New Graph

[6, 6, 6, 6, 6, 4], [5, 5, 1, 5, 2, 3]

π = [1, 1, 2, 2, 2, 4]
δ= [1, 1, 1, 1, 3, 5]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 33/256, .1289062500

NullSpace of Δ

{1, 2, 5, 6}, {3, 4}

Nullspace of A

[{3},{4}] `,` [{1, 2, 5},{6}]

1 . Coloring, {}

R: [6, 6, 6, 6, 6, 4]
B: [5, 5, 1, 5, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	2 *vs* 2	4 *vs* 4

Omega Rank for R :
$-t + t^3$
,
cycles: {{4, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 4 \end{pmatrix}$$

$[0, 0, 0, y_2, 0, y_1]$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{2, 5}} order: 4

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 4 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$[y_1, y_2, y_3, 0, y_4, 0]$$

2 . Coloring, {2}

R: [6, 5, 6, 6, 6, 4]
B: [5, 6, 1, 5, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	5 *vs* 5

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{4, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 1 & 7 \\ 0 & 0 & 0 & 7 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 7 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2, y_3]$$

Omega Rank for B :
$-t + t^6$
,
cycles: {{1, 2, 3, 5, 6}} order: 5

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 3 & 1 \\ 4 & 3 & 1 & 0 & 2 & 2 \\ 1 & 2 & 2 & 0 & 4 & 3 \\ 2 & 4 & 3 & 0 & 1 & 2 \\ 3 & 1 & 2 & 0 & 2 & 4 \end{pmatrix}$$

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

3 . Coloring, {3}

Ωp(Δ)=0: $p' = s^2 - 2s^3$ $p = s^2 - 4s^4$

R: [6, 6, 1, 6, 6, 4]
B: [5, 5, 6, 5, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 3	2 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{4, 6}} order: 2

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \end{pmatrix}$$

$$[-y_1 + y_2, 0, 0, y_1, 0, y_2]$$

$p = -s^2 + s^3$

Omega Rank for B :
$-t + t^3$
,
cycles: {{2, 5}, {3, 6}} order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 4 & 2 \\ 0 & 4 & 2 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 4 & 2 \\ 0 & 4 & 2 & 0 & 2 & 4 \end{pmatrix}$$

$$[0, y_2, y_1, 0, y_1, y_2]$$

$p = -s + s^3 \quad p' = -s + s^3$

M N

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 1 & 1 & 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 4 \\ 1 & 1 & 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 4 \end{pmatrix}$$

τ= 20 , r'= 1/2

R: [6, 6, 1, 6, 6, 4]
B: [5, 5, 6, 5, 2, 3]

Ranges

Action of R on ranges, [[4], [4], [1], [4]]
Action of B on ranges, [[3], [3], [2], [3]]

Cycles: R , {{4, 6}}, B , {{2, 5}, {3, 6}}

β({1, 6}) = 1/6
β({2, 6}) = 1/6
β({3, 5}) = 1/3
β({4, 6}) = 1/3

Partitions
α([{1, 2, 4, 5}, {3, 6}]) = 1/1

b1 = {1, 2, 4, 5} ` , ` b2 = {3, 6}

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]
with invariant measure [1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Right Group	
Coloring	{3}
Rank	2
R,B	[6, 6, 1, 6, 6, 4], [5, 5, 6, 5, 2, 3]
π_2	[0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 0]
u_2	[0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
wpp	[4, 4, 2, 4, 4, 2]

4 . Coloring, {4}

R: [6, 6, 6, 5, 6, 4]
B: [5, 5, 1, 6, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	4 *vs* 5	3 *vs* 3	4 *vs* 5

Omega Rank for R :
$-t + t^4$
,
cycles: {{4, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 6 & 4 & 2 \\ 0 & 0 & 0 & 2 & 6 & 4 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2, y_3]$$

Omega Rank for B :
$-t^4 + t^6$
,
cycles: {{2, 5}}   order: 4

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 2 & 2 \\ 4 & 2 & 2 & 0 & 4 & 0 \\ 2 & 4 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \end{pmatrix}$$

$$[y_3, y_4, y_3 + y_4 - y_1 + y_2, 0, y_1, y_2]$$

p = $-s^4 + s^5$

5 . Coloring, {5}

Ωp(Δ)=0:   p' = $s^2 - 2s^3$ p = $s^2 - 4s^4$

R: [6, 6, 6, 6, 2, 4]
B: [5, 5, 1, 5, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 3	2 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{4, 6}}   order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \end{pmatrix}$$

$$[0, -y_1 + y_2, 0, y_1, 0, y_2]$$

p = $-s^2 + s^3$

Omega Rank for B :
$-t + t^5$
,
cycles: {{1, 3, 5, 6}}   order: 4

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 4 & 2 \\ 4 & 0 & 2 & 0 & 2 & 4 \\ 2 & 0 & 4 & 0 & 4 & 2 \\ 4 & 0 & 2 & 0 & 2 & 4 \end{pmatrix}$$

