

New Graph

[6, 6, 6, 6, 6, 3], [4, 1, 5, 5, 4, 2]

$$\pi = [3, 6, 6, 4, 5, 12]$$

$$\delta = [1, 1, 1, 2, 2, 5]$$

POSSIBLE RANKS

- 1 x 36
- 2 x 18
- 3 x 12
- 4 x 9
- 6 x 6

BASE DETERMINANT  $31/256, .1210937500$

*NullSpace of  $\Delta$*

{2, 3}, {1, 4, 5, 6}

*Nullspace of A*

[{3},{2}] , [{6},{1, 4, 5}]

1 . Coloring, {}

R: [6, 6, 6, 6, 6, 3]  
 B: [4, 1, 5, 5, 4, 2]

` See graph

` ` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	2 vs 2	4 vs 4

Omega Rank for R :

$$-t^+ t^3$$

, cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 0 & 24 \\ 0 & 0 & 24 & 0 & 0 & 12 \end{pmatrix}$$

$$[0, 0, y_1, 0, 0, y_2]$$

Omega Rank for B :

$$-t^3 t^5$$

, cycles: {{4, 5}} order: 4

$$\begin{pmatrix} 6 & 12 & 0 & 8 & 10 & 0 \\ 12 & 0 & 0 & 16 & 8 & 0 \\ 0 & 0 & 0 & 20 & 16 & 0 \\ 0 & 0 & 0 & 16 & 20 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, 0]$$

2. Coloring, {2}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^3 + 4s^4$$

R: [6, 1, 6, 6, 6, 3]  
 B: [4, 6, 5, 5, 4, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
3 vs 4	3 vs 4	3 vs 4	2 vs 3	2 vs 4

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 6 & 0 & 12 & 0 & 0 & 18 \\ 0 & 0 & 18 & 0 & 0 & 18 \\ 0 & 0 & 18 & 0 & 0 & 18 \end{pmatrix}$$

$$[-y_1 + y_2, 0, y_1, 0, 0, y_2]$$

$$p = -s^2 + s^3$$

Omega Rank for B :

$$-t + t^3$$

, cycles: {{2, 6}, {4, 5}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 8 & 10 & 6 \\ 0 & 6 & 0 & 10 & 8 & 12 \\ 0 & 12 & 0 & 8 & 10 & 6 \\ 0 & 6 & 0 & 10 & 8 & 12 \end{pmatrix}$$

$$[0, 3y_1 - 2y_2, 0, y_1, 2y_1 - y_2, y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

M N

```

0 0 0 0 0 3 0 1 0 0 0 1
0 0 0 2 4 0 1 0 1 1 1 0
( 0 0 0 0 0 6 ) ( 0 1 0 0 0 1 )
( 0 2 0 0 0 2 ) ( 0 1 0 0 0 1 )
0 4 0 0 0 1 0 1 0 0 0 1
3 0 6 2 1 0 1 0 1 1 1 0
      NM
      3 0 6 4 5 0
      0 6 0 0 0 12
      ( 3 0 6 4 5 0 )
      ( 3 0 6 4 5 0 )
      3 0 6 4 5 0
      0 6 0 0 0 12

```

$\tau = 20, r' = 1/2$

R: [6, 1, 6, 6, 6, 3]  
 B: [4, 6, 5, 5, 4, 2]

Ranges

Action of R on ranges, [[4], [1], [1], [4], [4], [4]]  
 Action of B on ranges, [[2], [6], [5], [3], [3], [2]]

Cycles: R, {{3, 6}}, B, {{2, 6}, {4, 5}}

- $\beta(\{1, 6\}) = 1/6$
- $\beta(\{2, 4\}) = 1/9$
- $\beta(\{2, 5\}) = 2/9$
- $\beta(\{3, 6\}) = 1/3$
- $\beta(\{4, 6\}) = 1/9$
- $\beta(\{5, 6\}) = 1/18$

Partitions

$\alpha(\{\{2, 6\}, \{1, 3, 4, 5\}\}) = 1/1$

$b_1 = \{2, 6\}, b_2 = \{1, 3, 4, 5\}$

Action of R and B on the blocks of the partitions: = [2, 1][1, 2]  
 with invariant measure [1, 1]

N by blocks, check: true. See partition graph.

See level-2 partition graph.

Right Group	
Coloring	{2}
Rank	2
R,B	[6, 1, 6, 6, 6, 3], [4, 6, 5, 5, 4, 2]
$\pi_2$	[0, 0, 0, 0, 3, 0, 2, 4, 0, 0, 0, 6, 0, 2, 1]
$u_2$	[1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1] (dim 1)

wpp	[4, 2, 4, 4, 4, 2]
-----	--------------------

3 . Coloring, {3}

R: [6, 6, 5, 6, 6, 3]  
 B: [4, 1, 6, 5, 4, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	4 vs 5	3 vs 3	5 vs 5

Omega Rank for R :

$$-t^+ t^4$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 6 & 18 \\ 0 & 0 & 18 & 0 & 12 & 6 \\ 0 & 0 & 6 & 0 & 18 & 12 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

Omega Rank for B :

$$-t^4 t^6$$

, cycles: {{4, 5}} order: 4

$$\begin{pmatrix} 6 & 12 & 0 & 8 & 4 & 6 \\ 12 & 6 & 0 & 10 & 8 & 0 \\ 6 & 0 & 0 & 20 & 10 & 0 \\ 0 & 0 & 0 & 16 & 20 & 0 \\ 0 & 0 & 0 & 20 & 16 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

