New Graph

[5, 5, 5, 5, 2], [4, 3, 4, 3, 1]

π = [1, 1, 1, 1, 2]
δ= [1, 1, 2, 2, 4]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 35/256, .1367187500

NullSpace of Δ

{1, 2}, {3, 4, 5}

Nullspace of A

[{3, 4},{5}] `,` [{2},{1}]

1 . Coloring, {}

R: [5, 5, 5, 5, 2]
B: [4, 3, 4, 3, 1]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	2 *vs* 2	3 *vs* 3

Omega Rank for R :
$-t + t^3$
,
cycles: {{2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 0 & 4 \\ 0 & 4 & 0 & 0 & 2 \end{pmatrix}$$

$$[0, y_1, 0, 0, y_2]$$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{3, 4}} order: 2

$$\begin{pmatrix} 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 2 & 0 \end{pmatrix}$$

$$[y_3, 0, y_2, y_1, 0]$$

2 . Coloring, {2}

R: [5, 3, 5, 5, 2]
B: [4, 5, 4, 3, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	4 *vs* 4	3 *vs* 4	3 *vs* 3	3 *vs* 4

Omega Rank for R :
$-t + t^4$
,
cycles: {{2, 3, 5}} order: 3

$$\begin{pmatrix} 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 \end{pmatrix}$$

$$[0, y_1, y_2, 0, y_3]$$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{3, 4}} order: 4

$$\begin{pmatrix} 2 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{pmatrix}$$

$$[-y_1 + y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = -s^3 + s^4$

3 . Coloring, {3}

Ωp(Δ)=0: $p' = s - 2s^2$ $p = s - 4s^3$

R: [5, 5, 4, 5, 2]
B: [4, 3, 5, 3, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 3	2 *vs* 4	2 *vs* 4	2 *vs* 3	2 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 1 & 3 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

$$[0, y_2, 0, y_1, y_2 + y_1]$$

$p = s^2 - s^3$

Omega Rank for B :
$-t + t^5$
,
cycles: {{1, 3, 4, 5}} order: 4

$$\begin{pmatrix} 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 2 \\ 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 2 \end{pmatrix}$$

$$[y_1, 0, y_1, y_2, y_2]$$

$p = s - s^3$ $p' = -s + s^3$

M N

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 3 & 1 & 2 \\ 1 & 0 & 2 & 0 & 3 \\ 3 & 2 & 0 & 2 & 1 \\ 1 & 0 & 2 & 0 & 3 \\ 2 & 3 & 1 & 3 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 3 & 2 & 0 & 2 & 2 \\ 2 & 3 & 1 & 3 & 0 \\ 0 & 1 & 3 & 1 & 4 \\ 2 & 3 & 1 & 3 & 0 \\ 1 & 0 & 2 & 0 & 6 \end{pmatrix}$$

τ= 13 , r'= 1/2

R: [5, 5, 4, 5, 2]
B: [4, 3, 5, 3, 1]

Ranges

Action of R on ranges, [[3], [2], [2]]
Action of B on ranges, [[3], [1], [1]]

Cycles: R , {{2, 5}}, B , {{1, 3, 4, 5}}

β({1, 3}) = 1/3
β({2, 5}) = 1/3
β({4, 5}) = 1/3

Partitions

Action of R on partitions, [[2], [2]]
Action of B on partitions, [[2], [1]]

α([{1, 5}, {2, 3, 4}]) = 1/3
α([{1, 2, 4}, {3, 5}]) = 2/3

b1 = {1, 5} ` , ` b2 = {1, 2, 4} ` , ` b3 = {3, 5} ` , ` b4 = {2, 3, 4}

Action of R and B on the blocks of the partitions: = [2, 3, 2, 3] [3, 1, 4, 2]
with invariant measure [1, 2, 2, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Sandwich	
Coloring	{3}
Rank	2
R,B	[5, 5, 4, 5, 2], [4, 3, 5, 3, 1]
π_2	[0, 1, 0, 0, 0, 0, 1, 0, 0, 1]
u_2	[1, 3, 1, 2, 2, 0, 3, 2, 1, 3] (dim 1)
wpp	[8, 9, 7, 9, 6]

