, 0, 2, 0, 1], [0, 1, 2, 2, 0, 2, 0, 1], [0, 1, 1, 2, 0, 2, 0, 2], [0, 2, 1, 2, 0, 2, 0, 1], [0, 1, 2, 2, 0, 2, 0, 1]] \$$$

$$[0, y_1, -y_1 + 2y_3 - y_2, y_3, 0, y_3, 0, y_2]$$

$$p' = s - s^4 \quad p = -s + s^4$$

3. Coloring, $\{3\}$

R: $[3, 3, 8, 1, 7, 7, 5, 5]$

B: $[6, 8, 1, 6, 2, 4, 4, 2]$

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$$\$ [[1, 0, 2, 0, 2, 0, 2, 1], [0, 0, 1, 0, 3, 0, 2, 2], [0, 0, 0, 0, 4, 0, 3, 1], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$$

$$[y_1, 0, y_1 - y_2 + y_4 + y_3, 0, y_2, 0, y_4, y_3]$$

$$p = -s^4 + s^5$$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$\$ [[1, 2, 0, 2, 0, 2, 0, 1], [0, 1, 0, 2, 0, 3, 0, 2], [0, 2, 0, 3, 0, 2, 0, 1], [0, 1, 0, 2, 0, 3, 0, 2], [0, 2, 0, 3, 0, 2, 0, 1]] \$$$

$$[y_2, 3y_1 - 4y_3, 0, -y_2 + 4y_1 - 5y_3, 0, y_1, 0, y_3]$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^4$$

4. Coloring, {4}

R: [3, 3, 1, 6, 7, 7, 5, 5]

B: [6, 8, 8, 1, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	3 vs 5

Omega Rank for R : cycles: {{5, 7}, {1, 3}} order: 2

See Matrix

$$\$ [[1, 0, 2, 0, 2, 1, 2, 0], [2, 0, 1, 0, 2, 0, 3, 0], [1, 0, 2, 0, 3, 0, 2, 0], [2, 0, 1, 0, 2, 0, 3, 0], [1, 0, 2, 0, 3, 0, 2, 0]] \$$$

$$[y_1, 0, -4y_1 + 3y_2, 0, -5y_1 - y_3 + 4y_2, y_3, y_2, 0]$$

$$p = -s^2 + s^4 \quad p' = s^2 - s^4$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$\$ [[1, 2, 0, 2, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1, 0, 2], [1, 2, 0, 1, 0, 2, 0, 2], [1, 2, 0, 2, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1, 0, 2]] \$$$

$$[y_2, y_3, 0, -y_2 + 2y_3 - y_1, 0, y_1, 0, y_3]$$

$$p = -s + s^4 \quad p' = s - s^4$$

5. Coloring, {5}

R: [3, 3, 1, 1, 2, 7, 5, 5]

B: [6, 8, 8, 6, 7, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$\$ [[2, 1, 2, 0, 2, 0, 1, 0], [2, 2, 3, 0, 1, 0, 0, 0], [3, 1, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [0, 0, 0]] \$$$

$$[-y_1 + y_2 + y_3 - y_4, y_1, y_2, 0, y_3, 0, y_4, 0]$$

$$p = s^4 - s^5$$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$\$ [[0, 1, 0, 2, 0, 2, 1, 2], [0, 2, 0, 3, 0, 2, 0, 1], [0, 1, 0, 2, 0, 3, 0, 2], [0, 2, 0, 3, 0, 2, 0, 1], [0, 1, 0, 2, 0, 3, 0, 2]] \$$$

$$[0, y_1, 0, y_2, 0, y_3, -5y_1 + 4y_2 - y_3, -4y_1 + 3y_2]$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^4$$

6. Coloring, {6}

R: [3, 3, 1, 1, 7, 4, 5, 5]

B: [6, 8, 8, 6, 2, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[2, 0, 2, 1, 2, 0, 1, 0], [3, 0, 2, 0, 1, 0, 2, 0], [2, 0, 3, 0, 2, 0, 1, 0], [3, 0, 2, 0, 1, 0, 2, 0], [2, 0, 3, 0, 2, 0, 1, 0]] \$$

$$[y_2, 0, 4y_2 - y_1 - 5y_3, y_1, 3y_2 - 4y_3, 0, y_3, 0]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^4$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{4, 6, 7\}\}$ order: 6

See Matrix

$\$ [[0, 2, 0, 1, 0, 2, 1, 2], [0, 2, 0, 1, 0, 1, 2, 2], [0, 2, 0, 2, 0, 1, 1, 2], [0, 2, 0, 1, 0, 2, 1, 2], [0, 2, 0, 1, 0, 1, 2, 2]] \$$

$$[0, y_3, 0, y_2, 0, y_1, 2y_3 - y_2 - y_1, y_3]$$

$$p = -s + s^4 \quad p' = s - s^4$$

7 . Coloring, {7}

R: [3, 3, 1, 1, 7, 7, 4, 5]

B: [6, 8, 8, 6, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$$\$ [[2, 0, 2, 1, 1, 0, 2, 0], [3, 0, 2, 2, 0, 0, 1, 0], [4, 0, 3, 1, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [0, 0, 0]] \$$$

$$[y_1 + y_2 + y_3 - y_4, 0, y_1, y_2, y_3, 0, y_4, 0]$$

$$p = -s^4 + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{4, 6\}\}$ order: 2

See Matrix

$$\$ [[0, 2, 0, 1, 1, 2, 0, 2], [0, 3, 0, 2, 0, 1, 0, 2], [0, 2, 0, 1, 0, 2, 0, 3], [0, 3, 0, 2, 0, 1, 0, 2], [0, 2, 0, 1, 0, 2, 0, 3]] \$$$

$$[0, y_1, 0, y_2, 4y_1 - 5y_2 - y_3, 3y_1 - 4y_2, 0, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

8 . Coloring, $\{8\}$

R: [3, 3, 1, 1, 7, 7, 5, 2]

B: [6, 8, 8, 6, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$$\$ [[2, 1, 2, 0, 1, 0, 2, 0], [2, 0, 3, 0, 2, 0, 1, 0], [3, 0, 2, 0, 1, 0, 2, 0], [2, 0, 3, 0, 2, 0, 1, 0], [3, 0, 2, 0, 1, 0, 2, 0]] \$$$

$$[-y_1 + 4y_2 - 5y_3, y_1, y_2, 0, y_3, 0, 3y_2 - 4y_3, 0]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{2, 5, 8\}\}$ order: 6
See Matrix

$$\$ [[0, 1, 0, 2, 1, 2, 0, 2], [0, 1, 0, 2, 2, 2, 0, 1], [0, 2, 0, 2, 1, 2, 0, 1], [0, 1, 0, 2, 1, 2, 0, 2], [0, 1, 0, 2, 2, 2, 0, 1]] \$$$

$$[0, y_2, 0, y_3, y_1, y_3, 0, -y_2 - y_1 + 2y_3]$$

$$p = s - s^4 \quad p' = -s + s^4$$

9. Coloring, $\{2, 3\}$

R: [3, 8, 8, 1, 7, 7, 5, 5]

B: [6, 3, 1, 6, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	5 vs 5	5 vs 5

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4
See Matrix

$$\$ [[1, 0, 1, 0, 2, 0, 2, 2], [0, 0, 1, 0, 4, 0, 2, 1], [0, 0, 0, 0, 3, 0, 4, 1], [0, 0, 0, 0, 5, 0, 3, 0], [0, 0, 0, 0, 3, 0, 5, 0]] \$$$

$$[y_1, 0, y_2, 0, y_3, 0, y_4, y_5]$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 4
See Matrix

$$\$ [[1, 2, 1, 2, 0, 2, 0, 0], [1, 0, 2, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0], [0, 0, 0, 3, 0, 5, 0, 0], [0, 0, 0, 5, 0, 3, 0, 0]] \$$$

$$[y_5, y_1, y_2, y_3, 0, y_4, 0, 0]$$

10 . Coloring, {2, 4}

$$\Omega p(\Delta)=0: \quad p' = s + 4s^5 \quad p = s + 4s^5 \quad p' = s^2 - 2s^4 + 4s^5 \quad p' = s^3 - 2s^4 + 2s^5$$

R: [3, 8, 1, 6, 7, 7, 5, 5]

B: [6, 3, 8, 1, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 6	3 vs 7	3 vs 7	2 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$\$ [[1, 0, 1, 0, 2, 1, 2, 1], [1, 0, 1, 0, 3, 0, 3, 0], [1, 0, 1, 0, 3, 0, 3, 0], [1, 0, 1, 0, 3, 0, 3, 0], [1, 0, 1, 0, 3, 0, 3, 0], [1, 0, 1, 0, 3, 0, 3, 0], [1, 0, 1, 0, 3, 0, 3, 0]] \$$$

$$[y_1, 0, y_1, 0, 3y_1 - y_2, y_2, 3y_1 - y_2, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 3, 8}} order: 3

See Matrix

$$\$ [[1, 2, 1, 2, 0, 1, 0, 1], [2, 1, 2, 1, 0, 1, 0, 1], [1, 1, 1, 1, 0, 2, 0, 2], [1, 2, 1, 2, 0, 1, 0, 1], [2, 1, 2, 1, 0, 1, 0, 1], [1, 1, 1, 1, 0, 2, 0, 2]] \$$$

$$[y_1, y_2, y_1, y_2, 0, y_3, 0, y_3]$$

$$p = -s + s^4 \quad p' = -s^2 + s^5 \quad p' = -s + s^4$$

M \ ; N

$$\$ [[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 1, 1, 1, 1], [1, 0, 1, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 2, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 1, 1, 1, 1, 2, 1, 0]] \$$$

$$\tau = 32, r' = 1/2$$

R: [3, 8, 1, 6, 7, 7, 5, 5]

B: [6, 3, 8, 1, 2, 4, 4, 2]

Ranges

Action of R on ranges, [[1], [4], [3], [3]]

Action of B on ranges, [[4], [1], [2], [2]]

Cycles: R, {{1, 3}, {5, 7}}, B, {{1, 4, 6}, {2, 3, 8}}

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 4\}) = 1/4$$

$$\beta(\{5, 7\}) = 1/4$$

$$\beta(\{6, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[7], [4], [3], [5], [5], [4], [3], [7]]

Action of B on partitions, [[2], [8], [8], [2], [1], [6], [6], [1]]

$$\alpha(\{\{1, 2, 5, 8\}, \{3, 4, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 5, 8\}, \{1, 2, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 7, 8\}, \{2, 3, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{2, 3, 5, 8\}, \{1, 4, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{2, 3, 7, 8\}, \{1, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/8$$

b1 = {3, 4, 7, 8} ‘, ‘ b2 = {1, 2, 5, 6} ‘, ‘ b3 = {1, 4, 7, 8} ‘, ‘ b4 = {2, 3, 5, 6} ‘, ‘ b5 = {3, 4, 5, 8} ‘, ‘
b6 = {1, 2, 6, 7} ‘, ‘ b7 = {1, 2, 7, 8} ‘, ‘ b8 = {3, 4, 5, 6} ‘, ‘ b9 = {2, 3, 5, 8} ‘, ‘ b10 = {1, 4, 6, 7} ‘, ‘
b11 = {1, 4, 5, 8} ‘, ‘ b12 = {2, 3, 6, 7} ‘, ‘ b13 = {2, 3, 7, 8} ‘, ‘ b14 = {1, 4, 5, 6} ‘, ‘ b15 = {1, 2, 5, 8}
‘, ‘ b16 = {3, 4, 6, 7}

Action of R and B on the blocks of the partitions: = [2, 1, 4, 3, 7, 8, 4, 3, 7, 8, D, E, 2, 1, D, E] [C, B, 10, F, C, B, 5, 6, 9, A, 10, F, 9, A, 5, 6]

with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 4}
Rank	2
R,B	[3, 8, 1, 6, 7, 7, 5, 5], [6, 3, 8, 1, 2, 4, 4, 2]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

11 . Coloring, {2, 5}

R: [3, 8, 1, 1, 2, 7, 5, 5]

B: [6, 3, 8, 6, 7, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 5, 8}, {1, 3}} order: 6

See Matrix

\$ [[2, 1, 1, 0, 2, 0, 1, 1], [1, 2, 2, 0, 2, 0, 0, 1], [2, 2, 1, 0, 1, 0, 0, 2], [1, 1, 2, 0, 2, 0, 0, 2], [2, 2, 1, 0, 2, 0, 0, 1], [1, 2, 2, 0, 1, 0, 0, 2]] \$

$$[3 y_1, 5 y_1 + 5 y_2 - 3 y_3 - 3 y_4 - 3 y_5, 3 y_2, 0, 3 y_3, 0, 3 y_4, 3 y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

\$ [[0, 1, 1, 2, 0, 2, 1, 1], [0, 1, 1, 3, 0, 2, 0, 1], [0, 1, 1, 2, 0, 3, 0, 1], [0, 1, 1, 3, 0, 2, 0, 1], [0, 1, 1, 2, 0, 3, 0, 1], [0, 1, 1, 3, 0, 2, 0, 1]] \$

$$[0, y_3, y_3, 5y_3 - y_1 - y_2, 0, y_1, y_2, y_3]$$

$$p = s^2 - s^6 \quad p' = s^3 - s^5 \quad p'' = s^2 - s^4$$

12. Coloring, {2, 6}

$$\Omega p(\Delta)=0: \quad p = s + 4s^4 \quad p' = s + 4s^4 \quad p'' = s^2 + 4s^5$$

R: [3, 8, 1, 1, 7, 4, 5, 5]

B: [6, 3, 8, 6, 2, 7, 4, 2]

' See graph

' ' See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$\$ [[2, 0, 1, 1, 2, 0, 1, 1], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0]] \$$$

$$[y_2, 0, y_2 - y_1, y_1, y_2, 0, y_2 - y_1, y_1]$$

$$p = -s^2 + s^3 \quad p' = -s^2 + s^4 \quad p'' = -s^2 + s^5 \quad p''' = -s^2 + s^6$$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$\$ [[0, 2, 1, 1, 0, 2, 1, 1], [0, 1, 2, 1, 0, 1, 2, 1], [0, 1, 1, 2, 0, 1, 1, 2], [0, 2, 1, 1, 0, 2, 1, 1], [0, 1, 2, 1, 0, 1, 2, 1], [0, 1, 1, 2, 0, 1, 1, 2]] \$$$

$$[0, y_3, y_1, y_2, 0, y_3, y_1, y_2]$$

$$p = s - s^4 \quad p' = s - s^4 \quad p'' = s^2 - s^5$$

M \ ; N

$$\$ [[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]] \$ \quad \$ [[0, 2, 2, 1, 4, 2, 2, 3], [2, 0, 3, 2, 2, 4, 1, 2], [2, 3, 0, 1, 2, 1, 4, 3], [1, 2, 1, 0, 3, 2, 3, 4], [4, 2, 2, 3, 0, 2, 2, 1], [2, 4, 1, 2, 2, 0, 3,$$

2] , [2, 1, 4, 3, 2, 3, 0, 1] , [3, 2, 3, 4, 1, 2, 1, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 1, 1, 7, 4, 5, 5]

B: [6, 3, 8, 6, 2, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [4], [1], [1]]

Action of B on ranges, [[2], [3], [4], [2]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{4, 6, 7}, {2, 3, 8}}

$\beta(\{1, 5\}) = 1/4$

$\beta(\{2, 6\}) = 1/4$

$\beta(\{3, 7\}) = 1/4$

$\beta(\{4, 8\}) = 1/4$

Partitions

Action of R on partitions, [[3], [2], [2], [4], [4]]

Action of B on partitions, [[5], [4], [1], [1], [4]]

$\alpha(\{\{3, 5, 6, 8\}, \{1, 2, 4, 7\}\}) = 1/4$

$\alpha(\{\{2, 3, 4, 5\}, \{1, 6, 7, 8\}\}) = 1/8$

$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$

$\alpha(\{\{1, 3, 4, 6\}, \{2, 5, 7, 8\}\}) = 3/8$

$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/8$

$b_1 = \{3, 5, 6, 8\}$, , $b_2 = \{1, 2, 4, 7\}$, , $b_3 = \{2, 3, 4, 5\}$, , $b_4 = \{1, 6, 7, 8\}$, , $b_5 = \{1, 2, 7, 8\}$, ,
 $b_6 = \{3, 4, 5, 6\}$, , $b_7 = \{1, 3, 4, 6\}$, , $b_8 = \{2, 5, 7, 8\}$, , $b_9 = \{1, 2, 3, 4\}$, , $b_{10} = \{5, 6, 7, 8\}$

Action of R and B on the blocks of the partitions: = [5, 6, 4, 3, 3, 4, 7, 8, 7, 8] [9, A, 8, 7, 1, 2, 2, 1, 8, 7]
with invariant measure [2, 2, 1, 1, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

'

Sandwich	
Coloring	{2, 6}
Rank	2
R,B	[3, 8, 1, 1, 7, 4, 5, 5], [6, 3, 8, 6, 2, 7, 4, 2]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[2, 2, 1, 4, 2, 2, 3, 3, 2, 2, 4, 1, 2, 1, 2, 1, 4, 3, 3, 2, 3, 4, 2, 2, 1, 3, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

13 . Coloring, {2, 7}

R: [3, 8, 1, 1, 7, 7, 4, 5]

B: [6, 3, 8, 6, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	5 vs 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

\$ [[2, 0, 1, 1, 1, 0, 2, 1] , [2, 0, 2, 2, 1, 0, 1, 0] , [4, 0, 2, 1, 0, 0, 1, 0] , [3, 0, 4, 1, 0, 0, 0, 0] , [5, 0, 3, 0, 0, 0, 0, 0] , [3, 0, 5, 0, 0, 0, 0, 0]] \$

$[y_4, 0, y_5, y_6, y_1, 0, y_2, y_3]$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

\$ [[0, 2, 1, 1, 1, 2, 0, 1] , [0, 2, 2, 2, 0, 1, 0, 1] , [0, 1, 2, 1, 0, 2, 0, 2] , [0, 2, 1, 2, 0, 1, 0, 2] , [0, 2, 2, 1, 0, 2, 0, 1] , [0, 1, 2, 2, 0, 1, 0, 2]] \$

$$[0, -3y_3 + 5y_4 - 3y_2 + 5y_1 - 3y_5, 3y_3, 3y_4, 3y_2, 3y_1, 0, 3y_5]$$

$$p = s^2 + s^3 - s^5 - s^6$$

14 . Coloring, {2, 8}

$$\Omega p(\Delta)=0: \quad p = s + 4s^4 + 8s^5 - 16s^6 \quad p' = s - 4s^3 - 4s^4 + 8s^5 \quad p'' = s^2 + 2s^3 - 4s^5$$

R: [3, 8, 1, 1, 7, 7, 5, 2]

B: [6, 3, 8, 6, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	1 vs 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {5, 7}} order: 2

See Matrix

$$\$ [[2, 1, 1, 0, 1, 0, 2, 1], [1, 1, 2, 0, 2, 0, 1, 1], [2, 1, 1, 0, 1, 0, 2, 1], [1, 1, 2, 0, 2, 0, 1, 1], [2, 1, 1, 0, 1, 0, 2, 1], [1, 1, 2, 0, 2, 0, 1, 1]] \$$$

$$[3y_1 - y_2, y_1, y_2, 0, y_2, 0, 3y_1 - y_2, y_1]$$

$$p' = s - s^5 \quad p'' = s^3 - s^5 \quad p''' = -s + s^5 \quad p^{(4)} = -s + s^3$$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 5, 8}} order: 4

See Matrix

$$\$ [[0, 1, 1, 2, 1, 2, 0, 1], [0, 1, 1, 2, 1, 2, 0, 1], [0, 1, 1, 2, 1, 2, 0, 1], [0, 1, 1, 2, 1, 2, 0, 1], [0, 1, 1, 2, 1, 2, 0, 1], [0, 1, 1, 2, 1, 2, 0, 1]] \$$$

$$[0, y_1, y_1, 2y_1, y_1, 2y_1, 0, y_1]$$

$$p' = s - s^4 \quad p'' = s^2 - s^4 \quad p''' = s^3 - s^4 \quad p^{(4)} = -s^4 + s^5 \quad p^{(5)} = s - s^5$$

M \ ; N

$$\$ [[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]] \$ \quad \$ [[0, 7, 6, 3, 5, 8, 11, 4], [7, 0, 5, 6, 6, 5, 4, 11], [6, 5, 0, 3, 11, 8, 5, 6], [3, 6, 3, 0, 8, 11, 8, 5], [5, 6, 11, 8, 0, 3, 6, 5], [8, 5, 8, 11,$$

3, 0, 3, 6] , [11, 4, 5, 8, 6, 3, 0, 7] , [4, 11, 6, 5, 5, 6, 7, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 1, 1, 7, 7, 5, 2]

B: [6, 3, 8, 6, 2, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [1], [1]]

Action of B on ranges, [[4], [3], [2], [4]]

Cycles: R , {{2, 8}, {1, 3}, {5, 7}}, B , {{4, 6}, {2, 3, 5, 8}}

$\beta(\{1, 7\}) = 1/4$

$\beta(\{2, 8\}) = 1/4$

$\beta(\{3, 5\}) = 1/4$

$\beta(\{4, 6\}) = 1/4$

Partitions

Action of R on partitions, [[1], [4], [3], [2], [1], [3]]

Action of B on partitions, [[6], [6], [2], [2], [4], [5]]

$\alpha(\{\{1, 2, 5, 6\}, \{3, 4, 7, 8\}\}) = 1/11$

$\alpha(\{\{2, 5, 6, 7\}, \{1, 3, 4, 8\}\}) = 3/11$

$\alpha(\{\{1, 5, 6, 8\}, \{2, 3, 4, 7\}\}) = 2/11$

$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 2/11$

$\alpha(\{\{1, 2, 4, 5\}, \{3, 6, 7, 8\}\}) = 1/11$

$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 2/11$

b1 = {1, 5, 6, 8} ‘ , ‘ b2 = {2, 3, 4, 7} ‘ , ‘ b3 = {1, 2, 4, 5} ‘ , ‘ b4 = {3, 6, 7, 8} ‘ , ‘ b5 = {1, 4, 5, 8} ‘ , ‘
b6 = {2, 3, 6, 7} ‘ , ‘ b7 = {1, 2, 3, 4} ‘ , ‘ b8 = {5, 6, 7, 8} ‘ , ‘ b9 = {1, 2, 5, 6} ‘ , ‘ b10 = {2, 5, 6, 7} ‘ , ‘
b11 = {3, 4, 7, 8} ‘ , ‘ b12 = {1, 3, 4, 8}

Action of R and B on the blocks of the partitions: = [2, 1, B, 9, 2, 1, C, A, B, 8, 9, 7] [C, A, 8, 7, 4, 3, A,
C, 5, 5, 6, 6]

with invariant measure [2, 2, 1, 1, 2, 2, 2, 2, 1, 3, 1, 3]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 8}
Rank	2
R,B	[3, 8, 1, 1, 7, 7, 5, 2], [6, 3, 8, 6, 2, 4, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[7, 6, 3, 5, 8, 11, 4, 5, 6, 6, 5, 4, 11, 3, 11, 8, 5, 6, 8, 11, 8, 5, 3, 6, 5, 3, 6, 7] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

15 . Coloring, {3, 4}

R: [3, 3, 8, 6, 7, 7, 5, 5]

B: [6, 8, 1, 1, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 6	6 vs 6	6 vs 6	4 vs 5	4 vs 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

\$ [[0, 0, 2, 0, 2, 1, 2, 1] , [0, 0, 0, 0, 3, 0, 3, 2] , [0, 0, 0, 0, 5, 0, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0] , [0, 0, 0, 0, 5, 0, 3, 0]] \$

[0, 0, 2 y₂, 0, y₄, y₂, y₃, y₁]

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

\$ [[2, 2, 0, 2, 0, 1, 0, 1] , [2, 1, 0, 1, 0, 2, 0, 2] , [1, 2, 0, 2, 0, 2, 0, 1] , [2, 1, 0, 2, 0, 1, 0, 2] , [2, 2, 0, 1, 0, 2, 0, 1]] \$

$$[3 y_2, 3 y_1, 0, -3 y_2 + 5 y_1 - 3 y_3 + 5 y_4, 0, 3 y_3, 0, 3 y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

16 . Coloring, {3, 5}

$$\Omega p(\Delta)=0: \quad p = s - 4s^4 - 8s^5 \quad p' = s - 4s^4 - 8s^5 \quad p'' = s^2 + 2s^3 + 4s^4 + 4s^5$$

R: [3, 3, 8, 1, 2, 7, 5, 5]

B: [6, 8, 1, 6, 7, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	2 vs 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$\$ [[1, 1, 2, 0, 2, 0, 1, 1], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2]] \$$$

$$[y_1, y_2, y_1 + y_2, 0, y_1 + y_2, 0, y_1, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$\$ [[1, 1, 0, 2, 0, 2, 1, 1], [0, 1, 0, 3, 0, 3, 0, 1], [0, 1, 0, 3, 0, 3, 0, 1], [0, 1, 0, 3, 0, 3, 0, 1], [0, 1, 0, 3, 0, 3, 0, 1], [0, 1, 0, 3, 0, 3, 0, 1]] \$$$

$$[y_1, y_2, 0, -y_1 + 3y_2, 0, -y_1 + 3y_2, y_1, y_2]$$

$$p' = s^3 - s^4 \quad p' = -s^4 + s^5 \quad p = s^2 - s^5 \quad p' = s^2 - s^4$$

M \ ; N

$$\$ [[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]] \$ \quad \$ [[0, 2, 4, 2, 3, 5, 7, 5], [2, 0, 4, 4, 3, 3, 5, 7], [4, 4, 0, 5, 7, 2, 3, 3], [2, 4, 5, 0, 2, 7, 5, 3], [3, 3, 7, 2, 0, 5, 4, 4], [5, 3, 2, 7, 5, 0, 2,$$