4 . Coloring, {4}

R: [6, 6, 6, 5, 6, 3]  
 B: [4, 1, 5, 6, 4, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	5 vs 5

Omega Rank for R :

$$-t^2 + t^4$$

' cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 4 & 20 \\ 0 & 0 & 20 & 0 & 0 & 16 \\ 0 & 0 & 16 & 0 & 0 & 20 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

Omega Rank for B :

$$-t^2 + t^6$$

' cycles: {{1, 2, 4, 6}} order: 4

$$\begin{pmatrix} 6 & 12 & 0 & 8 & 6 & 4 \\ 12 & 4 & 0 & 12 & 0 & 8 \\ 4 & 8 & 0 & 12 & 0 & 12 \\ 8 & 12 & 0 & 4 & 0 & 12 \\ 12 & 12 & 0 & 8 & 0 & 4 \end{pmatrix}$$

$$[y_4, y_1, 0, y_5, y_2, y_3]$$

5 . Coloring, {5}

R: [6, 6, 6, 6, 4, 3]

B: [4, 1, 5, 5, 6, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	5 vs 5

Omega Rank for R :

$$-t^2 + t^4$$

' cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 5 & 0 & 19 \\ 0 & 0 & 19 & 0 & 0 & 17 \\ 0 & 0 & 17 & 0 & 0 & 19 \end{pmatrix}$$

$$[0, 0, y_1, y_2, 0, y_3]$$

Omega Rank for B :

$$-t + t^6$$

' cycles: {{1, 2, 4, 5, 6}} order: 5

6 12 0 3 10 5  
 12 5 0 6 3 10  
 ( 5 10 0 12 6 3 )  
 10 3 0 5 12 6  
 3 6 0 10 5 12

$[y_4, y_5, 0, y_1, y_2, y_3]$

6 . Coloring, {6}

R: [6, 6, 6, 6, 2]  
 B: [4, 1, 5, 5, 4, 3]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	2 vs 2	3 vs 4

Omega Rank for R :

$-t^+ t^3$

, cycles: {{2, 6}} order: 2

$\begin{pmatrix} 0 & 12 & 0 & 0 & 0 & 24 \\ 0 & 24 & 0 & 0 & 0 & 12 \end{pmatrix}$

$[0, y_1, 0, 0, 0, y_2]$

Omega Rank for B :

tailcheck  $-t^2^+ t^4$

, cycles: {{4, 5}} order: 2

$\begin{pmatrix} 6 & 0 & 12 & 8 & 10 & 0 \\ 0 & 0 & 0 & 16 & 20 & 0 \\ 0 & 0 & 0 & 20 & 16 & 0 \\ 0 & 0 & 0 & 16 & 20 & 0 \end{pmatrix}$

$[y_1, 0, 2y_1, y_2, y_3, 0]$

$p = -s^2^+ s^4$

7 . Coloring, {2, 3}

$\Omega p(\Delta)=0: p = s^3 - 2s^4$

R: [6, 1, 5, 6, 6, 3]  
 B: [4, 6, 6, 5, 4, 2]

\ See graph

\ \ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R :

$$-t^2 \quad t^5$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 0 & 6 & 12 \\ 0 & 0 & 12 & 0 & 12 & 12 \\ 0 & 0 & 12 & 0 & 12 & 12 \\ 0 & 0 & 12 & 0 & 12 & 12 \end{pmatrix}$$

$$[y_2 - y_1, 0, y_2, 0, y_1, y_2]$$

$$p = -s^2 \quad s^3 \quad p = -s^2 \quad s^4$$

Omega Rank for B :

$$-t \quad t^3$$

, cycles: {{2, 6}, {4, 5}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 8 & 4 & 12 \\ 0 & 12 & 0 & 4 & 8 & 12 \\ 0 & 12 & 0 & 8 & 4 & 12 \\ 0 & 12 & 0 & 4 & 8 & 12 \end{pmatrix}$$

$$[0, y_2, 0, y_1, y_2 - y_1, y_2]$$

$$p' = s - s^3 \quad p = s - s^3$$

\ See 3-level graph

,

$$\begin{matrix} & & & & & & M & N \\ \begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 & 2 & 6 \\ 3 & 0 & 0 & 0 & 3 & 6 \\ 0 & 4 & 0 & 0 & 0 & 4 \\ 0 & 2 & 3 & 0 & 0 & 5 \\ 3 & 6 & 6 & 4 & 5 & 0 \end{pmatrix} & & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \\ & & & & & & NM \end{matrix}$$

6 6 6 8 10 12  
 3 12 12 4 5 12  
 ( 3 12 12 4 5 12 )  
 6 6 6 8 10 12  
 3 6 6 4 5 24

$\tau = 14, r' = 2/3$

R: [6, 1, 5, 6, 6, 3]  
 B: [4, 6, 6, 5, 4, 2]

Ranges

Action of R on ranges, [[4], [1], [1], [4]]  
 Action of B on ranges, [[2], [3], [2], [2]]

Cycles: R, {{3, 5, 6}}, B, {{2, 6}, {4, 5}}

- $\beta(\{1, 3, 6\}) = 1/4$
- $\beta(\{2, 4, 6\}) = 1/3$
- $\beta(\{2, 5, 6\}) = 1/6$
- $\beta(\{3, 5, 6\}) = 1/4$