$$[y_2, 0, y_1, 0, y_1, y_2]$$

$p' = -s^{+}\ s^{3}$ $p = -s^{+}\ s^{3}$

M   N

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 2 & 3 \\ 1 & 1 & 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 0 & 2 & 3 \\ 2 & 2 & 3 & 2 & 0 & 1 \\ 3 & 3 & 2 & 3 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 3 & 3 & 4 & 6 & 2 & 0 \\ 3 & 3 & 4 & 6 & 2 & 0 \\ 2 & 2 & 6 & 4 & 0 & 4 \\ 3 & 3 & 4 & 6 & 2 & 0 \\ 1 & 1 & 0 & 2 & 6 & 8 \\ 0 & 0 & 2 & 0 & 4 & 12 \end{pmatrix}$$

τ= 20 , r'= 1/2

R: [6, 6, 6, 6, 2, 4]
B: [5, 5, 1, 5, 6, 3]

Ranges

Action of R on ranges, [[4], [4], [2], [4]]
Action of B on ranges, [[3], [3], [1], [3]]

Cycles: R , {{4, 6}}, B , {{1, 3, 5, 6}}

β({1, 6}) = 1/6
β({2, 6}) = 1/6
β({3, 5}) = 1/3
β({4, 6}) = 1/3

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 2, 3, 4}, {5, 6}]) = 2/3
α([{1, 2, 4, 5}, {3, 6}]) = 1/3

b1 = {1, 2, 3, 4} ` , ` b2 = {5, 6} ` , ` b3 = {1, 2, 4, 5} ` , ` b4 = {3, 6}

Action of R and B on the blocks of the partitions: = [2, 1, 2, 1] [4, 3, 1, 2]
with invariant measure [2, 2, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Sandwich	
Coloring	{5}
Rank	2
R,B	[6, 6, 6, 6, 2, 4], [5, 5, 1, 5, 6, 3]
π_2	[0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 0]
u_2	[0, 1, 0, 2, 3, 1, 0, 2, 3, 1, 3, 2, 2, 3, 1] (dim 1)
wpp	[12, 12, 10, 12, 8, 6]

6 . Coloring, {6}

R: [6, 6, 6, 6, 6, 3]
B: [5, 5, 1, 5, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	2 *vs* 2	3 *vs* 4

Omega Rank for R :
$-t^{+} t^{3}$

,
cycles: {{3, 6}}   order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 & 0 & 4 \end{pmatrix}$$

$$[0, 0, y_1, 0, 0, y_2]$$

Omega Rank for B :
tailcheck $-t^{2^{+}} t^{4}$

,
cycles: {{2, 5}}   order: 2

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 4 & 0 \\ 0 & 4 & 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 8 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, 2\, y_1, y_3, 0]$$

$p = -s^{2^{+}} s^{4}$

7 . Coloring, {2, 3}

R: [6, 5, 1, 6, 6, 4]
B: [5, 6, 6, 5, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	3 *vs* 4

Omega Rank for R :
tailcheck $-t^2 + t^4$

,
cycles: {{4, 6}}   order: 2

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 1 & 5 \\ 0 & 0 & 0 & 5 & 0 & 7 \\ 0 & 0 & 0 & 7 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 7 \end{pmatrix}$$

$$[2\,y_1, 0, 0, y_2, y_1, y_3]$$

$p = -s^2 + s^4$

Omega Rank for B :
$-t^3 + t^5$

,
cycles: {{3, 6}}   order: 4

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 3 & 3 \\ 0 & 3 & 3 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$$[0, y_2, y_3, 0, y_2 + y_3 - y_1, y_1]$$

$p = -s^3 + s^4$

8 . Coloring, {2, 4}

R: [6, 5, 6, 5, 6, 4]
B: [5, 6, 1, 6, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	5 *vs* 5

Omega Rank for R :

$-t^{'} \; t^4$

,
cycles: {{4, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 3 & 5 \\ 0 & 0 & 0 & 5 & 4 & 3 \\ 0 & 0 & 0 & 3 & 5 & 4 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2, y_3]$$

Omega Rank for B :
$-t^{+} \; t^6$

,
cycles: {{1, 2, 3, 5, 6}} order: 5

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 1 & 3 \\ 4 & 1 & 3 & 0 & 2 & 2 \\ 3 & 2 & 2 & 0 & 4 & 1 \\ 2 & 4 & 1 & 0 & 3 & 2 \\ 1 & 3 & 2 & 0 & 2 & 4 \end{pmatrix}$$

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

9 . Coloring, {2, 5}

R: [6, 5, 6, 6, 2, 4]
B: [5, 6, 1, 5, 6, 3]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	4 *vs* 4