4] , [7, 5, 3, 5, 4, 2, 0, 2] , [5, 7, 3, 3, 4, 4, 2, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 3, 8, 1, 2, 7, 5, 5]

B: [6, 8, 1, 6, 7, 4, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [2], [1]]

Action of B on ranges, [[4], [2], [1], [4]]

Cycles: R , {{2, 3, 5, 8}}, B , {{2, 8}, {4, 6}}

$\beta(\{1, 7\}) = 1/4$

$\beta(\{2, 8\}) = 1/4$

$\beta(\{3, 5\}) = 1/4$

$\beta(\{4, 6\}) = 1/4$

Partitions

Action of R on partitions, [[5], [4], [1], [5], [4], [2]]

Action of B on partitions, [[5], [3], [6], [3], [5], [6]]

$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/14$

$\alpha(\{\{1, 2, 5, 6\}, \{3, 4, 7, 8\}\}) = 1/14$

$\alpha(\{\{2, 5, 6, 7\}, \{1, 3, 4, 8\}\}) = 1/7$

$\alpha(\{\{1, 2, 3, 6\}, \{4, 5, 7, 8\}\}) = 3/14$

$\alpha(\{\{1, 2, 4, 5\}, \{3, 6, 7, 8\}\}) = 5/14$

$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/7$

b1 = {1, 2, 3, 6} ‘ , ‘ b2 = {4, 5, 7, 8} ‘ , ‘ b3 = {1, 2, 4, 5} ‘ , ‘ b4 = {3, 6, 7, 8} ‘ , ‘ b5 = {1, 4, 5, 8} ‘ , ‘
b6 = {2, 3, 6, 7} ‘ , ‘ b7 = {1, 2, 3, 4} ‘ , ‘ b8 = {5, 6, 7, 8} ‘ , ‘ b9 = {1, 2, 5, 6} ‘ , ‘ b10 = {2, 5, 6, 7} ‘ , ‘
b11 = {3, 4, 7, 8} ‘ , ‘ b12 = {1, 3, 4, 8}

Action of R and B on the blocks of the partitions: = [3, 4, 2, 1, B, 9, 3, 4, 2, 8, 1, 7] [C, A, 4, 3, 6, 5, 4, 3, C, 5, A, 6]

with invariant measure [3, 3, 5, 5, 2, 2, 1, 1, 1, 2, 1, 2]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{3, 5}
Rank	2
R,B	[3, 3, 8, 1, 2, 7, 5, 5], [6, 8, 1, 6, 7, 4, 4, 2]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[2, 4, 2, 3, 5, 7, 5, 4, 4, 3, 3, 5, 7, 5, 7, 2, 3, 3, 2, 7, 5, 3, 5, 4, 4, 2, 4, 2] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

17 . Coloring, {3, 6}

R: [3, 3, 8, 1, 7, 4, 5, 5]

B: [6, 8, 1, 6, 2, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	5 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

\$ [[1, 0, 2, 1, 2, 0, 1, 1] , [1, 0, 1, 0, 2, 0, 2, 2] , [0, 0, 1, 0, 4, 0, 2, 1] , [0, 0, 0, 0, 3, 0, 4, 1] , [0, 0, 0, 0, 5, 0, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0]] \$

[y₆, 0, y₃, y₄, y₅, 0, y₁, y₂]

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

\$ [[1, 2, 0, 1, 0, 2, 1, 1] , [0, 1, 0, 1, 0, 2, 2, 2] , [0, 2, 0, 2, 0, 1, 2, 1] , [0, 1, 0, 2, 0, 2, 1, 2] , [0, 2, 0, 1, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 1, 2, 2]] \$

$$[5y_1 - 3y_5 - 3y_4 - 3y_2 + 5y_3, 3y_1, 0, 3y_5, 0, 3y_4, 3y_2, 3y_3]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

18. Coloring, {3, 7}

$$\Omega p(\Delta)=0: \quad p = s - 4s^4 \quad p' = s - 4s^4 \quad p'' = s^2 - 4s^5$$

R: [3, 3, 8, 1, 7, 7, 4, 5]

B: [6, 8, 1, 6, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	3 vs 6	2 vs 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$\$ [[1, 0, 2, 1, 1, 0, 2, 1], [1, 0, 1, 2, 1, 0, 1, 2], [2, 0, 1, 1, 2, 0, 1, 1], [1, 0, 2, 1, 1, 0, 2, 1], [1, 0, 1, 2, 1, 0, 1, 2], [2, 0, 1, 1, 2, 0, 1, 1]] \$$$

$$[y_1, 0, y_2, y_3, y_1, 0, y_2, y_3]$$

$$p' = s^2 - s^5 \quad p'' = -s + s^4 \quad p''' = -s + s^4$$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$\$ [[1, 2, 0, 1, 1, 2, 0, 1], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2, 0, 2]] \$$$

$$[y_2, y_1, 0, -y_2 + y_1, y_2, y_1, 0, -y_2 + y_1]$$

$$p = -s^2 + s^6 \quad p' = -s^2 + s^3 \quad p'' = -s^2 + s^4 \quad p''' = -s^2 + s^5$$

M \ ; N

$$\$ [[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 4, 3, 2, 3], [1, 0, 2, 2, 3, 4, 2, 2], [2, 2, 0, 3, 2, 2, 4, 1], [1, 2, 3, 0, 3, 2, 1, 4], [4, 3, 2, 3, 0, 1, 2, 1], [3, 4, 2, 2, 1, 0, 2,$$

2] , [2, 2, 4, 1, 2, 2, 0, 3] , [3, 2, 1, 4, 1, 2, 3, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 3, 8, 1, 7, 7, 4, 5]

B: [6, 8, 1, 6, 2, 4, 5, 2]

Ranges

Action of R on ranges, [[3], [3], [4], [1]]

Action of B on ranges, [[2], [4], [1], [2]]

Cycles: R , {{1, 3, 4, 5, 7, 8}}, B , {{2, 8}, {4, 6}}

$\beta(\{1, 5\}) = 1/4$

$\beta(\{2, 6\}) = 1/4$

$\beta(\{3, 7\}) = 1/4$

$\beta(\{4, 8\}) = 1/4$

Partitions

Action of R on partitions, [[2], [4], [5], [5], [2]]

Action of B on partitions, [[3], [1], [3], [5], [5]]

$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 1/8$

$\alpha(\{\{1, 2, 3, 8\}, \{4, 5, 6, 7\}\}) = 1/4$

$\alpha(\{\{1, 3, 4, 6\}, \{2, 5, 7, 8\}\}) = 1/8$

$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/8$

$\alpha(\{\{1, 2, 4, 7\}, \{3, 5, 6, 8\}\}) = 3/8$

$b_1 = \{1, 2, 3, 8\}$ ‘ , ‘ $b_2 = \{4, 5, 6, 7\}$ ‘ , ‘ $b_3 = \{1, 2, 4, 7\}$ ‘ , ‘ $b_4 = \{3, 5, 6, 8\}$ ‘ , ‘ $b_5 = \{1, 3, 4, 6\}$ ‘ , ‘
 $b_6 = \{1, 4, 6, 7\}$ ‘ , ‘ $b_7 = \{2, 5, 7, 8\}$ ‘ , ‘ $b_8 = \{2, 3, 5, 8\}$ ‘ , ‘ $b_9 = \{1, 2, 3, 4\}$ ‘ , ‘ $b_{10} = \{5, 6, 7, 8\}$

Action of R and B on the blocks of the partitions: = [9, A, 2, 1, 3, 2, 4, 1, 3, 4] [8, 6, 4, 3, 5, 5, 7, 7, 4, 3]
with invariant measure [2, 2, 3, 3, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{3, 7}
Rank	2
R,B	[3, 3, 8, 1, 7, 7, 4, 5], [6, 8, 1, 6, 2, 4, 5, 2]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 2, 1, 4, 3, 2, 3, 2, 2, 3, 4, 2, 2, 3, 2, 2, 4, 1, 3, 2, 1, 4, 1, 2, 1, 2, 2, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

19 . Coloring, {3, 8}

R: [3, 3, 8, 1, 7, 7, 5, 2]

B: [6, 8, 1, 6, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

\$ [[1, 1, 2, 0, 1, 0, 2, 1], [0, 1, 2, 0, 2, 0, 1, 2], [0, 2, 1, 0, 1, 0, 2, 2], [0, 2, 2, 0, 2, 0, 1, 1], [0, 1, 2, 0, 1, 0, 2, 2], [0, 2, 1, 0, 2, 0, 1, 2]] \$

$$[-3 y_1 - 3 y_2 + 5 y_3 + 5 y_4 - 3 y_5, 3 y_1, 3 y_2, 0, 3 y_3, 0, 3 y_4, 3 y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

\$ [[1, 1, 0, 2, 1, 2, 0, 1], [0, 1, 0, 2, 1, 3, 0, 1], [0, 1, 0, 3, 1, 2, 0, 1], [0, 1, 0, 2, 1, 3, 0, 1], [0, 1, 0, 3, 1, 2, 0, 1], [0, 1, 0, 2, 1, 3, 0, 1]] \$

$$[5y_3 - y_1 - y_2, y_3, 0, y_1, y_3, y_2, 0, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6$$

20 . Coloring, {4, 5}

R: [3, 3, 1, 6, 2, 7, 5, 5]

B: [6, 8, 8, 1, 7, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	5 vs 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$\$ [[1, 1, 2, 0, 2, 1, 1, 0], [2, 2, 2, 0, 1, 0, 1, 0], [2, 1, 4, 0, 1, 0, 0, 0], [4, 1, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0]] \$$$

$$[y_2, y_1, y_5, 0, y_6, y_4, y_3, 0]$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$\$ [[1, 1, 0, 2, 0, 1, 1, 2], [2, 2, 0, 2, 0, 1, 0, 1], [2, 1, 0, 1, 0, 2, 0, 2], [1, 2, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 1, 0, 2], [2, 2, 0, 1, 0, 2, 0, 1]] \$$$

$$[5y_1 - 3y_2 - 3y_3 - 3y_4 + 5y_5, 3y_1, 0, 3y_2, 0, 3y_3, 3y_4, 3y_5]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

21 . Coloring, {4, 6}

$$\Omega p(\Delta)=0: \quad p = s + 4s^4 + 8s^5 - 16s^6 \quad p' = s - 4s^3 - 4s^4 + 8s^5 \quad p' = s^2 + 2s^3 - 4s^5$$

R: [3, 3, 1, 6, 7, 4, 5, 5]

B: [6, 8, 8, 1, 2, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	1 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[1, 0, 2, 1, 2, 1, 1, 0], [2, 0, 1, 1, 1, 1, 2, 0], [1, 0, 2, 1, 2, 1, 1, 0], [2, 0, 1, 1, 1, 1, 2, 0], [1, 0, 2, 1, 2, 1, 1, 0], [2, 0, 1, 1, 1, 1, 2, 0]] \$$

$$[y_2, 0, -y_2 + 3y_1, y_1, -y_2 + 3y_1, y_1, y_2, 0]$$

$$p = -s + s^5 \quad p' = s^3 - s^5 \quad p' = s - s^5 \quad p = -s + s^3$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[1, 2, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 1, 1, 2]] \$$

$$[y_1, 2y_1, 0, y_1, 0, y_1, y_1, 2y_1]$$

$$p = s - s^6 \quad p' = s^2 - s^5 \quad p' = s - s^5 \quad p' = s^3 - s^5 \quad p' = s^4 - s^5$$

M \ ; N

$\$ [[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]] \$ \quad \$ [[0, 3, 6, 5, 5, 6, 11, 8], [3, 0, 3, 6, 8, 5, 8, 11], [6, 3, 0, 7, 11, 4, 5, 8], [5, 6, 7, 0, 4, 11, 6, 5], [5, 8, 11, 4, 0, 7, 6, 3], [6, 5, 4, 11, 7, 0, 5, 6], [11, 8, 5, 6, 6, 5, 0, 3], [8, 11, 8, 5, 3, 6, 3, 0]] \$$

$$\tau = 32, r' = 1/2$$

R: [3, 3, 1, 6, 7, 4, 5, 5]

B: [6, 8, 8, 1, 2, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [1], [4]]

Action of B on ranges, [[4], [2], [2], [1]]

Cycles: R , {{4, 6}, {1, 3}, {5, 7}}, B , {{2, 8}, {1, 4, 6, 7}}

$$\beta(\{1, 7\}) = 1/4$$

$$\beta(\{2, 8\}) = 1/4$$

$$\beta(\{3, 5\}) = 1/4$$

$$\beta(\{4, 6\}) = 1/4$$

Partitions

Action of R on partitions, [[4], [5], [6], [4], [2], [6]]

Action of B on partitions, [[5], [3], [1], [3], [2], [2]]

$$\alpha(\{\{2, 3, 4, 7\}, \{1, 5, 6, 8\}\}) = 1/11$$

$$\alpha(\{\{1, 2, 3, 6\}, \{4, 5, 7, 8\}\}) = 3/11$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 2/11$$

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 1/11$$

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 2/11$$

$$\alpha(\{\{1, 2, 4, 5\}, \{3, 6, 7, 8\}\}) = 2/11$$

b1 = {1, 2, 3, 6} ‘ , ‘ b2 = {2, 3, 4, 7} ‘ , ‘ b3 = {1, 5, 6, 8} ‘ , ‘ b4 = {4, 5, 7, 8} ‘ , ‘ b5 = {1, 4, 5, 8} ‘ , ‘
 b6 = {2, 3, 6, 7} ‘ , ‘ b7 = {3, 4, 7, 8} ‘ , ‘ b8 = {1, 2, 5, 6} ‘ , ‘ b9 = {1, 2, 4, 5} ‘ , ‘ b10 = {1, 2, 3, 4} ‘ , ‘
 b11 = {3, 6, 7, 8} ‘ , ‘ b12 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [A, 8, 7, C, B, 9, 8, 7, B, 1, 9, 4] [5, C, A, 6, 2, 3, 6, 5, 4, 4, 1, 1]

with invariant measure [3, 1, 1, 3, 2, 2, 1, 1, 2, 2, 2, 2]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{4, 6}
Rank	2
R,B	[3, 3, 1, 6, 7, 4, 5, 5], [6, 8, 8, 1, 2, 7, 4, 2]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[3, 6, 5, 5, 6, 11, 8, 3, 6, 8, 5, 8, 11, 7, 11, 4, 5, 8, 4, 11, 6, 5, 7, 6, 3, 5, 6, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

22 . Coloring, {4, 7}

R: [3, 3, 1, 6, 7, 7, 4, 5]

B: [6, 8, 8, 1, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

$\$ [[1, 0, 2, 1, 1, 1, 2, 0], [2, 0, 1, 2, 0, 1, 2, 0], [1, 0, 2, 2, 0, 2, 1, 0], [2, 0, 1, 1, 0, 2, 2, 0], [1, 0, 2, 2, 0, 1, 2, 0], [2, 0, 1, 2, 0, 2, 1, 0]] \$$

$$[3y_4, 0, 3y_3, 3y_5, 5y_4 + 5y_3 - 3y_5 - 3y_1 - 3y_2, 3y_1, 3y_2, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$\$ [[1, 2, 0, 1, 1, 1, 0, 2], [1, 3, 0, 1, 0, 1, 0, 2], [1, 2, 0, 1, 0, 1, 0, 3], [1, 3, 0, 1, 0, 1, 0, 2], [1, 2, 0, 1, 0, 1, 0, 3], [1, 3, 0, 1, 0, 1, 0, 2]] \$$

$$[y_2, y_3, 0, y_2, y_1, y_2, 0, 5y_2 - y_3 - y_1]$$

$$p' = s^3 - s^5 \quad p = s^2 - s^6 \quad p' = s^2 - s^4$$

23 . Coloring, {4, 8}

$$\Omega p(\Delta)=0: \quad p' = s^2 + 4s^5 \quad p = s + 4s^4 \quad p' = s + 4s^4$$

R: [3, 3, 1, 6, 7, 7, 5, 2]

B: [6, 8, 8, 1, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	3 vs 6

Omega Rank for R : cycles: $\{\{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[1, 1, 2, 0, 1, 1, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0, 2, 0]] \$$

$$[y_2, -y_2 + y_1, y_1, 0, y_2, -y_2 + y_1, y_1, 0]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^3 \quad p' = -s^3 + s^4 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 5, 8\}, \{1, 4, 6\}\}$ order: 3

See Matrix

$\$ [[1, 1, 0, 2, 1, 1, 0, 2], [2, 1, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 2, 0, 1], [1, 1, 0, 2, 1, 1, 0, 2], [2, 1, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 2, 0, 1]] \$$

$$[y_1, y_2, 0, y_3, y_1, y_2, 0, y_3]$$

$$p' = -s^2 + s^5 \quad p' = -s + s^4 \quad p = s - s^4$$

M \ ; N

$\$ [[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]] \$ \quad \$ [[0, 1, 2, 3, 4, 3, 2, 1], [1, 0, 1, 2, 3, 4, 3, 2], [2, 1, 0, 2, 2, 3, 4, 2], [3, 2, 2, 0, 1, 2, 2, 4], [4, 3, 2, 1, 0, 1, 2, 3], [3, 4, 3, 2, 1, 0, 1, 2], [2, 3, 4, 2, 2, 1, 0, 2], [1, 2, 2, 4, 3, 2, 2, 0]] \$$

$\tau = 32, r' = 1/2$

R: [3, 3, 1, 6, 7, 7, 5, 2]

B: [6, 8, 8, 1, 2, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [3], [1], [2]]

Action of B on ranges, [[2], [4], [4], [1]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{2, 5, 8}, {1, 4, 6}}

$$\beta(\{1, 5\}) = 1/4$$

$$\beta(\{2, 6\}) = 1/4$$

$$\beta(\{3, 7\}) = 1/4$$

$$\beta(\{4, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[1], [3], [1], [4], [4]]

Action of B on partitions, [[4], [5], [2], [2], [4]]

$$\alpha(\{\{1, 2, 4, 7\}, \{3, 5, 6, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 6, 7, 8\}, \{2, 3, 4, 5\}\}) = 1/4$$

$$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 3, 8\}, \{4, 5, 6, 7\}\}) = 3/8$$

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/8$$

b1 = {1, 2, 4, 7} ‘ , ‘ b2 = {3, 5, 6, 8} ‘ , ‘ b3 = {1, 6, 7, 8} ‘ , ‘ b4 = {2, 3, 4, 5} ‘ , ‘ b5 = {1, 2, 7, 8} ‘ , ‘
 b6 = {3, 4, 5, 6} ‘ , ‘ b7 = {1, 2, 3, 8} ‘ , ‘ b8 = {4, 5, 6, 7} ‘ , ‘ b9 = {1, 2, 3, 4} ‘ , ‘ b10 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [2, 1, 6, 5, 2, 1, 7, 8, 7, 8] [8, 7, 9, A, 4, 3, 4, 3, 8, 7]
 with invariant measure [1, 1, 2, 2, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{4, 8}
Rank	2
R,B	[3, 3, 1, 6, 7, 7, 5, 2], [6, 8, 8, 1, 2, 4, 4, 5]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 2, 3, 4, 3, 2, 1, 1, 2, 3, 4, 3, 2, 2, 2, 3, 4, 2, 1, 2, 2, 4, 1, 2, 3, 1, 2, 2] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

24 . Coloring, {5, 6}

R: [3, 3, 1, 1, 2, 4, 5, 5]

B: [6, 8, 8, 6, 7, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	4 vs 5	4 vs 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$\$ [[2, 1, 2, 1, 2, 0, 0, 0], [3, 2, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0]] \$$

$[y_4, y_3, y_1, y_2, 2y_2, 0, 0, 0]$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$\$ [[0, 1, 0, 1, 0, 2, 2, 2], [0, 2, 0, 2, 0, 1, 2, 1], [0, 1, 0, 2, 0, 2, 1, 2], [0, 2, 0, 1, 0, 2, 2, 1], [0, 1, 0, 2, 0, 1, 2, 2], [0, 0, 0, 0, 0, 0, 0, 0]] \$$

$[0, 3y_1, 0, 5y_1 - 3y_4 - 3y_3 + 5y_2, 0, 3y_4, 3y_3, 3y_2]$

$$p = -s - s^2 + s^4 + s^5$$

25 . Coloring, {5, 7}

$$\Omega p(\Delta)=0: \quad p' = s^3 - 2s^5 \quad p = s - 4s^5 \quad p' = s - 4s^5 \quad p' = s^2 - 2s^4$$

R: [3, 3, 1, 1, 2, 7, 4, 5]

B: [6, 8, 8, 6, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 6	3 vs 7	3 vs 7	3 vs 6	2 vs 6

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4
See Matrix

$\$ [[2, 1, 2, 1, 1, 0, 1, 0], [3, 1, 3, 1, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0]] \$$

$[y_2, y_1, y_2, y_1, y_3, 0, y_3, 0]$

$$p' = s^4 - s^5 \quad p' = s^3 - s^5 \quad p = s^3 - s^6$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{4, 6\}, \{5, 7\}\}$ order: 2
See Matrix

$\$ [[0, 1, 0, 1, 1, 2, 1, 2], [0, 2, 0, 2, 1, 1, 1, 1], [0, 1, 0, 1, 1, 2, 1, 2], [0, 2, 0, 2, 1, 1, 1, 1], [0, 1, 0, 1, 1, 2, 1, 2], [0, 2, 0, 2, 1, 1, 1, 1]] \$$

$[0, 3y_1 - y_2, 0, 3y_1 - y_2, y_1, y_2, y_1, y_2]$

$$p' = -s + s^3 \quad p' = -s + s^5 \quad p = -s + s^3 \quad p = -s + s^5$$

M \ ; N

$\$ [[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 1, 1, 1, 1], [1, 0, 1, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 2, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 1, 1, 1, 1, 2, 1, 0]] \$$

$$\tau = 32, r' = 1/2$$

R: [3, 3, 1, 1, 2, 7, 4, 5]

B: [6, 8, 8, 6, 7, 4, 5, 2]

Ranges

Action of R on ranges, [[1], [1], [2], [3]]

Action of B on ranges, [[4], [4], [3], [2]]

Cycles: R, $\{\{1, 3\}\}$, B, $\{\{2, 8\}, \{4, 6\}, \{5, 7\}\}$

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 4\}) = 1/4$$

$$\beta(\{5, 7\}) = 1/4$$

$$\beta(\{6, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[3], [5], [7], [1], [3], [1], [7], [5]]

Action of B on partitions, [[8], [6], [4], [4], [6], [2], [2], [8]]

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 5, 8\}, \{1, 2, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 6, 7\}, \{1, 2, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 6\}, \{2, 3, 7, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 7, 8\}, \{2, 3, 5, 6\}\}) = 1/8$$

b1 = {1, 4, 7, 8} ‘ , ‘ b2 = {2, 3, 5, 6} ‘ , ‘ b3 = {1, 4, 5, 8} ‘ , ‘ b4 = {1, 4, 5, 6} ‘ , ‘ b5 = {2, 3, 6, 7} ‘ , ‘
 b6 = {2, 3, 7, 8} ‘ , ‘ b7 = {3, 4, 7, 8} ‘ , ‘ b8 = {1, 2, 7, 8} ‘ , ‘ b9 = {3, 4, 5, 6} ‘ , ‘ b10 = {1, 2, 5, 6} ‘ , ‘
 b11 = {3, 4, 5, 8} ‘ , ‘ b12 = {1, 2, 6, 7} ‘ , ‘ b13 = {3, 4, 6, 7} ‘ , ‘ b14 = {1, 4, 6, 7} ‘ , ‘ b15 = {2, 3, 5, 8}
 ‘ , ‘ b16 = {1, 2, 5, 8}

Action of R and B on the blocks of the partitions: = [D, 10, 7, 7, A, A, C, 9, 8, B, 8, 9, C, D, 10, B] [2, 1,
 5, E, 3, F, 2, F, E, 1, 5, 3, 4, 4, 6, 6]
 with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{5, 7}
Rank	2
R,B	[3, 3, 1, 1, 2, 7, 4, 5], [6, 8, 8, 6, 7, 4, 5, 2]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

26 . Coloring, {5, 8}

R: [3, 3, 1, 1, 2, 7, 5, 2]

B: [6, 8, 8, 6, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	5 vs 5	5 vs 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

\$ [[2, 2, 2, 0, 1, 0, 1, 0] , [2, 1, 4, 0, 1, 0, 0, 0] , [4, 1, 3, 0, 0, 0, 0, 0] , [3, 0, 5, 0, 0, 0, 0, 0] , [5, 0, 3, 0, 0, 0, 0, 0]] \$

$[y_1, y_4, y_3, 0, y_2, 0, y_5, 0]$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

\$ [[0, 0, 0, 2, 1, 2, 1, 2] , [0, 0, 0, 3, 2, 2, 1, 0] , [0, 0, 0, 3, 0, 3, 2, 0] , [0, 0, 0, 5, 0, 3, 0, 0] , [0, 0, 0, 3, 0, 5, 0, 0]] \$

$[0, 0, 0, y_4, y_5, y_2, y_1, y_3]$

27 . Coloring, {6, 7}

R: [3, 3, 1, 1, 7, 4, 4, 5]

B: [6, 8, 8, 6, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	5 vs 5	5 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4
See Matrix

$\$ [[2, 0, 2, 2, 1, 0, 1, 0], [4, 0, 2, 1, 0, 0, 1, 0], [3, 0, 4, 1, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0]] \$$

$[y_1, 0, y_5, y_2, y_3, 0, y_4, 0]$

Omega Rank for B : cycles: $\{\{2, 8\}\}$ order: 4
See Matrix

$\$ [[0, 2, 0, 0, 1, 2, 1, 2], [0, 3, 0, 0, 1, 0, 2, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 5, 0, 0, 0, 0, 0, 3], [0, 3, 0, 0, 0, 0, 0, 5]] \$$

