

New Graph

[3, 1, 1, 1, 2, 3], [5, 4, 4, 6, 4, 4]

$$\pi = [4, 1, 3, 4, 2, 2]$$

POSSIBLE RANKS

1 x 16

2 x 8

4 x 4

BASE DETERMINANT 2831/16384, .1727905273

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

[[{1, 2, 3},{4, 5, 6}]]

1 . Coloring, {}

$$\Omega p(\Delta)=0: \quad p = s^4 \quad p' = s^4$$

R: [3, 1, 1, 1, 2, 3]

B: [5, 4, 4, 6, 4, 4]

' See graph

' ' See pair graph

'

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 3 vs 5 | 3 vs 5 | 3 vs 5 | 2 vs 3 | 2 vs 3 |

Omega Rank for R : cycles: {{1, 3}} order: 2

See Matrix

\$ [[8, 2, 6, 0, 0, 0], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0]] \$

$$[y_1 + y_2, y_1, y_2, 0, 0, 0]$$

$$p = s^2 - s^3$$

Omega Rank for B : cycles: {{4, 6}} order: 2

See Matrix

$$\$ [[0, 0, 0, 8, 4, 4], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[0, 0, 0, y_1, y_1 - y_2, y_2]$$

$$p = s^2 - s^3$$

M N

$$\begin{aligned} & \$ [[0, 1, 3, 0, 0, 0], [1, 0, 0, 0, 0, 0], [3, 0, 0, 0, 0, 0], [0, 0, 0, 0, 2, 2], [0, 0, 0, 2, 0, 0], [0, 0, 0, 2, 0, 0]] \\ & \$ \quad \$ [[0, 2, 2, 1, 1, 1], [2, 0, 0, 1, 1, 1], [2, 0, 0, 1, 1, 1], [1, 1, 1, 0, 2, 2], [1, 1, 1, 2, 0, 0], [1, 1, 1, 2, 0, 0]] \$ \end{aligned}$$

$$\tau = 19, r' = 1/2$$

$$\mathbf{R}: [3, 1, 1, 1, 2, 3]$$

$$\mathbf{B}: [5, 4, 4, 6, 4, 4]$$

Ranges

Action of R on ranges, [[2], [2], [1], [2]]

Action of B on ranges, [[3], [3], [4], [4]]

Cycles: R, {{1, 3}}, B, {{4, 6}}

$$\beta(\{1, 2\}) = 1/8$$

$$\beta(\{1, 3\}) = 3/8$$

$$\beta(\{4, 5\}) = 1/4$$

$$\beta(\{4, 6\}) = 1/4$$

Partitions

Action of R on partitions, [[1], [1]]

Action of B on partitions, [[2], [2]]

$$\alpha(\{\{2, 3, 4\}, \{1, 5, 6\}\}) = 1/2$$

$$\alpha(\{\{1, 4\}, \{2, 3, 5, 6\}\}) = 1/2$$

$$b_1 = \{2, 3, 4\} ', ' b_2 = \{1, 5, 6\} ', ' b_3 = \{1, 4\} ', ' b_4 = \{2, 3, 5, 6\}$$

Action of R and B on the blocks of the partitions: = [2, 1, 1, 2] [4, 3, 4, 3]
 with invariant measure [1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

| Sandwich | |
|-----------------|---|
| Coloring | { } |
| Rank | 2 |
| R,B | [3, 1, 1, 1, 2, 3], [5, 4, 4, 6, 4, 4] |
| π_2 | [1, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0] |
| u_2 | [2, 2, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 2, 2, 0] (dim 1) |
| wpp | [5, 7, 7, 5, 7, 7] |

2 . Coloring, {2}

R: [3, 4, 1, 1, 2, 3]

B: [5, 1, 4, 6, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | A+(1/2) Δ | A-(1/2) Δ | R | B |
|----------------|------------------|------------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 3 vs 4 |

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

\$ [[7, 2, 6, 1, 0, 0] , [7, 0, 7, 2, 0, 0] , [9, 0, 7, 0, 0, 0] , [7, 0, 9, 0, 0, 0]] \$

$$[y_2, y_1, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[1, 0, 0, 7, 4, 4], [0, 0, 0, 8, 1, 7], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[-y_3 + y_1 + y_2, 0, 0, y_3, y_1, y_2]$$

$$p = -s^3 + s^4$$

3 . Coloring, {3}

R: [3, 1, 4, 1, 2, 3]