4 . Coloring, {4}

R: [5, 5, 5, 3, 2]
B: [4, 3, 4, 5, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 4	4 *vs* 4	2 *vs* 3	4 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

$$[0, -y_1 + y_2, y_1, 0, y_2]$$

$p = -s^2 + s^3$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{1, 4, 5}} order: 3

$$\begin{pmatrix} 2 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 3 & 2 \\ 2 & 0 & 0 & 1 & 3 \\ 3 & 0 & 0 & 2 & 1 \end{pmatrix}$$

$$[y_1, 0, y_2, y_3, y_4]$$

5 . Coloring, {5}

R: [5, 5, 5, 5, 1]
B: [4, 3, 4, 3, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	2 *vs* 2	3 *vs* 3

Omega Rank for R :
$-t + t^3$
,
cycles: {{1, 5}} order: 2

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$[y_1, 0, 0, 0, y_2]$$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{3, 4}} order: 2

$$\begin{pmatrix} 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 2 & 4 & 0 \end{pmatrix}$$

$$[0, y_3, y_2, y_1, 0]$$

6 . Coloring, {2, 3}

R: [5, 3, 4, 5, 2]
B: [4, 5, 5, 3, 1]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	4 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^{+} t^5$
,
cycles: {{2, 3, 4, 5}}   order: 4

$$\begin{pmatrix} 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 2 & 1 & 1 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$[0, -y_2 + y_1 + y_3, y_1, y_2, y_3]$

$p = -s^{+} s^2 - s^{3^{+}} s^4$

Omega Rank for B :
$-t^{+} t^5$
,
cycles: {{1, 3, 4, 5}}   order: 4

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 0 & 2 & 1 & 2 \end{pmatrix}$$

$[y_2, 0, -y_2 + y_1 + y_3, y_1, y_3]$

$p = -s^{+} s^2 - s^{3^{+}} s^4$

7 . Coloring, {2, 4}

$\Omega p(\Delta)=0$: $p = s^2$ $p' = s^2$

R: [5, 3, 5, 3, 2]
B: [4, 5, 4, 5, 1]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B

1 *vs* 3	1 *vs* 3	1 *vs* 3	1 *vs* 3	1 *vs* 3

Omega Rank for R :
$-t^{+} t^4$
,
cycles: {{2, 3, 5}} order: 3

$$\begin{pmatrix} 0 & 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \end{pmatrix}$$

$[0, y_1, y_1, 0, y_1]$

p = $-s^{+} s^3$ p = $-s^{+} s^2$

Omega Rank for B :
$-t^{+} t^4$
,
cycles: {{1, 4, 5}} order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 \end{pmatrix}$$

$[y_1, 0, 0, y_1, y_1]$

p = $-s^{+} s^2$ p = $-s^{+} s^3$

` See 3-level graph

`

M N

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 2 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 4 \end{pmatrix}$$

τ= 9 , r'= 2/3

R: [5, 3, 5, 3, 2]
B: [4, 5, 4, 5, 1]

Ranges

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

Cycles: R , {{2, 3, 5}}, B , {{1, 4, 5}}

β({1, 4, 5}) = 1/2

β({2, 3, 5}) = 1/2

Partitions
α([{1, 3}, {2, 4}, {5}]) = 1/1

b1 = {1, 3} ` , ` b2 = {2, 4} ` , ` b3 = {5}

Action of R and B on the blocks of the partitions: = [2, 3, 1] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{2, 4}
Rank	3
R,B	[5, 3, 5, 3, 2], [4, 5, 4, 5, 1]
π_2	[0, 0, 1, 1, 1, 0, 1, 0, 1, 1]
u_2	[1, 0, 1, 1, 1, 0, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
π_3	[0, 0, 0, 0, 0, 1, 0, 1, 0, 0]
u_3	[0, 0, 1, 0, 0, 1, 0, 1, 0, 1]

8 . Coloring, {2, 5}

Ωp(Δ)=0: $p = s - 4s^3$ $p' = s - 2s^2$

R: [5, 3, 5, 5, 1]
B: [4, 5, 4, 3, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 vs 3	2 vs 4	2 vs 4	2 vs 3	2 vs 4

Omega Rank for R :
$-t^2 + t^4$

,
cycles: {{1, 5}} order: 2

$$\begin{pmatrix} 2 & 0 & 1 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$[-y_1 + y_2, 0, y_1, 0, y_2]$

$p = -s^2 + s^3$

Omega Rank for B :
$-t + t^3$
,
cycles: {{2, 5}, {3, 4}}   order: 2

$$\begin{pmatrix} 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \end{pmatrix}$$

$[0, y_1, y_2, y_1, y_2]$

$p' = s - s^3 \quad p = s - s^3$

M   N

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 \end{pmatrix}$$

τ= 13 , r'= 1/2

R: [5, 3, 5, 5, 1]
B: [4, 5, 4, 3, 2]