$[0, y_1, 0, 0, y_4, y_3, y_2, y_5]$

28 . Coloring, $\{6, 8\}$

$\Omega p(\Delta)=0: p' = s + 4s^5 \quad p' = s^2 - 2s^4 + 4s^5 \quad p = s + 4s^5 \quad p' = s^3 - 2s^4 + 2s^5$

R: [3, 3, 1, 1, 7, 4, 5, 2]

B: [6, 8, 8, 6, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 6	3 vs 7	3 vs 7	2 vs 6	3 vs 6

Omega Rank for R : cycles: $\{\{1, 3\}, \{5, 7\}\}$ order: 2
See Matrix

$\$ [[2, 1, 2, 1, 1, 0, 1, 0], [3, 0, 3, 0, 1, 0, 1, 0], [3, 0, 3, 0, 1, 0, 1, 0], [3, 0, 3, 0, 1, 0, 1, 0], [3, 0, 3, 0, 1, 0, 1, 0], [3, 0, 3, 0, 1, 0, 1, 0]] \$$

$$[-y_2 + 3y_1, y_2, -y_2 + 3y_1, y_2, y_1, 0, y_1, 0]$$

$$p' = s^2 - s^4 \quad p' = s^3 - s^4 \quad p = s^2 - s^5 \quad p' = -s^4 + s^5$$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 5, 8}} order: 3

See Matrix

$$\$ [[0, 1, 0, 1, 1, 2, 1, 2], [0, 1, 0, 1, 2, 1, 2, 1], [0, 2, 0, 2, 1, 1, 1, 1], [0, 1, 0, 1, 1, 2, 1, 2], [0, 1, 0, 1, 2, 1, 2, 1], [0, 2, 0, 2, 1, 1, 1, 1]] \$$$

$$[0, y_1, 0, y_1, y_2, y_3, y_2, y_3]$$

$$p = -s + s^4 \quad p' = -s + s^4 \quad p' = -s^2 + s^5$$

M \ ; N

$$\$ [[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 1, 1, 1, 1], [1, 0, 1, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 2, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 1, 1, 1, 1, 2, 1, 0]] \$$$

$$\tau = 32, r' = 1/2$$

$$\mathbf{R}: [3, 3, 1, 1, 7, 4, 5, 2]$$

$$\mathbf{B}: [6, 8, 8, 6, 2, 7, 4, 5]$$

Ranges

Action of R on ranges, [[1], [1], [3], [2]]

Action of B on ranges, [[4], [4], [2], [3]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{4, 6, 7}, {2, 5, 8}}

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 4\}) = 1/4$$

$$\beta(\{5, 7\}) = 1/4$$

$$\beta(\{6, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[6], [5], [6], [4], [4], [7], [7], [5]]

Action of B on partitions, [[8], [2], [1], [1], [8], [2], [3], [3]]

$$\alpha(\{\{1, 4, 5, 6\}, \{2, 3, 7, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 5, 8\}, \{1, 2, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 6, 7\}, \{1, 2, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 7, 8\}, \{2, 3, 5, 6\}\}) = 1/8$$

$b_1 = \{1, 4, 7, 8\}$, , $b_2 = \{2, 3, 5, 6\}$, , $b_3 = \{1, 4, 5, 8\}$, , $b_4 = \{1, 4, 5, 6\}$, , $b_5 = \{2, 3, 6, 7\}$, ,
 $b_6 = \{2, 3, 7, 8\}$, , $b_7 = \{3, 4, 7, 8\}$, , $b_8 = \{1, 2, 7, 8\}$, , $b_9 = \{3, 4, 5, 6\}$, , $b_{10} = \{1, 2, 5, 6\}$, ,
 $b_{11} = \{3, 4, 5, 8\}$, , $b_{12} = \{1, 2, 6, 7\}$, , $b_{13} = \{3, 4, 6, 7\}$, , $b_{14} = \{1, 4, 6, 7\}$, , $b_{15} = \{2, 3, 5, 8\}$
 , , $b_{16} = \{1, 2, 5, 8\}$

Action of R and B on the blocks of the partitions: = [9, 8, D, D, 10, 10, A, B, C, 7, C, B, A, 9, 8, 7] [5, 3,
 6, 1, 4, 2, 5, 2, 1, 3, 6, 4, E, E, F, F]
 with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{6, 8}
Rank	2
R,B	[3, 3, 1, 1, 7, 4, 5, 2], [6, 8, 8, 6, 2, 7, 4, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

29 . Coloring, {7, 8}

R: [3, 3, 1, 1, 7, 7, 4, 2]

B: [6, 8, 8, 6, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	4 vs 5	4 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$\$ [[2, 1, 2, 1, 0, 0, 2, 0], [3, 0, 3, 2, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [0, 0, 0, 0]] \$$

$$[y_1, y_2, y_3, y_4, 0, 0, 2y_2, 0]$$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[0, 1, 0, 1, 2, 2, 0, 2], [0, 2, 0, 2, 2, 1, 0, 1], [0, 2, 0, 1, 1, 2, 0, 2], [0, 1, 0, 2, 2, 1, 0, 2], [0, 2, 0, 1, 2, 2, 0, 1]] \$$

$$[0, 3y_2, 0, 3y_3, -3y_2 + 5y_3 + 5y_1 - 3y_4, 3y_1, 0, 3y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

30 . Coloring, $\{2, 3, 4\}$

R: [3, 8, 8, 6, 7, 7, 5, 5]

B: [6, 3, 1, 1, 2, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	5 vs 5

Omega Rank for R : cycles: $\{\{5, 7\}\}$ order: 4

See Matrix

$\$ [[0, 0, 1, 0, 2, 1, 2, 2], [0, 0, 0, 0, 4, 0, 3, 1], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$

$$[0, 0, y_3, 0, y_2, y_3, 2y_3 + y_2 - y_1, y_1]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^4$$

Omega Rank for B : cycles: {{1, 4, 6}} order: 3

See Matrix

$$\$ [[2, 2, 1, 2, 0, 1, 0, 0], [3, 0, 2, 1, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0], [3, 0, 0, 3, 0, 2, 0, 0]] \$$$

$$[y_5, y_4, y_3, y_2, 0, y_1, 0, 0]$$

31 . Coloring, {2, 3, 5}

R: [3, 8, 8, 1, 2, 7, 5, 5]

B: [6, 3, 1, 6, 7, 4, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	4 vs 6

Omega Rank for R : cycles: {{2, 5, 8}} order: 3

See Matrix

$$\$ [[1, 1, 1, 0, 2, 0, 1, 2], [0, 2, 1, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3]] \$$$

$$[y_4, y_1, y_2, 0, y_3, 0, y_4, y_5]$$

$$p = -s^3 + s^6$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[1, 1, 1, 2, 0, 2, 1, 0], [1, 0, 1, 3, 0, 3, 0, 0], [1, 0, 0, 3, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$$

$$[y_1 - y_2 + y_3, y_4, y_1, y_2, 0, y_3, y_4, 0]$$

$$p = s^4 - s^6 \quad p' = s^4 - s^5$$

32 . Coloring, {2, 3, 6}

R: [3, 8, 8, 1, 7, 4, 5, 5]

B: [6, 3, 1, 6, 2, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

$$\$ [[1, 0, 1, 1, 2, 0, 1, 2], [1, 0, 1, 0, 3, 0, 2, 1], [0, 0, 1, 0, 3, 0, 3, 1], [0, 0, 0, 0, 4, 0, 3, 1], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$$

$$[y_5, 0, y_3, y_4, y_2, 0, y_1, -y_5 + y_3 + y_4 + y_2 - y_1]$$

$$p = s^5 - s^6$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$$\$ [[1, 2, 1, 1, 0, 2, 1, 0], [1, 0, 2, 1, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [0, 0, 0, 2, 0, 4, 2, 0], [0, 0, 0, 2, 0, 2, 4, 0], [0, 0, 0, 4, 0, 2, 2, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_6, y_5, 0]$$

33 . Coloring, {2, 3, 7}

R: [3, 8, 8, 1, 7, 7, 4, 5]

B: [6, 3, 1, 6, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{1, 3, 4, 5, 7, 8\}\}$ order: 6

See Matrix

$\$ [[1, 0, 1, 1, 1, 0, 2, 2], [1, 0, 1, 2, 2, 0, 1, 1], [2, 0, 1, 1, 1, 0, 2, 1], [1, 0, 2, 2, 1, 0, 1, 1], [2, 0, 1, 1, 1, 0, 1, 2], [1, 0, 2, 1, 2, 0, 1, 1]] \$$

$$[y_1, 0, y_2, y_3, y_4, 0, y_5, y_6]$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 6

See Matrix

$\$ [[1, 2, 1, 1, 1, 2, 0, 0], [1, 1, 2, 2, 0, 2, 0, 0], [2, 0, 1, 2, 0, 3, 0, 0], [1, 0, 0, 3, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$

$$[y_3, y_2, y_3 + y_2 + y_1 - y_4 - y_5, y_1, y_4, y_5, 0, 0]$$

$$p = -s^5 + s^6$$

34 . Coloring, $\{2, 3, 8\}$

R: [3, 8, 8, 1, 7, 7, 5, 2]

B: [6, 3, 1, 6, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[1, 1, 1, 0, 1, 0, 2, 2], [0, 2, 1, 0, 2, 0, 1, 2], [0, 2, 0, 0, 1, 0, 2, 3], [0, 3, 0, 0, 2, 0, 1, 2], [0, 2, 0, 0, 1, 0, 2, 3], [0, 3, 0, 0, 2, 0, 1, 2]] \$$

$$[4y_1 + 4y_2 - 5y_3 - y_4, y_1, y_2, 0, y_3, 0, 3y_1 + 3y_2 - 4y_3, y_4]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 2, 1, 2, 0, 0], [1, 1, 1, 2, 0, 3, 0, 0], [1, 0, 1, 3, 0, 3, 0, 0], [1, 0, 0, 3, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$

$$[-y_1 + y_4 - y_3 + y_2 + y_5, y_1, y_4, y_3, y_2, y_5, 0, 0]$$

$$p = -s^5 + s^6$$

35 . Coloring, $\{2, 4, 5\}$

R: $[3, 8, 1, 6, 2, 7, 5, 5]$

B: $[6, 3, 8, 1, 7, 4, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{1, 3\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 0, 2, 1, 1, 1], [1, 2, 1, 0, 2, 0, 1, 1], [1, 2, 1, 0, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2]] \$$

$$[y_1, 2y_1 - y_2, y_1, 0, 2y_1, y_2, 2y_1 - y_3, y_3]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^6 \quad p = -s^3 + s^7 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}, \{2, 3, 8\}\}$ order: 3
See Matrix

$$\$ [[1, 1, 1, 2, 0, 1, 1, 1], [2, 1, 1, 2, 0, 1, 0, 1], [2, 1, 1, 1, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [2, 1, 1, 2, 0, 1, 0, 1], [2, 1, 1, 1, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1]] \$$$

$$[5y_4 - y_1 - y_2 - y_3, y_4, y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^3 + s^6 \quad p' = -s^2 + s^5$$

36 . Coloring, $\{2, 4, 6\}$

R: $[3, 8, 1, 6, 7, 4, 5, 5]$

B: $[6, 3, 8, 1, 2, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	3 vs 7

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}, \{5, 7\}\}$ order: 2
See Matrix

$$\$ [[1, 0, 1, 1, 2, 1, 1, 1], [1, 0, 1, 1, 2, 1, 2, 0], [1, 0, 1, 1, 2, 1, 2, 0], [1, 0, 1, 1, 2, 1, 2, 0], [1, 0, 1, 1, 2, 1, 2, 0], [1, 0, 1, 1, 2, 1, 2, 0], [1, 0, 1, 1, 2, 1, 2, 0]] \$$$

$$[y_1, 0, y_1, y_1, 2y_1, y_1, 2y_1 - y_2, y_2]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^3 \quad p' = -s^3 + s^4 \quad p' = -s^3 + s^5 \quad p' = -s^3 + s^6$$

Omega Rank for B : cycles: $\{\{2, 3, 8\}, \{1, 4, 6, 7\}\}$
See Matrix

$$\$ [[1, 2, 1, 1, 0, 1, 1, 1], [1, 1, 2, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 2], [1, 2, 1, 1, 0, 1, 1, 1], [1, 1, 2, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 2], [1, 2, 1, 1, 0, 1, 1, 1]] \$$$

$$[y_2, y_1, 4y_2 - y_1 - y_3, y_2, 0, y_2, y_2, y_3]$$

$$p = -s + s^7 \quad p = -s + s^4 \quad p' = s - s^4 \quad p' = s^2 - s^5$$

37 . Coloring, {2, 4, 7}

R: [3, 8, 1, 6, 7, 7, 4, 5]

B: [6, 3, 8, 1, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 7	4 vs 7

Omega Rank for R : cycles: {{4, 6, 7}, {1, 3}} order: 6

See Matrix

$$\$ [[1, 0, 1, 1, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0]] \$$$

$$[y_1, 0, y_1, 2y_1 - y_3, 2y_1 - y_2, y_2, 2y_1, y_3]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5 \quad p = -s^3 + s^6 \quad p = -s^3 + s^7$$

Omega Rank for B : cycles: {{2, 3, 8}, {1, 4, 6}} order: 3

See Matrix

$$\$ [[1, 2, 1, 1, 1, 1, 0, 1], [1, 2, 2, 1, 0, 1, 0, 1], [1, 1, 2, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 2, 1, 0, 1, 0, 1], [1, 1, 2, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2]] \$$$

$$[y_3, 5y_3 - y_1 - y_2 - y_4, y_1, y_3, y_2, y_3, 0, y_4]$$

$$p' = s^2 - s^5 \quad p' = s^3 - s^6 \quad p = s^2 - s^5$$

38 . Coloring, {2, 4, 8}

R: [3, 8, 1, 6, 7, 7, 5, 2]

B: [6, 3, 8, 1, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	3 vs 7

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[1, 1, 1, 0, 1, 1, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1], [1, 1, 1, 0, 2, 0, 2, 1]] \$$

$[y_2, y_2, y_2, 0, y_1, 2y_2 - y_1, 2y_2, y_2]$

$p' = -s^2 + s^6 \quad p' = -s^2 + s^5 \quad p' = -s^2 + s^4 \quad p' = -s^2 + s^3 \quad p = s^2 - s^3$

Omega Rank for B : cycles: $\{\{1, 4, 6\}, \{2, 3, 5, 8\}\}$

See Matrix

$\$ [[1, 1, 1, 2, 1, 1, 0, 1], [2, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 2, 0, 1], [1, 1, 1, 2, 1, 1, 0, 1], [2, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 2, 0, 1], [1, 1, 1, 2, 1, 1, 0, 1]] \$$

$[y_3, y_2, y_2, y_1, y_2, -y_3 + 4y_2 - y_1, 0, y_2]$

$p = -s + s^4 \quad p' = -s + s^4 \quad p' = -s^2 + s^5 \quad p = -s + s^7$

39 . Coloring, $\{2, 5, 6\}$

R: $[3, 8, 1, 1, 2, 4, 5, 5]$

B: $[6, 3, 8, 6, 7, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	3 vs 6

Omega Rank for R : cycles: $\{\{1, 3\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 1, 2, 0, 0, 1], [2, 2, 2, 0, 1, 0, 0, 1], [2, 1, 2, 0, 1, 0, 0, 2], [2, 1, 2, 0, 2, 0, 0, 1], [2, 2, 2, 0, 1, 0, 0, 1], [2, 1, 2, 0, 1, 0, 0, 2]] \$$

$$[y_1 + y_3, 2y_1 + 2y_3 - y_2 - y_4, y_1, y_3, y_2, 0, 0, y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{4, 6, 7\}, \{2, 3, 8\}\}$ order: 3

See Matrix

$\$ [[0, 1, 1, 1, 0, 2, 2, 1], [0, 1, 1, 2, 0, 1, 2, 1], [0, 1, 1, 2, 0, 2, 1, 1], [0, 1, 1, 1, 0, 2, 2, 1], [0, 1, 1, 2, 0, 1, 2, 1], [0, 1, 1, 2, 0, 2, 1, 1]] \$$

$$[0, y_2, y_2, y_3, 0, 5y_2 - y_3 - y_1, y_1, y_2]$$

$$p' = s^2 - s^5 \quad p = -s + s^4 \quad p' = -s + s^4$$

40 . Coloring, $\{2, 5, 7\}$

R: $[3, 8, 1, 1, 2, 7, 4, 5]$

B: $[6, 3, 8, 6, 7, 4, 5, 2]$

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 7	2 vs 7

Omega Rank for R : cycles: $\{\{1, 3\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 1, 1, 0, 1, 1], [2, 1, 2, 1, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1], [2, 1, 3, 0, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1], [2, 1, 3, 0, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1]] \$$

$$[5y_4 - y_1 - y_2 - y_3, y_4, y_1, y_2, y_4, 0, y_3, y_4]$$

$$p = s^3 - s^5 \quad p' = -s^3 + s^5 \quad p' = -s^4 + s^6$$

Omega Rank for B : cycles: {{4, 6}, {5, 7}, {2, 3, 8}} order: 6

See Matrix

$$\$ [[0, 1, 1, 1, 1, 2, 1, 1], [0, 1, 1, 2, 1, 1, 1, 1], [0, 1, 1, 1, 1, 2, 1, 1], [0, 1, 1, 2, 1, 1, 1, 1], [0, 1, 1, 1, 1, 2, 1, 1], [0, 1, 1, 2, 1, 1, 1, 1], [0, 1, 1, 1, 1, 2, 1, 1]] \$$$

$$[0, y_2, y_2, y_1, y_2, 3y_2 - y_1, y_2, y_2]$$

$$p' = -s^4 + s^6 \quad p' = -s^3 + s^5 \quad p' = s - s^3 \quad p' = s^2 - s^4 \quad p = s - s^5$$

41 . Coloring, {2, 5, 8}

R: [3, 8, 1, 1, 2, 7, 5, 2]

B: [6, 3, 8, 6, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$\$ [[2, 2, 1, 0, 1, 0, 1, 1], [1, 2, 2, 0, 1, 0, 0, 2], [2, 3, 1, 0, 0, 0, 0, 2], [1, 2, 2, 0, 0, 0, 0, 3], [2, 3, 1, 0, 0, 0, 0, 2], [1, 2, 2, 0, 0, 0, 0, 3]] \$$$

$$[y_1, y_2, -4y_1 + 3y_2 + 3y_4, 0, -5y_1 + 4y_2 + 4y_4 - y_3, 0, y_4, y_3]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$\$ [[0, 0, 1, 2, 1, 2, 1, 1], [0, 0, 0, 3, 1, 2, 1, 1], [0, 0, 0, 3, 1, 3, 1, 0], [0, 0, 0, 4, 0, 3, 1, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$$

$$[0, 0, -y_1 - y_2 + y_3 + y_4 + y_5, y_1, y_2, y_3, y_4, y_5]$$

$$p = -s^5 + s^6$$

42 . Coloring, {2, 6, 7}

R: [3, 8, 1, 1, 7, 4, 4, 5]

B: [6, 3, 8, 6, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$\$ [[2, 0, 1, 2, 1, 0, 1, 1], [3, 0, 2, 1, 1, 0, 1, 0], [3, 0, 3, 1, 0, 0, 1, 0], [4, 0, 3, 1, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0]] \$$$

$$[y_3, 0, y_2, y_1, y_3 - y_2 - y_1 + y_5 + y_4, 0, y_5, y_4]$$

$$p = s^5 - s^6$$

Omega Rank for B : cycles: {{2, 3, 8}} order: 6

See Matrix

$$\$ [[0, 2, 1, 0, 1, 2, 1, 1], [0, 2, 2, 0, 1, 0, 2, 1], [0, 2, 2, 0, 2, 0, 0, 2], [0, 4, 2, 0, 0, 0, 0, 2], [0, 2, 4, 0, 0, 0, 0, 2], [0, 2, 2, 0, 0, 0, 0, 4]] \$$$

$$[0, y_6, y_5, 0, y_2, y_3, y_1, y_4]$$

43 . Coloring, {2, 6, 8}

R: [3, 8, 1, 1, 7, 4, 5, 2]

B: [6, 3, 8, 6, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	3 vs 7

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[2, 1, 1, 1, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1], [2, 1, 2, 0, 1, 0, 1, 1]] \$$

$$[2y_2, y_2, y_1, 2y_2 - y_1, y_2, 0, y_2, y_2]$$

$$p' = s^3 - s^4 \quad p = s^2 - s^5 \quad p' = -s^4 + s^6 \quad p' = -s^4 + s^5 \quad p' = s^2 - s^4$$

Omega Rank for B : cycles: $\{\{4, 6, 7\}, \{2, 3, 5, 8\}\}$

See Matrix

$\$ [[0, 1, 1, 1, 1, 2, 1, 1], [0, 1, 1, 1, 1, 2, 1], [0, 1, 1, 2, 1, 1, 1, 1], [0, 1, 1, 1, 1, 2, 1, 1], [0, 1, 1, 1, 1, 1, 2, 1], [0, 1, 1, 2, 1, 1, 1, 1], [0, 1, 1, 1, 1, 2, 1, 1]] \$$

$$[0, y_3, y_3, y_2, y_3, 4y_3 - y_2 - y_1, y_1, y_3]$$

$$p' = s^2 - s^5 \quad p' = s^3 - s^6 \quad p = s - s^7 \quad p' = s - s^4$$

44 . Coloring, $\{2, 7, 8\}$

R: $[3, 8, 1, 1, 7, 7, 4, 2]$

B: $[6, 3, 8, 6, 2, 4, 5, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}\}$ order: 4

See Matrix

$\$ [[2, 1, 1, 1, 0, 0, 2, 1], [2, 1, 2, 2, 0, 0, 0, 1], [4, 1, 2, 0, 0, 0, 0, 1], [2, 1, 4, 0, 0, 0, 0, 1], [4, 1, 2, 0, 0, 0, 0, 1], [2, 1, 4, 0, 0, 0, 0, 1]] \$$

$$[6y_2 - y_1 - y_3 - y_4, y_2, y_1, y_3, 0, 0, y_4, y_2]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{2, 3, 5, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 1, 1, 2, 2, 0, 1], [0, 2, 1, 2, 1, 1, 0, 1], [0, 1, 2, 1, 1, 2, 0, 1], [0, 1, 1, 2, 1, 1, 0, 2], [0, 1, 1, 1, 2, 2, 0, 1], [0, 2, 1, 2, 1, 1, 0, 1]] \$$

$$[0, y_1, y_4, y_3, 4y_1 - y_4 - 5y_3 + 4y_2, 3y_1 - 4y_3 + 3y_2, 0, y_2]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

45 . Coloring, $\{3, 4, 5\}$

R: $[3, 3, 8, 6, 2, 7, 5, 5]$

B: $[6, 8, 1, 1, 7, 4, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	2 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 5, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 2, 0, 2, 1, 1, 1], [0, 2, 1, 0, 2, 0, 1, 2], [0, 2, 2, 0, 3, 0, 0, 1], [0, 3, 2, 0, 1, 0, 0, 2], [0, 1, 3, 0, 2, 0, 0, 2], [0, 2, 1, 0, 2, 0, 0, 3]] \$$

$$[0, y_2, y_1, 0, y_6, y_5, y_4, y_3]$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 2, 0, 1, 1, 1], [2, 1, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 2, 0, 1], [2, 1, 0, 2, 0, 2, 0, 1]] \$$

$$[2y_2, y_2, 0, 2y_2, 0, 2y_2 - y_1, y_1, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

46 . Coloring, $\{3, 4, 6\}$

R: $[3, 3, 8, 6, 7, 4, 5, 5]$

B: $[6, 8, 1, 1, 2, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[0, 0, 2, 1, 2, 1, 1, 1], [0, 0, 0, 1, 2, 1, 2, 2], [0, 0, 0, 1, 4, 1, 2, 0], [0, 0, 0, 1, 2, 1, 4, 0], [0, 0, 0, 1, 4, 1, 2, 0], [0, 0, 0, 1, 2, 1, 4, 0]] \$$

$$[0, 0, 6y_2 - y_1 - y_3 - y_4, y_2, y_1, y_2, y_3, y_4]$$

$$p = s^3 - s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[2, 2, 0, 1, 0, 1, 1, 1], [1, 1, 0, 1, 0, 2, 1, 2], [1, 2, 0, 1, 0, 1, 2, 1], [1, 1, 0, 2, 0, 1, 1, 2], [2, 2, 0, 1, 0, 1, 1, 1], [1, 1, 0, 1, 0, 2, 1, 2]] \$$

$$[y_2, y_1, 0, 4y_2 - 5y_1 - y_3 + 4y_4, 0, y_3, y_4, 3y_2 - 4y_1 + 3y_4]$$

$$p' = -s + s^5 \quad p = -s + s^5$$

47 . Coloring, {3, 4, 7}

R: [3, 3, 8, 6, 7, 7, 4, 5]

B: [6, 8, 1, 1, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	4 vs 6

Omega Rank for R : cycles: {{4, 6, 7}} order: 6

See Matrix

$$\$ [[0, 0, 2, 1, 1, 1, 2, 1], [0, 0, 0, 2, 1, 1, 2, 2], [0, 0, 0, 2, 2, 2, 2, 0], [0, 0, 0, 2, 0, 2, 4, 0], [0, 0, 0, 4, 0, 2, 2, 0], [0, 0, 0, 2, 0, 4, 2, 0]] \$$$

$$[0, 0, y_1, y_3, y_4, y_2, y_5, y_6]$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$\$ [[2, 2, 0, 1, 1, 1, 0, 1], [1, 2, 0, 1, 0, 2, 0, 2], [1, 2, 0, 2, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1, 0, 2], [1, 2, 0, 1, 0, 2, 0, 2], [1, 2, 0, 2, 0, 1, 0, 2]] \$$$

$$[2y_3 + 2y_4 - y_2 - y_1, y_3 + y_4, 0, y_2, y_3, y_1, 0, y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