Partitions

$\alpha(\{\{2, 3\}, \{1, 4, 5\}, \{6\}\}) = 1/1$

$b_1 = \{2, 3\}, b_2 = \{1, 4, 5\}, b_3 = \{6\}$

Action of R and B on the blocks of the partitions: = [3, 1, 2] [3, 2, 1]  
 with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

,

Right Group	
Coloring	{2, 3}
Rank	3
R,B	[6, 1, 5, 6, 6, 3], [4, 6, 6, 5, 4, 2]
$\pi_2$	[0, 3, 0, 0, 3, 0, 4, 2, 6, 0, 3, 6, 0, 4, 5]
$u_2$	[1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1] (dim 1)
wpp	[3, 2, 2, 3, 3, 1]
$\pi_3$	[0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 4, 2, 0, 0, 3, 0]
$u_3$	[0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]

8 . Coloring, {2, 4}



R: [6, 1, 6, 5, 6, 3]  
 B: [4, 6, 5, 6, 4, 2]

[` See graph](#)

[`` See pair graph](#)

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 4	4 vs 4

Omega Rank for R :  
 tailcheck  $-t^2 + t^4$

, cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 6 & 0 & 12 & 0 & 4 & 14 \\ 0 & 0 & 14 & 0 & 0 & 22 \\ 0 & 0 & 22 & 0 & 0 & 14 \\ 0 & 0 & 14 & 0 & 0 & 22 \end{pmatrix}$$

$$[3 y_2, 0, 2 y_1, 0, 2 y_2, 2 y_3]$$

$$p = s^2 - s^4$$

Omega Rank for B :  
 tailcheck  $-t^3 + t^5$

, cycles: {{2, 6}} order: 4

$$\begin{pmatrix} 0 & 12 & 0 & 8 & 6 & 10 \\ 0 & 10 & 0 & 6 & 0 & 20 \\ 0 & 20 & 0 & 0 & 0 & 16 \\ 0 & 16 & 0 & 0 & 0 & 20 \end{pmatrix}$$

$$[0, y_4, 0, y_1, y_2, y_3]$$

9 . Coloring, {2, 5}

R: [6, 1, 6, 6, 4, 3]  
 B: [4, 6, 5, 5, 6, 2]

[` See graph](#)

[`` See pair graph](#)

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 4	4 vs 4

Omega Rank for R :  
 tailcheck  $-t^2 + t^4$

, cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 6 & 0 & 12 & 5 & 0 & 13 \\ 0 & 0 & 13 & 0 & 0 & 23 \\ 0 & 0 & 23 & 0 & 0 & 13 \\ 0 & 0 & 13 & 0 & 0 & 23 \end{pmatrix}$$

$$[6y_2, 0, 5y_1, 5y_2, 0, 5y_3]$$

$$p = -s^2 + s^4$$

Omega Rank for B :

$$-t^3 + t^5$$

, cycles: {{2, 6}} order: 4

$$\begin{pmatrix} 0 & 12 & 0 & 3 & 10 & 11 \\ 0 & 11 & 0 & 0 & 3 & 22 \\ 0 & 22 & 0 & 0 & 0 & 14 \\ 0 & 14 & 0 & 0 & 0 & 22 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3, y_4]$$

10 . Coloring, {2, 6}

R: [6, 1, 6, 6, 6, 2]

B: [4, 6, 5, 5, 4, 3]

` See graph

` ` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	3 vs 4	3 vs 3	4 vs 4

Omega Rank for R :

$$-t + t^4$$

, cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 0 & 0 & 18 \\ 12 & 18 & 0 & 0 & 0 & 6 \\ 18 & 6 & 0 & 0 & 0 & 12 \end{pmatrix}$$

$$[y_2, y_3, 0, 0, 0, y_1]$$

Omega Rank for B :

$$-t^3 + t^5$$

, cycles: {{4, 5}} order: 4

$$\begin{pmatrix} 0 & 0 & 12 & 8 & 10 & 6 \\ 0 & 0 & 6 & 10 & 20 & 0 \\ 0 & 0 & 0 & 20 & 16 & 0 \\ 0 & 0 & 0 & 16 & 20 & 0 \end{pmatrix}$$

$$[0, 0, y_1, y_2, y_3, y_4]$$

11 . Coloring, {3, 4}

R: [6, 6, 5, 5, 6, 3]  
 B: [4, 1, 6, 6, 4, 2]

[` See graph](#)

[`` See pair graph](#)

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	3 vs 3	4 vs 4

$\Omega_+$  Rank for R :

$$-t \quad t^4$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 10 & 14 \\ 0 & 0 & 14 & 0 & 12 & 10 \\ 0 & 0 & 10 & 0 & 14 & 12 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2, y_3]$$

$\Omega_+$  Rank for B :

$$-t \quad t^5$$

, cycles: {{1, 2, 4, 6}} order: 4

$$\begin{pmatrix} 6 & 12 & 0 & 8 & 0 & 10 \\ 12 & 10 & 0 & 6 & 0 & 8 \\ 10 & 8 & 0 & 12 & 0 & 6 \\ 8 & 6 & 0 & 10 & 0 & 12 \end{pmatrix}$$

$$[y_2, y_1, 0, y_4, 0, y_3]$$

12 . Coloring, {3, 5}

R: [6, 6, 5, 6, 4, 3]  
 B: [4, 1, 6, 5, 6, 2]

[` See graph](#)

[`` See pair graph](#)