B: [5, 4, 1, 6, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 3 vs 4 |

Omega Rank for R : cycles: {{1, 3, 4}} order: 3

See Matrix

$$\$ [[5, 2, 6, 3, 0, 0], [5, 0, 5, 6, 0, 0], [6, 0, 5, 5, 0, 0], [5, 0, 6, 5, 0, 0]] \$$$

$$[y_2, y_1, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[3, 0, 0, 5, 4, 4], [0, 0, 0, 8, 3, 5], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[y_1, 0, 0, -y_1 + y_3 + y_2, y_3, y_2]$$

$$p = -s^3 + s^4$$

4 . Coloring, {4}

R: [3, 1, 1, 6, 2, 3]

B: [5, 4, 4, 1, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 4 vs 5 | 2 vs 4 | 3 vs 3 |

Omega Rank for R : cycles: {{1, 3}} order: 2

See Matrix

$$\$ [[4, 2, 6, 0, 0, 4], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0]] \$$$

$$[-y_1 + y_2, y_1, y_2, 0, 0, 2 y_1]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

Omega Rank for B : cycles: {{1, 4, 5}} order: 3

See Matrix

$$\$ [[4, 0, 0, 8, 4, 0], [8, 0, 0, 4, 4, 0], [4, 0, 0, 4, 8, 0]] \$$$

$$[y_1, 0, 0, y_2, y_3, 0]$$

5 . Coloring, {5}

R: [3, 1, 1, 1, 4, 3]

B: [5, 4, 4, 6, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 2 vs 3 | 4 vs 4 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2
 See Matrix

$$\$ [[8, 0, 6, 2, 0, 0], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0]] \$$$

$$[y_1 + y_2, 0, y_1, y_2, 0, 0]$$

$$p = s^2 - s^3$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 4
 See Matrix

$$\$ [[0, 2, 0, 6, 4, 4], [0, 4, 0, 6, 0, 6], [0, 0, 0, 10, 0, 6], [0, 0, 0, 6, 0, 10]] \$$$

$$[0, y_2, 0, y_1, y_4, y_3]$$

6 . Coloring, $\{6\}$

R: [3, 1, 1, 1, 2, 4]

B: [5, 4, 4, 6, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 2 vs 4 | 4 vs 4 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2
 See Matrix

$$\$ [[8, 2, 4, 2, 0, 0], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0]] \$$$

$$[2y_1 + y_2, y_1, y_2, y_1, 0, 0]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^3$$

Omega Rank for B : cycles: $\{\{3, 4, 6\}\}$ order: 3
 See Matrix

$$\$ [[0, 0, 2, 6, 4, 4], [0, 0, 4, 6, 0, 6], [0, 0, 6, 4, 0, 6], [0, 0, 6, 6, 0, 4]] \$$$

$$[0, 0, y_4, y_3, y_2, y_1]$$

7. Coloring, {2, 3}

R: [3, 4, 4, 1, 2, 3]

B: [5, 1, 1, 6, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 4 vs 4 | 3 vs 4 |

Omega Rank for R : cycles: {{1, 3, 4}} order: 3

See Matrix

$$\$ [[4, 2, 6, 4, 0, 0], [4, 0, 4, 8, 0, 0], [8, 0, 4, 4, 0, 0], [4, 0, 8, 4, 0, 0]] \$$$

$$[y_3, y_4, y_2, y_1, 0, 0]$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[4, 0, 0, 4, 4, 4], [0, 0, 0, 8, 4, 4], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[-y_1 + y_2 + y_3, 0, 0, y_1, y_2, y_3]$$

$$p = -s^3 + s^4$$

8. Coloring, {2, 4}

R: [3, 4, 1, 6, 2, 3]

B: [5, 1, 4, 1, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 5 vs 5 | 3 vs 3 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 4
See Matrix

$$\$ [[3, 2, 6, 1, 0, 4], [6, 0, 7, 2, 0, 1], [7, 0, 7, 0, 0, 2], [7, 0, 9, 0, 0, 0], [9, 0, 7, 0, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

Omega Rank for B : cycles: $\{\{1, 4, 5\}\}$ order: 3
See Matrix

$$\$ [[5, 0, 0, 7, 4, 0], [7, 0, 0, 4, 5, 0], [4, 0, 0, 5, 7, 0]] \$$$

$$[y_2, 0, 0, y_1, y_3, 0]$$

9 . Coloring, $\{2, 5\}$

R: [3, 4, 1, 1, 4, 3]