48 . Coloring, {3, 4, 8}

R: [3, 3, 8, 6, 7, 7, 5, 2]

B: [6, 8, 1, 1, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	3 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[0, 1, 2, 0, 1, 1, 2, 1], [0, 1, 1, 0, 2, 0, 2, 2], [0, 2, 1, 0, 2, 0, 2, 1], [0, 1, 2, 0, 2, 0, 2, 1], [0, 1, 1, 0, 2, 0, 2, 2], [0, 2, 1, 0, 2, 0, 2, 1]] \$$

$$[0, -y_1 + 2y_2 - y_4, y_1, 0, y_3, -y_3 + y_2, y_2, y_4]$$

$$p' = -s^2 + s^5 \quad p = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{2, 5, 8\}, \{1, 4, 6\}\}$ order: 3

See Matrix

$\$ [[2, 1, 0, 2, 1, 1, 0, 1], [2, 1, 0, 1, 1, 2, 0, 1], [1, 1, 0, 2, 1, 2, 0, 1], [2, 1, 0, 2, 1, 1, 0, 1], [2, 1, 0, 1, 1, 2, 0, 1], [1, 1, 0, 2, 1, 2, 0, 1]] \$$

$$[-y_2 - y_1 + 5y_3, y_3, 0, y_2, y_3, y_1, 0, y_3]$$

$$p = -s + s^4 \quad p' = -s^2 + s^5 \quad p' = -s + s^4$$

49 . Coloring, $\{3, 5, 6\}$

R: $[3, 3, 8, 1, 2, 4, 5, 5]$

B: $[6, 8, 1, 6, 7, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	2 vs 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

\$ [[1, 1, 2, 1, 2, 0, 0, 1] , [1, 2, 2, 0, 1, 0, 0, 2] , [0, 1, 3, 0, 2, 0, 0, 2] , [0, 2, 1, 0, 2, 0, 0, 3] , [0, 2, 2, 0, 3, 0, 0, 1] , [0, 3, 2, 0, 1, 0, 0, 2]] \$

$[y_1, y_4, y_2, y_3, y_5, 0, 0, y_6]$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

\$ [[1, 1, 0, 1, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 2, 2, 1] , [0, 1, 0, 2, 0, 2, 2, 1]] \$

$[2y_2 - y_1, y_2, 0, y_1, 0, 2y_2, 2y_2, y_2]$

$p = -s^2 + s^3$ $p = -s^2 + s^4$ $p = -s^2 + s^5$ $p = -s^2 + s^6$

50 . Coloring, {3, 5, 7}

R: [3, 3, 8, 1, 2, 7, 4, 5]

B: [6, 8, 1, 6, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 7	2 vs 7

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

\$ [[1, 1, 2, 1, 1, 0, 1, 1] , [1, 1, 2, 1, 1, 0, 0, 2] , [1, 1, 2, 0, 2, 0, 0, 2] , [0, 2, 2, 0, 2, 0, 0, 2] , [0, 2, 2, 0, 2, 0, 0, 2] , [0, 2, 2, 0, 2, 0, 0, 2] , [0, 2, 2, 0, 2, 0, 0, 2]] \$

$[-y_2 + y_3 + y_4, y_2, y_3 + y_4, y_3, y_4, 0, y_3 + y_4 - y_1, y_1]$

$p = -s^4 + s^7$ $p = -s^4 + s^5$ $p = -s^4 + s^6$

Omega Rank for B : cycles: $\{\{2, 8\}, \{4, 6\}, \{5, 7\}\}$ order: 2

See Matrix

$$\$ [[1, 1, 0, 1, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2, 1, 1]] \$$$

$$[y_1, y_2, 0, -y_1 + 2y_2, y_2, 2y_2, y_2, y_2]$$

$$p' = -s^2 + s^3 \quad p' = -s^2 + s^4 \quad p' = -s^2 + s^5 \quad p' = -s^2 + s^6 \quad p = s^2 - s^3$$

51 . Coloring, $\{3, 5, 8\}$

R: $[3, 3, 8, 1, 2, 7, 5, 2]$

B: $[6, 8, 1, 6, 7, 4, 4, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 8\}\}$ order: 3

See Matrix

$$\$ [[1, 2, 2, 0, 1, 0, 1, 1], [0, 2, 3, 0, 1, 0, 0, 2], [0, 3, 2, 0, 0, 0, 0, 3], [0, 3, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3]] \$$$

$$[y_4, y_3, y_1, 0, y_2, 0, y_4, y_5]$$

$$p = -s^3 + s^6$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 4

See Matrix

$$\$ [[1, 0, 0, 2, 1, 2, 1, 1], [0, 0, 0, 3, 1, 3, 1, 0], [0, 0, 0, 4, 0, 3, 1, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$$

$$[y_1, 0, 0, -y_3 + y_4 + y_2, y_3, y_4, y_2, y_1]$$

$$p = -s^4 + s^6 \quad p = -s^4 + s^5$$

52 . Coloring, {3, 6, 7}

R: [3, 3, 8, 1, 7, 4, 4, 5]

B: [6, 8, 1, 6, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	5 vs 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$\$ [[1, 0, 2, 2, 1, 0, 1, 1], [2, 0, 1, 1, 1, 0, 1, 2], [1, 0, 2, 1, 2, 0, 1, 1], [1, 0, 1, 1, 1, 0, 2, 2], [1, 0, 1, 2, 2, 0, 1, 1], [2, 0, 1, 1, 1, 0, 2, 1]] \$$$

$$[y_1, 0, y_4, y_2, y_3, 0, y_5, y_6]$$

Omega Rank for B : cycles: {{2, 8}} order: 6

See Matrix

$$\$ [[1, 2, 0, 0, 1, 2, 1, 1], [0, 2, 0, 0, 1, 1, 2, 2], [0, 3, 0, 0, 2, 0, 1, 2], [0, 4, 0, 0, 1, 0, 0, 3], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4]] \$$$

$$[-y_1 + y_2 + y_3 - y_4 + y_5, y_1, 0, 0, y_2, y_3, y_4, y_5]$$

$$p = -s^5 + s^6$$

53 . Coloring, {3, 6, 8}

R: [3, 3, 8, 1, 7, 4, 5, 2]

B: [6, 8, 1, 6, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{2, 3, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 1, 2, 1, 1, 0, 1, 1], [1, 1, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2], [0, 2, 2, 0, 1, 0, 1, 2]] \$$

$$[-y_1 + 2y_3, y_1, 2y_3, 2y_3 - y_2, y_3, 0, y_3, y_2]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5 \quad p = -s^3 + s^7 \quad p = -s^3 + s^6$$

Omega Rank for B : cycles: $\{\{4, 6, 7\}, \{2, 5, 8\}\}$ order: 3

See Matrix

$\$ [[1, 1, 0, 1, 1, 2, 1, 1], [0, 1, 0, 1, 1, 2, 2, 1], [0, 1, 0, 2, 1, 1, 2, 1], [0, 1, 0, 2, 1, 2, 1, 1], [0, 1, 0, 1, 1, 2, 2, 1], [0, 1, 0, 2, 1, 1, 2, 1], [0, 1, 0, 2, 1, 2, 1, 1]] \$$

$$[-y_1 + 5y_4 - y_2 - y_3, y_4, 0, y_1, y_4, y_2, y_3, y_4]$$

$$p = s^2 - s^5 \quad p' = -s^2 + s^5 \quad p' = -s^3 + s^6$$

54 . Coloring, $\{3, 7, 8\}$

R: $[3, 3, 8, 1, 7, 7, 4, 2]$

B: $[6, 8, 1, 6, 2, 4, 5, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	4 vs 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

\$ [[1, 1, 2, 1, 0, 0, 2, 1] , [1, 1, 2, 2, 0, 0, 0, 2] , [2, 2, 2, 0, 0, 0, 0, 2] , [0, 2, 4, 0, 0, 0, 0, 2] , [0, 2, 2, 0, 0, 0, 0, 4] , [0, 4, 2, 0, 0, 0, 0, 2]] \$

$$[y_1, y_2, y_5, y_3, 0, 0, y_6, y_4]$$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

\$ [[1, 1, 0, 1, 2, 2, 0, 1] , [0, 2, 0, 2, 1, 2, 0, 1] , [0, 1, 0, 2, 1, 2, 0, 2] , [0, 1, 0, 2, 2, 2, 0, 1] , [0, 2, 0, 2, 1, 2, 0, 1] , [0, 1, 0, 2, 1, 2, 0, 2]] \$

$$[-y_3 + y_1, 2y_1 - y_2 - y_4, 0, y_3, y_2, y_1, 0, y_4]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

55 . Coloring, {4, 5, 6}

R: [3, 3, 1, 6, 2, 4, 5, 5]

B: [6, 8, 8, 1, 7, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

\$ [[1, 1, 2, 1, 2, 1, 0, 0] , [2, 2, 2, 1, 0, 1, 0, 0] , [2, 0, 4, 1, 0, 1, 0, 0] , [4, 0, 2, 1, 0, 1, 0, 0] , [2, 0, 4, 1, 0, 1, 0, 0] , [4, 0, 2, 1, 0, 1, 0, 0]] \$

$$[y_4, y_3, y_2, y_1, -y_4 - y_3 - y_2 + 6y_1, y_1, 0, 0]$$

$$p' = -s^3 + s^5 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6, 7\}\}$ order: 4

See Matrix

$$\$ [[1, 1, 0, 1, 0, 1, 2, 2], [1, 2, 0, 2, 0, 1, 1, 1], [2, 1, 0, 1, 0, 1, 1, 2], [1, 2, 0, 1, 0, 2, 1, 1], [1, 1, 0, 1, 0, 1, 2, 2], [1, 2, 0, 2, 0, 1, 1, 1]] \$$$

$$[-5y_1 + 4y_2 + 4y_3 - y_4, y_1, 0, y_2, 0, y_3, y_4, -4y_1 + 3y_2 + 3y_3]$$

$$p = -s + s^5 \quad p' = -s + s^5$$

56 . Coloring, $\{4, 5, 7\}$

R: $[3, 3, 1, 6, 2, 7, 4, 5]$

B: $[6, 8, 8, 1, 7, 4, 5, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 7	2 vs 7

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6, 7\}\}$ order: 6

See Matrix

$$\$ [[1, 1, 2, 1, 1, 1, 1, 0], [2, 1, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0], [3, 0, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0], [3, 0, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0]] \$$$

$$[y_1, y_2, y_3, y_4, -y_1 - y_2 - y_3 + 5y_4, y_4, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p = -s^3 + s^7$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6\}, \{5, 7\}\}$ order: 6

See Matrix

$$\$ [[1, 1, 0, 1, 1, 1, 1, 2], [1, 2, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 2], [1, 2, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 2], [1, 2, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 2]] \$$$

$$[y_2, 3y_2 - y_1, 0, y_2, y_2, y_2, y_2, y_1]$$

$$p = -s + s^5 \quad p = -s + s^7 \quad p = -s + s^3 \quad p' = -s + s^5 \quad p' = -s + s^3$$

57 . Coloring, {4, 5, 8}

R: [3, 3, 1, 6, 2, 7, 5, 2]

B: [6, 8, 8, 1, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$\$ [[1, 2, 2, 0, 1, 1, 1, 0], [2, 1, 3, 0, 1, 0, 1, 0], [3, 1, 3, 0, 1, 0, 0, 0], [3, 1, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0]] \$$$

$$[y_1, -y_1 + y_2 + y_3 + y_4 - y_5, y_2, 0, y_3, y_4, y_5, 0]$$

$$p = -s^5 + s^6$$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$$\$ [[1, 0, 0, 2, 1, 1, 1, 2], [2, 0, 0, 2, 2, 1, 1, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 4, 0, 2, 0, 0], [4, 0, 0, 2, 0, 2, 0, 0], [2, 0, 0, 2, 0, 4, 0, 0]] \$$$

$$[y_3, 0, 0, y_2, y_1, y_4, y_5, y_6]$$

58 . Coloring, {4, 6, 7}

R: [3, 3, 1, 6, 7, 4, 4, 5]

B: [6, 8, 8, 1, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}\}$ order: 4

See Matrix

$\$ [[1, 0, 2, 2, 1, 1, 1, 0], [2, 0, 1, 2, 0, 2, 1, 0], [1, 0, 2, 3, 0, 2, 0, 0], [2, 0, 1, 2, 0, 3, 0, 0], [1, 0, 2, 3, 0, 2, 0, 0], [2, 0, 1, 2, 0, 3, 0, 0]] \$$

$$[y_1, 0, -4y_1 + 3y_3 + 3y_4, -5y_1 - y_2 + 4y_3 + 4y_4, y_2, y_3, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}\}$ order: 6

See Matrix

$\$ [[1, 2, 0, 0, 1, 1, 1, 2], [0, 3, 0, 0, 1, 1, 1, 2], [0, 3, 0, 0, 1, 0, 1, 3], [0, 4, 0, 0, 1, 0, 0, 3], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4]] \$$

$$[-y_1 + y_2 + y_3 - y_4 + y_5, y_1, 0, 0, y_2, y_3, y_4, y_5]$$

$$p = -s^5 + s^6$$

59 . Coloring, $\{4, 6, 8\}$

R: [3, 3, 1, 6, 7, 4, 5, 2]

B: [6, 8, 8, 1, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	3 vs 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {5, 7}} order: 2

See Matrix

\$ [[1, 1, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0], [2, 0, 2, 1, 1, 1, 1, 0]] \$

$$[y_1, -y_1 + 2y_2, 2y_2, y_2, y_2, y_2, y_2, 0]$$

$$p' = -s^2 + s^3 \quad p' = -s^2 + s^4 \quad p' = -s^2 + s^5 \quad p' = -s^2 + s^6 \quad p = s^2 - s^3$$

Omega Rank for B : cycles: {{2, 5, 8}, {1, 4, 6, 7}}

See Matrix

\$ [[1, 1, 0, 1, 1, 1, 1, 2], [1, 1, 0, 1, 2, 1, 1, 1], [1, 2, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 2], [1, 1, 0, 1, 2, 1, 1, 1], [1, 2, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 2]] \$

$$[y_2, 4y_2 - y_1 - y_3, 0, y_2, y_1, y_2, y_2, y_3]$$

$$p = -s + s^4 \quad p' = -s + s^4 \quad p' = -s^2 + s^5 \quad p = -s + s^7$$

60 . Coloring, {4, 7, 8}

R: [3, 3, 1, 6, 7, 7, 4, 2]

B: [6, 8, 8, 1, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

\$ [[1, 1, 2, 1, 0, 1, 2, 0], [2, 0, 2, 2, 0, 1, 1, 0], [2, 0, 2, 1, 0, 2, 1, 0], [2, 0, 2, 1, 0, 1, 2, 0], [2, 0, 2, 2, 0, 1, 1, 0], [2, 0, 2, 1, 0, 2, 1, 0]] \$

$$[y_1, y_2, y_1 + y_2, 2y_1 + 2y_2 - y_3 - y_4, 0, y_3, y_4, 0]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{2, 5, 8\}, \{1, 4, 6\}\}$ order: 3

See Matrix

$$\$ [[1, 1, 0, 1, 2, 1, 0, 2], [1, 2, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 1, 0, 2], [1, 1, 0, 1, 2, 1, 0, 2], [1, 2, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 1, 0, 2]] \$$$

$$[y_2, 5y_2 - y_1 - y_3, 0, y_2, y_1, y_2, 0, y_3]$$

$$p' = s^2 - s^5 \quad p = -s + s^4 \quad p' = -s + s^4$$

61 . Coloring, $\{5, 6, 7\}$

R: [3, 3, 1, 1, 2, 4, 4, 5]

B: [6, 8, 8, 6, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$$\$ [[2, 1, 2, 2, 1, 0, 0, 0], [4, 1, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [0, 0, 0]] \$$$

$$[y_4, y_3, y_2, 2y_1, y_1, 0, 0, 0]$$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{5, 7\}\}$ order: 2

See Matrix

$$\$ [[0, 1, 0, 0, 1, 2, 2, 2], [0, 2, 0, 0, 2, 0, 3, 1], [0, 1, 0, 0, 3, 0, 2, 2], [0, 2, 0, 0, 2, 0, 3, 1], [0, 1, 0, 0, 3, 0, 2, 2]] \$$$

$$[0, y_1, 0, 0, -5 y_1 - y_2 + 4 y_3, y_2, y_3, -4 y_1 + 3 y_3]$$

$$p' = s^2 - s^4 \quad p = s^2 - s^4$$

62 . Coloring, {5, 6, 8}

R: [3, 3, 1, 1, 2, 4, 5, 2]

B: [6, 8, 8, 6, 7, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	5 vs 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$\$ [[2, 2, 2, 1, 1, 0, 0, 0], [3, 1, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [0, 0, 0]] \$$$

$$[-y_1 + y_2 + 2 y_3, y_1, y_2, y_3, y_3, 0, 0, 0]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

$$\$ [[0, 0, 0, 1, 1, 2, 2, 2], [0, 0, 0, 2, 2, 1, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0], [0, 0, 0, 3, 0, 3, 2, 0], [0, 0, 0, 2, 0, 3, 3, 0]] \$$$

$$[0, 0, 0, y_4, y_5, y_1, y_2, y_3]$$

63 . Coloring, {5, 7, 8}

R: [3, 3, 1, 1, 2, 7, 4, 2]

B: [6, 8, 8, 6, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$\$ [[2, 2, 2, 1, 0, 0, 1, 0], [3, 0, 4, 1, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0], [3, 0, 5, 0, 0, 0, 0, 0], [5, 0, 3, 0, 0, 0, 0, 0]] \$$

$[y_3, 2y_4, y_1, y_2, 0, 0, y_4, 0]$

$$p = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{4, 6\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[0, 0, 0, 1, 2, 2, 1, 2], [0, 0, 0, 2, 3, 1, 2, 0], [0, 0, 0, 1, 2, 2, 3, 0], [0, 0, 0, 2, 3, 1, 2, 0], [0, 0, 0, 1, 2, 2, 3, 0]] \$$

$[0, 0, 0, y_1, y_2, -4y_1 + 3y_2, -5y_1 + 4y_2 - y_3, y_3]$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

64 . Coloring, $\{6, 7, 8\}$

R: $[3, 3, 1, 1, 7, 4, 4, 2]$

B: $[6, 8, 8, 6, 2, 7, 5, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	5 vs 5

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$\$ [[2, 1, 2, 2, 0, 0, 1, 0], [4, 0, 3, 1, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [0, 0, 0]] \$$

$$[y_1, y_2, y_3, y_1 + 2y_2 - y_3, 0, 0, y_2, 0]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: $\{\{2, 5, 8\}\}$ order: 3

See Matrix

$\$ [[0, 1, 0, 0, 2, 2, 1, 2], [0, 2, 0, 0, 3, 0, 2, 1], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3]] \$$

$$[0, y_1, 0, 0, y_2, y_3, y_4, y_5]$$

65 . Coloring, $\{2, 3, 4, 5\}$

R: $[3, 8, 8, 6, 2, 7, 5, 5]$

B: $[6, 3, 1, 1, 7, 4, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{2, 5, 8\}\}$ order: 3

See Matrix

$\$ [[0, 1, 1, 0, 2, 1, 1, 2], [0, 2, 0, 0, 3, 0, 1, 2], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3], [0, 3, 0, 0, 3, 0, 0, 2]] \$$

$$[0, y_1, y_5, 0, y_2, y_5, y_3, y_4]$$

$$p = s^3 - s^6$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}\}$ order: 3

See Matrix

$\$ [[2, 1, 1, 2, 0, 1, 1, 0], [3, 0, 1, 2, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0], [3, 0, 0, 3, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0]] \$$

$[y_3, y_4, y_2, y_1, 0, y_5, y_4, 0]$

$$p = s^3 - s^6$$

66 . Coloring, $\{2, 3, 4, 6\}$

R: $[3, 8, 8, 6, 7, 4, 5, 5]$

B: $[6, 3, 1, 1, 2, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[0, 0, 1, 1, 2, 1, 1, 2], [0, 0, 0, 1, 3, 1, 2, 1], [0, 0, 0, 1, 3, 1, 3, 0], [0, 0, 0, 1, 3, 1, 3, 0], [0, 0, 0, 1, 3, 1, 3, 0], [0, 0, 0, 1, 3, 1, 3, 0]] \$$

$[0, 0, y_3, y_2, -y_3 + 3y_2, y_2, y_1, 3y_2 - y_1]$

$$p' = s^4 - s^5 \quad p = s^3 - s^6 \quad p' = s^3 - s^5$$

Omega Rank for B : cycles: $\{\{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[2, 2, 1, 1, 0, 1, 1, 0], [2, 0, 2, 1, 0, 2, 1, 0], [3, 0, 0, 1, 0, 2, 2, 0], [1, 0, 0, 2, 0, 3, 2, 0], [2, 0, 0, 2, 0, 1, 3, 0], [2, 0, 0, 3, 0, 2, 1, 0]] \$$

$[y_1, y_2, y_3, y_4, 0, y_5, y_6, 0]$

67 . Coloring, {2, 3, 4, 7}

R: [3, 8, 8, 6, 7, 7, 4, 5]

B: [6, 3, 1, 1, 2, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	6 vs 6

Omega Rank for R : cycles: {{4, 6, 7}} order: 6

See Matrix

\$ [[0, 0, 1, 1, 1, 1, 2, 2] , [0, 0, 0, 2, 2, 1, 2, 1] , [0, 0, 0, 2, 1, 2, 3, 0] , [0, 0, 0, 3, 0, 2, 3, 0] , [0, 0, 0, 3, 0, 3, 2, 0] , [0, 0, 0, 2, 0, 3, 3, 0]] \$

[0, 0, y_5 , y_6 , y_2 , y_3 , y_4 , y_1]

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

\$ [[2, 2, 1, 1, 1, 1, 0, 0] , [2, 1, 2, 1, 0, 2, 0, 0] , [3, 0, 1, 2, 0, 2, 0, 0] , [3, 0, 0, 2, 0, 3, 0, 0] , [2, 0, 0, 3, 0, 3, 0, 0] , [3, 0, 0, 3, 0, 2, 0, 0]] \$

[y_3 , y_2 , y_1 , y_6 , y_4 , y_5 , 0, 0]

68 . Coloring, {2, 3, 4, 8}

R: [3, 8, 8, 6, 7, 7, 5, 2]

B: [6, 3, 1, 1, 2, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[0, 1, 1, 0, 1, 1, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 2, 0, 2, 2]] \$$

$$[0, -y_1 + y_2, y_1, 0, -y_1 + y_2, y_1, y_2, y_2]$$

$$p = s^2 - s^5 \quad p' = s^2 - s^4 \quad p'' = -s^4 + s^5 \quad p''' = s^3 - s^4$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 2, 1, 1, 0, 0], [3, 1, 1, 1, 0, 2, 0, 0], [2, 0, 1, 2, 0, 3, 0, 0], [3, 0, 0, 3, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0]] \$$

$$[y_1, y_2, y_3, y_4, y_5, y_6, 0, 0]$$

69 . Coloring, $\{2, 3, 5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^2 \quad p' = s^3 \quad p'' = s^2 \quad p''' = s^4 \quad p^{(4)} = s^5$$

R: $[3, 8, 8, 1, 2, 4, 5, 5]$

B: $[6, 3, 1, 6, 7, 7, 4, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
1 vs 6	6 vs 6	6 vs 6	6 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 1, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2], [0, 2, 1, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3]] \$$

$$[y_4, y_5, y_1, y_2, y_3, 0, 0, y_6]$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6
See Matrix

\$ [[1, 1, 1, 1, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 0, 2, 0, 3, 2, 0], [0, 0, 0, 2, 0, 3, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0], [0, 0, 0, 3, 0, 3, 2, 0]] \$

[y₂, y₁, y₅, y₆, 0, y₃, y₄, 0]

70 . Coloring, {2, 3, 5, 7}

$\Omega p(\Delta)=0: p = s - 2s^3 + 8s^6$

R: [3, 8, 8, 1, 2, 7, 4, 5]

B: [6, 3, 1, 6, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
5 vs 6	7 vs 7	7 vs 7	7 vs 7	4 vs 7

Omega Rank for R : cycles: {{2, 5, 8}} order: 6
See Matrix

\$ [[1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 0, 0, 2], [1, 2, 1, 0, 2, 0, 0, 2], [0, 2, 1, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3]] \$

[y₄, y₂, y₃, y₁, y₇, 0, y₆, y₅]

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4
See Matrix

\$ [[1, 1, 1, 1, 1, 2, 1, 0], [1, 0, 1, 2, 1, 2, 1, 0], [1, 0, 0, 2, 1, 3, 1, 0], [0, 0, 0, 3, 1, 3, 1, 0], [0, 0, 0, 3, 1, 3, 1, 0], [0, 0, 0, 3, 1, 3, 1, 0], [0, 0, 0, 3, 1, 3, 1, 0]] \$

[-y₁ - y₄ + 3 y₃, y₁, -y₂ + 3 y₃, y₄, y₃, y₂, y₃, 0]

$p' = s^4 - s^6$ $p' = s^5 - s^6$ $p = s^4 - s^7$

71 . Coloring, {2, 3, 5, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^6 \quad p' = s^3 + 2s^5 \quad p'' = s^2 + 2s^4$$

R: [3, 8, 8, 1, 2, 7, 5, 2]