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	5 vs 5

Omega Rank for R :

$$-t^+ t^5$$

, cycles: {{3, 4, 5, 6}} order: 4

$$\begin{pmatrix} 0 & 0 & 12 & 5 & 6 & 13 \\ 0 & 0 & 13 & 6 & 12 & 5 \\ 0 & 0 & 5 & 12 & 13 & 6 \\ 0 & 0 & 6 & 13 & 5 & 12 \end{pmatrix}$$

$$[0, 0, y_1, y_2, y_3, y_4]$$

Omega Rank for B :

$$-t^+ t^6$$

, cycles: {{1, 2, 4, 5, 6}} order: 5

$$\begin{pmatrix} 6 & 12 & 0 & 3 & 4 & 11 \\ 12 & 11 & 0 & 6 & 3 & 4 \\ 11 & 4 & 0 & 12 & 6 & 3 \\ 4 & 3 & 0 & 11 & 12 & 6 \\ 3 & 6 & 0 & 4 & 11 & 12 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

13 . Coloring, {3, 6}

$$\Omega p(\Delta)=0: p = s^2 - 4s^3 + 4s^4$$

$$R: [6, 6, 5, 6, 6, 2]$$

$$B: [4, 1, 6, 5, 4, 3]$$

` See graph

` ` See pair graph

,

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 3	3 vs 5

Omega Rank for R :

$$-t^{2+} t^4$$

, cycles: {{2, 6}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 0 & 6 & 18 \\ 0 & 18 & 0 & 0 & 0 & 18 \\ 0 & 18 & 0 & 0 & 0 & 18 \end{pmatrix}$$

$$[0, -y_1 + y_2, 0, 0, y_1, y_2]$$

$$p = -s^{2+} s^3$$

Omega Rank for B :

$$-t^{2^+} t^4$$

'  
cycles: {{4, 5}, {3, 6}} order: 2

$$\begin{pmatrix} 6 & 0 & 12 & 8 & 4 & 6 \\ 0 & 0 & 6 & 10 & 8 & 12 \\ 0 & 0 & 12 & 8 & 10 & 6 \\ 0 & 0 & 6 & 10 & 8 & 12 \\ 0 & 0 & 12 & 8 & 10 & 6 \end{pmatrix}$$

$$p = -s^{2^+} s^4 \quad p' = -s^{2^+} s^4$$

$$[2y_2 - y_3 - y_1, 0, 3y_2 - 2y_1, y_2, y_3, y_1]$$

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 3 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 6 & 0 & 2 & 4 & 0 \end{pmatrix} \begin{matrix} M & N \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{matrix}$$

$$\begin{matrix} NM \\ 3 & 6 & 0 & 4 & 5 & 0 \\ 3 & 6 & 0 & 4 & 5 & 0 \\ 0 & 0 & 6 & 0 & 0 & 12 \\ 3 & 6 & 0 & 4 & 5 & 0 \\ 3 & 6 & 0 & 4 & 5 & 0 \\ 0 & 0 & 6 & 0 & 0 & 12 \end{matrix}$$

$$\tau = 20, r' = 1/2$$

$$\begin{matrix} R: [6, 6, 5, 6, 6, 2] \\ B: [4, 1, 6, 5, 4, 3] \end{matrix}$$

Ranges

$$\text{Action of R on ranges, } [[6], [2], [6], [6], [2], [2]]$$

$$\text{Action of B on ranges, } [[5], [1], [6], [5], [4], [3]]$$

Cycles: R , {{2, 6}}, B , {{4, 5}, {3, 6}}

$$\begin{matrix} \beta(\{1, 3\}) = 1/6 \\ \beta(\{2, 6\}) = 1/3 \\ \beta(\{3, 4\}) = 1/9 \\ \beta(\{3, 5\}) = 1/18 \\ \beta(\{4, 6\}) = 1/9 \\ \beta(\{5, 6\}) = 2/9 \end{matrix}$$

Partitions

$$\alpha(\{1, 2, 4, 5\}, \{3, 6\}) = 1/1$$

$$b1 = \{1, 2, 4, 5\}, b2 = \{3, 6\}$$

Action of R and B on the blocks of the partitions: = [2, 1][1, 2]  
with invariant measure [1, 1]

N by blocks, check: true .` See partition graph.

`` See level-2 partition graph.

,

Right Group	
Coloring	{3, 6}
Rank	2
R,B	[6, 6, 5, 6, 6, 2], [4, 1, 6, 5, 4, 3]
$\pi_2$	[0, 3, 0, 0, 0, 0, 0, 6, 2, 1, 0, 0, 2, 4]
$u_2$	[0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
wpp	[4, 4, 2, 4, 4, 2]

14 . Coloring, {4, 5}

R: [6, 6, 6, 5, 4, 3]  
 B: [4, 1, 5, 6, 6, 2]

` See graph

`` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	5 vs 5

Omega Rank for R :

$$-t^+ \quad t^3$$

, cycles: {{4, 5}, {3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 5 & 4 & 15 \\ 0 & 0 & 15 & 4 & 5 & 12 \\ 0 & 0 & 12 & 5 & 4 & 15 \\ 0 & 0 & 15 & 4 & 5 & 12 \end{pmatrix}$$

$$[0, 0, 3 y_2, y_1, y_2, 3 y_1]$$

$$p = -s^+ \quad s^3 \quad p' = -s^+ \quad s^3$$

Omega Rank for B :

$$-t^2 \quad t^6$$

, cycles: {{1, 2, 4, 6}} order: 4

6 12 0 3 6 9  
 12 9 0 6 0 9  
 (9 9 0 12 0 6)  
 9 6 0 9 0 12  
 6 12 0 9 0 9

$[y_1, y_2, 0, y_3, y_4, y_5]$

15 . Coloring, {4, 6}

R: [6, 6, 6, 5, 6, 2]  
 B: [4, 1, 5, 6, 4, 3]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	5 vs 5