Omega Rank for R :
$-t^{+} \; t^3$

,
cycles: {{4, 6}, {2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 1 & 5 \\ 0 & 1 & 0 & 5 & 2 & 4 \\ 0 & 2 & 0 & 4 & 1 & 5 \\ 0 & 1 & 0 & 5 & 2 & 4 \end{pmatrix}$$

$$[0, y_1 - 2\, y_2, 0, y_1, y_2, 2\, y_1 - 3\, y_2]$$

$p = -s^{+} \; s^3 \quad p' = -s^{+} \; s^3$

Omega Rank for B :
$-t^{+} \; t^5$

,
cycles: {{1, 3, 5, 6}} order: 4

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 3 & 3 \\ 4 & 0 & 3 & 0 & 2 & 3 \\ 3 & 0 & 3 & 0 & 4 & 2 \\ 3 & 0 & 2 & 0 & 3 & 4 \end{pmatrix}$$

$$[y_1, 0, y_2, 0, y_3, y_4]$$

10 . Coloring, {2, 6}

R: [6, 5, 6, 6, 6, 3]
B: [5, 6, 1, 5, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	5 *vs* 5

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 1 & 7 \\ 0 & 0 & 7 & 0 & 0 & 5 \\ 0 & 0 & 5 & 0 & 0 & 7 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

Omega Rank for B :
$-t^2 + t^6$
,
cycles: {{2, 4, 5, 6}} order: 4

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 3 & 1 \\ 0 & 3 & 0 & 1 & 6 & 2 \\ 0 & 6 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & 3 & 2 & 6 \\ 0 & 2 & 0 & 6 & 3 & 1 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

11 . Coloring, {3, 4}

Ωp(Δ)=0: $p = s^3 - 2s^4$

R: [6, 6, 1, 5, 6, 4]
B: [5, 5, 6, 6, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	2 *vs* 4	1 *vs* 4

Omega Rank for R :
$-t^2 + t^5$
,
cycles: {{4, 5, 6}} order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 2 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \end{pmatrix}$$

$[y_2, 0, 0, y_1, -y_2 + y_1, y_1]$

$p = s^2 - s^3$ $p' = -s^2 + s^3$

Omega Rank for B :
$-t + t^3$
,
cycles: {{2, 5}, {3, 6}} order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 2 & 4 \end{pmatrix}$$

$[0, y_1, 2 y_1, 0, y_1, 2 y_1]$

$p = -s + s^3$ $p = -s + s^4$ $p = -s + s^2$

` See 3-level graph

`

M N

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 2 & 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 2 & 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 1 & 1 & 4 & 4 & 2 & 4 \\ 1 & 1 & 4 & 4 & 2 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 1 & 1 & 2 & 2 & 2 & 8 \end{pmatrix}$$

τ= 14 , r'= 2/3

R: [6, 6, 1, 5, 6, 4]
B: [5, 5, 6, 6, 2, 3]

Ranges

Action of R on ranges, [[4], [1], [1], [4]]
Action of B on ranges, [[3], [3], [2], [2]]

Cycles: R , {{4, 5, 6}}, B , {{2, 5}, {3, 6}}

β({1, 4, 6}) = 1/4
β({2, 3, 6}) = 1/4
β({3, 5, 6}) = 1/4
β({4, 5, 6}) = 1/4

Partitions
α([{1, 2, 5}, {6}, {3, 4}]) = 1/1

b1 = {1, 2, 5} ` , ` b2 = {6} ` , ` b3 = {3, 4}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3, 4}
Rank	3
R,B	[6, 6, 1, 5, 6, 4], [5, 5, 6, 6, 2, 3]
π_2	[0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 2, 1, 2, 2]
u_2	[0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1] (dim 1)
wpp	[3, 3, 2, 2, 3, 1]
π_3	[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1]
u_3	[0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]

12 . Coloring, {3, 5}

R: [6, 6, 1, 6, 2, 4]
B: [5, 5, 6, 5, 6, 3]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 3

Omega Rank for R :
tailcheck $-t^2 + t^4$

,
cycles: {{4, 6}}   order: 2

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 4 \\ 0 & 0 & 0 & 4 & 0 & 8 \end{pmatrix}$$

$$[y_1, y_1, 0, y_2, 0, y_3]$$

$p = -s^2 + s^4$

Omega Rank for B :
$-t^2 + t^4$

,
cycles: {{3, 6}}   order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 4 & 4 \\ 0 & 0 & 4 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 & 0 & 4 \end{pmatrix}$$

$$[0, 0, y_3, 0, y_2, y_1]$$

13 . Coloring, {3, 6}

R: [6, 6, 1, 6, 6, 3]
B: [5, 5, 6, 5, 2, 4]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 3	3 *vs* 4

Omega Rank for R :
$-t + t^4$

,
cycles: {{1, 3, 6}}   order: 3

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 0 & 6 \\ 4 & 0 & 6 & 0 & 0 & 2 \\ 6 & 0 & 2 & 0 & 0 & 4 \end{pmatrix}$$

$$[y_1, 0, y_2, 0, 0, y_3]$$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{2, 5}}   order: 4