Ranges

Action of R on ranges, [[1], [3], [1]]
Action of B on ranges, [[2], [3], [2]]

Cycles: R , {{1, 5}}, B , {{2, 5}, {3, 4}}

β({1, 5}) = 1/3
β({2, 4}) = 1/3
β({3, 5}) = 1/3

Partitions
α([{1, 3, 4}, {2, 5}]) = 1/1

b1 = {1, 3, 4} ` , ` b2 = {2, 5}

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]
with invariant measure [1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Right Group	
Coloring	{2, 5}
Rank	2
R,B	[5, 3, 5, 5, 1], [4, 5, 4, 3, 2]
π_2	[0, 0, 0, 1, 0, 1, 0, 0, 1, 0]
u_2	[1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
wpp	[3, 2, 3, 3, 2]

9 . Coloring, {3, 4}

Ωp(Δ)=0: $p = s^2 + 2s^3$

R: [5, 5, 4, 3, 2]
B: [4, 3, 5, 5, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 3	2 *vs* 4	2 *vs* 4	1 *vs* 4	2 *vs* 4

Omega Rank for R :
$-t + t^3$
,
cycles: {{2, 5}, {3, 4}} order: 2

$$\begin{pmatrix} 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 & 2 \end{pmatrix}$$

$[0, 2y_1, y_1, y_1, 2y_1]$

$p = -s + s^2$ $p = -s + s^3$ $p = -s + s^4$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{1, 4, 5}} order: 3

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 \end{pmatrix}$$

$[y_2, 0, y_2 - y_1, y_1, y_2]$

$p = -s^{2^+} s^3 \quad p = -s^{2^+} s^4$

` See 3-level graph

`

$$M = \begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$NM = \begin{pmatrix} 4 & 4 & 2 & 2 & 4 \\ 4 & 4 & 2 & 2 & 4 \\ 2 & 2 & 4 & 4 & 4 \\ 2 & 2 & 4 & 4 & 4 \\ 2 & 2 & 2 & 2 & 8 \end{pmatrix}$$

τ= 9 , r'= 2/3

R: [5, 5, 4, 3, 2]
B: [4, 3, 5, 5, 1]

Ranges

Action of R on ranges, [[4], [3], [4], [3]]
Action of B on ranges, [[2], [2], [1], [1]]

Cycles: R , {{2, 5}, {3, 4}}, B , {{1, 4, 5}}

β({1, 3, 5}) = 1/4
β({1, 4, 5}) = 1/4
β({2, 3, 5}) = 1/4
β({2, 4, 5}) = 1/4

Partitions
α([{1, 2}, {5}, {3, 4}]) = 1/1

b1 = {1, 2} ` , ` b2 = {5} ` , ` b3 = {3, 4}

Action of R and B on the blocks of the partitions: = [2, 1, 3] [2, 3, 1]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3, 4}
Rank	3

R,B	[5, 5, 4, 3, 2], [4, 3, 5, 5, 1]
π_2	[0, 1, 1, 2, 1, 1, 2, 0, 2, 2]
u_2	[0, 1, 1, 1, 1, 1, 1, 0, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
π_3	[0, 0, 0, 0, 1, 1, 0, 1, 1, 0]
u_3	[0, 0, 0, 0, 1, 1, 0, 1, 1, 0]

10 . Coloring, {3, 5}

R: [5, 5, 4, 5, 1]
B: [4, 3, 5, 3, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 4	4 *vs* 4	2 *vs* 3	4 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{1, 5}}   order: 2

$$\begin{pmatrix} 2 & 0 & 0 & 1 & 3 \\ 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$[-y_1 + y_2, 0, 0, y_1, y_2]$$

p = $s^2 - s^3$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{2, 3, 5}}   order: 3

$$\begin{pmatrix} 0 & 2 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 2 & 0 & 1 \end{pmatrix}$$

$$[0, y_1, y_2, y_3, y_4]$$

11 . Coloring, {4, 5}

$\Omega p(\Delta)=0$: $p = s - 4s^3$ $p' = s - 2s^2$

R: [5, 5, 5, 3, 1]
B: [4, 3, 4, 5, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 3	2 *vs* 4	2 *vs* 4	2 *vs* 3	2 *vs* 4

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{1, 5}} order: 2

$$\begin{pmatrix} 2 & 0 & 1 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$[-y_1 + y_2, 0, y_1, 0, y_2]$$