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{2, 6}} order: 2

0 12 0 0 4 20  
 (0 20 0 0 0 16)  
 0 16 0 0 0 20

$[0, y_1, 0, 0, y_2, y_3]$

Omega Rank for B :

$$-t^2 + t^6$$

, cycles: {{3, 4, 5, 6}} order: 4

6 0 12 8 6 4  
 0 0 4 12 12 8  
 (0 0 8 12 4 12)  
 0 0 12 4 8 12  
 0 0 12 8 12 4

$[y_1, 0, y_2, y_3, y_4, y_5]$

16 . Coloring, {5, 6}

R: [6, 6, 6, 6, 4, 2]  
 B: [4, 1, 5, 5, 6, 3]

\ See graph

\ \ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	5 vs 5

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{2, 6}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 5 & 0 & 19 \\ 0 & 19 & 0 & 0 & 0 & 17 \\ 0 & 17 & 0 & 0 & 0 & 19 \end{pmatrix}$$

$$[0, y_2, 0, y_3, 0, y_1]$$

Omega Rank for B :

$$-t^3 + t^6$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 3 & 10 & 5 \\ 0 & 0 & 5 & 6 & 15 & 10 \\ 0 & 0 & 10 & 0 & 11 & 15 \\ 0 & 0 & 15 & 0 & 10 & 11 \\ 0 & 0 & 11 & 0 & 15 & 10 \end{pmatrix}$$

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

17 . Coloring, {2, 3, 4}

$$\begin{matrix} R: [6, 1, 5, 5, 6, 3] \\ B: [4, 6, 6, 6, 4, 2] \end{matrix}$$

\ See graph

\ \ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	4 vs 4	3 vs 3

Omega Rank for R :

$$-t^2 + t^5$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 0 & 10 & 8 \\ 0 & 0 & 8 & 0 & 12 & 16 \\ 0 & 0 & 16 & 0 & 8 & 12 \\ 0 & 0 & 12 & 0 & 16 & 8 \end{pmatrix}$$



$$[y_4, 0, y_3, 0, y_1, y_2]$$

Omega Rank for B :

$$-t^2 + t^4$$

' cycles: {{2, 6}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 8 & 0 & 16 \\ 0 & 16 & 0 & 0 & 0 & 20 \\ 0 & 20 & 0 & 0 & 0 & 16 \end{pmatrix}$$

$$[0, y_1, 0, y_2, 0, y_3]$$

18 . Coloring, {2, 3, 5}

R: [6, 1, 5, 6, 4, 3]

B: [4, 6, 6, 5, 6, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	5 vs 5	4 vs 4

Omega Rank for R :

$$-t^2 + t^6$$

' cycles: {{3, 4, 5, 6}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 5 & 6 & 7 \\ 0 & 0 & 7 & 6 & 12 & 11 \\ 0 & 0 & 11 & 12 & 7 & 6 \\ 0 & 0 & 6 & 7 & 11 & 12 \\ 0 & 0 & 12 & 11 & 6 & 7 \end{pmatrix}$$

$$[y_5, 0, y_4, y_3, y_2, y_1]$$

Omega Rank for B :

$$-t^3 + t^5$$

' cycles: {{2, 6}} order: 4

$$\begin{pmatrix} 0 & 12 & 0 & 3 & 4 & 17 \\ 0 & 17 & 0 & 0 & 3 & 16 \\ 0 & 16 & 0 & 0 & 0 & 20 \\ 0 & 20 & 0 & 0 & 0 & 16 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3, y_4]$$

19 . Coloring, {2, 3, 6}

$$\Omega p(\Delta)=0: \quad p = s^3 - 2s^4$$

R: [6, 1, 5, 6, 6, 2]  
 B: [4, 6, 6, 5, 4, 3]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R :

$$-t^{2+} \quad t^5$$

' cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 0 & 6 & 12 \\ 12 & 12 & 0 & 0 & 0 & 12 \\ 12 & 12 & 0 & 0 & 0 & 12 \\ 12 & 12 & 0 & 0 & 0 & 12 \end{pmatrix}$$

$$[y_2, y_1, 0, 0, -y_2 + y_1, y_1]$$

$$p = -s^{2+} s^4 \quad p = -s^{2+} s^3$$

Omega Rank for B :

$$-t \quad t^3$$

' cycles: {{4, 5}, {3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 8 & 4 & 12 \\ 0 & 0 & 12 & 4 & 8 & 12 \\ 0 & 0 & 12 & 8 & 4 & 12 \\ 0 & 0 & 12 & 4 & 8 & 12 \end{pmatrix}$$

$$[0, 0, y_2, y_2 - y_1, y_1, y_2]$$

$$p = -s^+ s^3 \quad p' = -s^+ s^3$$

` See 3-level graph

`

$$\begin{matrix} & & & & & & M & N \\ \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 4 & 2 & 6 \\ 0 & 0 & 4 & 0 & 0 & 4 \\ 0 & 3 & 2 & 0 & 0 & 5 \\ 3 & 6 & 6 & 4 & 5 & 0 \end{pmatrix} & & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \\ & & & & & & NM & \end{matrix}$$