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 4 & 2 \\ 0 & 4 & 0 & 2 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \end{pmatrix}$$

$$[0, y_3, 0, -y_3 + y_1 + y_2, y_1, y_2]$$

$p = -s^3 + s^4$

14 . Coloring, {4, 5}

R: [6, 6, 6, 5, 2, 4]
B: [5, 5, 1, 6, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t + t^5$
,
cycles: {{2, 4, 5, 6}}   order: 4

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 2 & 4 \\ 0 & 2 & 0 & 4 & 4 & 2 \\ 0 & 4 & 0 & 2 & 4 & 2 \\ 0 & 4 & 0 & 2 & 2 & 4 \end{pmatrix}$$

$$[0, -y_1 + y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = -s + s^2 - s^3 + s^4$

Omega Rank for B :
$-t + t^5$
,
cycles: {{1, 3, 5, 6}}   order: 4

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 2 & 4 \\ 4 & 0 & 4 & 0 & 2 & 2 \\ 4 & 0 & 2 & 0 & 4 & 2 \\ 2 & 0 & 2 & 0 & 4 & 4 \end{pmatrix}$$

$$[y_1 + y_2 - y_3, 0, y_1, 0, y_2, y_3]$$

p = $s - s^2 + s^3 - s^4$

15 . Coloring, {4, 6}

Ωp(Δ)=0: p = $s^2 - 4s^4$ p' = $s^2 - 2s^3$

R: [6, 6, 6, 5, 6, 3]
B: [5, 5, 1, 6, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	3 *vs* 5	3 *vs* 5	2 *vs* 3	3 *vs* 5

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 2 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$$[0, 0, -y_1 + y_2, 0, y_1, y_2]$$

p = $-s^2 + s^3$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{4, 6}, {2, 5}} order: 2

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 2 & 2 \\ 0 & 2 & 0 & 2 & 4 & 4 \\ 0 & 4 & 0 & 4 & 2 & 2 \\ 0 & 2 & 0 & 2 & 4 & 4 \\ 0 & 4 & 0 & 4 & 2 & 2 \end{pmatrix}$$

$$[y_3, y_2, 0, y_3 + y_2, y_1, y_1]$$

p = $s^2 - s^4$ p' = $-s^2 + s^4$

M N

$$\begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&0&2 \\ 1&1&0&0&0&0 \\ 0&0&0&0&0&2 \\ 0&0&2&0&2&0 \end{pmatrix} \begin{pmatrix} 0&0&0&1&0&1 \\ 0&0&0&1&0&1 \\ 0&0&0&1&0&1 \\ 1&1&1&0&1&0 \\ 0&0&0&1&0&1 \\ 1&1&1&0&1&0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 1&1&2&0&2&0 \\ 1&1&2&0&2&0 \\ 1&1&2&0&2&0 \\ 0&0&0&2&0&4 \\ 1&1&2&0&2&0 \\ 0&0&0&2&0&4 \end{pmatrix}$$

τ= 20 , r'= 1/2

R: [6, 6, 6, 5, 6, 3]
B: [5, 5, 1, 6, 2, 4]

Ranges

Action of R on ranges, [[4], [4], [3], [3]]
Action of B on ranges, [[4], [4], [1], [2]]

Cycles: R , {{3, 6}}, B , {{4, 6}, {2, 5}}

β({1, 4}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3
β({5, 6}) = 1/3

Partitions
α([{4, 6}, {1, 2, 3, 5}]) = 1/1

b1 = {4, 6} ` , ` b2 = {1, 2, 3, 5}

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]
with invariant measure [1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Right Group	
Coloring	{4, 6}
Rank	2
R,B	[6, 6, 6, 5, 6, 3], [5, 5, 1, 6, 2, 4]
π_2	[0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0, 2]
u_2	[0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1] (dim 1)

wpp	[4, 4, 4, 2, 4, 2]

16 . Coloring, {5, 6}

R: [6, 6, 6, 6, 2, 3]
B: [5, 5, 1, 5, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	3 *vs* 4	4 *vs* 4	2 *vs* 3	4 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$$[0, -y_1 + y_2, y_1, 0, 0, y_2]$$

$p = -s^2 + s^3$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{4, 5, 6}} order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 4 & 2 \\ 0 & 0 & 0 & 2 & 6 & 4 \\ 0 & 0 & 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 6 & 4 & 2 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_3, y_4]$$

17 . Coloring, {2, 3, 4}

R: [6, 5, 1, 5, 6, 4]
B: [5, 6, 6, 6, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^{2^{+}} \; t^5$
,
cycles: {{4, 5, 6}}   order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 3 & 3 \\ 0 & 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 5 & 3 & 4 \\ 0 & 0 & 0 & 4 & 5 & 3 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_3, y_4]$$

Omega Rank for B :
$-t^{3^{+}} \; t^5$
,
cycles: {{3, 6}}   order: 4

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 1 & 5 \\ 0 & 1 & 5 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$$[0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3]$$