B: [5, 1, 4, 6, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 3 | 4 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2
See Matrix

$$\$ [[7, 0, 6, 3, 0, 0], [9, 0, 7, 0, 0, 0], [7, 0, 9, 0, 0, 0]] \$$$

$$[y_1, 0, y_2, y_3, 0, 0]$$

Omega Rank for B : cycles: {{1, 2, 5}, {4, 6}} order: 6

See Matrix

$$\$ [[1, 2, 0, 5, 4, 4], [2, 4, 0, 4, 1, 5], [4, 1, 0, 5, 2, 4], [1, 2, 0, 4, 4, 5], [2, 4, 0, 5, 1, 4]] \$$$

$$[7y_1, 7y_4, 0, 7y_3, 7y_2, 9y_1 + 9y_4 - 7y_3 + 9y_2]$$

$$p = -s - s^2 + s^4 + s^5$$

10 . Coloring, {2, 6}

R: [3, 4, 1, 1, 2, 4]

B: [5, 1, 4, 6, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 5 vs 5 |

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$\$ [[7, 2, 4, 3, 0, 0], [7, 0, 7, 2, 0, 0], [9, 0, 7, 0, 0, 0], [7, 0, 9, 0, 0, 0]] \$$$

$$[y_4, y_2, y_3, y_1, 0, 0]$$

Omega Rank for B : cycles: {{3, 4, 6}} order: 3

See Matrix

$$\$ [[1, 0, 2, 5, 4, 4], [0, 0, 4, 6, 1, 5], [0, 0, 5, 5, 0, 6], [0, 0, 6, 5, 0, 5], [0, 0, 5, 6, 0, 5]] \$$$

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

11 . Coloring, {3, 4}

R: [3, 1, 4, 6, 2, 3]

B: [5, 4, 1, 1, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 5 vs 5 | 3 vs 3 |

Omega Rank for R : cycles: {{3, 4, 6}} order: 3

See Matrix

\$ [[1, 2, 6, 3, 0, 4] , [2, 0, 5, 6, 0, 3] , [0, 0, 5, 5, 0, 6] , [0, 0, 6, 5, 0, 5] , [0, 0, 5, 6, 0, 5]] \$

[$y_1, y_4, y_3, y_2, 0, y_5$]

Omega Rank for B : cycles: {{1, 4, 5}} order: 3

See Matrix

\$ [[7, 0, 0, 5, 4, 0] , [5, 0, 0, 4, 7, 0] , [4, 0, 0, 7, 5, 0]] \$

[$y_1, 0, 0, y_2, y_3, 0$]

12 . Coloring, {3, 5}

R: [3, 1, 4, 1, 4, 3]

B: [5, 4, 1, 6, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 3 | 5 vs 5 |

Omega Rank for R : cycles: {{1, 3, 4}} order: 3

See Matrix

$$\$ [[5, 0, 6, 5, 0, 0], [5, 0, 5, 6, 0, 0], [6, 0, 5, 5, 0, 0]] \$$$

$$[y_3, 0, y_1, y_2, 0, 0]$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$ order: 4

See Matrix

$$\$ [[3, 2, 0, 3, 4, 4], [0, 4, 0, 6, 3, 3], [0, 3, 0, 7, 0, 6], [0, 0, 0, 9, 0, 7], [0, 0, 0, 7, 0, 9]] \$$$

$$[y_5, y_4, 0, y_3, y_1, y_2]$$

13 . Coloring, $\{3, 6\}$

R: [3, 1, 4, 1, 2, 4]

B: [5, 4, 1, 6, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 5 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3, 4\}\}$ order: 3

See Matrix

$$\$ [[5, 2, 4, 5, 0, 0], [7, 0, 5, 4, 0, 0], [4, 0, 7, 5, 0, 0], [5, 0, 4, 7, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: $\{\{1, 3, 4, 5, 6\}\}$ order: 5

See Matrix

$$\$ [[3, 0, 2, 3, 4, 4], [2, 0, 4, 4, 3, 3], [4, 0, 3, 3, 2, 4], [3, 0, 4, 2, 4, 3], [4, 0, 3, 4, 3, 2]] \$$$

$$[y_5, 0, y_4, y_3, y_1, y_2]$$

14 . Coloring, $\{4, 5\}$

R: [3, 1, 1, 6, 4, 3]