6 6 6 8 10 12  
 3 12 12 4 5 12  
 ( 3 12 12 4 5 12 )  
 6 6 6 8 10 12  
 3 6 6 4 5 24

$\tau = 14, r' = 2/3$

R: [6, 1, 5, 6, 6, 2]  
 B: [4, 6, 6, 5, 4, 3]

Ranges

Action of R on ranges, [[1], [1], [2], [2]]  
 Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R, {{1, 2, 6}}, B, {{4, 5}, {3, 6}}

$\beta(\{1, 2, 6\}) = 1/4$   
 $\beta(\{2, 5, 6\}) = 1/4$   
 $\beta(\{3, 4, 6\}) = 1/3$   
 $\beta(\{3, 5, 6\}) = 1/6$

Partitions

$\alpha(\{\{2, 3\}, \{1, 4, 5\}, \{6\}\}) = 1/1$

$b_1 = \{2, 3\}, b_2 = \{1, 4, 5\}, b_3 = \{6\}$

Action of R and B on the blocks of the partitions: = [3, 1, 2] [3, 2, 1]  
 with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

,

Right Group	
Coloring	{2, 3, 6}
Rank	3
R,B	[6, 1, 5, 6, 6, 2], [4, 6, 6, 5, 4, 3]
$\pi_2$	[3, 0, 0, 0, 3, 0, 0, 3, 6, 4, 2, 6, 0, 4, 5]
$u_2$	[1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1] (dim 1)
wpp	[3, 2, 2, 3, 3, 1]
$\pi_3$	[0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 4, 2, 0]
$u_3$	[0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0]

20 . Coloring, {2, 4, 5}

R: [6, 1, 6, 5, 4, 3]  
 B: [4, 6, 5, 6, 6, 2]

` See graph  
 `` See pair graph  
 ,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 5	3 vs 4

Omega Rank for R :  
 $-t^{2+} t^4$

, cycles: {{4, 5}, {3, 6}} order: 2

6 0 12 5 4 9  
 0 0 9 4 5 18  
 (0 0 18 5 4 9)  
 0 0 9 4 5 18  
 0 0 18 5 4 9

$[-y_1 + 6y_2 - 3y_3, 0, y_1, y_2, y_3, -3y_2 + 6y_3]$

$p = -s^{2+} s^4 \quad p' = -s^{2+} s^4$

Omega Rank for B :  
 tailcheck  $-t^{2+} t^4$

, cycles: {{2, 6}} order: 2

0 12 0 3 6 15  
 0 15 0 0 0 21  
 (0 21 0 0 0 15)  
 0 15 0 0 0 21

$[0, y_3, 0, y_2, 2y_2, y_1]$

$p = -s^{2+} s^4$

21 . Coloring, {2, 4, 6}

R: [6, 1, 6, 5, 6, 2]  
 B: [4, 6, 5, 6, 4, 3]

` See graph  
 `` See pair graph  
 ,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R :  
 $-t^{2+} t^5$

' cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 0 & 4 & 14 \\ 12 & 14 & 0 & 0 & 0 & 10 \\ 14 & 10 & 0 & 0 & 0 & 12 \\ 10 & 12 & 0 & 0 & 0 & 14 \end{pmatrix}$$

$$[y_2, y_3, 0, 0, y_4, y_1]$$

Omega Rank for B :  
 $-t^+ t^5$

' cycles: {{3, 4, 5, 6}} order: 4

$$\begin{pmatrix} 0 & 0 & 12 & 8 & 6 & 10 \\ 0 & 0 & 10 & 6 & 12 & 8 \\ 0 & 0 & 8 & 12 & 10 & 6 \\ 0 & 0 & 6 & 10 & 8 & 12 \end{pmatrix}$$

$$[0, 0, y_4, y_3, y_1, y_2]$$

22 . Coloring, {2, 5, 6}

R: [6, 1, 6, 6, 4, 2]  
 B: [4, 6, 5, 5, 6, 3]

` See graph

` ` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R :  
 $-t^{2+} t^5$

' cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 5 & 0 & 13 \\ 12 & 13 & 0 & 0 & 0 & 11 \\ 13 & 11 & 0 & 0 & 0 & 12 \\ 11 & 12 & 0 & 0 & 0 & 13 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, 0, y_4]$$

Omega Rank for B :  
 $-t^{2+} t^5$

' cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 12 & 3 & 10 & 11 \\ 0 & 0 & 11 & 0 & 15 & 10 \\ 0 & 0 & 10 & 0 & 11 & 15 \\ 0 & 0 & 15 & 0 & 10 & 11 \end{pmatrix}$$

$$[0, 0, y_2, y_1, y_3, y_4]$$

23 . Coloring, {3, 4, 5}

R: [6, 6, 5, 5, 4, 3]  
 B: [4, 1, 6, 6, 6, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	4 vs 4

Omega Rank for R :

$$-t^3 + t^5$$

,  
 cycles: {{4, 5}} order: 4

$$\begin{pmatrix} 0 & 0 & 12 & 5 & 10 & 9 \\ 0 & 0 & 9 & 10 & 17 & 0 \\ 0 & 0 & 0 & 17 & 19 & 0 \\ 0 & 0 & 0 & 19 & 17 & 0 \end{pmatrix}$$

$$[0, 0, y_1, y_2, y_3, y_4]$$

Omega Rank for B :

$$-t + t^5$$

,  
 cycles: {{1, 2, 4, 6}} order: 4

$$\begin{pmatrix} 6 & 12 & 0 & 3 & 0 & 15 \\ 12 & 15 & 0 & 6 & 0 & 3 \\ 15 & 3 & 0 & 12 & 0 & 6 \\ 3 & 6 & 0 & 15 & 0 & 12 \end{pmatrix}$$

$$[y_1, y_3, 0, y_2, 0, y_4]$$

24 . Coloring, {3, 4, 6}

R: [6, 6, 5, 5, 6, 2]  
 B: [4, 1, 6, 6, 4, 3]