$p = -s^{3^{+}} \; s^4$

18 . Coloring, {2, 3, 5}

R: [6, 5, 1, 6, 2, 4]
B: [5, 6, 6, 5, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 5	3 *vs* 3

Omega Rank for R :
$-t^{2^{+}} \; t^4$
,
cycles: {{4, 6}, {2, 5}}   order: 2

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 1 & 3 \\ 0 & 1 & 0 & 3 & 2 & 6 \\ 0 & 2 & 0 & 6 & 1 & 3 \\ 0 & 1 & 0 & 3 & 2 & 6 \\ 0 & 2 & 0 & 6 & 1 & 3 \end{pmatrix}$$

$$[y_3, y_2, 0, -y_3 + 3\, y_2, y_1, 3\, y_1]$$

$p' = -s^{2^{+}} \; s^4$ $p = -s^{2^{+}} \; s^4$

Omega Rank for B :

$-t^{2}$ ' t^{4}

,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 3 & 5 \\ 0 & 0 & 5 & 0 & 0 & 7 \\ 0 & 0 & 7 & 0 & 0 & 5 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

19 . Coloring, {2, 3, 6}

R: [6, 5, 1, 6, 6, 3]
B: [5, 6, 6, 5, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^{2}$ + t^{5}

,
cycles: {{1, 3, 6}} order: 3

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 1 & 5 \\ 4 & 0 & 5 & 0 & 0 & 3 \\ 5 & 0 & 3 & 0 & 0 & 4 \\ 3 & 0 & 4 & 0 & 0 & 5 \end{pmatrix}$$

$$[y_1, 0, y_3, 0, y_2, y_4]$$

Omega Rank for B :
$-t$ + t^{5}

,
cycles: {{2, 4, 5, 6}} order: 4

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 3 & 3 \\ 0 & 3 & 0 & 3 & 4 & 2 \\ 0 & 4 & 0 & 2 & 3 & 3 \\ 0 & 3 & 0 & 3 & 2 & 4 \end{pmatrix}$$

$$[0, -y_1 + y_2 + y_3, 0, y_1, y_2, y_3]$$

p = $-s$ + $s^{2} - s^{3}$ + s^{4}

20 . Coloring, {2, 4, 5}

R: [6, 5, 6, 5, 2, 4]

B: [5, 6, 1, 6, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R :
$-t^3 + t^5$
,
cycles: {{2, 5}}   order: 4

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 3 & 3 \\ 0 & 3 & 0 & 3 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3, y_1 + y_2 - y_3]$$

p = $-s^3 + s^4$

Omega Rank for B :
$-t + t^5$
,
cycles: {{1, 3, 5, 6}}   order: 4

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 1 & 5 \\ 4 & 0 & 5 & 0 & 2 & 1 \\ 5 & 0 & 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 0 & 5 & 4 \end{pmatrix}$$

$$[y_1, 0, y_4, 0, y_2, y_3]$$

21 . Coloring, {2, 4, 6}

R: [6, 5, 6, 5, 6, 3]
B: [5, 6, 1, 6, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 3	5 *vs* 5

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{3, 6}}   order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 3 & 5 \\ 0 & 0 & 5 & 0 & 0 & 7 \\ 0 & 0 & 7 & 0 & 0 & 5 \end{pmatrix}$$

$$[0, 0, y_3, 0, y_2, y_1]$$

Omega Rank for B :
$-t^4 + t^6$
,
cycles: {{4, 6}}   order: 4

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 1 & 3 \\ 0 & 1 & 0 & 3 & 2 & 6 \\ 0 & 2 & 0 & 6 & 0 & 4 \\ 0 & 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 4 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

22 . Coloring, {2, 5, 6}

$\Omega p(\Delta)=0$: $p = s - 2s^3 + 4s^4$

R: [6, 5, 6, 6, 2, 3]
B: [5, 6, 1, 5, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 5	4 *vs* 5	2 *vs* 4	4 *vs* 4

Omega Rank for R :
$-t + t^3$
,
cycles: {{2, 5}, {3, 6}}   order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 1 & 5 \\ 0 & 1 & 5 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 1 & 5 \\ 0 & 1 & 5 & 0 & 2 & 4 \end{pmatrix}$$

$$[0, y_1, -3 y_1 + 2 y_2, 0, -2 y_1 + y_2, y_2]$$

$p' = s - s^3$ $p = s - s^3$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{4, 5, 6}}   order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 3 & 3 \\ 0 & 0 & 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 3 & 3 & 6 \\ 0 & 0 & 0 & 6 & 3 & 3 \end{pmatrix}$$

$[y_4, 0, 0, y_1, y_2, y_3]$

23 . Coloring, {3, 4, 5}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^{2^+}\ 2s^3$

R: [6, 6, 1, 5, 2, 4]
B: [5, 5, 6, 6, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 5	2 *vs* 3