B: [6, 3, 1, 6, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	3 vs 6	3 vs 6	3 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

$$\$ [[1, 2, 1, 0, 1, 0, 1, 2], [0, 3, 1, 0, 1, 0, 0, 3], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4]] \$$$

$$[y_2, y_3, y_1, 0, y_1, 0, y_2, y_3]$$

$$p = -s^3 + s^6 \quad p' = -s^3 + s^4 \quad p'' = -s^3 + s^5$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[1, 0, 1, 2, 1, 2, 1, 0], [1, 0, 0, 3, 0, 3, 1, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$$

$$[y_3, 0, y_1, y_2, y_1, y_2, y_3, 0]$$

$$p = s^3 - s^6 \quad p' = s^3 - s^5 \quad p'' = s^4 - s^5$$

M \ ; N

$$\$ [[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]] \$ \quad \$ [[0, 5, 6, 3, 3, 6, 9, 4], [5, 0, 3, 4, 6, 5, 4, 9], [6, 3, 0, 5, 9, 4, 3, 6], [3, 4, 5, 0, 4, 9, 6, 5], [3, 6, 9, 4, 0, 5, 6, 3], [6, 5, 4, 9, 5, 0, 3, 4], [9, 4, 3, 6, 6, 3, 0, 5], [4, 9, 6, 5, 3, 4, 5, 0]] \$$$

$$\tau = 32, r' = 1/2$$

R: [3, 8, 8, 1, 2, 7, 5, 2]

B: [6, 3, 1, 6, 7, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [2], [1]]

Action of B on ranges, [[4], [3], [1], [4]]

Cycles: R, {{2, 8}}, B, {{4, 6}}

$$\beta(\{1, 7\}) = 1/4$$

$$\beta(\{2, 8\}) = 1/4$$

$$\beta(\{3, 5\}) = 1/4$$

$$\beta(\{4, 6\}) = 1/4$$

Partitions

Action of R on partitions, [[6], [5], [3], [3], [4], [4], [5], [6]]

Action of B on partitions, [[6], [2], [8], [2], [6], [4], [8], [4]]

$$\alpha(\{\{2, 4, 5, 7\}, \{1, 3, 6, 8\}\}) = 0/1$$

$$\alpha(\{\{1, 2, 4, 5\}, \{3, 6, 7, 8\}\}) = 2/9$$

$$\alpha(\{\{2, 3, 4, 7\}, \{1, 5, 6, 8\}\}) = 2/9$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 2/9$$

$$\alpha(\{\{1, 2, 3, 6\}, \{4, 5, 7, 8\}\}) = 1/9$$

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/9$$

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 0/1$$

$$\alpha(\{\{1, 3, 4, 8\}, \{2, 5, 6, 7\}\}) = 1/9$$

b1 = {1, 2, 3, 6} ‘, ‘ b2 = {2, 4, 5, 7} ‘, ‘ b3 = {1, 3, 6, 8} ‘, ‘ b4 = {1, 2, 4, 5} ‘, ‘ b5 = {3, 6, 7, 8} ‘, ‘
b6 = {2, 3, 4, 7} ‘, ‘ b7 = {1, 5, 6, 8} ‘, ‘ b8 = {1, 4, 5, 8} ‘, ‘ b9 = {2, 3, 6, 7} ‘, ‘ b10 = {1, 2, 3, 4} ‘, ‘
b11 = {5, 6, 7, 8} ‘, ‘ b12 = {3, 4, 7, 8} ‘, ‘ b13 = {4, 5, 7, 8} ‘, ‘ b14 = {1, 2, 5, 6} ‘, ‘ b15 = {1, 3, 4, 8}
‘, ‘ b16 = {2, 5, 6, 7}

Action of R and B on the blocks of the partitions: = [8, B, A, D, 1, 7, 6, 6, 7, 8, 9, 1, 9, D, A, B] [A, B, A,
5, 4, 10, F, 5, 4, 9, 8, 10, B, F, 9, 8]

with invariant measure [1, 0, 0, 2, 2, 2, 2, 2, 1, 1, 0, 1, 0, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 3, 5, 8}
Rank	2
R,B	[3, 8, 8, 1, 2, 7, 5, 2], [6, 3, 1, 6, 7, 4, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[5, 6, 3, 3, 6, 9, 4, 3, 4, 6, 5, 4, 9, 5, 9, 4, 3, 6, 4, 9, 6, 5, 5, 6, 3, 3, 4, 5] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

72 . Coloring, {2, 3, 6, 7}

$$\Omega p(\Delta)=0: \quad p' = s^2 + 2s^4 \quad p' = s^3 + 2s^5 \quad p = s^2 - 4s^6$$

R: [3, 8, 8, 1, 7, 4, 4, 5]

B: [6, 3, 1, 6, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
3 vs 6	3 vs 6	3 vs 6	3 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$\$ [[1, 0, 1, 2, 1, 0, 1, 2], [2, 0, 1, 1, 2, 0, 1, 1], [1, 0, 2, 1, 1, 0, 2, 1], [1, 0, 1, 2, 1, 0, 1, 2], [2, 0, 1, 1, 2, 0, 1, 1], [1, 0, 2, 1, 1, 0, 2, 1]] \$$$

$$[y_1, 0, y_2, y_3, y_1, 0, y_2, y_3]$$

$$p' = -s + s^4 \quad p' = -s^2 + s^5 \quad p = -s + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$\$ [[1, 2, 1, 0, 1, 2, 1, 0], [1, 1, 2, 0, 1, 1, 2, 0], [2, 1, 1, 0, 2, 1, 1, 0], [1, 2, 1, 0, 1, 2, 1, 0], [1, 1, 2, 0, 1, 1, 2, 0], [2, 1, 1, 0, 2, 1, 1, 0]] \$$

$$[y_2, y_3, y_1, 0, y_2, y_3, y_1, 0]$$

$$p = -s + s^4 \quad p' = -s + s^4 \quad p'' = -s^2 + s^5$$

M \ ; N

$\$ [[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]] \$$ $\$ [[0, 3, 2, 1, 5, 2, 3, 4], [3, 0, 1, 4, 2, 5, 4, 1], [2, 1, 0, 3, 3, 4, 5, 2], [1, 4, 3, 0, 4, 1, 2, 5], [5, 2, 3, 4, 0, 3, 2, 1], [2, 5, 4, 1, 3, 0, 1, 4], [3, 4, 5, 2, 2, 1, 0, 3], [4, 1, 2, 5, 1, 4, 3, 0]] \$$

$$\tau = 32, r' = 1/2$$

R: [3, 8, 8, 1, 7, 4, 4, 5]

B: [6, 3, 1, 6, 2, 7, 5, 2]

Ranges

Action of R on ranges, [[3], [4], [4], [1]]

Action of B on ranges, [[2], [3], [1], [2]]

Cycles: R, {{1, 3, 4, 5, 7, 8}}, B, {{1, 2, 3, 5, 6, 7}}

$$\beta(\{1, 5\}) = 1/4$$

$$\beta(\{2, 6\}) = 1/4$$

$$\beta(\{3, 7\}) = 1/4$$

$$\beta(\{4, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[3], [1], [2], [2]]

Action of B on partitions, [[2], [4], [2], [3]]

$$\alpha(\{\{4, 5, 6, 7\}, \{1, 2, 3, 8\}\}) = 1/5$$

$$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 2/5$$

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/5$$

$$\alpha(\{\{2, 5, 7, 8\}, \{1, 3, 4, 6\}\}) = 1/5$$

b1 = {1, 4, 6, 7} ‘, ‘ b2 = {1, 2, 3, 4} ‘, ‘ b3 = {5, 6, 7, 8} ‘, ‘ b4 = {4, 5, 6, 7} ‘, ‘ b5 = {1, 2, 3, 8} ‘, ‘ b6 = {2, 3, 5, 8} ‘, ‘ b7 = {2, 5, 7, 8} ‘, ‘ b8 = {1, 3, 4, 6}

Action of R and B on the blocks of the partitions: = [4, 1, 6, 3, 2, 5, 6, 1] [8, 6, 1, 1, 6, 7, 3, 2]
with invariant measure [2, 1, 1, 1, 1, 2, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 3, 6, 7}
Rank	2
R,B	[3, 8, 8, 1, 7, 4, 4, 5], [6, 3, 1, 6, 2, 7, 5, 2]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[3, 2, 1, 5, 2, 3, 4, 1, 4, 2, 5, 4, 1, 3, 3, 4, 5, 2, 4, 1, 2, 5, 3, 2, 1, 1, 4, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

73 . Coloring, {2, 3, 6, 8}

$$\Omega p(\Delta)=0: \quad p = s - 2s^3 - 8s^6$$

R: [3, 8, 8, 1, 7, 4, 5, 2]

B: [6, 3, 1, 6, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 6	7 vs 7	7 vs 7	4 vs 7	7 vs 7

Omega Rank for R : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$\$ [[1, 1, 1, 1, 1, 0, 1, 2], [1, 2, 1, 0, 1, 0, 1, 2], [0, 2, 1, 0, 1, 0, 1, 3], [0, 3, 0, 0, 1, 0, 1, 3], [0, 3, 0, 0, 1, 0, 1, 3], [0, 3, 0, 0, 1, 0, 1, 3], [0, 3, 0, 0, 1, 0, 1, 3]] \$$$

$$[3y_3 - y_4, -y_1 - y_2 + 3y_3, y_1, y_2, y_3, 0, y_3, y_4]$$

$$p = -s^4 + s^5 \quad p = -s^4 + s^6 \quad p = -s^4 + s^7$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$$\$ [[1, 1, 1, 1, 1, 2, 1, 0], [1, 1, 1, 1, 0, 2, 2, 0], [1, 0, 1, 2, 0, 2, 2, 0], [1, 0, 0, 2, 0, 3, 2, 0], [0, 0, 0, 2, 0, 3, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0], [0, 0, 0, 3, 0, 3, 2, 0]] \$$$

$$[y_1, y_2, y_7, y_4, y_5, y_6, y_3, 0]$$

74 . Coloring, {2, 3, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s^4 \quad p' = s^4 \quad p'' = s^5$$

R: [3, 8, 8, 1, 7, 7, 4, 2]

B: [6, 3, 1, 6, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	6 vs 6	6 vs 6	6 vs 6	6 vs 6

Omega Rank for R : cycles: {{2, 8}} order: 6

See Matrix

$$\$ [[1, 1, 1, 1, 0, 0, 2, 2], [1, 2, 1, 2, 0, 0, 0, 2], [2, 2, 1, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3], [0, 3, 0, 0, 0, 0, 0, 5], [0, 5, 0, 0, 0, 0, 0, 3]] \$$$

$$[y_3, y_2, y_1, y_6, 0, 0, y_5, y_4]$$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$\$ [[1, 1, 1, 1, 2, 2, 0, 0], [1, 2, 1, 2, 0, 2, 0, 0], [1, 0, 2, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0], [0, 0, 0, 3, 0, 5, 0, 0], [0, 0, 0, 5, 0, 3, 0, 0]] \$$$

$$[y_6, y_5, y_4, y_3, y_2, y_1, 0, 0]$$

75 . Coloring, {2, 4, 5, 6}

$$\Omega p(\Delta)=0: \quad p = s^2 + s^3 - 4s^6$$

R: [3, 8, 1, 6, 2, 4, 5, 5]

B: [6, 3, 8, 1, 7, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 6	7 vs 7	7 vs 7	3 vs 7	4 vs 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {2, 5, 8}} order: 6

See Matrix

$$\$ [[1, 1, 1, 1, 2, 1, 0, 1], [1, 2, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 0, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 2, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 0, 2], [1, 1, 1, 1, 2, 1, 0, 1]] \$$$

$$[y_2, 4y_2 - y_1 - y_3, y_2, y_2, y_1, y_2, 0, y_3]$$

$$p' = s^3 - s^6 \quad p' = s^2 - s^5 \quad p' = s - s^4 \quad p = s - s^7$$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 3, 8}}

See Matrix

$$\$ [[1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 2, 0, 1, 1, 1], [2, 1, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 2, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 2, 0, 1, 1, 1], [2, 1, 1, 1, 0, 1, 1, 1]] \$$$

$$[5y_3 - y_1 - y_2 - y_4, y_3, y_3, y_1, 0, y_2, y_4, y_3]$$

$$p = -s + s^5 \quad p' = -s + s^5 \quad p' = -s^2 + s^6$$

76 . Coloring, {2, 4, 5, 7}

$$\Omega p(\Delta)=0: \quad p' = s^5 \quad p = s \quad p' = s \quad p' = s^2 \quad p' = s^4 \quad p' = s^3$$

R: [3, 8, 1, 6, 2, 7, 4, 5]

B: [6, 3, 8, 1, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
0 vs 6	1 vs 8	1 vs 8	1 vs 8	1 vs 8

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6, 7\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1]] \$$

$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$

$p' = -s + s^4 \quad p' = -s + s^7 \quad p' = -s + s^6 \quad p' = -s + s^5 \quad p' = -s + s^3 \quad p' = -s + s^2 \quad p' = 1 - s$

Omega Rank for B : cycles: $\{\{1, 4, 6\}, \{5, 7\}, \{2, 3, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1]] \$$

$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$

$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5 \quad p' = -1 + s^6 \quad p' = -1 + s^7$

‘ See 8-level graph

‘

M \ ; N

$\$ [[0, 1, 1, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 1], [1, 1, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 1, 0]] \$ \quad \$ [[0, 1, 1, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 1], [1, 1, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 1, 0]] \$$

$\tau = 8, r' = 7/8$

R: [3, 8, 1, 6, 2, 7, 4, 5]
B: [6, 3, 8, 1, 7, 4, 5, 2]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R, {{1, 3}, {4, 6, 7}, {2, 5, 8}}, B, {{1, 4, 6}, {5, 7}, {2, 3, 8}}

$\beta(\{1, 2, 3, 4, 5, 6, 7, 8\}) = 1/1$

Partitions

$\alpha(\{\{8\}, \{1\}, \{3\}, \{4\}, \{2\}, \{5\}, \{6\}, \{7\}\}) = 1/1$

b1 = {8} , , b2 = {1} , , b3 = {3} , , b4 = {4} , , b5 = {2} , , b6 = {5} , , b7 = {6} , , b8 = {7}

Action of R and B on the blocks of the partitions: = [5, 3, 2, 8, 6, 1, 4, 7] [3, 4, 5, 7, 1, 8, 2, 6]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-8 partition graph.

‘

Right Group	
Coloring	{2, 4, 5, 7}
Rank	8
R,B	[3, 8, 1, 6, 2, 7, 4, 5], [6, 3, 8, 1, 7, 4, 5, 2]
π_2	[1, 1]
u_2	[1, 1] (dim 2)
wpp	[1, 1, 1, 1, 1, 1, 1, 1]
π_8	[1]
u_8	[1]

77 . Coloring, {2, 4, 5, 8}

$$\Omega p(\Delta)=0: \quad p = s - 2s^3 - 8s^6$$

R: [3, 8, 1, 6, 2, 7, 5, 2]

B: [6, 3, 8, 1, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 6	7 vs 7	7 vs 7	4 vs 7	7 vs 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

\$ [[1, 2, 1, 0, 1, 1, 1, 1], [1, 2, 1, 0, 1, 0, 1, 2], [1, 3, 1, 0, 1, 0, 0, 2], [1, 3, 1, 0, 0, 0, 0, 3], [1, 3, 1, 0, 0, 0, 0, 3], [1, 3, 1, 0, 0, 0, 0, 3], [1, 3, 1, 0, 0, 0, 0, 3]] \$

$$[y_3, y_2, y_3, 0, y_4, 3y_3 - y_4 - y_1, 3y_3 - y_2, y_1]$$

$$p = -s^4 + s^6 \quad p = -s^4 + s^7 \quad p = -s^4 + s^5$$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

\$ [[1, 0, 1, 2, 1, 1, 1, 1], [2, 0, 0, 2, 1, 1, 1, 1], [2, 0, 0, 2, 1, 2, 1, 0], [2, 0, 0, 3, 0, 2, 1, 0], [3, 0, 0, 3, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0]] \$

$$[y_1, 0, y_2, y_3, y_6, y_4, y_5, y_7]$$

78 . Coloring, {2, 4, 6, 7}

$$\Omega p(\Delta)=0: \quad p = s - 2s^3 - 8s^6$$

R: [3, 8, 1, 6, 7, 4, 4, 5]

B: [6, 3, 8, 1, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 6	7 vs 7	7 vs 7	4 vs 7	7 vs 7

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}\}$ order: 4
See Matrix

$\$ [[1, 0, 1, 2, 1, 1, 1, 1], [1, 0, 1, 2, 1, 2, 1, 0], [1, 0, 1, 3, 0, 2, 1, 0], [1, 0, 1, 3, 0, 3, 0, 0], [1, 0, 1, 3, 0, 3, 0, 0], [1, 0, 1, 3, 0, 3, 0, 0], [1, 0, 1, 3, 0, 3, 0, 0]] \$$

$$[y_4, 0, y_4, y_3, 3y_4 - y_3, y_2, 3y_4 - y_2 - y_1, y_1]$$

$$p' = -s^5 + s^6 \quad p = s^4 - s^6 \quad p' = s^4 - s^5$$

Omega Rank for B : cycles: $\{\{2, 3, 8\}\}$ order: 6
See Matrix

$\$ [[1, 2, 1, 0, 1, 1, 1, 1], [0, 2, 2, 0, 1, 1, 1, 1], [0, 2, 2, 0, 1, 0, 1, 2], [0, 3, 2, 0, 1, 0, 0, 2], [0, 3, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3]] \$$

$$[y_1, y_2, y_4, 0, y_3, y_5, y_6, y_7]$$

79 . Coloring, $\{2, 4, 6, 8\}$

$$\Omega p(\Delta)=0: \quad p' = s^3 \quad p' = s^4 \quad p' = s^5 \quad p' = s^2 \quad p' = s \quad p = s$$

R: [3, 8, 1, 6, 7, 4, 5, 2]

B: [6, 3, 8, 1, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
0 vs 6	1 vs 8	1 vs 8	1 vs 8	1 vs 8

Omega Rank for R : cycles: $\{\{2, 8\}, \{4, 6\}, \{1, 3\}, \{5, 7\}\}$ order: 2
See Matrix

\$ [[1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1]] \$

[y₁, y₁, y₁, y₁, y₁, y₁, y₁, y₁]

$$p' = -s + s^7 \quad p' = -s + s^6 \quad p' = -s + s^5 \quad p' = -s + s^3 \quad p' = -s + s^4 \quad p' = -s + s^2 \quad p' = 1 - s$$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 3, 5, 8}} order: 4

See Matrix

\$ [[1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1]] \$

[y₁, y₁, y₁, y₁, y₁, y₁, y₁, y₁]

$$p' = -1 + s^4 \quad p' = -1 + s^5 \quad p' = -1 + s^6 \quad p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3 \quad p' = -1 + s^7$$

‘ See 8-level graph

‘

M \ ; N

\$ [[0, 1, 1, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 1], [1, 1, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 1, 0]] \$ \$ [[0, 1, 1, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1, 1, 1], [1, 1, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 1, 0]] \$

$$\tau = 8, r' = 7/8$$

R: [3, 8, 1, 6, 7, 4, 5, 2]

B: [6, 3, 8, 1, 2, 7, 4, 5]

Ranges

Action of R on ranges, [[1]]

Action of B on ranges, [[1]]

Cycles: R, {{2, 8}, {4, 6}, {1, 3}, {5, 7}}, B, {{1, 4, 6, 7}, {2, 3, 5, 8}}

$$\beta(\{1, 2, 3, 4, 5, 6, 7, 8\}) = 1/1$$

Partitions

$$\alpha(\{\{8\}, \{1\}, \{3\}, \{4\}, \{2\}, \{5\}, \{6\}, \{7\}\}) = 1/1$$

b1 = {8} ‘ , ‘ b2 = {1} ‘ , ‘ b3 = {3} ‘ , ‘ b4 = {4} ‘ , ‘ b5 = {2} ‘ , ‘ b6 = {5} ‘ , ‘ b7 = {6} ‘ , ‘ b8 = {7}

Action of R and B on the blocks of the partitions: = [5, 3, 2, 7, 1, 8, 4, 6] [3, 4, 5, 8, 6, 1, 2, 7]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-8 partition graph.

‘

Right Group	
Coloring	{2, 4, 6, 8}
Rank	8
R,B	[3, 8, 1, 6, 7, 4, 5, 2], [6, 3, 8, 1, 2, 7, 4, 5]
π_2	[1, 1]
u_2	[1, 1] (dim 4)
wpp	[1, 1, 1, 1, 1, 1, 1, 1]
π_8	[1]
u_8	[1]

80 . Coloring, {2, 4, 7, 8}

$$\Omega p(\Delta)=0: p = s^2 + s^3 - 4s^6$$

R: [3, 8, 1, 6, 7, 7, 4, 2]

B: [6, 3, 8, 1, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
5 vs 6	7 vs 7	7 vs 7	3 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{2, 8\}, \{4, 6, 7\}, \{1, 3\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 2, 0, 1, 1, 1], [1, 1, 1, 1, 0, 2, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 2, 0, 1, 1, 1], [1, 1, 1, 1, 0, 2, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1]] \$$

$$[y_3, y_3, y_3, 4y_3 - y_2 - y_1, 0, y_2, y_1, y_3]$$

$$p = -s + s^4 \quad p' = -s + s^4 \quad p'' = -s^2 + s^5 \quad p''' = -s + s^7$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}, \{2, 3, 5, 8\}\}$

See Matrix

$\$ [[1, 1, 1, 1, 2, 1, 0, 1], [1, 2, 1, 1, 1, 1, 0, 1], [1, 1, 2, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 1, 0, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 2, 1, 1, 1, 1, 0, 1], [1, 1, 2, 1, 1, 1, 0, 1]] \$$

$$[y_3, y_4, y_2, y_3, y_1, y_3, 0, 5y_3 - y_4 - y_2 - y_1]$$

$$p = -s + s^5 \quad p' = -s + s^5 \quad p'' = -s^2 + s^6$$

81 . Coloring, $\{2, 5, 6, 7\}$

R: $[3, 8, 1, 1, 2, 4, 4, 5]$

B: $[6, 3, 8, 6, 7, 7, 5, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	3 vs 6

Omega Rank for R : cycles: $\{\{1, 3\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 2, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1], [2, 1, 3, 0, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1], [2, 1, 3, 0, 1, 0, 0, 1], [3, 1, 2, 0, 1, 0, 0, 1]] \$$

$$[y_2, y_3, -y_2 + 5y_3 - y_1, y_1, y_3, 0, 0, y_3]$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{2, 3, 8\}, \{5, 7\}\}$ order: 6
See Matrix

$$\$ [[0, 1, 1, 0, 1, 2, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1], [0, 1, 1, 0, 3, 0, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1], [0, 1, 1, 0, 3, 0, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1]] \$$$

$$[0, y_3, y_3, 0, y_1, y_2, 5y_3 - y_1 - y_2, y_3]$$

$$p = s^2 - s^4 \quad p' = -s^3 + s^5 \quad p' = s^2 - s^4$$

82 . Coloring, $\{2, 5, 6, 8\}$

R: $[3, 8, 1, 1, 2, 4, 5, 2]$

B: $[6, 3, 8, 6, 7, 7, 4, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}\}$ order: 2
See Matrix

$$\$ [[2, 2, 1, 1, 1, 0, 0, 1], [2, 2, 2, 0, 0, 0, 0, 2], [2, 2, 2, 0, 0, 0, 0, 2], [2, 2, 2, 0, 0, 0, 0, 2], [2, 2, 2, 0, 0, 0, 0, 2], [2, 2, 2, 0, 0, 0, 0, 2]] \$$$

$$[y_2, y_2, y_1, y_2 - y_1, y_2 - y_1, 0, 0, y_1]$$

$$p' = -s^2 + s^5 \quad p' = -s^2 + s^4 \quad p' = -s^2 + s^3 \quad p = s^2 - s^3$$

Omega Rank for B : cycles: $\{\{4, 6, 7\}\}$ order: 6
See Matrix

$$\$ [[0, 0, 1, 1, 1, 2, 2, 1], [0, 0, 0, 2, 1, 1, 3, 1], [0, 0, 0, 3, 1, 2, 2, 0], [0, 0, 0, 2, 0, 3, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0], [0, 0, 0, 3, 0, 3, 2, 0]] \$$$

$$[0, 0, y_6, y_5, y_4, y_3, y_2, y_1]$$

83 . Coloring, {2, 5, 7, 8}

R: [3, 8, 1, 1, 2, 7, 4, 2]

B: [6, 3, 8, 6, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$\$ [[2, 2, 1, 1, 0, 0, 1, 1], [2, 1, 2, 1, 0, 0, 0, 2], [3, 2, 2, 0, 0, 0, 0, 1], [2, 1, 3, 0, 0, 0, 0, 2], [3, 2, 2, 0, 0, 0, 0, 1], [2, 1, 3, 0, 0, 0, 0, 2]] \$$$

$$[y_1, y_2, 4y_1 - 5y_2 - y_3 + 4y_4, y_3, 0, 0, y_4, 3y_1 - 4y_2 + 3y_4]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$\$ [[0, 0, 1, 1, 2, 2, 1, 1], [0, 0, 0, 2, 2, 1, 2, 1], [0, 0, 0, 1, 3, 2, 2, 0], [0, 0, 0, 2, 2, 1, 3, 0], [0, 0, 0, 1, 3, 2, 2, 0], [0, 0, 0, 2, 2, 1, 3, 0]] \$$$

$$[0, 0, -5y_1 - y_3 + 4y_2 + 4y_4, y_1, y_3, -4y_1 + 3y_2 + 3y_4, y_2, y_4]$$

$$p = s^3 - s^5 \quad p' = s^3 - s^5$$

84 . Coloring, {2, 6, 7, 8}

R: [3, 8, 1, 1, 7, 4, 4, 2]