` See graph

`` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	3 vs 3	4 vs 4

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{2, 6}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 0 & 10 & 14 \\ 0 & 14 & 0 & 0 & 0 & 22 \\ 0 & 22 & 0 & 0 & 0 & 14 \end{pmatrix}$$

$$[0, y_1, 0, 0, y_2, y_3]$$

Omega Rank for B :

$$-t^3 + t^5$$

, cycles: {{3, 6}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 8 & 0 & 10 \\ 0 & 0 & 10 & 6 & 0 & 20 \\ 0 & 0 & 20 & 0 & 0 & 16 \\ 0 & 0 & 16 & 0 & 0 & 20 \end{pmatrix}$$

$$[y_1, 0, y_4, y_2, 0, y_3]$$

25 . Coloring, {3, 5, 6}

$$R: [6, 6, 5, 6, 4, 2]$$

$$B: [4, 1, 6, 5, 6, 3]$$

` See graph

`` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	5 vs 5

Omega Rank for R :

$$-t^3 + t^5$$

, cycles: {{2, 6}} order: 4

$$\begin{pmatrix} 0 & 12 & 0 & 5 & 6 & 13 \\ 0 & 13 & 0 & 6 & 0 & 17 \\ 0 & 17 & 0 & 0 & 0 & 19 \\ 0 & 19 & 0 & 0 & 0 & 17 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3, y_4]$$

Omega Rank for B :

$$-t^4 + t^6$$

cycles: {{3, 6}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 3 & 4 & 11 \\ 0 & 0 & 11 & 6 & 3 & 16 \\ 0 & 0 & 16 & 0 & 6 & 14 \\ 0 & 0 & 14 & 0 & 0 & 22 \\ 0 & 0 & 22 & 0 & 0 & 14 \end{pmatrix}$$

$$[y_5, 0, y_1, y_2, y_3, y_4]$$

26 . Coloring, {4, 5, 6}

R: [6, 6, 6, 5, 4, 2]

B: [4, 1, 5, 6, 6, 3]

[See graph](#)

[See pair graph](#)

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	5 vs 5

Omega Rank for R :

$$-t + t^3$$

cycles: {{4, 5}, {2, 6}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 5 & 4 & 15 \\ 0 & 15 & 0 & 4 & 5 & 12 \\ 0 & 12 & 0 & 5 & 4 & 15 \\ 0 & 15 & 0 & 4 & 5 & 12 \end{pmatrix}$$

$$[0, 3 y_2, 0, y_1, y_2, 3 y_1]$$

$$p' = -s + s^3 \quad p = -s + s^3$$

Omega Rank for B :

$$-t^3 + t^6$$

cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 3 & 6 & 9 \\ 0 & 0 & 9 & 6 & 12 & 9 \\ 0 & 0 & 9 & 0 & 9 & 18 \\ 0 & 0 & 18 & 0 & 9 & 9 \\ 0 & 0 & 9 & 0 & 18 & 9 \end{pmatrix}$$

$$[y_1, 0, y_2, y_3, y_4, y_5]$$



27 . Coloring, {2, 3, 4, 5}

R: [6, 1, 5, 5, 4, 3]  
 B: [4, 6, 6, 6, 6, 2]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	5 vs 5	3 vs 3

Omega Rank for R :

$$-t^4 + t^6$$

, cycles: {{4, 5}} order: 4

```

6 0 12 5 10 3
0 0 3 10 17 6
(0 0 6 17 13 0)
0 0 0 13 23 0
0 0 0 23 13 0
    
```

$$[y_1, 0, y_2, y_5, y_3, y_4]$$

Omega Rank for B :

$$-t^2 + t^4$$

, cycles: {{2, 6}} order: 2

```

0 12 0 3 0 21
(0 21 0 0 0 15)
0 15 0 0 0 21
    
```

$$[0, y_1, 0, y_2, 0, y_3]$$

28 . Coloring, {2, 3, 4, 6}

R: [6, 1, 5, 5, 6, 2]  
 B: [4, 6, 6, 6, 4, 3]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	4 vs 4	3 vs 3

Omega Rank for R :

$$-t^2 + t^5$$

, cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 0 & 10 & 8 \\ 12 & 8 & 0 & 0 & 0 & 16 \\ 8 & 16 & 0 & 0 & 0 & 12 \\ 16 & 12 & 0 & 0 & 0 & 8 \end{pmatrix}$$

$$[y_2, y_1, 0, 0, y_4, y_3]$$

Omega Rank for B :

$$-t^2 + t^4$$

' cycles: {{3, 6}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 8 & 0 & 16 \\ 0 & 0 & 16 & 0 & 0 & 20 \\ 0 & 0 & 20 & 0 & 0 & 16 \end{pmatrix}$$

$$[0, 0, y_1, y_2, 0, y_3]$$

29 . Coloring, {2, 3, 5, 6}

R: [6, 1, 5, 6, 4, 2]  
 B: [4, 6, 6, 5, 6, 3]