$p = -s^2 + s^3$

Omega Rank for B :
$-t + t^5$
,
cycles: {{2, 3, 4, 5}} order: 4

$$\begin{pmatrix} 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \end{pmatrix}$$

$$[0, y_2, y_1, y_2, y_1]$$

$p = s - s^3$ $p' = s - s^3$

M N

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 2 & 3 \\ 2 & 3 & 2 & 0 & 1 \\ 3 & 2 & 3 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 3 & 2 & 3 & 1 & 0 \\ 2 & 3 & 2 & 0 & 2 \\ 3 & 2 & 3 & 1 & 0 \\ 1 & 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & 2 & 6 \end{pmatrix}$$

$\tau = 13$, $r' = 1/2$

R: [5, 5, 5, 3, 1]
B: [4, 3, 4, 5, 2]

Ranges

Action of R on ranges, [[1], [3], [1]]
Action of B on ranges, [[2], [3], [2]]

Cycles: R , {{1, 5}}, B , {{2, 3, 4, 5}}

β({1, 5}) = 1/3
β({2, 4}) = 1/3
β({3, 5}) = 1/3

Partitions

Action of R on partitions, [[2], [2]]
Action of B on partitions, [[2], [1]]

α([{1, 3, 4}, {2, 5}]) = 1/3
α([{1, 2, 3}, {4, 5}]) = 2/3

b1 = {1, 2, 3} ` , ` b2 = {1, 3, 4} ` , ` b3 = {2, 5} ` , ` b4 = {4, 5}

Action of R and B on the blocks of the partitions: = [4, 4, 1, 1] [3, 1, 4, 2]
with invariant measure [2, 1, 1, 2]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Sandwich	
Coloring	{4, 5}
Rank	2
R,B	[5, 5, 5, 3, 1], [4, 3, 4, 5, 2]
π_2	[0, 0, 0, 1, 0, 1, 0, 0, 1, 0]
u_2	[1, 0, 2, 3, 1, 3, 2, 2, 3, 1] (dim 1)
wpp	[9, 8, 9, 7, 6]

12 . Coloring, {2, 3, 4}

R: [5, 3, 4, 3, 2]
B: [4, 5, 5, 5, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 3

Omega Rank for R :
$-t^3 + t^5$
,
cycles: {{3, 4}}   order: 4

$$\begin{pmatrix} 0 & 2 & 2 & 1 & 1 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{pmatrix}$$

$$[0, y_1 - y_2 + y_3, y_1, y_2, y_3]$$

$p = -s^3 + s^4$

Omega Rank for B :
$-t + t^4$
,
cycles: {{1, 4, 5}}   order: 3

$$\begin{pmatrix} 2 & 0 & 0 & 1 & 3 \\ 3 & 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 3 & 2 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_3]$$

13 . Coloring, {2, 3, 5}

R: [5, 3, 4, 5, 1]
B: [4, 5, 5, 3, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	4 *vs* 4	4 *vs* 4	3 *vs* 4	3 *vs* 4

Omega Rank for R :
$-t^3 + t^5$
,
cycles: {{1, 5}}   order: 4

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 1 & 3 \\ 3 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = s^3 - s^4$

Omega Rank for B :
$-t^3 + t^5$
,
cycles: {{2, 5}}   order: 4

$$\begin{pmatrix} 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

$$[0, y_2, y_3, y_2 + y_3 - y_1, y_1]$$

$p = -s^3 + s^4$

14 . Coloring, {2, 4, 5}

R: [5, 3, 5, 3, 1]
B: [4, 5, 4, 5, 2]

` See graph

`` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{1, 5}}   order: 2

$$\begin{pmatrix} 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$[y_1, 0, y_2, 0, y_3]$$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{2, 5}}   order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 2 & 2 \\ 0 & 2 & 0 & 0 & 4 \\ 0 & 4 & 0 & 0 & 2 \end{pmatrix}$$

$$[0, y_1, 0, y_3, y_2]$$

15 . Coloring, {3, 4, 5}

Ωp(Δ)=0: $p = s^2 + 2s^3$

R: [5, 5, 4, 3, 1]
B: [4, 3, 5, 5, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 3	2 *vs* 4	2 *vs* 4	1 *vs* 4	2 *vs* 4

Omega Rank for R :
$-t + t^3$
,
cycles: {{1, 5}, {3, 4}}   order: 2

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 2 \end{pmatrix}$$

$$[2 y_1, 0, y_1, y_1, 2 y_1]$$

$p = -s + s^2$ $p = -s + s^4$ $p = -s + s^3$

Omega Rank for B :
$-t^2 + t^5$
,
cycles: {{2, 3, 5}}   order: 3

$$\begin{pmatrix} 0 & 2 & 1 & 1 & 2 \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \end{pmatrix}$$

$$[0, y_1 + y_2, y_1, y_2, y_1 + y_2]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$