Omega Rank for R :
$-t^{2^+}\ t^6$
,
cycles: {{2, 4, 5, 6}} order: 4

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 2 & 2 \\ 0 & 2 & 0 & 2 & 4 & 4 \\ 0 & 4 & 0 & 4 & 2 & 2 \\ 0 & 2 & 0 & 2 & 4 & 4 \\ 0 & 4 & 0 & 4 & 2 & 2 \end{pmatrix}$$

$[y_2, y_3, 0, y_2 + y_3, y_1, y_1]$

$p = -s^{2^+}\ s^4$ $p' = -s^{2^+}\ s^4$

Omega Rank for B :
$-t^{2^+}\ t^4$
,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 2 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$[0, 0, -y_1 + y_2, 0, y_1, y_2]$

$p = -s^{2^+}\ s^3$

M N

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 & 3 & 2 & 1 \\ 0 & 0 & 2 & 3 & 2 & 1 \\ 2 & 2 & 0 & 1 & 0 & 3 \\ 3 & 3 & 1 & 0 & 1 & 2 \\ 2 & 2 & 0 & 1 & 0 & 3 \\ 1 & 1 & 3 & 2 & 3 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 3 & 3 & 2 & 0 & 2 & 8 \\ 3 & 3 & 2 & 0 & 2 & 8 \\ 1 & 1 & 6 & 4 & 6 & 0 \\ 0 & 0 & 4 & 6 & 4 & 4 \\ 1 & 1 & 6 & 4 & 6 & 0 \\ 2 & 2 & 0 & 2 & 0 & 12 \end{pmatrix}$$

τ= 56 -- 3 , r'= 1/2

R: [6, 6, 1, 5, 2, 4]
B: [5, 5, 6, 6, 6, 3]

Ranges

Action of R on ranges, [[4], [4], [1], [2]]
Action of B on ranges, [[4], [4], [3], [3]]

Cycles: R , {{2, 4, 5, 6}}, B , {{3, 6}}

β({1, 4}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3
β({5, 6}) = 1/3

Partitions

Action of R on partitions, [[2], [1]]
Action of B on partitions, [[2], [2]]

α([{4, 6}, {1, 2, 3, 5}]) = 1/3
α([{1, 2, 6}, {3, 4, 5}]) = 2/3

b1 = {4, 6} ` , ` b2 = {1, 2, 6} ` , ` b3 = {3, 4, 5} ` , ` b4 = {1, 2, 3, 5}

Action of R and B on the blocks of the partitions: = [2, 4, 1, 3] [3, 3, 2, 2]
with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Sandwich	
Coloring	{3, 4, 5}
Rank	2
R,B	[6, 6, 1, 5, 2, 4], [5, 5, 6, 6, 6, 3]
π_2	[0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0, 2]

u_2	[0, 2, 3, 2, 1, 2, 3, 2, 1, 1, 0, 3, 1, 2, 3] (dim 1)
wpp	[10, 10, 10, 8, 10, 8]

24 . Coloring, {3, 4, 6}

Ωp(Δ)=0: $p = s^3 - 2s^4$

R: [6, 6, 1, 5, 6, 3]
B: [5, 5, 6, 6, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	2 *vs* 4	1 *vs* 4

Omega Rank for R :
$-t^2 + t^5$
,
cycles: {{1, 3, 6}} order: 3

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 2 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \end{pmatrix}$$

$$[-y_1 + y_2, 0, y_2, 0, y_1, y_2]$$

$p' = s^2 - s^3$ $p = s^2 - s^4$

Omega Rank for B :
$-t + t^3$
,
cycles: {{4, 6}, {2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 2 & 4 \\ 0 & 2 & 0 & 4 & 2 & 4 \\ 0 & 2 & 0 & 4 & 2 & 4 \\ 0 & 2 & 0 & 4 & 2 & 4 \end{pmatrix}$$

$$[0, y_1, 0, 2\,y_1, y_1, 2\,y_1]$$

$p = -s + s^2$ $p = -s + s^3$ $p = -s + s^4$

` See 3-level graph

`

M N

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 2 & 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 1 & 1 & 4 & 4 & 2 & 4 \\ 1 & 1 & 4 & 4 & 2 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 1 & 1 & 2 & 2 & 2 & 8 \end{pmatrix}$$

τ= 14 , r'= 2/3

R: [6, 6, 1, 5, 6, 3]
B: [5, 5, 6, 6, 2, 4]

Ranges

Action of R on ranges, [[1], [3], [1], [3]]
Action of B on ranges, [[4], [4], [2], [2]]

Cycles: R , {{1, 3, 6}}, B , {{4, 6}, {2, 5}}

β({1, 3, 6}) = 1/4
β({2, 4, 6}) = 1/4
β({3, 5, 6}) = 1/4
β({4, 5, 6}) = 1/4

Partitions
α([{1, 2, 5}, {6}, {3, 4}]) = 1/1

b1 = {1, 2, 5} ` , ` b2 = {6} ` , ` b3 = {3, 4}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3, 4, 6}
Rank	3
R,B	[6, 6, 1, 5, 6, 3], [5, 5, 6, 6, 2, 4]
π_2	[0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 2, 1, 2, 2]
u_2	[0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1] (dim 1)
wpp	[3, 3, 2, 2, 3, 1]