B: [6, 3, 8, 6, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}\}$ order: 4

See Matrix

$\$ [[2, 1, 1, 2, 0, 0, 1, 1], [3, 1, 2, 1, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1], [3, 1, 3, 0, 0, 0, 0, 1]] \$$

$[3y_3 - y_2, y_3, 3y_3 - y_1, y_1, 0, 0, y_2, y_3]$

$p = -s^3 + s^4 \quad p = -s^3 + s^6 \quad p = -s^3 + s^5$

Omega Rank for B : cycles: $\{\{2, 3, 5, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 1, 0, 2, 2, 1, 1], [0, 2, 1, 0, 2, 0, 2, 1], [0, 2, 2, 0, 3, 0, 0, 1], [0, 3, 2, 0, 1, 0, 0, 2], [0, 1, 3, 0, 2, 0, 0, 2], [0, 2, 1, 0, 2, 0, 0, 3]] \$$

$[0, y_4, y_1, 0, y_2, y_3, y_5, y_6]$

85 . Coloring, $\{3, 4, 5, 6\}$

$\Omega p(\Delta)=0: \quad p' = s^3 - 2s^5 \quad p = s^2 - 4s^6 \quad p' = s^2 - 2s^4$

R: [3, 3, 8, 6, 2, 4, 5, 5]

B: [6, 8, 1, 1, 7, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	3 vs 6	3 vs 6	2 vs 6	2 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{2, 3, 5, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 2, 1, 2, 1, 0, 1], [0, 2, 1, 1, 1, 1, 0, 2], [0, 1, 2, 1, 2, 1, 0, 1], [0, 2, 1, 1, 1, 1, 0, 2], [0, 1, 2, 1, 2, 1, 0, 1], [0, 2, 1, 1, 1, 1, 0, 2]] \$$

$$[0, y_2, -y_2 + 3y_1, y_1, -y_2 + 3y_1, y_1, 0, y_2]$$

$$p' = -s + s^3 \quad p = -s + s^5 \quad p = -s + s^3 \quad p' = -s + s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 1, 0, 1, 2, 1], [1, 1, 0, 2, 0, 2, 1, 1], [2, 1, 0, 1, 0, 1, 2, 1], [1, 1, 0, 2, 0, 2, 1, 1], [2, 1, 0, 1, 0, 1, 2, 1], [1, 1, 0, 2, 0, 2, 1, 1]] \$$

$$[y_2, y_1, 0, -y_2 + 3y_1, 0, -y_2 + 3y_1, y_2, y_1]$$

$$p = s - s^3 \quad p' = -s + s^3 \quad p' = -s^2 + s^4 \quad p' = -s + s^5$$

M \ ; N

$\$ [[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]] \$ \quad \$ [[0, 3, 4, 5, 5, 4, 9, 6], [3, 0, 5, 6, 4, 3, 6, 9], [4, 5, 0, 3, 9, 6, 5, 4], [5, 6, 3, 0, 6, 9, 4, 3], [5, 4, 9, 6, 0, 3, 4, 5], [4, 3, 6, 9, 3, 0, 5, 6], [9, 6, 5, 4, 4, 5, 0, 3], [6, 9, 4, 3, 5, 6, 3, 0]] \$$

$$\tau = 32, r' = 1/2$$

R: [3, 3, 8, 6, 2, 4, 5, 5]

B: [6, 8, 1, 1, 7, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [2], [4]]

Action of B on ranges, [[4], [2], [1], [1]]

Cycles: R, $\{\{4, 6\}, \{2, 3, 5, 8\}\}$, B, $\{\{2, 8\}, \{1, 4, 6, 7\}\}$

$$\beta(\{1, 7\}) = 1/4$$

$$\beta(\{2, 8\}) = 1/4$$

$$\beta(\{3, 5\}) = 1/4$$

$$\beta(\{4, 6\}) = 1/4$$

Partitions

Action of R on partitions, [[4], [6], [5], [3], [6], [4]]
 Action of B on partitions, [[2], [5], [6], [1], [6], [1]]

$$\alpha(\{\{1, 3, 4, 8\}, \{2, 5, 6, 7\}\}) = 2/9$$

$$\alpha(\{\{2, 3, 4, 7\}, \{1, 5, 6, 8\}\}) = 1/9$$

$$\alpha(\{\{1, 2, 4, 5\}, \{3, 6, 7, 8\}\}) = 1/9$$

$$\alpha(\{\{1, 2, 3, 6\}, \{4, 5, 7, 8\}\}) = 2/9$$

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/9$$

$$\alpha(\{\{3, 4, 7, 8\}, \{1, 2, 5, 6\}\}) = 2/9$$

b1 = {1, 2, 3, 6} ‘, ‘ b2 = {1, 3, 4, 8} ‘, ‘ b3 = {2, 5, 6, 7} ‘, ‘ b4 = {2, 3, 4, 7} ‘, ‘ b5 = {1, 5, 6, 8} ‘, ‘
 b6 = {4, 5, 7, 8} ‘, ‘ b7 = {1, 2, 4, 5} ‘, ‘ b8 = {3, 6, 7, 8} ‘, ‘ b9 = {1, 2, 3, 4} ‘, ‘ b10 = {5, 6, 7, 8} ‘, ‘
 b11 = {3, 4, 7, 8} ‘, ‘ b12 = {1, 2, 5, 6}

Action of R and B on the blocks of the partitions: = [7, 1, 6, C, B, 8, A, 9, C, B, 1, 6] [2, 4, 5, A, 9, 3, B, C, B, C, 3, 2]
 with invariant measure [2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 2, 2]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{3, 4, 5, 6}
Rank	2
R,B	[3, 3, 8, 6, 2, 4, 5, 5], [6, 8, 1, 1, 7, 7, 4, 2]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[3, 4, 5, 5, 4, 9, 6, 5, 6, 4, 3, 6, 9, 3, 9, 6, 5, 4, 6, 9, 4, 3, 3, 4, 5, 5, 6, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

86 . Coloring, {3, 4, 5, 7}

$$\Omega p(\Delta)=0: \quad p = -s^2 + s^3 + 4s^6$$

R: [3, 3, 8, 6, 2, 7, 4, 5]

B: [6, 8, 1, 1, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 6	7 vs 7	7 vs 7	4 vs 7	3 vs 7

Omega Rank for R : cycles: $\{\{4, 6, 7\}, \{2, 3, 5, 8\}\}$

See Matrix

$\$ [[0, 1, 2, 1, 1, 1, 1, 1], [0, 1, 1, 1, 1, 1, 1, 2], [0, 1, 1, 1, 2, 1, 1, 1], [0, 2, 1, 1, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1, 1, 1], [0, 1, 1, 1, 1, 1, 1, 2], [0, 1, 1, 1, 2, 1, 1, 1]] \$$

$$[0, -y_2 + 5y_4 - y_3 - y_1, y_2, y_4, y_3, y_4, y_4, y_1]$$

$$p = -s + s^5 \quad p' = -s + s^5 \quad p' = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{1, 4, 6\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[2, 1, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 2, 1, 1], [1, 1, 0, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 2, 1, 1], [1, 1, 0, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1]] \$$

$$[y_1, y_3, 0, y_2, y_3, -y_1 - y_2 + 4y_3, y_3, y_3]$$

$$p = s - s^4 \quad p' = -s + s^4 \quad p' = -s^2 + s^5 \quad p' = -s^3 + s^6$$

87 . Coloring, $\{3, 4, 5, 8\}$

$$\Omega p(\Delta)=0: \quad p' = s^2 \quad p' = s^3 \quad p' = s^4 \quad p = s^2 \quad p' = s^5$$

R: [3, 3, 8, 6, 2, 7, 5, 2]

B: [6, 8, 1, 1, 7, 4, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
1 vs 6	6 vs 6	6 vs 6	6 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 8\}\}$ order: 6
 See Matrix

\$ [[0, 2, 2, 0, 1, 1, 1, 1] , [0, 2, 2, 0, 1, 0, 1, 2] , [0, 3, 2, 0, 1, 0, 0, 2] , [0, 3, 3, 0, 0, 0, 0, 2] , [0, 2, 3, 0, 0, 0, 0, 3] , [0, 3, 2, 0, 0, 0, 0, 3]] \$

[0, y_1 , y_2 , 0, y_3 , y_4 , y_5 , y_6]

Omega Rank for B : cycles: $\{\{1, 4, 6\}\}$ order: 6
 See Matrix

\$ [[2, 0, 0, 2, 1, 1, 1, 1] , [2, 0, 0, 2, 1, 2, 1, 0] , [2, 0, 0, 3, 0, 2, 1, 0] , [3, 0, 0, 3, 0, 2, 0, 0] , [3, 0, 0, 2, 0, 3, 0, 0] , [2, 0, 0, 3, 0, 3, 0, 0]] \$

[y_2 , 0, 0, y_3 , y_1 , y_6 , y_4 , y_5]

88 . Coloring, {3, 4, 6, 7}

$\Omega p(\Delta)=0$: $p = s^4$ $p' = s^4$ $p'' = s^5$

R: [3, 3, 8, 6, 7, 4, 4, 5]

B: [6, 8, 1, 1, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	6 vs 6	6 vs 6	6 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}\}$ order: 6
 See Matrix

\$ [[0, 0, 2, 2, 1, 1, 1, 1] , [0, 0, 0, 2, 1, 2, 1, 2] , [0, 0, 0, 3, 2, 2, 1, 0] , [0, 0, 0, 3, 0, 3, 2, 0] , [0, 0, 0, 5, 0, 3, 0, 0] , [0, 0, 0, 3, 0, 5, 0, 0]] \$

[0, 0, y_2 , y_1 , y_4 , y_5 , y_6 , y_3]

Omega Rank for B : cycles: $\{\{2, 8\}\}$ order: 6
 See Matrix

\$ [[2, 2, 0, 0, 1, 1, 1, 1], [0, 2, 0, 0, 1, 2, 1, 2], [0, 3, 0, 0, 1, 0, 2, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 5, 0, 0, 0, 0, 0, 3], [0, 3, 0, 0, 0, 0, 0, 5]] \$

[$y_2, y_1, 0, 0, y_3, y_6, y_5, y_4$]

89 . Coloring, {3, 4, 6, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 + s^3 - 4s^6$$

R: [3, 3, 8, 6, 7, 4, 5, 2]

B: [6, 8, 1, 1, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
5 vs 6	7 vs 7	7 vs 7	3 vs 7	4 vs 7

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}, {5, 7}} order: 6

See Matrix

\$ [[0, 1, 2, 1, 1, 1, 1, 1], [0, 1, 1, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1, 1, 1], [0, 1, 1, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1, 1, 1]] \$

[$0, y_1, y_2, y_3, y_3, y_3, y_3, -y_1 - y_2 + 4y_3$]

$$p = s - s^4 \quad p' = -s + s^4 \quad p'' = -s^2 + s^5 \quad p''' = -s^3 + s^6$$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 5, 8}}

See Matrix

\$ [[2, 1, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 2, 1, 1], [1, 1, 0, 1, 1, 1, 2, 1], [1, 1, 0, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 1, 0, 1, 1, 2, 1, 1], [1, 1, 0, 1, 1, 1, 2, 1]] \$

[$5y_3 - y_4 - y_1 - y_2, y_3, 0, y_4, y_3, y_1, y_2, y_3$]

$$p' = -s + s^5 \quad p'' = -s^2 + s^6 \quad p''' = -s + s^5$$

90 . Coloring, {3, 4, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^6 \quad p' = s^3 + 2s^5 \quad p'' = s^2 + 2s^4$$

R: [3, 3, 8, 6, 7, 7, 4, 2]

B: [6, 8, 1, 1, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	3 vs 6	3 vs 6	3 vs 6	3 vs 6

Omega Rank for R : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$\$ [[0, 1, 2, 1, 0, 1, 2, 1], [0, 1, 1, 2, 0, 1, 1, 2], [0, 2, 1, 1, 0, 2, 1, 1], [0, 1, 2, 1, 0, 1, 2, 1], [0, 1, 1, 2, 0, 1, 1, 2], [0, 2, 1, 1, 0, 2, 1, 1]] \$$$

$$[0, y_1, y_3, y_2, 0, y_1, y_3, y_2]$$

$$p' = s^2 - s^5 \quad p'' = s - s^4 \quad p''' = s - s^4$$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 5, 8}} order: 3

See Matrix

$$\$ [[2, 1, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 2, 0, 1], [1, 1, 0, 2, 1, 1, 0, 2], [2, 1, 0, 1, 2, 1, 0, 1], [1, 2, 0, 1, 1, 2, 0, 1], [1, 1, 0, 2, 1, 1, 0, 2]] \$$$

$$[y_1, y_2, 0, y_3, y_1, y_2, 0, y_3]$$

$$p' = s - s^4 \quad p'' = s^2 - s^5 \quad p''' = -s + s^4$$

M \ ; N

$$\$ [[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]] \$ \quad \$ [[0, 1, 4, 3, 5, 4, 1, 2], [1, 0, 3, 2, 4, 5, 2, 3], [4, 3, 0, 1, 1, 2, 5, 4], [3, 2, 1, 0, 2, 3, 4, 5], [5, 4, 1, 2, 0, 1, 4, 3], [4, 5, 2, 3, 1, 0, 3, 2], [1, 2, 5, 4, 4, 3, 0, 1], [2, 3, 4, 5, 3, 2, 1, 0]] \$$$

$$\tau = 32, r' = 1/2$$

R: [3, 3, 8, 6, 7, 7, 4, 2]

B: [6, 8, 1, 1, 2, 4, 5, 5]

Ranges

Action of R on ranges, [[3], [3], [4], [2]]

Action of B on ranges, [[2], [4], [1], [1]]

Cycles: R, {{4, 6, 7}, {2, 3, 8}}, B, {{1, 4, 6}, {2, 5, 8}}

$$\beta(\{1, 5\}) = 1/4$$

$$\beta(\{2, 6\}) = 1/4$$

$$\beta(\{3, 7\}) = 1/4$$

$$\beta(\{4, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[2], [3], [1], [2]]

Action of B on partitions, [[2], [4], [2], [1]]

$$\alpha(\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}) = 1/5$$

$$\alpha(\{\{3, 4, 5, 6\}, \{1, 2, 7, 8\}\}) = 2/5$$

$$\alpha(\{\{1, 2, 4, 7\}, \{3, 5, 6, 8\}\}) = 1/5$$

$$\alpha(\{\{1, 6, 7, 8\}, \{2, 3, 4, 5\}\}) = 1/5$$

b1 = {1, 6, 7, 8} ‘, ‘ b2 = {2, 3, 4, 5} ‘, ‘ b3 = {3, 4, 5, 6} ‘, ‘ b4 = {1, 2, 3, 4} ‘, ‘ b5 = {5, 6, 7, 8} ‘, ‘

b6 = {1, 2, 7, 8} ‘, ‘ b7 = {1, 2, 4, 7} ‘, ‘ b8 = {3, 5, 6, 8}

Action of R and B on the blocks of the partitions: = [3, 6, 7, 6, 3, 8, 5, 4] [4, 5, 1, 3, 6, 2, 3, 6]

with invariant measure [1, 1, 2, 1, 1, 2, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{3, 4, 7, 8}
Rank	2
R,B	[3, 3, 8, 6, 7, 7, 4, 2], [6, 8, 1, 1, 2, 4, 5, 5]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 4, 3, 5, 4, 1, 2, 3, 2, 4, 5, 2, 3, 1, 1, 2, 5, 4, 2, 3, 4, 5, 1, 4, 3, 3, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

91 . Coloring, {3, 5, 6, 7}

R: [3, 3, 8, 1, 2, 4, 4, 5]

B: [6, 8, 1, 6, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

\$ [[1, 1, 2, 2, 1, 0, 0, 1], [2, 1, 2, 0, 1, 0, 0, 2], [0, 1, 3, 0, 2, 0, 0, 2], [0, 2, 1, 0, 2, 0, 0, 3], [0, 2, 2, 0, 3, 0, 0, 1], [0, 3, 2, 0, 1, 0, 0, 2]] \$

[$y_4, y_3, y_2, y_1, y_6, 0, 0, y_5$]

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

\$ [[1, 1, 0, 0, 1, 2, 2, 1], [0, 1, 0, 0, 2, 1, 3, 1], [0, 1, 0, 0, 3, 0, 3, 1], [0, 1, 0, 0, 3, 0, 3, 1], [0, 1, 0, 0, 3, 0, 3, 1], [0, 1, 0, 0, 3, 0, 3, 1]] \$

$$[y_3, y_2, 0, 0, 3y_2 - y_1, y_1, -y_3 + 3y_2, y_2]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5 \quad p = -s^3 + s^6$$

92 . Coloring, {3, 5, 6, 8}

R: [3, 3, 8, 1, 2, 4, 5, 2]

B: [6, 8, 1, 6, 7, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	5 vs 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 3

See Matrix

$$\$ [[1, 2, 2, 1, 1, 0, 0, 1], [1, 2, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3], [0, 3, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3]] \$$$

$$[y_1, y_3, y_2, y_5, y_5, 0, 0, y_4]$$

$$p = s^3 - s^6$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

$$\$ [[1, 0, 0, 1, 1, 2, 2, 1], [0, 0, 0, 2, 1, 2, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0], [0, 0, 0, 3, 0, 3, 2, 0], [0, 0, 0, 2, 0, 3, 3, 0], [0, 0, 0, 3, 0, 2, 3, 0]] \$$$

$$[y_4, 0, 0, y_2, y_1, y_5, y_3, y_4]$$

$$p = -s^3 + s^6$$

93 . Coloring, {3, 5, 7, 8}

R: [3, 3, 8, 1, 2, 7, 4, 2]

B: [6, 8, 1, 6, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	2 vs 6

Omega Rank for R : cycles: $\{\{2, 3, 8\}\}$ order: 6

See Matrix

$\$ [[1, 2, 2, 1, 0, 0, 1, 1], [1, 1, 3, 1, 0, 0, 0, 2], [1, 2, 2, 0, 0, 0, 0, 3], [0, 3, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3]] \$$

$[y_1, y_2, y_3, y_4, 0, 0, y_5, y_6]$

Omega Rank for B : cycles: $\{\{4, 6\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[1, 0, 0, 1, 2, 2, 1, 1], [0, 0, 0, 2, 2, 2, 2, 0], [0, 0, 0, 2, 2, 2, 2, 0], [0, 0, 0, 2, 2, 2, 2, 0], [0, 0, 0, 2, 2, 2, 2, 0], [0, 0, 0, 2, 2, 2, 2, 0]] \$$

$[y_2, 0, 0, y_1 - y_2, y_1, y_1, y_1 - y_2, y_2]$

$p' = s^3 - s^5 \quad p' = s^4 - s^5 \quad p' = s^2 - s^5 \quad p = s^2 - s^6$

94 . Coloring, $\{3, 6, 7, 8\}$

R: $[3, 3, 8, 1, 7, 4, 4, 2]$

B: $[6, 8, 1, 6, 2, 7, 5, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	6 vs 6	6 vs 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

\$ [[1, 1, 2, 2, 0, 0, 1, 1], [2, 1, 2, 1, 0, 0, 0, 2], [1, 2, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3], [0, 3, 3, 0, 0, 0, 0, 2]] \$

$[y_1, y_2, y_3, y_4, 0, 0, y_5, y_6]$

Omega Rank for B : cycles: {{2, 5, 8}} order: 6

See Matrix

\$ [[1, 1, 0, 0, 2, 2, 1, 1], [0, 2, 0, 0, 2, 1, 2, 1], [0, 2, 0, 0, 3, 0, 1, 2], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3], [0, 2, 0, 0, 3, 0, 0, 3]] \$

$[y_6, y_5, 0, 0, y_4, y_3, y_2, y_1]$

95 . Coloring, {4, 5, 6, 7}

R: [3, 3, 1, 6, 2, 4, 4, 5]

B: [6, 8, 8, 1, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 6	4 vs 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

\$ [[1, 1, 2, 2, 1, 1, 0, 0], [2, 1, 2, 1, 0, 2, 0, 0], [2, 0, 3, 2, 0, 1, 0, 0], [3, 0, 2, 1, 0, 2, 0, 0], [2, 0, 3, 2, 0, 1, 0, 0], [3, 0, 2, 1, 0, 2, 0, 0]] \$

$[y_2, y_1, y_4, 3y_2 + 3y_1 - 4y_3, 4y_2 + 4y_1 - y_4 - 5y_3, y_3, 0, 0]$

$p' = s^3 - s^5 \quad p = -s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

\$ [[1, 1, 0, 0, 1, 1, 2, 2] , [0, 2, 0, 0, 2, 1, 2, 1] , [0, 1, 0, 0, 2, 0, 3, 2] , [0, 2, 0, 0, 3, 0, 2, 1] , [0, 1, 0, 0, 2, 0, 3, 2] , [0, 2, 0, 0, 3, 0, 2, 1]] \$

$$[-5 y_1 + 4 y_3 + 4 y_2 - y_4, y_1, 0, 0, y_3, y_2, y_4, -4 y_1 + 3 y_3 + 3 y_2]$$

$$p = s^3 - s^5 \quad p' = -s^3 + s^5$$

96 . Coloring, {4, 5, 6, 8}

R: [3, 3, 1, 6, 2, 4, 5, 2]

B: [6, 8, 8, 1, 7, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	6 vs 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

\$ [[1, 2, 2, 1, 1, 1, 0, 0] , [2, 1, 3, 1, 0, 1, 0, 0] , [3, 0, 3, 1, 0, 1, 0, 0] , [3, 0, 3, 1, 0, 1, 0, 0] , [3, 0, 3, 1, 0, 1, 0, 0] , [3, 0, 3, 1, 0, 1, 0, 0]] \$

$$[y_3, -y_3 + 3 y_1, y_2, y_1, -y_2 + 3 y_1, y_1, 0, 0]$$

$$p' = -s^3 + s^4 \quad p' = -s^3 + s^5 \quad p = s^3 - s^4$$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

\$ [[1, 0, 0, 1, 1, 1, 2, 2] , [1, 0, 0, 2, 2, 1, 2, 0] , [2, 0, 0, 2, 0, 1, 3, 0] , [2, 0, 0, 3, 0, 2, 1, 0] , [3, 0, 0, 1, 0, 2, 2, 0] , [1, 0, 0, 2, 0, 3, 2, 0]] \$

$$[y_1, 0, 0, y_4, y_5, y_6, y_2, y_3]$$

97 . Coloring, {4, 5, 7, 8}

R: [3, 3, 1, 6, 2, 7, 4, 2]

B: [6, 8, 8, 1, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	3 vs 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

\$ [[1, 2, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0], [3, 0, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0], [3, 0, 2, 1, 0, 1, 1, 0], [2, 0, 3, 1, 0, 1, 1, 0]] \$

$$[-y_1 - y_2 + 5 y_3, y_1, y_2, y_3, 0, y_3, y_3, 0]$$

$$p = s^2 - s^6 \quad p' = s^3 - s^5 \quad p'' = s^2 - s^4$$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

\$ [[1, 0, 0, 1, 2, 1, 1, 2], [1, 0, 0, 1, 3, 1, 2, 0], [1, 0, 0, 1, 2, 1, 3, 0], [1, 0, 0, 1, 3, 1, 2, 0], [1, 0, 0, 1, 2, 1, 3, 0], [1, 0, 0, 1, 3, 1, 2, 0]] \$

$$[y_1, 0, 0, y_1, 5 y_1 - y_2 - y_3, y_1, y_2, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p'' = -s^2 + s^6$$

98 . Coloring, {4, 6, 7, 8}

R: [3, 3, 1, 6, 7, 4, 4, 2]

B: [6, 8, 8, 1, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}\}$ order: 2

See Matrix

$\$ [[1, 1, 2, 2, 0, 1, 1, 0], [2, 0, 2, 2, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0]] \$$

$$[y_2, -y_2 + y_1, y_1, y_1, 0, y_2, -y_2 + y_1, 0]$$

$$p' = s^3 - s^5 \quad p' = s^4 - s^5 \quad p = s^2 - s^6 \quad p' = s^2 - s^5$$

Omega Rank for B : cycles: $\{\{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 0, 0, 2, 1, 1, 2], [0, 2, 0, 0, 3, 1, 1, 1], [0, 3, 0, 0, 2, 0, 1, 2], [0, 2, 0, 0, 3, 0, 0, 3], [0, 3, 0, 0, 3, 0, 0, 2], [0, 3, 0, 0, 2, 0, 0, 3]] \$$

$$[y_3, y_2, 0, 0, y_1, y_5, y_6, y_4]$$

99 . Coloring, $\{5, 6, 7, 8\}$

$$\Omega p(\Delta)=0: \quad p = s^3 \quad p' = s^3 \quad p' = s^4 \quad p' = s^5$$

R: [3, 3, 1, 1, 2, 4, 4, 2]

B: [6, 8, 8, 6, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 6	2 vs 6	2 vs 6	2 vs 4	2 vs 4

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2

See Matrix

$\$ [[2, 2, 2, 2, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0], [4, 0, 4, 0, 0, 0, 0, 0]] \$$

$$[y_2, y_1, y_2, y_1, 0, 0, 0, 0]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles: {{5, 7}} order: 2

See Matrix

$$\$ [[0, 0, 0, 0, 2, 2, 2, 2], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$$

$$[0, 0, 0, 0, y_2, y_1, y_2, y_1]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

M \ ; N

$$\$ [[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 1, 1, 1, 1], [1, 0, 1, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 2, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 1, 1, 1, 1, 2, 1, 0]] \$$$

$$\tau = 32, r' = 1/2$$

$$\mathbf{R}: [3, 3, 1, 1, 2, 4, 4, 2]$$

$$\mathbf{B}: [6, 8, 8, 6, 7, 7, 5, 5]$$

Ranges

Action of R on ranges, [[1], [1], [2], [2]]