` See 3-level graph

`

M   N

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 4 & 4 & 2 & 2 & 4 \\ 4 & 4 & 2 & 2 & 4 \\ 2 & 2 & 4 & 4 & 4 \\ 2 & 2 & 4 & 4 & 4 \\ 2 & 2 & 2 & 2 & 8 \end{pmatrix}$$

τ= 9 , r'= 2/3

R: [5, 5, 4, 3, 1]
B: [4, 3, 5, 5, 2]

Ranges

Action of R on ranges, [[2], [1], [2], [1]]
Action of B on ranges, [[4], [4], [3], [3]]

Cycles: R , {{1, 5}, {3, 4}}, B , {{2, 3, 5}}

β({1, 3, 5}) = 1/4
β({1, 4, 5}) = 1/4
β({2, 3, 5}) = 1/4
β({2, 4, 5}) = 1/4

Partitions
α([{1, 2}, {5}, {3, 4}]) = 1/1

b1 = {1, 2} ` , ` b2 = {5} ` , ` b3 = {3, 4}

Action of R and B on the blocks of the partitions: = [2, 1, 3] [2, 3, 1]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3, 4, 5}
Rank	3
R,B	[5, 5, 4, 3, 1], [4, 3, 5, 5, 2]
π_2	[0, 1, 1, 2, 1, 1, 2, 0, 2, 2]
u_2	[0, 1, 1, 1, 1, 1, 1, 0, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
π_3	[0, 0, 0, 0, 1, 1, 0, 1, 1, 0]
u_3	[0, 0, 0, 0, 1, 1, 0, 1, 1, 0]

16 . Coloring, {2, 3, 4, 5}

Ωp(Δ)=0: $p = s - 4s^3$ $p' = s^+ 2s^2$

R: [5, 3, 4, 3, 1]
B: [4, 5, 5, 5, 2]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B

1 *vs* 3	2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 3

Omega Rank for R :
$-t + t^3$
,
cycles: {{1, 5}, {3, 4}} order: 2

$$\begin{pmatrix} 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 2 \\ 2 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 2 \end{pmatrix}$$

$$[y_1, 0, y_1, y_2, y_2]$$

$p = -s + s^3$ $p' = -s + s^3$

Omega Rank for B :
$-t^2 + t^4$
,
cycles: {{2, 5}} order: 2

$$\begin{pmatrix} 0 & 2 & 0 & 1 & 3 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

$$[0, -y_2 + y_1, 0, y_2, y_1]$$

$p = -s^2 + s^3$

M N

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

τ= 13 , r'= 1/2

R: [5, 3, 4, 3, 1]
B: [4, 5, 5, 5, 2]

Ranges

Action of R on ranges, [[3], [1], [1]]
Action of B on ranges, [[3], [2], [2]]

Cycles: R , {{1, 5}, {3, 4}}, B , {{2, 5}}

$\beta(\{1, 3\}) = 1/3$
$\beta(\{2, 5\}) = 1/3$

β({4, 5}) = 1/3

Partitions
α([{1, 5}, {2, 3, 4}]) = 1/1

b1 = {1, 5} ` , ` b2 = {2, 3, 4}

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Right Group	
Coloring	{2, 3, 4, 5}
Rank	2
R,B	[5, 3, 4, 3, 1], [4, 5, 5, 5, 2]
π_2	[0, 1, 0, 0, 0, 0, 1, 0, 0, 1]
u_2	[1, 1, 1, 0, 0, 0, 1, 0, 1, 1] (dim 1)
wpp	[2, 3, 3, 3, 2]

SUMMARY	
Graph Type	CC
ν(A)	2
ν(Δ)	2
π	[1, 1, 1, 1, 2]
Dbly Stoch	false

SANDWICH		Total 2
No .	Coloring	Rank
1	{3}	2
2	{4, 5}	2

RT GROUPS		Total 5	
No .	Coloring	Rank	Solv

1	{3, 4, 5}	3	Not Solvable
2	{2, 3, 4, 5}	2	Solvable
3	{3, 4}	3	Not Solvable
4	{2, 5}	2	Solvable
5	{2, 4}	3	Solvable

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
9	0	6 , 8	4 , 6	7	16	16