B: [5, 4, 4, 1, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 4 | 4 vs 4 |

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$\$ [[4, 0, 6, 2, 0, 4], [6, 0, 8, 0, 0, 2], [8, 0, 8, 0, 0, 0], [8, 0, 8, 0, 0, 0]] \$$$

$$[y_2 + y_3 - y_1, 0, y_2, y_3, 0, y_1]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$\$ [[4, 2, 0, 6, 4, 0], [6, 4, 0, 2, 4, 0], [2, 4, 0, 4, 6, 0], [4, 6, 0, 4, 2, 0]] \$$$

$$[y_3, y_4, 0, y_1, y_2, 0]$$

15 . Coloring, {4, 6}

R: [3, 1, 1, 6, 2, 4]

B: [5, 4, 4, 1, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 5 | 4 vs 4 |

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6\}\}$ order: 2
 See Matrix

$$\$ [[4, 2, 4, 2, 0, 4] , [6, 0, 4, 4, 0, 2] , [4, 0, 6, 2, 0, 4] , [6, 0, 4, 4, 0, 2] , [4, 0, 6, 2, 0, 4]] \$$$

$$[y_3, 4y_3 - y_2 - 5y_1, y_2, y_1, 0, 3y_3 - 4y_1]$$

$$p' = s^2 - s^4 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{1, 4, 5\}\}$ order: 3
 See Matrix

$$\$ [[4, 0, 2, 6, 4, 0] , [6, 0, 0, 6, 4, 0] , [6, 0, 0, 4, 6, 0] , [4, 0, 0, 6, 6, 0]] \$$$

$$[y_1, 0, y_2, y_3, y_4, 0]$$

16 . Coloring, $\{5, 6\}$

R: [3, 1, 1, 1, 4, 4]

B: [5, 4, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 2 vs 3 | 5 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2
 See Matrix

$$\$ [[8, 0, 4, 4, 0, 0] , [8, 0, 8, 0, 0, 0] , [8, 0, 8, 0, 0, 0]] \$$$

$$[y_1 + y_2, 0, y_1, y_2, 0, 0]$$

$$p = -s^2 + s^3$$

Omega Rank for B : cycles: $\{\{3, 4, 6\}\}$ order: 3
 See Matrix

\$ [[0, 2, 2, 4, 4, 4] , [0, 4, 4, 4, 0, 4] , [0, 0, 4, 8, 0, 4] , [0, 0, 4, 4, 0, 8] , [0, 0, 8, 4, 0, 4]] \$

[0, y₁, y₂, y₃, y₄, y₅]

17 . Coloring, {2, 3, 4}

R: [3, 4, 4, 6, 2, 3]

B: [5, 1, 1, 1, 4, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | A+(1/2) Δ | A-(1/2) Δ | R | B |
|----------------|------------------|------------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 4 vs 4 | 3 vs 3 |

Omega Rank for R : cycles: {{3, 4, 6}} order: 3

See Matrix

\$ [[0, 2, 6, 4, 0, 4] , [0, 0, 4, 8, 0, 4] , [0, 0, 4, 4, 0, 8] , [0, 0, 8, 4, 0, 4]] \$

[0, y₃, y₄, y₂, 0, y₁]

Omega Rank for B : cycles: {{1, 4, 5}} order: 3

See Matrix

\$ [[8, 0, 0, 4, 4, 0] , [4, 0, 0, 4, 8, 0] , [4, 0, 0, 8, 4, 0]] \$

[y₁, 0, 0, y₂, y₃, 0]

18 . Coloring, {2, 3, 5}

R: [3, 4, 4, 1, 4, 3]

B: [5, 1, 1, 6, 2, 4]

‘ See graph

‘ ‘ See pair graph

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 3 | 4 vs 5 |

Omega Rank for R : cycles: {{1, 3, 4}} order: 3
See Matrix

$$\$ [[4, 0, 6, 6, 0, 0], [6, 0, 4, 6, 0, 0], [6, 0, 6, 4, 0, 0]] \$$$

$$[y_1, 0, y_2, y_3, 0, 0]$$

Omega Rank for B : cycles: {{1, 2, 5}, {4, 6}} order: 6
See Matrix

$$\$ [[4, 2, 0, 2, 4, 4], [2, 4, 0, 4, 4, 2], [4, 4, 0, 2, 2, 4], [4, 2, 0, 4, 4, 2], [2, 4, 0, 2, 4, 4]] \$$$

$$[-3 y_2 + 5 y_1 - 3 y_3 + 5 y_4, 3 y_2, 0, 3 y_1, 3 y_3, 3 y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