Action of B on ranges, [[4], [4], [3], [3]]

Cycles: R, {{1, 3}}, B, {{5, 7}}

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 4\}) = 1/4$$

$$\beta(\{5, 7\}) = 1/4$$

$$\beta(\{6, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[8], [8], [6], [8], [6], [6], [8], [6]]

Action of B on partitions, [[2], [7], [7], [2], [7], [2], [7], [2]]

$$\alpha(\{1, 4, 5, 8\}, \{2, 3, 6, 7\}) = 0/1$$

$$\alpha(\{1, 4, 5, 6\}, \{2, 3, 7, 8\}) = 1/4$$

$$\alpha(\{1, 2, 5, 6\}, \{3, 4, 7, 8\}) = 0/1$$

$$\alpha(\{1, 4, 6, 7\}, \{2, 3, 5, 8\}) = 0/1$$

$$\alpha(\{3, 4, 5, 6\}, \{1, 2, 7, 8\}) = 0/1$$

$$\alpha(\{1, 2, 6, 7\}, \{3, 4, 5, 8\}) = 1/4$$

$$\alpha(\{\{2, 3, 5, 6\}, \{1, 4, 7, 8\}\}) = 1/4$$

$$\alpha(\{\{3, 4, 6, 7\}, \{1, 2, 5, 8\}\}) = 1/4$$

$b_1 = \{1, 4, 5, 8\}$, $b_2 = \{3, 4, 5, 6\}$, $b_3 = \{1, 2, 6, 7\}$, $b_4 = \{1, 4, 6, 7\}$, $b_5 = \{1, 2, 5, 6\}$, $b_6 = \{3, 4, 5, 8\}$, $b_7 = \{2, 3, 5, 6\}$, $b_8 = \{1, 4, 7, 8\}$, $b_9 = \{3, 4, 6, 7\}$, $b_{10} = \{1, 2, 5, 8\}$, $b_{11} = \{2, 3, 6, 7\}$, $b_{12} = \{1, 2, 7, 8\}$, $b_{13} = \{3, 4, 7, 8\}$, $b_{14} = \{2, 3, 5, 8\}$, $b_{15} = \{1, 4, 5, 6\}$, $b_{16} = \{2, 3, 7, 8\}$

Action of R and B on the blocks of the partitions: = [9, 3, 6, 9, 6, 3, A, 9, 3, 6, A, 6, 3, A, 9, A] [10, 8, F, F, 8, 10, 8, 7, F, 10, F, 7, 7, 10, 8, 7]
 with invariant measure [0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{5, 6, 7, 8}
Rank	2
R,B	[3, 3, 1, 1, 2, 4, 4, 2], [6, 8, 8, 6, 7, 7, 5, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

100 . Coloring, {2, 3, 4, 5, 6}

R: [3, 8, 8, 6, 2, 4, 5, 5]

B: [6, 3, 1, 1, 7, 7, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
6 vs 6	7 vs 7	7 vs 7	2 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 5, 8\}, \{4, 6\}\}$ order: 6

See Matrix

$$\$ [[0, 1, 1, 1, 2, 1, 0, 2], [0, 2, 0, 1, 2, 1, 0, 2], [0, 2, 0, 1, 2, 1, 0, 2], [0, 2, 0, 1, 2, 1, 0, 2], [0, 2, 0, 1, 2, 1, 0, 2], [0, 2, 0, 1, 2, 1, 0, 2]] \$$$

$$[0, y_2, -y_2 + 2y_1, y_1, 2y_1, y_1, 0, 2y_1]$$

$$p = s^2 - s^3 \quad p' = -s^2 + s^3 \quad p'' = -s^2 + s^4 \quad p''' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{1, 4, 6, 7\}\}$ order: 4

See Matrix

$$\$ [[2, 1, 1, 1, 0, 1, 2, 0], [2, 0, 1, 2, 0, 2, 1, 0], [3, 0, 0, 1, 0, 2, 2, 0], [1, 0, 0, 2, 0, 3, 2, 0], [2, 0, 0, 2, 0, 1, 3, 0], [2, 0, 0, 3, 0, 2, 1, 0]] \$$$

$$[y_3, y_4, y_1, y_2, 0, y_6, y_5, 0]$$

101 . Coloring, $\{2, 3, 4, 5, 7\}$

R: [3, 8, 8, 6, 2, 7, 4, 5]

B: [6, 3, 1, 1, 7, 4, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	4 vs 7	3 vs 7

Omega Rank for R : cycles: $\{\{2, 5, 8\}, \{4, 6, 7\}\}$ order: 3

See Matrix

$$\$ [[0, 1, 1, 1, 1, 1, 1, 2], [0, 1, 0, 1, 2, 1, 1, 2], [0, 2, 0, 1, 2, 1, 1, 1], [0, 2, 0, 1, 1, 1, 1, 2], [0, 1, 0, 1, 2, 1, 1, 2], [0, 2, 0, 1, 2, 1, 1, 1], [0, 2, 0, 1, 1, 1, 1, 2]] \$$$

$$[0, -y_1 + 5y_3 - y_2 - y_4, y_1, y_3, y_2, y_3, y_3, y_4]$$

$$p' = s^2 - s^5 \quad p = -s^2 + s^5 \quad p'' = -s^3 + s^6$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 1, 1, 1, 1, 0], [2, 0, 1, 1, 1, 2, 1, 0], [2, 0, 0, 2, 1, 2, 1, 0], [2, 0, 0, 2, 1, 2, 1, 0], [2, 0, 0, 2, 1, 2, 1, 0], [2, 0, 0, 2, 1, 2, 1, 0], [2, 0, 0, 2, 1, 2, 1, 0]] \$$

$$[2 y_2, -y_1 + 2 y_2, -y_3 + 2 y_2, y_3, y_2, y_1, y_2, 0]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^6 \quad p = -s^3 + s^7 \quad p = -s^3 + s^5$$

102 . Coloring, $\{2, 3, 4, 5, 8\}$

R: $[3, 8, 8, 6, 2, 7, 5, 2]$

B: $[6, 3, 1, 1, 7, 4, 4, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}\}$ order: 4

See Matrix

$\$ [[0, 2, 1, 0, 1, 1, 1, 2], [0, 3, 0, 0, 1, 0, 1, 3], [0, 4, 0, 0, 1, 0, 0, 3], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4]] \$$

$$[0, y_1 - y_3 + y_4, y_2, 0, y_1, y_2, y_3, y_4]$$

$$p = -s^4 + s^5 \quad p = -s^4 + s^6$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}\}$ order: 3

See Matrix

$\$ [[2, 0, 1, 2, 1, 1, 1, 0], [3, 0, 0, 2, 0, 2, 1, 0], [2, 0, 0, 3, 0, 3, 0, 0], [3, 0, 0, 3, 0, 2, 0, 0], [3, 0, 0, 2, 0, 3, 0, 0], [2, 0, 0, 3, 0, 3, 0, 0]] \$$

$$[y_1, 0, y_5, y_4, y_5, y_2, y_3, 0]$$

$$p = -s^3 + s^6$$

103 . Coloring, {2, 3, 4, 6, 7}

R: [3, 8, 8, 6, 7, 4, 4, 5]

B: [6, 3, 1, 1, 2, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{4, 6}} order: 6

See Matrix

$$\$ [[0, 0, 1, 2, 1, 1, 1, 2], [0, 0, 0, 2, 2, 2, 1, 1], [0, 0, 0, 3, 1, 2, 2, 0], [0, 0, 0, 4, 0, 3, 1, 0], [0, 0, 0, 4, 0, 4, 0, 0], [0, 0, 0, 4, 0, 4, 0, 0]] \$$$

$$[0, 0, y_3, y_4, y_5, y_3 + y_4 + y_5 - y_1 - y_2, y_1, y_2]$$

$$p = s^5 - s^6$$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$$\$ [[2, 2, 1, 0, 1, 1, 1, 0], [1, 1, 2, 0, 1, 2, 1, 0], [2, 1, 1, 0, 1, 1, 2, 0], [1, 1, 1, 0, 2, 2, 1, 0], [1, 2, 1, 0, 1, 1, 2, 0], [1, 1, 2, 0, 2, 1, 1, 0]] \$$$

$$[y_4, y_3, y_2, 0, y_1, y_5, y_6, 0]$$

104 . Coloring, {2, 3, 4, 6, 8}

R: [3, 8, 8, 6, 7, 4, 5, 2]

B: [6, 3, 1, 1, 2, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{2, 8\}, \{4, 6\}, \{5, 7\}\}$ order: 2

See Matrix

$\$ [[0, 1, 1, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2], [0, 2, 0, 1, 1, 1, 1, 2]] \$$

$$[0, -y_1 + 2y_2, y_1, y_2, y_2, y_2, y_2, 2y_2]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6 \quad p = -s^2 + s^7$$

Omega Rank for B : cycles: $\{\{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 1, 1, 1, 1, 1, 0], [2, 1, 1, 1, 0, 2, 1, 0], [2, 0, 1, 1, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0]] \$$

$$[y_2 + y_3, y_2 + y_3 - y_4, -y_1 + y_2 + y_3, y_1, y_2, y_3, y_4, 0]$$

$$p = s^4 - s^7 \quad p' = s^4 - s^6 \quad p' = s^5 - s^6$$

105 . Coloring, $\{2, 3, 4, 7, 8\}$

R: [3, 8, 8, 6, 7, 7, 4, 2]

B: [6, 3, 1, 1, 2, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}, \{4, 6, 7\}\}$ order: 6

See Matrix

$\$ [[0, 1, 1, 1, 0, 1, 2, 2], [0, 2, 0, 2, 0, 1, 1, 2], [0, 2, 0, 1, 0, 2, 1, 2], [0, 2, 0, 1, 0, 1, 2, 2], [0, 2, 0, 2, 0, 1, 1, 2], [0, 2, 0, 1, 0, 2, 1, 2]] \$$

$$[0, -y_1 + y_4, y_1, 2y_4 - y_2 - y_3, 0, y_2, y_3, y_4]$$

$$p = s^2 - s^5 \quad p' = -s^2 + s^5$$

Omega Rank for B : cycles: $\{\{1, 4, 6\}\}$ order: 6

See Matrix

$\$ [[2, 1, 1, 1, 2, 1, 0, 0], [2, 2, 1, 1, 0, 2, 0, 0], [2, 0, 2, 2, 0, 2, 0, 0], [4, 0, 0, 2, 0, 2, 0, 0], [2, 0, 0, 2, 0, 4, 0, 0], [2, 0, 0, 4, 0, 2, 0, 0]] \$$

$$[y_1, y_2, y_3, y_5, y_6, y_4, 0, 0]$$

106 . Coloring, $\{2, 3, 5, 6, 7\}$

R: [3, 8, 8, 1, 2, 4, 4, 5]

B: [6, 3, 1, 6, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	6 vs 6	5 vs 6

Omega Rank for R : cycles: $\{\{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 2, 1, 0, 0, 2], [2, 1, 1, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 0, 0, 2, 0, 0, 4], [0, 2, 0, 0, 4, 0, 0, 2], [0, 4, 0, 0, 2, 0, 0, 2]] \$$

$$[y_5, y_6, y_4, y_3, y_2, 0, 0, y_1]$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 6

See Matrix

\$ [[1, 1, 1, 0, 1, 2, 2, 0] , [1, 0, 1, 0, 2, 1, 3, 0] , [1, 0, 0, 0, 3, 1, 3, 0] , [0, 0, 0, 0, 3, 1, 4, 0] , [0, 0, 0, 0, 4, 0, 4, 0] , [0, 0, 0, 0, 4, 0, 4, 0]] \$

$$[y_2, y_1, y_2 + y_1 - y_5 - y_4 + y_3, 0, y_5, y_4, y_3, 0]$$

$$p = s^5 - s^6$$

107 . Coloring, {2, 3, 5, 6, 8}

R: [3, 8, 8, 1, 2, 4, 5, 2]

B: [6, 3, 1, 6, 7, 7, 4, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	5 vs 6

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

\$ [[1, 2, 1, 1, 1, 0, 0, 2] , [1, 3, 1, 0, 0, 0, 0, 3] , [0, 3, 1, 0, 0, 0, 0, 4] , [0, 4, 0, 0, 0, 0, 0, 4] , [0, 4, 0, 0, 0, 0, 0, 4] , [0, 4, 0, 0, 0, 0, 0, 4]] \$

$$[y_1 + y_2 - y_4, y_1, y_2, y_3, y_3, 0, 0, y_4]$$

$$p' = -s^4 + s^5 \quad p = s^4 - s^5$$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

\$ [[1, 0, 1, 1, 1, 2, 2, 0] , [1, 0, 0, 2, 0, 2, 3, 0] , [0, 0, 0, 3, 0, 3, 2, 0] , [0, 0, 0, 2, 0, 3, 3, 0] , [0, 0, 0, 3, 0, 2, 3, 0] , [0, 0, 0, 3, 0, 3, 2, 0]] \$

$$[y_1, 0, y_2, y_3, y_2, y_5, y_4, 0]$$

$$p = -s^3 + s^6$$

108 . Coloring, {2, 3, 5, 7, 8}

R: [3, 8, 8, 1, 2, 7, 4, 2]

B: [6, 3, 1, 6, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	4 vs 6

Omega Rank for R : cycles: {{2, 8}} order: 6

See Matrix

\$ [[1, 2, 1, 1, 0, 0, 1, 2] , [1, 2, 1, 1, 0, 0, 0, 3] , [1, 3, 1, 0, 0, 0, 0, 3] , [0, 3, 1, 0, 0, 0, 0, 4] , [0, 4, 0, 0, 0, 0, 0, 4] , [0, 4, 0, 0, 0, 0, 0, 4]] \$

$$[y_2 + y_1 + y_3 - y_5 - y_4, y_2, y_1, y_3, 0, 0, y_5, y_4]$$

$$p = s^5 - s^6$$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

\$ [[1, 0, 1, 1, 2, 2, 1, 0] , [1, 0, 0, 2, 1, 2, 2, 0] , [0, 0, 0, 2, 2, 3, 1, 0] , [0, 0, 0, 3, 1, 2, 2, 0] , [0, 0, 0, 2, 2, 3, 1, 0] , [0, 0, 0, 3, 1, 2, 2, 0]] \$

$$[y_2, 0, 4y_2 + 4y_1 - y_3 - 5y_4, y_1, 3y_2 + 3y_1 - 4y_4, y_3, y_4, 0]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

109 . Coloring, {2, 3, 6, 7, 8}

R: [3, 8, 8, 1, 7, 4, 4, 2]

B: [6, 3, 1, 6, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{2, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 2, 0, 0, 1, 2], [2, 2, 1, 1, 0, 0, 0, 2], [1, 2, 2, 0, 0, 0, 0, 3], [0, 3, 1, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4], [0, 4, 0, 0, 0, 0, 0, 4]] \$$

$[y_2, y_3, y_1, y_2 - y_3 - y_1 + y_5 + y_4, 0, 0, y_5, y_4]$

$$p = -s^5 + s^6$$

Omega Rank for B : cycles: $\{\{1, 2, 3, 5, 6, 7\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 0, 2, 2, 1, 0], [1, 2, 1, 0, 1, 1, 2, 0], [1, 1, 2, 0, 2, 1, 1, 0], [2, 2, 1, 0, 1, 1, 1, 0], [1, 1, 2, 0, 1, 2, 1, 0], [2, 1, 1, 0, 1, 1, 2, 0]] \$$

$[y_1, y_2, y_6, 0, y_5, y_4, y_3, 0]$

110 . Coloring, $\{2, 4, 5, 6, 7\}$

R: $[3, 8, 1, 6, 2, 4, 4, 5]$

B: $[6, 3, 8, 1, 7, 7, 5, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}, \{2, 5, 8\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 2, 1, 1, 0, 1], [1, 1, 1, 1, 1, 2, 0, 1], [1, 1, 1, 2, 1, 1, 0, 1], [1, 1, 1, 1, 1, 2, 0, 1], [1, 1, 1, 2, 1, 1, 0, 1], [1, 1, 1, 1, 1, 2, 0, 1], [1, 1, 1, 2, 1, 1, 0, 1]] \$$

$$[y_2, y_2, y_2, 3y_2 - y_1, y_2, y_1, 0, y_2]$$

$$p = s - s^3 \quad p' = -s + s^5 \quad p'' = -s^2 + s^6 \quad p''' = -s + s^3 \quad p^{(4)} = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{2, 3, 8\}, \{5, 7\}\}$ order: 6

See Matrix

$\$ [[1, 1, 1, 0, 1, 1, 2, 1], [0, 1, 1, 0, 2, 1, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1], [0, 1, 1, 0, 3, 0, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1], [0, 1, 1, 0, 3, 0, 2, 1], [0, 1, 1, 0, 2, 0, 3, 1]] \$$

$$[5y_4 - y_2 - y_3 - y_1, y_4, y_4, 0, y_2, y_3, y_1, y_4]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5 \quad p'' = -s^3 + s^7$$

111 . Coloring, $\{2, 4, 5, 6, 8\}$

R: $[3, 8, 1, 6, 2, 4, 5, 2]$

B: $[6, 3, 8, 1, 7, 7, 4, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	4 vs 7

Omega Rank for R : cycles: $\{\{2, 8\}, \{4, 6\}, \{1, 3\}\}$ order: 2

See Matrix

$\$ [[1, 2, 1, 1, 1, 1, 0, 1], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2], [1, 2, 1, 1, 0, 1, 0, 2]] \$$

$$[y_1, 2y_1, y_1, y_1, 2y_1 - y_2, y_1, 0, y_2]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4 \quad p'' = -s^3 + s^4 \quad p''' = -s^3 + s^6 \quad p^{(4)} = -s^3 + s^5$$

Omega Rank for B : cycles: $\{\{1, 4, 6, 7\}\}$ order: 4

See Matrix

$\$ [[1, 0, 1, 1, 1, 1, 2, 1], [1, 0, 0, 2, 1, 1, 2, 1], [2, 0, 0, 2, 1, 1, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0]] \$$

$$[y_3 + y_2 - y_4, 0, y_3, y_2, y_1, y_3 + y_2 - y_1, y_3 + y_2, y_4]$$

$$p = -s^4 + s^6 \quad p = -s^4 + s^7 \quad p = -s^4 + s^5$$

112 . Coloring, {2, 4, 5, 7, 8}

R: [3, 8, 1, 6, 2, 7, 4, 2]

B: [6, 3, 8, 1, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	4 vs 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {4, 6, 7}} order: 6

See Matrix

$\$ [[1, 2, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 2], [1, 2, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 2], [1, 2, 1, 1, 0, 1, 1, 1], [1, 1, 1, 1, 0, 1, 1, 2], [1, 2, 1, 1, 0, 1, 1, 1]] \$$

$$[y_2, 3y_2 - y_1, y_2, y_2, 0, y_2, y_2, y_1]$$

$$p' = s^2 - s^4 \quad p' = -s + s^3 \quad p' = -s^4 + s^6 \quad p' = -s + s^5 \quad p = s - s^5$$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

$\$ [[1, 0, 1, 1, 2, 1, 1, 1], [1, 0, 0, 1, 2, 1, 2, 1], [1, 0, 0, 1, 3, 1, 2, 0], [1, 0, 0, 1, 2, 1, 3, 0], [1, 0, 0, 1, 3, 1, 2, 0], [1, 0, 0, 1, 2, 1, 3, 0], [1, 0, 0, 1, 3, 1, 2, 0]] \$$

$$[y_3, 0, 5y_3 - y_1 - y_2 - y_4, y_3, y_1, y_3, y_2, y_4]$$

$$p = s^3 - s^7 \quad p' = s^3 - s^5 \quad p' = s^4 - s^6$$

113 . Coloring, {2, 4, 6, 7, 8}

R: [3, 8, 1, 6, 7, 4, 4, 2]

B: [6, 3, 8, 1, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	2 vs 7	4 vs 7

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}} order: 2

See Matrix

$\$ [[1, 1, 1, 2, 0, 1, 1, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1], [1, 1, 1, 2, 0, 2, 0, 1]] \$$

$$[y_2, y_2, y_2, 2y_2, 0, y_1, 2y_2 - y_1, y_2]$$

$$p' = -s^3 + s^6 \quad p' = -s^3 + s^5 \quad p = s^2 - s^4 \quad p' = -s^3 + s^4 \quad p' = s^2 - s^3$$

Omega Rank for B : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$\$ [[1, 1, 1, 0, 2, 1, 1, 1], [0, 2, 1, 0, 2, 1, 1, 1], [0, 2, 2, 0, 2, 0, 1, 1], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2], [0, 2, 2, 0, 2, 0, 0, 2]] \$$

$$[y_1, y_2 + y_4 - y_1, y_3, 0, y_2 + y_4, y_2 - y_3 + y_4, y_2, y_4]$$

$$p = -s^4 + s^7 \quad p = -s^4 + s^6 \quad p = -s^4 + s^5$$

114 . Coloring, {2, 5, 6, 7, 8}

R: [3, 8, 1, 1, 2, 4, 4, 2]

B: [6, 3, 8, 6, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	4 vs 5

Omega Rank for R : cycles: $\{\{2, 8\}, \{1, 3\}\}$ order: 2
See Matrix

$\$ [[2, 2, 1, 2, 0, 0, 0, 1], [3, 1, 2, 0, 0, 0, 0, 2], [2, 2, 3, 0, 0, 0, 0, 1], [3, 1, 2, 0, 0, 0, 0, 2], [2, 2, 3, 0, 0, 0, 0, 1]] \$$

$$[-5 y_2 + 4 y_3 + 4 y_1, y_2, y_3, y_1, 0, 0, 0, -4 y_2 + 3 y_3 + 3 y_1]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4
See Matrix

$\$ [[0, 0, 1, 0, 2, 2, 2, 1], [0, 0, 0, 0, 3, 0, 4, 1], [0, 0, 0, 0, 5, 0, 3, 0], [0, 0, 0, 0, 3, 0, 5, 0], [0, 0, 0, 0, 5, 0, 3, 0]] \$$

$$[0, 0, y_1, 0, y_2, 2 y_1, y_3, y_4]$$

$$p = s^3 - s^5$$

115 . Coloring, $\{3, 4, 5, 6, 7\}$

R: [3, 3, 8, 6, 2, 4, 4, 5]

B: [6, 8, 1, 1, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{2, 3, 5, 8\}\}$ order: 4

See Matrix

$\$ [[0, 1, 2, 2, 1, 1, 0, 1], [0, 1, 1, 1, 1, 2, 0, 2], [0, 1, 1, 2, 2, 1, 0, 1], [0, 2, 1, 1, 1, 2, 0, 1], [0, 1, 2, 2, 1, 1, 0, 1], [0, 1, 1, 1, 1, 2, 0, 2]] \$$

$$[0, y_4, y_3, y_1, y_2, 3y_3 - 4y_1 + 3y_2, 0, -y_4 + 4y_3 - 5y_1 + 4y_2]$$

$$p = s - s^5 \quad p' = s - s^5$$

Omega Rank for B : cycles: $\{\{2, 8\}, \{5, 7\}\}$ order: 4

See Matrix

$\$ [[2, 1, 0, 0, 1, 1, 2, 1], [0, 1, 0, 0, 2, 2, 2, 1], [0, 1, 0, 0, 2, 0, 4, 1], [0, 1, 0, 0, 4, 0, 2, 1], [0, 1, 0, 0, 2, 0, 4, 1], [0, 1, 0, 0, 4, 0, 2, 1]] \$$

$$[y_3, y_4, 0, 0, -y_3 + 6y_4 - y_1 - y_2, y_1, y_2, y_4]$$

$$p = -s^3 + s^5 \quad p' = -s^3 + s^5$$

116 . Coloring, $\{3, 4, 5, 6, 8\}$

R: $[3, 3, 8, 6, 2, 4, 5, 2]$

B: $[6, 8, 1, 1, 7, 7, 4, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	2 vs 6	6 vs 6

Omega Rank for R : cycles: $\{\{4, 6\}, \{2, 3, 8\}\}$ order: 6

See Matrix

$\$ [[0, 2, 2, 1, 1, 1, 0, 1], [0, 2, 2, 1, 0, 1, 0, 2], [0, 2, 2, 1, 0, 1, 0, 2], [0, 2, 2, 1, 0, 1, 0, 2], [0, 2, 2, 1, 0, 1, 0, 2], [0, 2, 2, 1, 0, 1, 0, 2]] \$$

$$[0, 2y_1, 2y_1, y_1, 2y_1 - y_2, y_1, 0, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: $\{\{1, 4, 6, 7\}\}$ order: 4

See Matrix

$$\$ [[2, 0, 0, 1, 1, 1, 2, 1], [1, 0, 0, 2, 1, 2, 2, 0], [2, 0, 0, 2, 0, 1, 3, 0], [2, 0, 0, 3, 0, 2, 1, 0], [3, 0, 0, 1, 0, 2, 2, 0], [1, 0, 0, 2, 0, 3, 2, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6]$$

117 . Coloring, $\{3, 4, 5, 7, 8\}$

R: [3, 3, 8, 6, 2, 7, 4, 2]