π_3	[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1]
u_3	[0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]

25 . Coloring, {3, 5, 6}

Ωp(Δ)=0: p = s^3 p' = s^3

R: [6, 6, 1, 6, 2, 3]
B: [5, 5, 6, 5, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4	1 *vs* 3

Omega Rank for R :
$-t^2 + t^5$

,
cycles: {{1, 3, 6}} order: 3

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \\ 4 & 0 & 4 & 0 & 0 & 4 \end{pmatrix}$$

$$[-y_1 + y_2, y_1, y_2, 0, 0, y_2]$$

p = $s^2 - s^4$ p' = $s^2 - s^3$

Omega Rank for B :
$-t + t^4$

,
cycles: {{4, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_1, y_1]$$

p = $-s + s^2$ p = $-s + s^3$

` See 3-level graph

`

M N

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 1 & 1 & 2 & 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 2 & 2 & 2 & 4 & 2 & 4 \\ 2 & 2 & 2 & 4 & 2 & 4 \\ 1 & 1 & 4 & 2 & 4 & 4 \\ 2 & 2 & 2 & 4 & 2 & 4 \\ 1 & 1 & 4 & 2 & 4 & 4 \\ 1 & 1 & 2 & 2 & 2 & 8 \end{pmatrix}$$

τ= 14 , r'= 2/3

R: [6, 6, 1, 6, 2, 3]
B: [5, 5, 6, 5, 6, 4]

Ranges

Action of R on ranges, [[1], [1], [2]]
Action of B on ranges, [[3], [3], [3]]

Cycles: R , {{1, 3, 6}}, B , {{4, 5, 6}}

β({1, 3, 6}) = 1/4
β({2, 3, 6}) = 1/4
β({4, 5, 6}) = 1/2

Partitions
α([{1, 2, 4}, {3, 5}, {6}]) = 1/1

b1 = {1, 2, 4} ` , ` b2 = {3, 5} ` , ` b3 = {6}

Action of R and B on the blocks of the partitions: = [2, 3, 1] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3, 5, 6}
Rank	3
R,B	[6, 6, 1, 6, 2, 3], [5, 5, 6, 5, 6, 4]
π_2	[0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2]
u_2	[0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1] (dim 1)
wpp	[3, 3, 2, 3, 2, 1]

π_3	[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2]
u_3	[0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1]

26 . Coloring, {4, 5, 6}

R: [6, 6, 6, 5, 2, 3]
B: [5, 5, 1, 6, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^3 + t^5$
,
cycles: {{3, 6}} order: 4

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 & 0 & 6 \end{pmatrix}$$

$$[0, -y_1 + y_2 + y_3, y_1, 0, y_2, y_3]$$

$p = -s^3 + s^4$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{4, 6}} order: 4

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 2 & 4 \\ 0 & 0 & 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \end{pmatrix}$$

$$[y_3, 0, 0, y_3 - y_1 + y_2, y_1, y_2]$$

$p = -s^3 + s^4$

27 . Coloring, {2, 3, 4, 5}

Ωp(Δ)=0: $p = s - 2s^3 + 4s^4$

R: [6, 5, 1, 5, 2, 4]
B: [5, 6, 6, 6, 6, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	3 *vs* 3

Omega Rank for R :
$-t^4 + t^6$
,
cycles: {{2, 5}} order: 4

$$\begin{pmatrix} 2 & 2 & 0 & 4 & 3 & 1 \\ 0 & 3 & 0 & 1 & 6 & 2 \\ 0 & 6 & 0 & 2 & 4 & 0 \\ 0 & 4 & 0 & 0 & 8 & 0 \\ 0 & 8 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 0 & 1 & 7 \\ 0 & 0 & 7 & 0 & 0 & 5 \\ 0 & 0 & 5 & 0 & 0 & 7 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

28 . Coloring, {2, 3, 4, 6}

R: [6, 5, 1, 5, 6, 3]
B: [5, 6, 6, 6, 2, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^2 + t^5$
,
cycles: {{1, 3, 6}} order: 3

$$\begin{pmatrix} 2 & 0 & 4 & 0 & 3 & 3 \\ 4 & 0 & 3 & 0 & 0 & 5 \\ 3 & 0 & 5 & 0 & 0 & 4 \\ 5 & 0 & 4 & 0 & 0 & 3 \end{pmatrix}$$

$$[y_1, 0, y_2, 0, y_3, y_4]$$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{4, 6}}   order: 4

$$\begin{pmatrix} 0 & 2 & 0 & 4 & 1 & 5 \\ 0 & 1 & 0 & 5 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \end{pmatrix}$$

$$[0, y_3, 0, -y_3 + y_1 + y_2, y_1, y_2]$$

p = $-s^3 + s^4$

29 . Coloring, {2, 3, 5, 6}

R: [6, 5, 1, 6, 2, 3]
B: [5, 6, 6, 5, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	3 *vs* 3