19 . Coloring, {2, 3, 6}

R: [3, 4, 4, 1, 2, 4]

B: [5, 1, 1, 6, 4, 3]

‘ See graph

‘ ‘ See pair graph

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 4 vs 4 | 5 vs 5 |

Omega Rank for R : cycles: {{1, 3, 4}} order: 3
See Matrix

$$\$ [[4, 2, 4, 6, 0, 0], [6, 0, 4, 6, 0, 0], [6, 0, 6, 4, 0, 0], [4, 0, 6, 6, 0, 0]] \$$$

$$[y_1, y_2, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

$$\$ [[4, 0, 2, 2, 4, 4], [2, 0, 4, 4, 4, 2], [4, 0, 2, 4, 2, 4], [2, 0, 4, 2, 4, 4], [4, 0, 4, 4, 2, 2]] \$$$

$$[y_2, 0, y_1, y_3, y_4, y_5]$$

20 . Coloring, {2, 4, 5}

R: [3, 4, 1, 6, 4, 3]

B: [5, 1, 4, 1, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 4 vs 4 |

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$\$ [[3, 0, 6, 3, 0, 4], [6, 0, 7, 0, 0, 3], [7, 0, 9, 0, 0, 0], [9, 0, 7, 0, 0, 0]] \$$$

$$[y_1, 0, y_2, y_3, 0, y_4]$$

Omega Rank for B : cycles: {{1, 2, 5}} order: 3

See Matrix

$$\$ [[5, 2, 0, 5, 4, 0], [7, 4, 0, 0, 5, 0], [4, 5, 0, 0, 7, 0], [5, 7, 0, 0, 4, 0]] \$$$

$$[y_4, y_3, 0, y_2, y_1, 0]$$

21 . Coloring, {2, 4, 6}

R: [3, 4, 1, 6, 2, 4]

B: [5, 1, 4, 1, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 5 | 4 vs 4 |

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6\}\}$ order: 2

See Matrix

$$\$ [[3, 2, 4, 3, 0, 4], [4, 0, 3, 6, 0, 3], [3, 0, 4, 3, 0, 6], [4, 0, 3, 6, 0, 3], [3, 0, 4, 3, 0, 6]] \$$$

$$[-7 y_1 + 15 y_2 - 7 y_3, 6 y_1, 6 y_2, -15 y_1 + 27 y_2 - 15 y_3, 0, 6 y_3]$$

$$p = -s^2 + s^4 \quad p' = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{1, 4, 5\}\}$ order: 3

See Matrix

$$\$ [[5, 0, 2, 5, 4, 0], [5, 0, 0, 6, 5, 0], [6, 0, 0, 5, 5, 0], [5, 0, 0, 5, 6, 0]] \$$$

$$[y_2, 0, y_1, y_3, y_4, 0]$$

22 . Coloring, $\{2, 5, 6\}$

R: [3, 4, 1, 1, 4, 4]

B: [5, 1, 4, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 3 | 3 vs 6 |

Omega Rank for R : cycles: $\{\{1, 3\}\}$ order: 2

See Matrix

$$\$ [[7, 0, 4, 5, 0, 0], [9, 0, 7, 0, 0, 0], [7, 0, 9, 0, 0, 0]] \$$$

$$[y_3, 0, y_1, y_2, 0, 0]$$

Omega Rank for B : cycles: {{1, 2, 5}, {3, 4, 6}} order: 3

See Matrix

$$\$ [[1, 2, 2, 3, 4, 4], [2, 4, 4, 2, 1, 3], [4, 1, 3, 4, 2, 2], [1, 2, 2, 3, 4, 4], [2, 4, 4, 2, 1, 3], [4, 1, 3, 4, 2, 2]] \$$$

$$[28 y_1 + 25 y_2 - 35 y_3, -20 y_2 + 28 y_3, 16 y_1, 20 y_1 + 27 y_2 - 25 y_3, 16 y_2, 16 y_3]$$

$$p' = 1 - s^3 \quad p' = s - s^4 \quad p' = s^2 - s^5$$

23 . Coloring, {3, 4, 5}

R: [3, 1, 4, 6, 4, 3]