B: [6, 8, 1, 1, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 6	4 vs 6

Omega Rank for R : cycles: $\{\{4, 6, 7\}, \{2, 3, 8\}\}$ order: 3

See Matrix

$$\$ [[0, 2, 2, 1, 0, 1, 1, 1], [0, 1, 2, 1, 0, 1, 1, 2], [0, 2, 1, 1, 0, 1, 1, 2], [0, 2, 2, 1, 0, 1, 1, 1], [0, 1, 2, 1, 0, 1, 1, 2], [0, 2, 1, 1, 0, 1, 1, 2]] \$$$

$$[0, y_1, -y_1 + 5y_3 - y_2, y_3, 0, y_3, y_3, y_2]$$

$$p' = -s + s^4 \quad p' = -s^2 + s^5 \quad p = -s + s^4$$

Omega Rank for B : cycles: $\{\{5, 7\}, \{1, 4, 6\}\}$ order: 6

See Matrix

$$\$ [[2, 0, 0, 1, 2, 1, 1, 1], [1, 0, 0, 1, 2, 2, 2, 0], [1, 0, 0, 2, 2, 1, 2, 0], [2, 0, 0, 1, 2, 1, 2, 0], [1, 0, 0, 1, 2, 2, 2, 0], [1, 0, 0, 2, 2, 1, 2, 0]] \$$$

$$[y_4, 0, 0, y_3, y_2, -y_4 - y_3 + 2y_2, y_2 - y_1, y_1]$$

$$p = -s^2 + s^5 \quad p' = -s^2 + s^5$$

118 . Coloring, {3, 4, 6, 7, 8}

R: [3, 3, 8, 6, 7, 4, 4, 2]

B: [6, 8, 1, 1, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	4 vs 6	6 vs 6

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$\$ [[0, 1, 2, 2, 0, 1, 1, 1], [0, 1, 1, 2, 0, 2, 0, 2], [0, 2, 1, 2, 0, 2, 0, 1], [0, 1, 2, 2, 0, 2, 0, 1], [0, 1, 1, 2, 0, 2, 0, 2], [0, 2, 1, 2, 0, 2, 0, 1]] \$$$

$$[0, y_1, -y_1 + 2y_4 + 2y_3 - y_2, y_4 + y_3, 0, y_4, y_3, y_2]$$

$$p' = s^2 - s^5 \quad p = s^2 - s^5$$

Omega Rank for B : cycles: {{2, 5, 8}} order: 6

See Matrix

$$\$ [[2, 1, 0, 0, 2, 1, 1, 1], [0, 2, 0, 0, 2, 2, 1, 1], [0, 2, 0, 0, 2, 0, 2, 2], [0, 2, 0, 0, 4, 0, 0, 2], [0, 4, 0, 0, 2, 0, 0, 2], [0, 2, 0, 0, 2, 0, 0, 4]] \$$$

$$[y_1, y_2, 0, 0, y_4, y_5, y_6, y_3]$$

119 . Coloring, {3, 5, 6, 7, 8}

R: [3, 3, 8, 1, 2, 4, 4, 2]

B: [6, 8, 1, 6, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	5 vs 5	3 vs 5

Omega Rank for R : cycles: $\{\{2, 3, 8\}\}$ order: 3
See Matrix

$\$ [[1, 2, 2, 2, 0, 0, 0, 1], [2, 1, 3, 0, 0, 0, 0, 2], [0, 2, 3, 0, 0, 0, 0, 3], [0, 3, 2, 0, 0, 0, 0, 3], [0, 3, 3, 0, 0, 0, 0, 2]] \$$

$$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5]$$

Omega Rank for B : cycles: $\{\{5, 7\}\}$ order: 4
See Matrix

$\$ [[1, 0, 0, 0, 2, 2, 2, 1], [0, 0, 0, 0, 3, 1, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$

$$[y_1, 0, 0, 0, y_2, y_3, -2y_1 + y_2 + y_3, y_1]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

120 . Coloring, $\{4, 5, 6, 7, 8\}$

R: $[3, 3, 1, 6, 2, 4, 4, 2]$

B: $[6, 8, 8, 1, 7, 7, 5, 5]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	4 vs 5

Omega Rank for R : cycles: $\{\{4, 6\}, \{1, 3\}\}$ order: 2
See Matrix

\$ [[1, 2, 2, 2, 0, 1, 0, 0] , [2, 0, 3, 1, 0, 2, 0, 0] , [3, 0, 2, 2, 0, 1, 0, 0] , [2, 0, 3, 1, 0, 2, 0, 0] , [3, 0, 2, 2, 0, 1, 0, 0]] \$

$$[-y_1 + 4y_2 - 5y_3, y_1, y_2, 3y_2 - 4y_3, 0, y_3, 0, 0]$$

$$p' = s^2 - s^4 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{5, 7}} order: 4
See Matrix

\$ [[1, 0, 0, 0, 2, 1, 2, 2] , [0, 0, 0, 0, 4, 1, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0] , [0, 0, 0, 0, 5, 0, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0]] \$

$$[y_4, 0, 0, 0, y_3, y_2, y_1, 2y_4]$$

$$p = -s^3 + s^5$$

121 . Coloring, {2, 3, 4, 5, 6, 7}

R: [3, 8, 8, 6, 2, 4, 4, 5]

B: [6, 3, 1, 1, 7, 7, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	5 vs 6	6 vs 6

Omega Rank for R : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

\$ [[0, 1, 1, 2, 1, 1, 0, 2] , [0, 1, 0, 1, 2, 2, 0, 2] , [0, 2, 0, 2, 2, 1, 0, 1] , [0, 2, 0, 1, 1, 2, 0, 2] , [0, 1, 0, 2, 2, 1, 0, 2] , [0, 2, 0, 1, 2, 2, 0, 1]] \$

$$[0, -3y_1 + 5y_2 - 3y_5 + 5y_3 - 3y_4, 3y_1, 3y_2, 3y_5, 3y_3, 0, 3y_4]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

Omega Rank for B : cycles: {{5, 7}} order: 6
See Matrix

\$ [[2, 1, 1, 0, 1, 1, 2, 0], [1, 0, 1, 0, 2, 2, 2, 0], [1, 0, 0, 0, 2, 1, 4, 0], [0, 0, 0, 0, 4, 1, 3, 0], [0, 0, 0, 0, 3, 0, 5, 0], [0, 0, 0, 0, 5, 0, 3, 0]] \$

$$[y_4, y_1, y_3, 0, y_2, y_5, y_6, 0]$$

122 . Coloring, {2, 3, 4, 5, 6, 8}

$$\Omega p(\Delta)=0: \quad p' = s + 4s^4 - 8s^5 \quad p' = s^2 - 2s^3 + 4s^4 - 4s^5 \quad p = s + 4s^4 - 8s^5$$

R: [3, 8, 8, 6, 2, 4, 5, 2]

B: [6, 3, 1, 1, 7, 7, 4, 5]

' See graph

' ' See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	2 vs 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

\$ [[0, 2, 1, 1, 1, 1, 0, 2], [0, 3, 0, 1, 0, 1, 0, 3], [0, 3, 0, 1, 0, 1, 0, 3], [0, 3, 0, 1, 0, 1, 0, 3], [0, 3, 0, 1, 0, 1, 0, 3], [0, 3, 0, 1, 0, 1, 0, 3], [0, 3, 0, 1, 0, 1, 0, 3]] \$

$$[0, y_2, 3y_1 - y_2, y_1, 3y_1 - y_2, y_1, 0, y_2]$$

$$p = s^2 - s^6 \quad p' = s^2 - s^5 \quad p' = s^4 - s^5 \quad p' = s^3 - s^5$$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

\$ [[2, 0, 1, 1, 1, 1, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0], [2, 0, 0, 2, 0, 2, 2, 0]] \$

$$[y_2, 0, y_2 - y_1, y_1, y_2 - y_1, y_1, y_2, 0]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^6 \quad p = -s^2 + s^5$$

M \ ; N

\$ [[0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0]] \$ \$ [[0, 5, 4, 4, 3, 3, 7, 2] , [5, 0, 2, 4, 5, 3, 2, 7] , [4, 2, 0, 2, 7, 5, 3, 5] , [4, 4, 2, 0, 5, 7, 3, 3] , [3, 5, 7, 5, 0, 2, 4, 2] , [3, 3, 5, 7, 2, 0, 4, 4] , [7, 2, 3, 3, 4, 4, 0, 5] , [2, 7, 5, 3, 2, 4, 5, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 8, 6, 2, 4, 5, 2]

B: [6, 3, 1, 1, 7, 7, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [2], [4]]

Action of B on ranges, [[4], [3], [1], [1]]

Cycles: R , {{2, 8}, {4, 6}}, B , {{1, 4, 6, 7}}

$\beta(\{1, 7\}) = 1/4$

$\beta(\{2, 8\}) = 1/4$

$\beta(\{3, 5\}) = 1/4$

$\beta(\{4, 6\}) = 1/4$

Partitions

Action of R on partitions, [[4], [6], [6], [4], [5], [5]]

Action of B on partitions, [[4], [3], [4], [3], [2], [1]]

$\alpha(\{\{5, 6, 7, 8\}, \{1, 2, 3, 4\}\}) = 1/14$

$\alpha(\{\{1, 2, 5, 6\}, \{3, 4, 7, 8\}\}) = 1/14$

$\alpha(\{\{1, 3, 4, 8\}, \{2, 5, 6, 7\}\}) = 3/14$

$\alpha(\{\{2, 3, 4, 7\}, \{1, 5, 6, 8\}\}) = 5/14$

$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/7$

$\alpha(\{\{4, 5, 7, 8\}, \{1, 2, 3, 6\}\}) = 1/7$

b1 = {4, 5, 7, 8} ‘ , ‘ b2 = {1, 4, 5, 8} ‘ , ‘ b3 = {1, 2, 3, 6} ‘ , ‘ b4 = {1, 2, 5, 6} ‘ , ‘ b5 = {2, 3, 6, 7} ‘ , ‘
b6 = {1, 3, 4, 8} ‘ , ‘ b7 = {2, 5, 6, 7} ‘ , ‘ b8 = {5, 6, 7, 8} ‘ , ‘ b9 = {2, 3, 4, 7} ‘ , ‘ b10 = {3, 4, 7, 8} ‘ , ‘
b11 = {1, 5, 6, 8} ‘ , ‘ b12 = {1, 2, 3, 4}

Action of R and B on the blocks of the partitions: = [5, 5, 2, 1, 2, 3, 1, 9, B, 3, 9, B] [8, A, C, 6, 4, 9, B, B, 7, 7, 6, 9]

with invariant measure [2, 2, 2, 1, 2, 3, 3, 1, 5, 1, 5, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 4, 5, 6, 8}
Rank	2
R,B	[3, 8, 8, 6, 2, 4, 5, 2], [6, 3, 1, 1, 7, 7, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
u_2	[5, 4, 4, 3, 3, 7, 2, 2, 4, 5, 3, 2, 7, 2, 7, 5, 3, 5, 5, 7, 3, 3, 2, 4, 2, 4, 4, 5] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

123 . Coloring, {2, 3, 4, 5, 7, 8}

R: [3, 8, 8, 6, 2, 7, 4, 2]

B: [6, 3, 1, 1, 7, 4, 5, 5]

‘ See graph

‘ ‘ See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	8 vs 8	8 vs 8	3 vs 6	5 vs 6

Omega Rank for R : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

\$ [[0, 2, 1, 1, 0, 1, 1, 2], [0, 2, 0, 1, 0, 1, 1, 3], [0, 3, 0, 1, 0, 1, 1, 2], [0, 2, 0, 1, 0, 1, 1, 3], [0, 3, 0, 1, 0, 1, 1, 2], [0, 2, 0, 1, 0, 1, 1, 3]] \$

$$[0, y_1, -y_1 + 5y_2 - y_3, y_2, 0, y_2, y_2, y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4 \quad p = -s^2 + s^6$$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

\$ [[2, 0, 1, 1, 2, 1, 1, 0] , [2, 0, 0, 1, 1, 2, 2, 0] , [1, 0, 0, 2, 2, 2, 1, 0] , [2, 0, 0, 2, 1, 1, 2, 0] , [2, 0, 0, 1, 2, 2, 1, 0] , [1, 0, 0, 2, 1, 2, 2, 0]] \$

$$[-3 y_1 - 3 y_2 + 5 y_5 - 3 y_3 + 5 y_4, 0, 3 y_1, 3 y_2, 3 y_5, 3 y_3, 3 y_4, 0]$$

$$p = -s^2 - s^3 + s^5 + s^6$$

124 . Coloring, {2, 3, 4, 6, 7, 8}

$$\Omega p(\Delta)=0: \quad p = s + 4s^4 \quad p' = s + 4s^4 \quad p'' = s^2 + 4s^5$$

R: [3, 8, 8, 6, 7, 4, 4, 2]

B: [6, 3, 1, 1, 2, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 6	4 vs 8	4 vs 8	2 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

\$ [[0, 1, 1, 2, 0, 1, 1, 2] , [0, 2, 0, 2, 0, 2, 0, 2] , [0, 2, 0, 2, 0, 2, 0, 2] , [0, 2, 0, 2, 0, 2, 0, 2] , [0, 2, 0, 2, 0, 2, 0, 2] , [0, 2, 0, 2, 0, 2, 0, 2]] \$

$$[0, -y_1 + y_2, y_1, y_2, 0, -y_1 + y_2, y_1, y_2]$$

$$p' = -s^2 + s^5 \quad p = s^2 - s^3 \quad p' = -s^2 + s^3 \quad p'' = -s^2 + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

\$ [[2, 1, 1, 0, 2, 1, 1, 0] , [1, 2, 1, 0, 1, 2, 1, 0] , [1, 1, 2, 0, 1, 1, 2, 0] , [2, 1, 1, 0, 2, 1, 1, 0] , [1, 2, 1, 0, 1, 2, 1, 0] , [1, 1, 2, 0, 1, 1, 2, 0]] \$

$$[y_1, y_2, y_3, 0, y_1, y_2, y_3, 0]$$

$$p = -s + s^4 \quad p' = -s + s^4 \quad p'' = -s^2 + s^5$$

M \ ; N

\$ [[0, 0, 0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0, 0, 0] , [0, 0, 0, 1, 0, 0, 0, 0]] \$ \$ [[0, 3, 2, 2, 4, 1, 2, 2] , [3, 0, 1, 2, 1, 4, 3, 2] , [2, 1, 0, 1, 2, 3, 4, 3] , [2, 2, 1, 0, 2, 2, 3, 4] , [4, 1, 2, 2, 0, 3, 2, 2] , [1, 4, 3, 2, 3, 0, 1, 2] , [2, 3, 4, 3, 2, 1, 0, 1] , [2, 2, 3, 4, 2, 2, 1, 0]] \$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 8, 6, 7, 4, 4, 2]

B: [6, 3, 1, 1, 2, 7, 5, 5]

Ranges

Action of R on ranges, [[3], [4], [4], [2]]

Action of B on ranges, [[2], [3], [1], [1]]

Cycles: R , {{2, 8}, {4, 6}}, B , {{1, 2, 3, 5, 6, 7}}

$\beta(\{1, 5\}) = 1/4$

$\beta(\{2, 6\}) = 1/4$

$\beta(\{3, 7\}) = 1/4$

$\beta(\{4, 8\}) = 1/4$

Partitions

Action of R on partitions, [[3], [5], [4], [4], [5]]

Action of B on partitions, [[2], [5], [1], [5], [1]]

$\alpha(\{\{2, 5, 7, 8\}, \{1, 3, 4, 6\}\}) = 1/4$

$\alpha(\{\{5, 6, 7, 8\}, \{1, 2, 3, 4\}\}) = 1/8$

$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 1/8$

$\alpha(\{\{1, 2, 3, 8\}, \{4, 5, 6, 7\}\}) = 1/8$

$\alpha(\{\{2, 3, 4, 5\}, \{1, 6, 7, 8\}\}) = 3/8$

b1 = {2, 5, 7, 8} ‘ , ‘ b2 = {1, 3, 4, 6} ‘ , ‘ b3 = {1, 4, 6, 7} ‘ , ‘ b4 = {1, 2, 3, 8} ‘ , ‘ b5 = {4, 5, 6, 7} ‘ , ‘

b6 = {5, 6, 7, 8} ‘ , ‘ b7 = {2, 3, 4, 5} ‘ , ‘ b8 = {1, 6, 7, 8} ‘ , ‘ b9 = {2, 3, 5, 8} ‘ , ‘ b10 = {1, 2, 3, 4}

Action of R and B on the blocks of the partitions: = [9, 3, 5, 4, 5, 7, 8, 7, 4, 8] [6, A, 2, 7, 8, 8, 1, 2, 1, 7]
with invariant measure [2, 2, 1, 1, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 3, 4, 6, 7, 8}
Rank	2
R,B	[3, 8, 8, 6, 7, 4, 4, 2], [6, 3, 1, 1, 2, 7, 5, 5]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[3, 2, 2, 4, 1, 2, 2, 1, 2, 1, 4, 3, 2, 1, 2, 3, 4, 3, 2, 2, 3, 4, 3, 2, 2, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

125 . Coloring, {2, 3, 5, 6, 7, 8}

R: [3, 8, 8, 1, 2, 4, 4, 2]

B: [6, 3, 1, 6, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	5 vs 5	5 vs 5

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

\$ [[1, 2, 1, 2, 0, 0, 0, 2] , [2, 2, 1, 0, 0, 0, 0, 3] , [0, 3, 2, 0, 0, 0, 0, 3] , [0, 3, 0, 0, 0, 0, 0, 5] , [0, 5, 0, 0, 0, 0, 0, 3]] \$

$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5]$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

\$ [[1, 0, 1, 0, 2, 2, 2, 0] , [1, 0, 0, 0, 2, 1, 4, 0] , [0, 0, 0, 0, 4, 1, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0] , [0, 0, 0, 0, 5, 0, 3, 0]] \$

$$[y_1, 0, y_5, 0, y_4, y_2, y_3, 0]$$

126 . Coloring, {2, 4, 5, 6, 7, 8}

$$\Omega p(\Delta)=0: \quad p' = s - 4s^5 \quad p = s - 4s^5 \quad p' = s^2 - 2s^4 \quad p' = s^3 - 2s^5$$

R: [3, 8, 1, 6, 2, 4, 4, 2]

B: [6, 3, 8, 1, 7, 7, 5, 5]

' See graph

' ' See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 6	3 vs 7	3 vs 7	2 vs 6	3 vs 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}} order: 2

See Matrix

$$\$ [[1, 2, 1, 2, 0, 1, 0, 1], [1, 1, 1, 1, 0, 2, 0, 2], [1, 2, 1, 2, 0, 1, 0, 1], [1, 1, 1, 1, 0, 2, 0, 2], [1, 2, 1, 2, 0, 1, 0, 1], [1, 1, 1, 1, 0, 2, 0, 2]] \$$$

$$[y_1, 3y_1 - y_2, y_1, 3y_1 - y_2, 0, y_2, 0, y_2]$$

$$p' = -s^2 + s^4 \quad p' = -s + s^5 \quad p' = -s + s^3 \quad p = s - s^3$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[1, 0, 1, 0, 2, 1, 2, 1], [0, 0, 0, 0, 3, 1, 3, 1], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$$

$$[y_1, 0, y_1, 0, y_3, y_2, y_3, y_2]$$

$$p' = s^4 - s^5 \quad p = s^3 - s^6 \quad p' = s^3 - s^5$$

M \ ; N

$$\$ [[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]] \$ \quad \$ [[0, 1, 2, 1, 1, 1, 1, 1], [1, 0, 1, 2, 1, 1, 1, 1], [2, 1, 0, 1, 1, 1, 1, 1], [1, 2, 1, 0, 1, 1, 1, 1], [1, 1, 1, 1, 0, 1, 2, 1], [1, 1, 1, 1, 1, 0, 1, 2], [1, 1, 1, 1, 2, 1, 0, 1], [1, 1, 1, 1, 1, 2, 1, 0]] \$$$

$$\tau = 32, r' = 1/2$$

R: [3, 8, 1, 6, 2, 4, 4, 2]

B: [6, 3, 8, 1, 7, 7, 5, 5]

Ranges

Action of R on ranges, [[1], [4], [2], [2]]

Action of B on ranges, [[4], [1], [3], [3]]

Cycles: R, {{2, 8}, {4, 6}, {1, 3}}, B, {{5, 7}}

$$\beta(\{1, 3\}) = 1/4$$

$$\beta(\{2, 4\}) = 1/4$$

$$\beta(\{5, 7\}) = 1/4$$

$$\beta(\{6, 8\}) = 1/4$$

Partitions

Action of R on partitions, [[7], [1], [3], [6], [7], [6], [1], [3]]

Action of B on partitions, [[8], [4], [2], [5], [5], [8], [2], [4]]

$$\alpha(\{\{3, 4, 6, 7\}, \{1, 2, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 6\}, \{2, 3, 7, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 6, 7\}, \{3, 4, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{2, 3, 5, 6\}, \{1, 4, 7, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 7, 8\}, \{3, 4, 5, 6\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 5, 8\}, \{2, 3, 6, 7\}\}) = 1/8$$

$$\alpha(\{\{1, 4, 6, 7\}, \{2, 3, 5, 8\}\}) = 1/8$$

$$\alpha(\{\{1, 2, 5, 6\}, \{3, 4, 7, 8\}\}) = 1/8$$

b1 = {3, 4, 6, 7} ‘, ‘ b2 = {1, 2, 5, 8} ‘, ‘ b3 = {1, 2, 7, 8} ‘, ‘ b4 = {1, 4, 5, 6} ‘, ‘ b5 = {3, 4, 5, 6} ‘, ‘
b6 = {2, 3, 7, 8} ‘, ‘ b7 = {1, 4, 5, 8} ‘, ‘ b8 = {1, 4, 6, 7} ‘, ‘ b9 = {2, 3, 6, 7} ‘, ‘ b10 = {1, 2, 6, 7} ‘, ‘
b11 = {3, 4, 5, 8} ‘, ‘ b12 = {2, 3, 5, 6} ‘, ‘ b13 = {1, 4, 7, 8} ‘, ‘ b14 = {1, 2, 5, 6} ‘, ‘ b15 = {3, 4, 7, 8}
‘, ‘ b16 = {2, 3, 5, 8}

Action of R and B on the blocks of the partitions: = [8, 10, 10, 1, 8, 2, 9, 1, 7, B, A, 7, 9, B, A, 2] [E, F, 5, D, 3, C, F, 4, E, 4, 6, 3, 5, D, C, 6]

with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 4, 5, 6, 7, 8}
Rank	2
R,B	[3, 8, 1, 6, 2, 4, 4, 2], [6, 3, 8, 1, 7, 7, 5, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

127 . Coloring, {3, 4, 5, 6, 7, 8}

R: [3, 3, 8, 6, 2, 4, 4, 2]

B: [6, 8, 1, 1, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	6 vs 6	6 vs 6	4 vs 5	4 vs 5

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

\$ [[0, 2, 2, 2, 0, 1, 0, 1] , [0, 1, 2, 1, 0, 2, 0, 2] , [0, 2, 1, 2, 0, 1, 0, 2] , [0, 2, 2, 1, 0, 2, 0, 1] , [0, 1, 2, 2, 0, 1, 0, 2]] \$

$$[0, -3 y_1 + 5 y_3 + 5 y_2 - 3 y_4, 3 y_1, 3 y_3, 0, 3 y_2, 0, 3 y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

\$ [[2, 0, 0, 0, 2, 1, 2, 1] , [0, 0, 0, 0, 3, 2, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0] , [0, 0, 0, 0, 5, 0, 3, 0] , [0, 0, 0, 0, 3, 0, 5, 0]] \$

$$[2 y_4, 0, 0, 0, y_1, y_2, y_3, y_4]$$

$$p = -s^3 + s^5$$

128 . Coloring, {2, 3, 4, 5, 6, 7, 8}

R: [3, 8, 8, 6, 2, 4, 4, 2]

B: [6, 3, 1, 1, 7, 7, 5, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
6 vs 6	7 vs 7	7 vs 7	3 vs 5	4 vs 5

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$\$ [[0, 2, 1, 2, 0, 1, 0, 2], [0, 2, 0, 1, 0, 2, 0, 3], [0, 3, 0, 2, 0, 1, 0, 2], [0, 2, 0, 1, 0, 2, 0, 3], [0, 3, 0, 2, 0, 1, 0, 2]] \$$$

$$[0, y_1, -y_1 - 5 y_3 + 4 y_2, -4 y_3 + 3 y_2, 0, y_3, 0, y_2]$$

$$p' = s^2 - s^4 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$\$ [[2, 0, 1, 0, 2, 1, 2, 0], [1, 0, 0, 0, 2, 2, 3, 0], [0, 0, 0, 0, 3, 1, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0], [0, 0, 0, 0, 4, 0, 4, 0]] \$$$

$$[y_1 + y_2 + y_3 - y_4, 0, y_1, 0, y_2, y_3, y_4, 0]$$

$$p = s^4 - s^5$$

SUMMARY	
Graph Type	CC
$v(\mathbf{A})$	2
$v(\Delta)$	2
π	[1, 1, 1, 1, 1, 1, 1, 1]
Dbly Stoch	true

SANDWICH		Total 18
No .	Coloring	Rank
1	{2, 6}	2
2	{3, 7}	2
3	{3, 4, 5, 6}	2
4	{3, 4, 7, 8}	2
5	{}	4
6	{2, 8}	2
7	{2, 3, 6, 7}	2
8	{2, 3, 4, 5, 6, 8}	2
9	{3, 5}	2
10	{2, 3, 4, 6, 7, 8}	2
11	{4, 8}	2
12	{5, 6, 7, 8}	2
13	{2, 3, 5, 8}	2
14	{2, 4}	2
15	{6, 8}	2
16	{5, 7}	2
17	{2, 4, 5, 6, 7, 8}	2
18	{4, 6}	2

RT GROUPS		Total 2	
No .	Coloring	Rank	Solv
1	{2, 4, 5, 7}	8	["group", Not Solvable]
2	{2, 4, 6, 8}	8	["group", Not Solvable]

CC Colorings		Total 2
No .	Coloring	Sandwich,Rank
1	{}	true, 4
2	{5, 6, 7, 8}	true, 2

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
96	0	108 , 108	24 , 32	20	128	128