Omega Rank for R :
$-t - t^2 + t^4 + t^5$
,
cycles: {{1, 3, 6}, {2, 5}}   order: 6

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 1 & 3 \\ 4 & 1 & 3 & 0 & 2 & 2 \\ 3 & 2 & 2 & 0 & 1 & 4 \\ 2 & 1 & 4 & 0 & 2 & 3 \\ 4 & 2 & 3 & 0 & 1 & 2 \end{pmatrix}$$

$$[3 y_1 - y_2 + 3 y_3 - y_4, y_1, y_2, 0, y_3, y_4]$$

p = $-s - s^2 + s^4 + s^5$

Omega Rank for B :
$-t + t^4$
,
cycles: {{4, 5, 6}}   order: 3

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 3 & 5 \\ 0 & 0 & 0 & 5 & 4 & 3 \\ 0 & 0 & 0 & 3 & 5 & 4 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2, y_3]$$

30 . Coloring, {2, 4, 5, 6}

R: [6, 5, 6, 5, 2, 3]
B: [5, 6, 1, 6, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	4 *vs* 4

Omega Rank for R :
$-t^{+} \; t^3$

,
cycles: {{2, 5}, {3, 6}}   order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 & 3 & 3 \\ 0 & 3 & 3 & 0 & 2 & 4 \\ 0 & 2 & 4 & 0 & 3 & 3 \\ 0 & 3 & 3 & 0 & 2 & 4 \end{pmatrix}$$

$$[0, y_1, -7\,y_1 + 6\,y_2, 0, -6\,y_1 + 5\,y_2, y_2]$$

$p' = -s^{+} \; s^3 \quad p = -s^{+} \; s^3$

Omega Rank for B :
$-t^{3^{+}} \; t^5$

,
cycles: {{4, 6}}   order: 4

$$\begin{pmatrix} 2 & 0 & 0 & 4 & 1 & 5 \\ 0 & 0 & 0 & 5 & 2 & 5 \\ 0 & 0 & 0 & 5 & 0 & 7 \\ 0 & 0 & 0 & 7 & 0 & 5 \end{pmatrix}$$

$$[y_4, 0, 0, y_1, y_2, y_3]$$

31 . Coloring, {3, 4, 5, 6}

R: [6, 6, 1, 5, 2, 3]
B: [5, 5, 6, 6, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	4 *vs* 5	5 *vs* 5	2 *vs* 3

Omega Rank for R :

$-t^3 + t^6$
,
cycles: {{1, 3, 6}} order: 3

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 2 & 2 \\ 4 & 2 & 2 & 0 & 0 & 4 \\ 2 & 0 & 4 & 0 & 0 & 6 \\ 4 & 0 & 6 & 0 & 0 & 2 \\ 6 & 0 & 2 & 0 & 0 & 4 \end{pmatrix}$$

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{4, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 6 & 0 & 6 \end{pmatrix}$$

$$[0, 0, 0, y_2, y_1, y_2 + y_1]$$

$p = -s^2 + s^3$

32 . Coloring, {2, 3, 4, 5, 6}

R: [6, 5, 1, 5, 2, 3]
B: [5, 6, 6, 6, 6, 4]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	3 *vs* 3

Omega Rank for R :
$-t - t^2 + t^4 + t^5$
,
cycles: {{1, 3, 6}, {2, 5}} order: 6

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 3 & 1 \\ 4 & 3 & 1 & 0 & 2 & 2 \\ 1 & 2 & 2 & 0 & 3 & 4 \\ 2 & 3 & 4 & 0 & 2 & 1 \\ 4 & 2 & 1 & 0 & 3 & 2 \end{pmatrix}$$

$$[7 y_1 - 5 y_2 + 7 y_3 - 5 y_4, 5 y_1, 5 y_2, 0, 5 y_3, 5 y_4]$$

$p = -s - s^2 + s^4 + s^5$

Omega Rank for B :
$-t^2 + t^4$

,
cycles: {{4, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 1 & 7 \\ 0 & 0 & 0 & 7 & 0 & 5 \\ 0 & 0 & 0 & 5 & 0 & 7 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2, y_3]$$

SUMMARY	
Graph Type	CC
ν(A)	2
ν(Δ)	2
π	[1, 1, 2, 2, 2, 4]
Dbly Stoch	false

SANDWICH		Total 2
No .	Coloring	Rank
1	{3, 4, 5}	2
2	{5}	2

RT GROUPS		Total 5	
No .	Coloring	Rank	Solv
1	{3, 4}	3	Not Solvable
2	{3, 4, 6}	3	Not Solvable
3	{4, 6}	2	Solvable
4	{3}	2	Solvable
5	{3, 5, 6}	3	Solvable

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
23	0	23 , 21	13 , 15	7	32	32