B: [5, 4, 1, 1, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 4 vs 4 | 4 vs 4 |

Omega Rank for R : cycles: {{3, 4, 6}} order: 3

See Matrix

$$\$ [[1, 0, 6, 5, 0, 4], [0, 0, 5, 6, 0, 5], [0, 0, 5, 5, 0, 6], [0, 0, 6, 5, 0, 5]] \$$$

$$[y_1, 0, y_2, y_3, 0, y_4]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$\$ [[7, 2, 0, 3, 4, 0], [3, 4, 0, 2, 7, 0], [2, 7, 0, 4, 3, 0], [4, 3, 0, 7, 2, 0]] \$$$

$$[y_1, y_2, 0, y_3, y_4, 0]$$

24 . Coloring, {3, 4, 6}

R: [3, 1, 4, 6, 2, 4]

B: [5, 4, 1, 1, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 5 vs 5 | 4 vs 4 |

Omega Rank for R : cycles: {{4, 6}} order: 4

See Matrix

$$\$ [[1, 2, 4, 5, 0, 4], [2, 0, 1, 8, 0, 5], [0, 0, 2, 6, 0, 8], [0, 0, 0, 10, 0, 6], [0, 0, 0, 6, 0, 10]] \$$$

$$[y_4, y_1, y_2, y_3, 0, y_5]$$

Omega Rank for B : cycles: {{1, 4, 5}} order: 3

See Matrix

$$\$ [[7, 0, 2, 3, 4, 0], [5, 0, 0, 4, 7, 0], [4, 0, 0, 7, 5, 0], [7, 0, 0, 5, 4, 0]] \$$$

$$[y_3, 0, y_1, y_2, y_4, 0]$$

25 . Coloring, {3, 5, 6}

R: [3, 1, 4, 1, 4, 4]

B: [5, 4, 1, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 3 | 6 vs 6 |

Omega Rank for R : cycles: $\{\{1, 3, 4\}\}$ order: 3
 See Matrix

$$\$ [[5, 0, 4, 7, 0, 0], [7, 0, 5, 4, 0, 0], [4, 0, 7, 5, 0, 0]] \$$$

$$[y_3, 0, y_1, y_2, 0, 0]$$

Omega Rank for B : cycles: $\{\{1, 2, 3, 4, 5, 6\}\}$ order: 6
 See Matrix

$$\$ [[3, 2, 2, 1, 4, 4], [2, 4, 4, 2, 3, 1], [4, 3, 1, 4, 2, 2], [1, 2, 2, 3, 4, 4], [2, 4, 4, 2, 1, 3], [4, 1, 3, 4, 2, 2]] \$$$

$$[y_1, y_2, y_3, y_4, y_5, y_6]$$

26 . Coloring, $\{4, 5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^3 \quad p' = s^3 \quad p'' = s^4$$

R: [3, 1, 1, 6, 4, 4]

B: [5, 4, 4, 1, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 2 vs 5 | 2 vs 5 | 2 vs 5 | 1 vs 4 | 2 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6\}\}$ order: 2
 See Matrix

$$\$ [[4, 0, 4, 4, 0, 4], [4, 0, 4, 4, 0, 4], [4, 0, 4, 4, 0, 4], [4, 0, 4, 4, 0, 4]] \$$$

$$[y_1, 0, y_1, y_1, 0, y_1]$$

$$p' = -s + s^3 \quad p = s - s^2 \quad p'' = -s + s^2$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 5\}\}$ order: 4
 See Matrix

$\$ [[4, 2, 2, 4, 4, 0] , [4, 4, 0, 4, 4, 0] , [4, 4, 0, 4, 4, 0] , [4, 4, 0, 4, 4, 0] , [4, 4, 0, 4, 4, 0]] \$$

$$[y_2, y_2 - y_1, y_1, y_2, y_2, 0]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4 \quad p = -s^2 + s^5$$

‘ See 4-level graph

‘

M N

$\$ [[0, 8, 24, 36, 18, 18] , [8, 0, 0, 9, 9, 0] , [24, 0, 0, 27, 9, 18] , [36, 9, 27, 0, 16, 16] , [18, 9, 9, 16, 0, 0] , [18, 0, 18, 16, 0, 0]] \$$ $\$ [[0, 1, 1, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 0] , [1, 1, 1, 1, 0, 0]] \$$

$$\tau = 10, r' = 3/4$$

R: [3, 1, 1, 6, 4, 4]