` See graph

` ` See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	5 vs 5	4 vs 4

Omega Rank for R :

$$-t^3 + t^6$$

' cycles: {{1, 2, 6}} order: 3

$$\begin{pmatrix} 6 & 12 & 0 & 5 & 6 & 7 \\ 12 & 7 & 0 & 6 & 0 & 11 \\ 7 & 11 & 0 & 0 & 0 & 18 \\ 11 & 18 & 0 & 0 & 0 & 7 \\ 18 & 7 & 0 & 0 & 0 & 11 \end{pmatrix}$$

$$[y_4, y_5, 0, y_1, y_2, y_3]$$

Omega Rank for B :

$$-t^3 + t^5$$

' cycles: {{3, 6}} order: 4

$$\begin{pmatrix} 0 & 0 & 12 & 3 & 4 & 17 \\ 0 & 0 & 17 & 0 & 3 & 16 \\ 0 & 0 & 16 & 0 & 0 & 20 \\ 0 & 0 & 20 & 0 & 0 & 16 \end{pmatrix}$$

$$[0, 0, y_1, y_3, y_4, y_2]$$

30 . Coloring, {2, 4, 5, 6}

R: [6, 1, 6, 5, 4, 2]  
 B: [4, 6, 5, 6, 6, 3]

` See graph

` ` See pair graph

`

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 5	4 vs 4

Omega Rank for R :

$$-t - t^{2+} t^{4+} t^5$$

, cycles: {{4, 5}, {1, 2, 6}} order: 6

$$\begin{pmatrix} 6 & 12 & 0 & 5 & 4 & 9 \\ 12 & 9 & 0 & 4 & 5 & 6 \\ 9 & 6 & 0 & 5 & 4 & 12 \\ 6 & 12 & 0 & 4 & 5 & 9 \\ 12 & 9 & 0 & 5 & 4 & 6 \end{pmatrix}$$

$$[-y_1 + 3y_2 + 3y_3 - y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = s^+ s^2 - s^4 - s^5$$

Omega Rank for B :

$$-t^2 t^5$$

, cycles: {{3, 5, 6}} order: 3

$$\begin{pmatrix} 0 & 0 & 12 & 3 & 6 & 15 \\ 0 & 0 & 15 & 0 & 12 & 9 \\ 0 & 0 & 9 & 0 & 15 & 12 \\ 0 & 0 & 12 & 0 & 9 & 15 \end{pmatrix}$$

$$[0, 0, y_1, y_4, y_2, y_3]$$

31 . Coloring, {3, 4, 5, 6}

R: [6, 6, 5, 5, 4, 2]

B: [4, 1, 6, 6, 6, 3]

\ See graph

\ \ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	4 vs 4

Omega Rank for R :

$$-t^+ t^3$$

, cycles: {{2, 6}, {4, 5}} order: 2

$$\begin{pmatrix} 0 & 12 & 0 & 5 & 10 & 9 \\ 0 & 9 & 0 & 10 & 5 & 12 \\ 0 & 12 & 0 & 5 & 10 & 9 \\ 0 & 9 & 0 & 10 & 5 & 12 \end{pmatrix}$$

$$[0, 10 y_1, 0, 25 y_1 - 25 y_2, 10 y_2, 25 y_1 - 21 y_2]$$

$$p = -s^+ s^3 \quad p' = -s^+ s^3$$

Omega Rank for B :

$$-t^3 t^5$$

, cycles: {{3, 6}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 3 & 0 & 15 \\ 0 & 0 & 15 & 6 & 0 & 15 \\ 0 & 0 & 15 & 0 & 0 & 21 \\ 0 & 0 & 21 & 0 & 0 & 15 \end{pmatrix}$$

$$[y_1, 0, y_2, y_3, 0, y_4]$$

32 . Coloring, {2, 3, 4, 5, 6}

R: [6, 1, 5, 5, 4, 2]

B: [4, 6, 6, 6, 6, 3]

\ See graph

\ \ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 5	3 vs 3

Omega Rank for R :

$$-t - t^2 t^4 t^5$$

, cycles: {{4, 5}, {1, 2, 6}} order: 6

6 12 0 5 10 3  
 12 3 0 10 5 6  
 (3 6 0 5 10 12)  
 6 12 0 10 5 3  
 12 3 0 5 10 6

$$[5y_2, -5y_2 + 7y_1 + 7y_4 - 5y_3, 0, 5y_1, 5y_4, 5y_3]$$

$$p = s^+ s^2 - s^4 - s^5$$

Omega Rank for B :

$$-t^2 + t^4$$

' cycles: {{3, 6}} order: 2

0 0 12 3 0 21  
 (0 0 21 0 0 15)  
 0 0 15 0 0 21

$$[0, 0, y_1, y_2, 0, y_3]$$

SUMMARY	
Graph Type	CC
v(A)	2
v(Δ)	2
π	[3, 6, 6, 4, 5, 12]
Dbly Stoch	false

SANDWICH		Total 0
No .	Coloring	Rank

RT GROUPS		Total 4	
No .	Coloring	Rank	Solv
1	{3, 6}	2	Solvable
2	{2, 3}	3	Not Solvable
3	{2}	2	Solvable
4	{2, 3, 6}	3	Not Solvable

$\Delta$ -RANK'D	SC'D !RK'D	$\tau$ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	$2^{n-1}$
28	0	28 , 26	20 , 26	4	32	32

---