B: [5, 4, 4, 1, 2, 3]

Ranges

Action of R on ranges, [[3], [3], [3]]

Action of B on ranges, [[1], [1], [2]]

Cycles: R, {{1, 3}, {4, 6}}, B, {{1, 2, 4, 5}}

$$\beta(\{1, 2, 4, 5\}) = 1/4$$

$$\beta(\{1, 3, 4, 5\}) = 1/4$$

$$\beta(\{1, 3, 4, 6\}) = 1/2$$

Partitions

$$\alpha(\{\{1\}, \{5, 6\}, \{4\}, \{2, 3\}\}) = 1/1$$

$$b_1 = \{1\} \text{ ‘ , ‘ } b_2 = \{5, 6\} \text{ ‘ , ‘ } b_3 = \{4\} \text{ ‘ , ‘ } b_4 = \{2, 3\}$$

Action of R and B on the blocks of the partitions: = [4, 3, 2, 1] [3, 1, 4, 2]

with invariant measure [1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-4 partition graph.

‘

| Right Group | |
|-----------------|---|
| Coloring | {4, 5, 6} |
| Rank | 4 |
| R,B | [3, 1, 1, 6, 4, 4], [5, 4, 4, 1, 2, 3] |
| π_2 | [8, 24, 36, 18, 18, 0, 9, 9, 0, 27, 9, 18, 16, 16, 0] |
| u_2 | [1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 2) |
| wpp | [1, 2, 2, 1, 2, 2] |
| π_4 | [0, 0, 0, 1, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0] |
| u_4 | [0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0] |

27 . Coloring, {2, 3, 4, 5}

R: [3, 4, 4, 6, 4, 3]

B: [5, 1, 1, 1, 2, 4]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 3 | 4 vs 4 |

Omega Rank for R : cycles: {{3, 4, 6}} order: 3

See Matrix

$$\$ [[0, 0, 6, 6, 0, 4] , [0, 0, 4, 6, 0, 6] , [0, 0, 6, 4, 0, 6]] \$$$

$$[0, 0, y_1, y_2, 0, y_3]$$

Omega Rank for B : cycles: {{1, 2, 5}} order: 3

See Matrix

$$\$ [[8, 2, 0, 2, 4, 0] , [4, 4, 0, 0, 8, 0] , [4, 8, 0, 0, 4, 0] , [8, 4, 0, 0, 4, 0]] \$$$

$$[y_4, y_3, 0, y_2, y_1, 0]$$

28 . Coloring, {2, 3, 4, 6}

R: [3, 4, 4, 6, 2, 4]

B: [5, 1, 1, 1, 4, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 4 | 4 vs 4 |

Omega Rank for R : cycles: {{4, 6}} order: 2

See Matrix

$$\$ [[0, 2, 4, 6, 0, 4], [0, 0, 0, 10, 0, 6], [0, 0, 0, 6, 0, 10], [0, 0, 0, 10, 0, 6]] \$$$

$$[0, y_1, 2y_1, y_3, 0, y_2]$$

$$p = s^2 - s^4$$

Omega Rank for B : cycles: {{1, 4, 5}} order: 3

See Matrix

$$\$ [[8, 0, 2, 2, 4, 0], [4, 0, 0, 4, 8, 0], [4, 0, 0, 8, 4, 0], [8, 0, 0, 4, 4, 0]] \$$$

$$[y_1, 0, y_4, y_2, y_3, 0]$$

29 . Coloring, {2, 3, 5, 6}

R: [3, 4, 4, 1, 4, 4]

B: [5, 1, 1, 6, 2, 3]

‘ See graph

‘ ‘ See pair graph

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 5 vs 5 | 5 vs 5 | 3 vs 3 | 5 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3, 4\}\}$ order: 3
See Matrix

$$\$ [[4, 0, 4, 8, 0, 0], [8, 0, 4, 4, 0, 0], [4, 0, 8, 4, 0, 0]] \$$$

$$[y_1, 0, y_3, y_2, 0, 0]$$

Omega Rank for B : cycles: $\{\{1, 2, 5\}\}$ order: 3
See Matrix

$$\$ [[4, 2, 2, 0, 4, 4], [4, 4, 4, 0, 4, 0], [8, 4, 0, 0, 4, 0], [4, 4, 0, 0, 8, 0], [4, 8, 0, 0, 4, 0]] \$$$

$$[y_5, y_3, y_4, 0, y_1, y_2]$$

30 . Coloring, $\{2, 4, 5, 6\}$

R: [3, 4, 1, 6, 4, 4]

B: [5, 1, 4, 1, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 2 vs 4 | 5 vs 5 |

Omega Rank for R : cycles: $\{\{1, 3\}, \{4, 6\}\}$ order: 2
See Matrix

$$\$ [[3, 0, 4, 5, 0, 4], [4, 0, 3, 4, 0, 5], [3, 0, 4, 5, 0, 4], [4, 0, 3, 4, 0, 5]] \$$$

$$[-8 y_2 + 7 y_1, 0, y_2, y_1, 0, -9 y_2 + 8 y_1]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

Omega Rank for B : cycles: $\{\{1, 2, 5\}\}$ order: 3
 See Matrix

$$\$ [[5, 2, 2, 3, 4, 0], [5, 4, 0, 2, 5, 0], [6, 5, 0, 0, 5, 0], [5, 5, 0, 0, 6, 0], [5, 6, 0, 0, 5, 0]] \$$$

$$[y_4, y_5, y_2, y_1, y_3, 0]$$

31 . Coloring, $\{3, 4, 5, 6\}$

R: [3, 1, 4, 6, 4, 4]

B: [5, 4, 1, 1, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 6 vs 6 | 6 vs 6 | 3 vs 4 | 5 vs 5 |

Omega Rank for R : cycles: $\{\{4, 6\}\}$ order: 4
 See Matrix

$$\$ [[1, 0, 4, 7, 0, 4], [0, 0, 1, 8, 0, 7], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[y_3, 0, y_3 + y_1 - y_2, y_1, 0, y_2]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: $\{\{1, 2, 4, 5\}\}$ order: 4
 See Matrix

$$\$ [[7, 2, 2, 1, 4, 0], [3, 4, 0, 2, 7, 0], [2, 7, 0, 4, 3, 0], [4, 3, 0, 7, 2, 0], [7, 2, 0, 3, 4, 0]] \$$$

$$[y_2, y_1, y_4, y_5, y_3, 0]$$

32 . Coloring, $\{2, 3, 4, 5, 6\}$

R: [3, 4, 4, 6, 4, 4]

B: [5, 1, 1, 1, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

| Δ -Rank | $A+(1/2)\Delta$ | $A-(1/2)\Delta$ | R | B |
|----------------|-----------------|-----------------|----------|----------|
| 5 vs 5 | 4 vs 5 | 5 vs 5 | 2 vs 3 | 4 vs 4 |

Omega Rank for R : cycles: {{4, 6}} order: 2

See Matrix

$$\$ [[0, 0, 4, 8, 0, 4], [0, 0, 0, 8, 0, 8], [0, 0, 0, 8, 0, 8]] \$$$

$$[0, 0, y_1 - y_2, y_1, 0, y_2]$$

$$p = -s^2 + s^3$$

Omega Rank for B : cycles: {{1, 2, 5}} order: 3

See Matrix

$$\$ [[8, 2, 2, 0, 4, 0], [4, 4, 0, 0, 8, 0], [4, 8, 0, 0, 4, 0], [8, 4, 0, 0, 4, 0]] \$$$

$$[y_3, y_2, y_1, 0, y_4, 0]$$

| SUMMARY | |
|-------------------|--------------------|
| Graph Type | CC |
| $v(A)$ | 1 |
| $v(\Delta)$ | 1 |
| π | [4, 1, 3, 4, 2, 2] |
| Dbly Stoch | false |

| SANDWICH | | Total 1 |
|-----------------|-----------------|-------------|
| No . | Coloring | Rank |
| 1 | {} | 2 |

| RT GROUPS | | Total 1 | |
|------------------|-----------------|-------------|-------------|
| No . | Coloring | Rank | Solv |
| 1 | {4, 5, 6} | 4 | Solvable |

| CC Colorings | | Total 1 |
|---------------------|-----------------|----------------------|
| No . | Coloring | Sandwich,Rank |
| 1 | {} | true, 2 |

| Δ-RANK'D | SC'D !RK'D | τ-RANK'D | R/B RANK'D | NOT SYNC'D | Total Runs | 2^{n-1} |
|-----------------------------------|-------------------|---------------------------------|-------------------|-------------------|-------------------|-----------------------------|
| 30 | 0 | 29 , 29 | 19 , 24 | 2 | 32 | 32 |
